

BAYESAIN ESTIMATION OF SIZE BIASED CLASSICAL GAMMA DISTRIBUTION

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Abstract

In this paper, we present Bayes' estimator of the parameter of Size biased Gamma distribution (SBGMD), that stems from an extension of Jeffery's prior (Al-Kutubi (2005)) with a new loss function (Al-Bayyati (2002)). We are proposing four different types of estimators. Under squared error loss function, there are two estimators formed by using Jaffrey prior and an extension of Jaffrey's prior. The two remaining estimators are derived using the same Jeffrey's prior and extension of Jeffrey's prior under a new loss function. We are also deriving the survival function of the size biased Gamma distribution. These methods are compared by using mean square error through simulation study with varying sample sizes.

Key Words: Size Biased Gamma Distribution, Jeffrey's Prior and Extension of Jeffrey's Prior, Loss Functions, Software.

1. Introduction

The gamma distribution is used as a lifetime model Gupta and Groll (1961), though not, nearly as much as the Weibull distribution. It does fit a widely variety of lifetime adequately, besides failure process models that leads to it. It also arises in some situations involving the exponential distribution; because of the well known results that the sum of independently and identically distributed exponential random variables has a gamma distribution. Inference for gamma model has been considered by Engelhard and Bain (1978), Chao and Glaser (1978), Jamali *et al* (2006), Lawless (2003) and Kalbfleisch and Prentice (2002) have made significant contributions. The size biased classical gamma distribution was introduced by Ahmed *et.al* (2013). The size biased gamma (SBGMD) distribution that is a flexible distribution in statistical literature, and has size biased exponential and exponential distribution as a subfamilies are introduced. Consider the two parameter size biased gamma distribution with having the probability density function of the form:

$$f(x; \alpha, \beta + 1) = \frac{\alpha^{\beta+1}}{\Gamma(\beta+1)} e^{-\alpha x} x^{\beta} \quad (1.1)$$

Where $\alpha > 0$ and $\beta \geq 0$ are parameters; α is a scale parameter and β is sometimes called the index or shape parameter.

The objective of this article is to estimate the parameters of size biased Gamma distribution. We are proposing four different types of estimator. Under squared error loss function, there are two estimates formed by using Jeffrey's prior and an

extension of Jeffrey's prior. The other two estimators are derived using the same Jeffrey's prior and extension of Jeffrey's prior under a new loss function introduced by Al-Bayyati (2002).

2. Materials and Methods

Recently Bayesian estimation approach has received great attention by most researchers. Bayesian analysis is an important approach to statistics, which formally seeks use of prior information and Bayes' Theorem provides the formal basis for using this information. In this approach, parameters are treated as random variables and data is treated fixed. Ghafoor *et al* (2005), Ali *et al* (2011) and Rahul *et al* (2009) have discussed the application of Bayesian methods. An important pre-requisite in Bayesian estimation is the appropriate choice of prior(s) for the parameters. However, Bayesian analysts have pointed out that there is no clear cut way from which one can conclude that one prior is better than the other. Very often, priors are chosen according to ones subjective knowledge and beliefs. However, if one has adequate information about the parameter(s) one should use informative prior(s), otherwise it is preferable to use non informative prior(s). In this paper we consider the extended Jeffrey's prior proposed by Al-Kutubi (2005) as:

$$g(\alpha) \propto [I(\alpha)]^{c_1}, c_1 \in R^+ \quad (2.1)$$

Where $[I(\alpha)] = -nE\left[\frac{\partial^2 \log f(x; \alpha, \beta + 1)}{\partial \alpha^2}\right]$ is the Fisher's information matrix.

$$\text{For the model (1.1), } g(\alpha) = k \left[\frac{n(\beta + 1)}{\alpha^2} \right]^{c_1} \quad (2.2)$$

Where k is a constant. With the above prior, we use two different loss functions for the model (1.1), first is the squared error loss function which is symmetric, second is the precautionary loss which is a simple asymmetric function. It is well known that choice of loss function is an integral part of Bayesian inference. As there is no specific analytical procedure that allows us to identify the appropriate loss function to be used, most of the works on point estimation and point prediction assume the underlying loss function to be squared error which is symmetric in nature. However, in-discriminate use of SELF is not appropriate particularly in these cases, where the losses are not symmetric. Thus in order to make the statistical inferences more practical and applicable, we often needs to choose an asymmetric loss function. A number of asymmetric loss functions have been shown to be functional, see Varian (1975), Zellner (1986), Spiring and Yeung (1998) etc. In the present work, we consider symmetric as well as asymmetric loss functions for better comprehension of Bayesian analysis.

