Linear Consecutive-*k*-out-of-*n*: G System Reliability Analysis

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Abstract

The concerned study pertains to the development of a new stochastic model for the reliability analysis of linear consecutive-*k*-out-of-*n*: G system, where $k > \frac{n}{2}$. In the developed model, system may collapse as a result of common cause failure or hardware failure in its units. The system has exponentially distributed failure rates, and in case of breakdown, it is repaired with the copula method. The developed model has been examined through supplementary variable technique (SVT) along with Laplace transform. The current paper has specifically studied consecutive-(n-1)-out-of-n: G system. The performance of such system having ten components is explored and its various reliability measures have been obtained and discussed with the help of graphs. The originality of this work lies in incorporating common cause failure in conjunction with copula repair in the reliability modeling of consecutive systems through the SVT. The study confirms that an increase in failure rates

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and the number of components of the concerned system decreases mean time to failure (MTTF). The profit of linear consecutive-9-out-of-10: G system is examined with the help of a numerical example.

Keywords: Consecutive-*k*-out-of-*n*: G system, SVT, common cause failure, copula, reliability measures.

1 Introduction

The reliability of deployed systems and manufactured goods in factories is of profound importance, as it directly affects the consumer's choice and the organization's business. The users' and environmental safety are two serious issues closely linked to system reliability. Any breach in critical systems like nuclear power plant systems, aircraft systems, chemical manufacturing plant systems, medical devices, and weapon systems may be menacing and lead to massive loss. In their study, Kumar et al. (2018) listed some catastrophic accidents that occurred during 2006–2016. They have also mentioned the reasons for accidents, along with levels of their severity and losses in terms of life and property.

Hence, it has become quintessential to have safe systems capable of producing efficient and long-running quality products. The ever-increasing and necessary use of emerging technologies such as robotics, nanotechnology, 3D printing, artificial intelligence, etc., strongly impact today's manufacturing process. In modern industries, highly intricate systems are being employed for advanced manufacturing. Effective system design, proper maintenance, and repair schemes ensure system reliability and company profitability.

SVT was first used by Cox (1955) in investigating the non-markovian system. Several researchers have applied this method for evaluating metrics like availability, reliability, and MTTF. In this technique, the non-markovian process is changed to markovian by adding some supplementary variables as elapsed repair time. As far as the system structure is concerned, most critical systems are based on k-out-of-n redundancy. A k-out-of-n: G/F system is in a working/failed state provided its not less than k units among n are working/failed. If $k = 1$ and n, a k-out-of-n: G system reduces to parallel and series systems, respectively.

Linton and Saw (1974), Gupta and Goyal (1992), Singh and Dayal (1991), Mishra and Jain (2013), Wu et al. (2014), Ram and Kumar (2014, 2015), Singh and Poonia (2022), Poonia (2022) and Singh et al. (2022) have used SVT to develop stochastic models to evaluate the reliability of k-out-of-n: G/F systems. These studies have addressed various situations, such as different types of failures (common cause, human, partial, catastrophic), repairs and maintenance strategies. Linton and Saw (1974) applied the SVT approach to *k*-out-of-*n*: F systems ($k = 2$; $k = 3$) and evaluated Laplace transform of the time to system failure distributions. Gupta and Goyal (1992) studied the profit function of the *k*-out-of-*n* trichotomous system by assuming failure and repair times to be exponentially and general distributed, respectively. Singh and Dayal (1991) investigated the effect of common cause failure, critical human errors, and *r* repair facilities on the performance of 1-out-of-*n*: G system. Mishra and Jain (2013) considered imperfect repair, shut-off rule, and arbitrary distributed failure and repair time in their model and analyzed a system consisting of a major unit (*k*-out-of-*n*: F) and a minor unit. Wu et al. (2014) incorporated the concept of a single vacation into their study. Ram and Kumar (2014) inspected the reliability measures of the complex series system comprising of sub-unit A (2-out-of-3: F) and sub-unit B in the presence of human failure. Ram and Kumar (2015) also investigated a hybrid system involving 1-out-of-2: G subsystem with flawless reworking strategy. Goyal et al. (2017) examined sensitivity of the reliability metrics of a complicated system possessing of three subsystems linked in series for *k*-out-of-*n* redundancy.