a) The first is the common squared error loss function given by:

$$l_1(\hat{\alpha}, \alpha) = c(\hat{\alpha} - \alpha)^2 \quad (2.3)$$

which is symmetric, α and $\hat{\alpha}$ represent the true and estimated values of the parameter. This loss function is frequently used because of its analytical tractability in Bayesian analysis.

b) The second is the precautionary loss function given by:

$$l_2(\hat{\alpha}, \alpha) = \frac{c(\hat{\alpha} - \alpha)^2}{\hat{\alpha}}$$

Which is an asymmetric loss function, for details, see Norstrom (1996). This loss function is interesting in the sense that a slight modification of squared error loss introduces asymmetry.

3. Parameter estimation under squared error loss function.

In this section, two different prior distributions are used for estimating the parameter of the size biased Gamma distribution namely; Jeffery’s prior and extension of Jeffrey’s prior information.

3.1 Bayes’ estimation of parameter of size biased Gamma distribution under Jeffrey’s prior

Consider there are n recorded values, $\underline{x} = (x_1, \dots, x_n)$ from (1.1). We consider the extended Jeffrey’s prior as: $g(\theta) \propto \sqrt{[I\alpha]}$

Where $[I(\alpha)] = -nE\left[\frac{\partial^2 \log f(x; \theta, k)}{\partial \alpha^2}\right]$ is the Fisher’s information matrix. For the model (1.1),

$$g(\alpha) = k \frac{\sqrt{n(\beta + 1)}}{\alpha} \tag{3.1.1}$$

Then the joint probability density function is given by:

$$f(\underline{x}, \alpha) = L(x; \alpha)g(\alpha)$$

$$f(\underline{x}, \alpha) = \frac{k\sqrt{n(\beta + 1)}}{[\Gamma(\beta + 1)]^n} e^{-\alpha \sum_{i=1}^n x_i} \prod_{i=1}^n x_i^\beta \alpha^{n\beta + n - 1}$$

And the corresponding marginal PDF of $\underline{x} = (x_1, \dots, x_n)$ is obtained as:

$$p(\underline{x}) = \int_0^\infty f(\underline{x}, \alpha) d\alpha$$

$$p(\underline{x}) = \int_0^\infty \frac{k\sqrt{n(\beta + 1)}}{[\Gamma(\beta + 1)]^n} \prod_{i=1}^n x_i^\beta e^{-\alpha \sum_{i=1}^n x_i} \alpha^{n\beta + n - 1} d\alpha$$

$$p(\underline{x}) = \frac{k\sqrt{n(\beta + 1)}}{[\Gamma(\beta + 1)]^n} \prod_{i=1}^n x_i^\beta \frac{\Gamma(n\beta + n)}{\left(\sum_{i=1}^n x_i\right)^{n\beta + n}} \tag{3.1.2}$$

The posterior PDF of α has the following form

$$\pi_1(\alpha/\underline{x}) = \frac{f(\underline{x}, \alpha)}{p(\underline{x})}$$

$$\pi_1(\alpha/\underline{x}) = \frac{k\sqrt{n(\beta+1)} e^{-\alpha \sum_{i=1}^n x_i} \prod_{i=1}^n x_i^\beta \alpha^{n\beta+n-1}}{[\Gamma(\beta+1)]^n}$$

$$\pi_1(\alpha/\underline{x}) = \frac{k\sqrt{n(\beta+1)} \prod_{i=1}^n x_i^\beta \frac{\Gamma(n\beta+n)}{\left(\sum_{i=1}^n x_i\right)^{n\beta+n}}}{\Gamma(\beta+1)^n}$$

$$\pi_1(\alpha/\underline{x}) = \frac{e^{-\alpha \sum_{i=1}^n x_i} \alpha^{n\beta+n-1} \left(\sum_{i=1}^n x_i\right)^{n\beta+n}}{\Gamma(n\beta+n)} \tag{3.1.3}$$

By using a squared error loss function $l_1(\hat{\alpha}, \alpha) = c(\hat{\alpha} - \alpha)^2$ for some constant c, the risk function is:

$$R(\hat{\alpha}) = \int_0^\infty c(\hat{\alpha} - \alpha)^2 \pi_1(\alpha/\underline{x}) d\alpha$$

$$R(\hat{\alpha}) = \int_0^\infty c(\hat{\alpha} - \alpha)^2 \frac{e^{-\alpha \sum_{i=1}^n x_i} \alpha^{n\beta+n-1} \left(\sum_{i=1}^n x_i\right)^{n\beta+n}}{\Gamma(n\beta+n)} d\alpha$$

$$R(\hat{\alpha}) = c\hat{\alpha}^2 + \frac{c(n\beta+n)(n\beta+n+1)}{\left(\sum_{i=1}^n x_i\right)^2} - \frac{2c\hat{\alpha}(n\beta+n)}{\sum_{i=1}^n x_i} \tag{3.1.4}$$

Now $\frac{\partial R(\hat{\alpha})}{\partial \hat{\alpha}} = 0$, Then the Bayes' estimator is

$$\hat{\alpha}_1 = \frac{n(\beta+1)}{\sum_{i=1}^n x_i}$$

$$\hat{\alpha}_1 = \frac{\beta+1}{\bar{x}} \tag{3.1.5}$$

Estimation of Survival function: By using posterior probability density function, we can found the Survival function, such that

$$\hat{S}_1(\underline{x}) = \int_0^\infty e^{-\alpha x} \pi_1(\alpha/\underline{x}) d\alpha$$

$$\hat{S}_2(\underline{x}) = \int_0^\infty e^{-\alpha \left(x + \sum_{i=1}^n x_i\right)} \alpha^{(n\beta+n-1)} \frac{\left(\sum_{i=1}^n x_i\right)^{n\beta+n}}{\Gamma(n\beta+n)} d\alpha$$

$$\hat{S}_1(\underline{x}) = \left(\frac{\sum_{i=1}^n x_i}{x + \sum_{i=1}^n x_i} \right)^{n\beta+n} \tag{3.1.6}$$

3.2 Bayes' estimation of parameter of size biased gamma distribution using extension of Jeffrey's prior

We consider the extended Jeffrey's prior are given as: $g(\alpha) \propto [I(\alpha)]^{c_1}$, $c_1 \in R^+$

Where $[I(\alpha)] = -nE\left[\frac{\partial^2 \log f(x; \alpha, \beta + 1)}{\partial \alpha^2}\right]$ is the Fisher's information matrix. For the model (1.1), we have

$$g(\alpha) = k \left[\frac{n(\beta + 1)}{\alpha^2} \right]^{c_1} \tag{3.2.1}$$

Then the joint probability density function is given by:

$$f(\underline{x}, \alpha) = L(x; \alpha)g(\alpha)$$

$$f(\underline{x}, \alpha) = \frac{k(n(\beta + 1))^{c_1}}{[\Gamma(\beta + 1)]^n} e^{-\alpha \sum_{i=1}^n x_i} \prod_{i=1}^n x_i^\beta \alpha^{n\beta+n-2c_1} \tag{3.2.2}$$

And the corresponding marginal PDF of $\underline{x} = (x_1, \dots, x_n)$ is obtained as:

$$p(\underline{x}) = \int_0^\infty f(\underline{x}, \alpha) d\alpha$$

$$p(\underline{x}) = \int_0^\infty \frac{k(n(\beta + 1))^{c_1}}{[\Gamma(\beta + 1)]^n} \prod_{i=1}^n x_i^\beta e^{-\alpha \sum_{i=1}^n x_i} \alpha^{n\beta+n-2c_1+1} d\alpha$$

$$p(\underline{x}) = \frac{k(n(\beta + 1))^{c_1}}{[\Gamma(\beta + 1)]^n} \prod_{i=1}^n x_i^\beta \frac{\Gamma(n\beta + n - 2c_1 + 1)}{\left(\sum_{i=1}^n x_i\right)^{n\beta+n-2c_1+1}} \tag{3.2.3}$$

The posterior PDF of α has the following form

$$\pi_1(\alpha/\underline{x}) = \frac{f(\underline{x}, \alpha)}{p(\underline{x})}$$

$$\pi_2(\alpha/\underline{x}) = \frac{e^{-\alpha \sum_{i=1}^n x_i} \alpha^{n\beta+n-2c_1} \left(\sum_{i=1}^n x_i\right)^{n\beta+n-2c_1+1}}{\Gamma(n\beta + n - 2c_1 + 1)} \tag{3.2.4}$$

By using a squared error loss function $l_1(\hat{\alpha}, \alpha) = c(\hat{\alpha} - \alpha)^2$ for some constant c , the risk function is:

$$\begin{aligned}
 R(\hat{\alpha}) &= \int_0^{\infty} c(\hat{\alpha} - \alpha)^2 \pi_1(\alpha/x) d\alpha \\
 R(\hat{\alpha}) &= \int_0^{\infty} c(\hat{\alpha} - \alpha)^2 \frac{e^{-\alpha \sum_{i=1}^n x_i} \alpha^{n\beta+n-2c_1} \left(\sum_{i=1}^n x_i\right)^{n\beta+n-2c_1+1}}{\Gamma(n\beta+n-2c_1+1)} d\alpha \\
 R(\hat{\alpha}) &= c\hat{\alpha}^2 + \frac{c(n\beta+n-2c_1+2)(n\beta+n-2c_1+1)}{\left(\sum_{i=1}^n x_i\right)^2} - \frac{2c\hat{\alpha}(n\beta+n-2c_1+1)}{\sum_{i=1}^n x_i} \tag{3.2.5}
 \end{aligned}$$

Now $\frac{\partial R(\hat{\alpha})}{\partial \hat{\alpha}} = 0$, Then the Bayes' estimator is

$$\hat{\alpha}_2 = \frac{n(\beta+1) - 2c_1 + 1}{\sum_{i=1}^n x_i} \tag{3.2.6}$$

The Bayes' estimator under a precautionary loss function is denoted by $\hat{\alpha}$, and is given by the following equation:

$\hat{\alpha}_p = E\left[\alpha^2\right]^{\frac{1}{2}}$ and the corresponding Bayes' estimator comes out to be:

$$\hat{\alpha}_2 = \frac{n(\beta+1) - 2c_1 + 1}{\sum_{i=1}^n x_i}$$

The risk function under precautionary loss function is given by:

$$R(\hat{\alpha}) = c\hat{\alpha} + \frac{c(n\beta+n-2c_1+2)(n\beta+n-2c_1+1)}{\hat{\alpha} \left(\sum_{i=1}^n x_i\right)^2} - \frac{2c(n\beta+n-2c_1+1)}{\sum_{i=1}^n x_i} \tag{3.2.7}$$

Remark 1: Replacing $c_1 = 1/2$ in (3.2.6), the same Bayes' estimator is obtained as in (3.1.5) corresponding to the Jeffrey's prior. By Replacing $c_1 = 3/2$ in (3.2.6), the Bayes' estimator becomes the estimator under Hartigan's prior (Hartigan (1964)). By Replacing $c_1 = 0$ in (3.2.6), thus we get uniform prior.

Estimation of Survival function: By using posterior probability density function, we can find the Survival function, such that

$$\hat{S}_2(x) = \int_0^{\infty} e^{-\alpha x} \pi_1(\alpha/x) d\alpha$$

$$\hat{S}_2(\underline{x}) = \int_0^\infty e^{-\alpha \left(x + \sum_{i=1}^n x_i \right)} \alpha^{(n\beta+n-2c_1+1)-1} \frac{\left(\sum_{i=1}^n x_i \right)^{n\beta+n-2c_1+1}}{\Gamma(n\beta+n-2c_1+1)} d\alpha$$

$$\hat{S}_1(\underline{x}) = \left(\frac{\sum_{i=1}^n x_i}{x + \sum_{i=1}^n x_i} \right)^{n\beta+n-2c_1+1} \tag{3.2.8}$$

4. Parameter estimation under a new loss function

This section uses a new loss function introduced by Al-Bayyati (2002). Employing this loss function, we obtain Bayes’ estimators using Jeffrey’s and extension of Jeffrey’s prior information.

Al-Bayyati introduced a new loss function of the form:

$$l_A(\hat{\alpha}, \alpha) = \alpha^{c_2} (\hat{\alpha} - \alpha)^2; c_2 \in R. \tag{4.1}$$

Here, this loss function is used to obtain the estimator of the parameter of the size biased Gamma distribution.

4.1 Bayes’ estimation of parameter of size biased Gamma distribution under Jeffrey’s prior

By using the loss function in the form given in (4.1), we obtained the following risk function:

$$R(\hat{\alpha}) = \int_0^\infty \alpha^{c_2} (\hat{\alpha} - \alpha)^2 \pi_1(\alpha/\underline{x}) d\alpha$$

$$R(\hat{\alpha}) = \int_0^\infty \alpha^{c_2} (\hat{\alpha} - \alpha)^2 e^{-\alpha \sum_{i=1}^n x_i} \alpha^{n\beta+n-1} \frac{\left(\sum_{i=1}^n x_i \right)^{n\beta+n}}{\Gamma(n\beta+n)} d\alpha$$

$$R(\hat{\alpha}) = \frac{\left(\sum_{i=1}^n x_i \right)^{n\beta+n+1}}{\Gamma(n\beta+n)} \frac{1}{\left(\sum_{i=1}^n x_i \right)^{n\beta+n+c_2}} \left[\hat{\alpha}^2 \Gamma(n\beta+n+c_2) + \frac{\Gamma(n\beta+n+c_2+2)}{\left(\sum_{i=1}^n x_i \right)^2} - \frac{2\hat{\alpha} \Gamma(n\beta+n+c_2+1)}{\sum_{i=1}^n x_i} \right]$$

Now $\frac{\partial R(\hat{\alpha})}{\partial \hat{\alpha}} = 0$, Then the Bayes’ estimator is

$$\hat{\alpha}_3 = \frac{n(\beta+1)+c_2}{\sum_{i=1}^n x_i} \tag{4.1.1}$$

The Bayes' estimator under a precautionary loss function is denoted by $\hat{\alpha}$, and is given by the following equation:

$\hat{\alpha}_p = E[\alpha^2]^{1/2}$ and the corresponding Bayes' estimator comes out to be:

$$\hat{\alpha}_3 = \frac{n(\beta + 1) + c_2}{\sum_{i=1}^n x_i} \tag{4.1.2}$$

The risk function under precautionary loss function is given by:

$$R_p(\hat{\alpha}_p) = \frac{\left(\sum_{i=1}^n x_i\right)^{n\beta+n}}{\Gamma(n\beta+n)} \frac{1}{\left(\sum_{i=1}^n x_i\right)^{n\beta+n+c_2}} \left[\hat{\alpha}\Gamma(n\beta+n+c_2) + \frac{\Gamma(n\beta+n+c_2+2)}{\hat{\alpha}\left(\sum_{i=1}^n x_i\right)^2} - \frac{2\Gamma(n\beta+n+c_2+1)}{\sum_{i=1}^n x_i} \right] \tag{4.1.3}$$

Remark 2: Replacing $c_2 = 0$ in (4.1.2), the same Bayes' estimator is obtained as in (3.1.5) corresponding to the Jeffrey's prior. By Replacing $c_1= 3/2$ and $c_2 =-2$ in (4.1.2), the Bayes' estimator becomes the estimator under Hartigan's prior (Hartigan (1964)). By Replacing $c_2 =1$ in (4.1, 2), we get uniform prior.

4.2 Bayes' estimation of parameter of size biased Gamma distribution using extension of Jeffrey's prior.

By using the loss function in the form given in (4.1), we obtained the following risk function:

$$R(\hat{\alpha}) = \int_0^{\infty} \alpha^{c_2} (\hat{\alpha} - \alpha)^2 \pi_2(\alpha/x) d\alpha$$

$$R(\hat{\alpha}) = \int_0^{\infty} \alpha^{c_2} (\hat{\alpha} - \alpha)^2 \frac{e^{-\alpha \sum_{i=1}^n x_i} \alpha^{n\beta+n-2c_1} \left(\sum_{i=1}^n x_i\right)^{n\beta+n-2c_1+1}}{\Gamma(n\beta+n-2c_1+1)} d\alpha$$

$$R(\hat{\alpha}) = \frac{\left(\sum_{i=1}^n x_i\right)^{n\beta+n-2c_1+1}}{\Gamma(n\beta+n-2c_1+1)} \frac{1}{\left(\sum_{i=1}^n x_i\right)^{n\beta+n+c_2-2c_1+1}} \left[\hat{\alpha}^2\Gamma(n\beta+n+c_2-2c_1+1) + \frac{\Gamma(n\beta+n+c_2-2c_1+3)}{\left(\sum_{i=1}^n x_i\right)^2} - \frac{2\hat{\alpha}\Gamma(n\beta+n+c_2-2c_1+2)}{\sum_{i=1}^n x_i} \right] \tag{4.2.1}$$

Now $\frac{\partial R(\hat{\alpha})}{\partial \hat{\alpha}} = 0$, Then the Bayes' estimator is

$$\hat{\alpha}_4 = \frac{n(\beta + 1) + c_2 - 2c_1 + 1}{\sum_{i=1}^n x_i} \tag{4.2.2}$$

The Bayes' estimator under a precautionary loss function is denoted by $\hat{\alpha}$, and is given by the following equation:

$\hat{\alpha}_p = E[\alpha^2]^{\frac{1}{2}}$ and the corresponding Bayes' estimator comes out to be:

$$\hat{\alpha}_4 = \frac{n(\beta + 1) + c_2 - 2c_1 + 1}{\sum_{i=1}^n x_i}$$

The risk function under precautionary loss function is given by:

$$R(\hat{\alpha}) = \frac{\left(\sum_{i=1}^n x_i\right)^{n\beta+n-2c_1+1}}{\Gamma(n\beta+n-2c_1+1)} \frac{1}{\left(\sum_{i=1}^n x_i\right)^{n\beta+n+c_2-2c_1+1}} \left[\frac{\hat{\alpha}\Gamma(n\beta+n+c_2-2c_1+1) + \frac{\Gamma(n\beta+n+c_2-2c_1+3)}{\hat{\alpha}\left(\sum_{i=1}^n x_i\right)^2}}{2\Gamma(n\beta+n+c_2-2c_1+2)} - \frac{\sum_{i=1}^n x_i}{\sum_{i=1}^n x_i} \right] \tag{4.2.3}$$

Remark 3: Replacing $c_1 = 1/2$ and $c_2 = 0$ in (4.2.2), the same Bayes' estimator is obtained as in (3.5.1) corresponding to the Jeffrey's prior. By Replacing $c_1 = 3/2$ and $c_2 = 0$ in (4.2.2), the Bayes' estimator becomes the estimator under Hartigan's prior (Hartigan (1964)). By Replacing $c_1 = 0$ and $c_2 = 0$ in (4.2.2), we get uniform prior.

4. Simulation Study of Size biased Gamma Distribution

In our simulation study, we choose a sample size of $n=25, 50$ and 100 to represent small, medium and large data set. The scale parameter is estimated for Size biased Gamma Distribution with Maximum Likelihood and Bayesian using Jeffrey's & extension of Jeffrey's prior methods. For the scale parameter we have considered $\alpha = 1.5, .2.0$ and 2.5 . The values of Jeffrey's extension were $c_1 = 0.5, 1.0, 1.5$ and 2.0 . The value for the loss parameter $c_2 = -1, 0$ and $+1$. This was iterated 5000 times and the scale parameter for each method was calculated. A simulation study was conducted using R-software to examine and compare the performance of the estimates for different sample sizes with different values for the Extension of Jeffrey's' prior and the loss functions. The results are presented in tables (5.1) and (5.2) for different selections of the parameters and c extension of Jeffrey's prior.

n	α	$\beta + 1$	α_{ML}	α_{sl}	α_{NI}		
					C2=-1.0	C2=0	C2=1.0
25	1.0	1.5	0.694831	0.694831	0.598016	0.694831	0.544739
	1.5	2.0	1.616179	1.616179	1.349812	1.585304	1.436083
	2.0	2.5	2.485953	2.485953	2.597593	2.456618	2.389094
50	1.0	1.5	0.610410	0.610409	0.5293161	0.530716	0.513424
	1.5	2.0	1.311115	1.311115	1.185384	1.311115	1.209764
	2.0	2.5	2.359689	2.379689	2.282167	2.359689	2.331872
100	1.0	1.5	0.466204	0.466204	0.3765582	0.466204	0.378032
	1.5	2.0	1.204456	1.204456	1.138956	1.127385	1.136965
	2.0	2.5	2.274085	2.274065	2.144278	2.274065	2.284561

Table 5.1: Mean Squared Error for under $\hat{\alpha}$ Jeffrey’s prior

N	α	$\beta + 1$	C_1	α_{ML}	α_{sl}	α_{NI}		
						C2=-1.0	C2=0	C2=1.0
25	1.0	1.5	0.5	1.751815	0.605711	0.491789	0.605711	0.675668
			1.0	1.751815	0.678151	0.785964	0.678151	0.715066
			1.5	1.751815	0.756965	0.796710	0.707501	0.761006
			2.0	1.751815	1.743262	1.551585	1.743262	1.873616
	2.0	2.5	0.5	1.465009	1.465009	1.530603	1.478005	1.525623
			1.0	1.465009	1.664499	1.393877	1.154286	1.600484
			1.5	1.465009	1.579438	1.178935	1.336005	0.743068
			2.0	1.465009	1.685124	1.469333	1.685124	1.753295
50	1.0	1.5	0.5	0.523688	0.523688	0.442794	0.523688	0.621982
			1.0	0.523688	0.581831	0.591908	0.581831	0.553100
			1.5	0.523688	0.580325	0.357746	0.441632	0.602326
			2.0	0.523688	0.718147	0.737926	0.471831	0.421187
	2.0	2.5	0.5	1.170480	1.170480	1.203897	1.132317	1.095543
			1.0	1.170480	1.275611	1.309292	1.181767	0.993167
			1.5	1.170480	1.421131	1.148665	1.221135	0.904535
			2.0	1.170480	1.656525	1.365212	1.656525	1.532779
100	1.0	1.5	0.5	0.494552	0.494552	0.357457	0.494552	0.560155
			1.0	0.494552	0.522866	0.532341	0.574154	0.410739
			1.5	0.494552	0.523018	0.332578	0.405039	0.480748
			2.0	0.494552	0.499482	0.459488	0.469988	0.344415
	2.0	2.5	0.5	1.069772	1.069772	1.192434	1.069772	0.981817
			1.0	1.069772	1.195533	1.222817	1.164875	0.879562
			1.5	1.069772	1.129912	1.146871	0.8618621	1.321517
			2.0	1.069772	1.346683	1.317127	1.346683	1.250574

Table 5.2: Mean Squared Error for $\hat{\alpha}$ under extension of Jeffrey’s prior

ML= Maximum Likelihood, SL=Squared Error Loss Function, NL= New Loss Function,

In table 5.1, Bayes' estimation with New Loss function under Jeffrey's prior provides the smallest values in most cases especially when loss parameter C_2 is ± 1 . Similarly, in table 5.2, Bayes' estimation with New Loss function under extension of Jeffrey's prior provides the smallest values in most cases especially when loss parameter C_2 is ± 1 whether the extension of Jeffrey's prior is 0.5, 1.0, 1.5 or 2.0.

6. Concluding Remarks

In this article, we have primarily studied the Bayes' estimator of the parameter of the size biased Gamma distribution under the extended Jeffrey's prior assuming two different loss functions. The extended Jeffrey's prior gives the opportunity of covering wide spectrum of priors to get Bayes' estimates of the parameter - particular cases of which are Jeffrey's prior and Hartigan's prior. A comparative study has been done between the MLE and the estimates of two loss functions (SELF and Al-Bayyati's new loss function). From the results, we observe that in most cases, Bayesian Estimator under New Loss function (Al-Bayyati's Loss function) has the smallest Mean Squared Error values for both prior's i.e, Jeffrey's and an extension of Jeffrey's prior information. Moreover, when the sample size increases from 25 to 100, the MSE decreases quite significantly. The future research may be consider to estimate the parameters using different loss functions especially Linlex loss function and Generalized Entropy Loss function under different prior distributions like Gamma prior distributions, Conjugate priors and double priors etc.

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