A literature survey reveals that the consecutive-*k*-out-of-*n* systems have drawn the attention of many researchers. Linear consecutive-*k*-out-of-*n*: G/F systems have linearly arranged *n* units, and these systems are in a working/failed state, provided that not less than its *k* consecutive units among *n* are working/failed. These systems have wide uses in telecommunication systems, oil pipeline systems, water supply systems, the series-parallel systems in the electrical circuit designs, parking system as well as in the spacecraft relay stations.

Kontoleon (1980) determined the reliability of an *r*-successive-out-of-*n*: F system. A tropological formula was derived by Kossow and Preuss (1989) to assess reliability concerning linear as well as circular consecutive-*k*-out-of-*n*: F systems, wherein units had different reliabilities. Zhang and Wang (1996) specifically studied these systems for $k = 2$. Zhang and Lam (1998) derived the state transition probabilities of the consecutive-*k*-out-of-*n*: G system, where $k > \frac{n}{2}$ using generalized transition probability. They assumed that the working as well as repair times are exponentially distributed and obtained expressions for system reliability and MTTF. Cheng and Zhang (2001) extended this work to study consecutive-*k*-out-of-*n*: F system by considering key component, whose repair has been given the highest priority. They evaluated state transition probabilities and important reliability measures using a generalized transition probability definition. Krishnan and Somasundaram (2011) assessed the performance metrics of consecutive-*k*-out-of-*n*: G system having sensor and *r* repair persons. They used the Laplace transform and concluded that a use of sensor improves the system performance. Yuan and Cui (2013) inspected reliability indexes of the consecutive-*k*-out-of-*n*: F system attended by *r* repair persons who are allowed to take multiple vacations. The authors specifically investigated the performance of consecutive-3-out-of-8: F system handled by a single repairman who can take many rests through the Runge-Kutta method. Zhou et al. (2020) focused on the optimization problem for a consecutive-2-out-of-*n*: G system. Through minimizing expected costs rates, authors have obtained ideal number of components as well as ideal replacement time. Kuo et al. (1990), Zhang et al. (2000), Yam et al. (2003), Guan and Wu (2006), Özbey and Gökdere (2021), Gökdere and Tony (2022), Wu et al. (2022) and Wang et al. (2022) also investigated consecutive-*k*-out-of-*n*: F/G systems in their studies.

Many researchers, including Ram and Singh (2010), Rawal et al. (2015), Chopra and Ram (2019), Yusuf et al. (2021), Rawal et al. (2022), and Sanusi et al. (2022) have applied the copula approach in their models. Poonia (2021) studied the performance of a multi-state computer network system incorporating copula modeling. Recently, Singh et al. (2022) used copula in the stochastic modeling of complex repairable *k*-out-of-*n*: G system with controllers. Copula repair, employed in many reliability models, has emerged as an effective approach for better performance of the systems. However, this approach has not yet been applied in the study of reliability of linear consecutive-*k*-out-of-*n*: F/G systems.

Common cause failure occurs when multiple system components stop working due to a single shared cause. These failures are relevant in the study of redundant systems, as their excessive presence impedes the benefits of redundant configured systems. The current study has been carried out to develop a stochastic model concerning linear consecutive-*k*-out-of-*n*: G system for $k > \frac{n}{2}$ via SVT, wherein all *n* units are independent, and their failure times are distributed exponentially. A specific case of consecutive-(*n*-1)-out-of-*n*: G system has been worked out, considering common cause failure and copula repair. Gumbel-Hougaard copula is used to model two types of repair rates between failed and good states. The contribution of this work lies in the fact that no such reliability model has been worked out earlier by incorporating common cause failure and copula repair for the considered redundant system using the SVT technique.

2 Notations, Model Description and Assumptions

All notations employed in this model are described as follows:

t/s	Time/Laplace transform variable
$\bar{}$	Symbol for Laplace Transform
λ/λ_{cc}	Each unit/Common cause failure rate
M_{-l}	Number of cases in state S_{-l}, where $l = 1, 2, \ldots, n-k$
λ_0	Degradation rate from the state S_0 to S_{-1}
λ_l	Degradation rate from the state S_{-l} to $S_{-(l+1)}$, where $l = 1, 2, \ldots, n-k-1$
f_0	Failure rate from the state S_0 to S_1
f_l	Failure rate from the state S_{-l} to $S_{(l+1)}$, where $l = 1, 2, \ldots, n-k$
μ	Each unit Repair rate
$\phi_l(x)$	Repair rate for the state S_l, for $l = cc, 1, 2, \ldots, n-k+1$
$C_{\theta_l}(u(x), \phi_l(x))$	Joint Pdf as per Gumbel-Hougaard copula for repair rate of the state S_l, $l = cc, 1, 2, \ldots, n-k+1, u(x) = e^x$ and $C_{\theta_l}(u(x), \phi_l(x)) = exp[x^\theta + (\log(\phi_l(x)))^\theta]^{1/\theta}$
$P_{-m}(t)/P_l(t)$	Probability that at any time t system is in state S_{-m}/S_l, $m = 1, 2, \ldots, n-k$ and $l = cc, 0, 1, 2, \ldots, n-k+1$
$P_l(x, t)$	Probability that at any moment t system is in the failed state S_l and elapsed repair time is x, where $l = cc, 1, 2, \ldots, n-k+1$
$\overline{q_l}(s)$	Laplace Transform of $C_{\theta_l}(u(x), \phi_l(x))$, for $l = cc, 1, 2, \ldots, n-k+1$
$\overline{P_{-m}}(s)/\overline{P_l}(s)$	Laplace transform of $P_{-m}(t)/P_l(t)$, where $m = 1, 2, \ldots, n-k$ and $l = cc, 0, 1, 2, \ldots, n-k+1$
$E_p(t)$	Expected system profit in the interval $[0\ t)$

The developed reliability model for linear consecutive-*k*-out-of-*n*: G system ($k > \frac{n}{2}$) has total $2(n-k)+3$ states, among which the number of good, degraded, and failed states are 1, $n-k$, and $n-k+2$, respectively. These states are explained in Table 1 and shown in the transition state diagram in Figure 1. In state S_l ($l = 1, 2, \ldots, n-k+1$), number of consecutively functioning units in the system is less than *k*, and hence the system is in a failed state.

Referring to states, $S_{-l}(l = 1, 2, \ldots, n-k)$, as discussed by Krishnan and Somasundaram (2011), there are M_{-l} cases, which are given by:

$$M_{-l} = \binom{n}{l} - N(n-l, l+1, k-1)$$

Table 1 Various states of the system

S_0	Good state as all *n* units are functioning
S_{-l} $(l = 1, 2, \ldots, n-k)$	Degraded working state as l $(l = 1, 2, \ldots, n-k)$ units are non-operational, but at least k consecutive units are operational
S_l $(l = 1, 2, \ldots, n-k+1)$	Failed state as l $(l = 1, 2, \ldots, n-k+1)$ units are non-operational and the number of consecutively functioning units is less than k
S_{cc}	Failed state owing to common cause failure

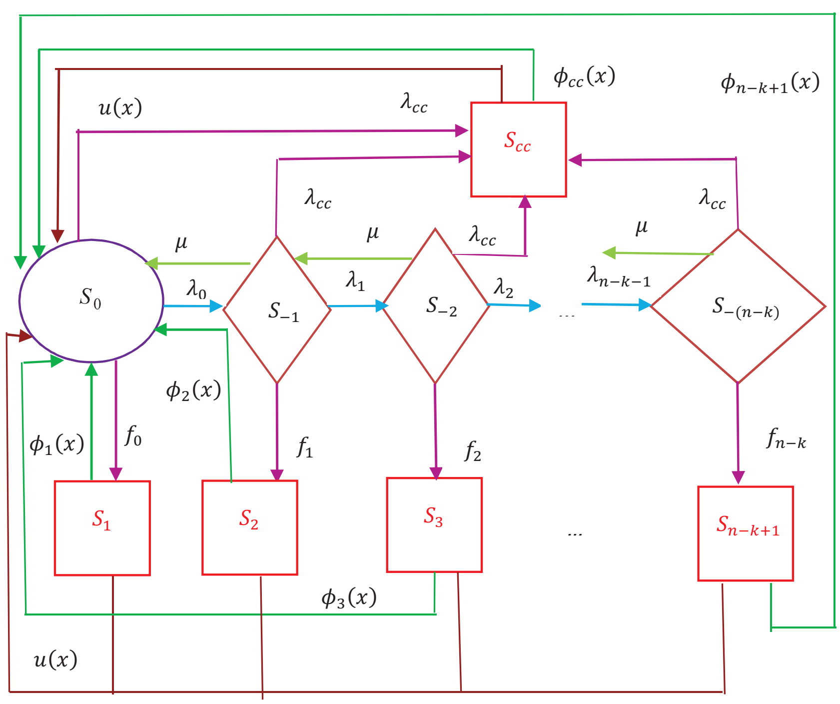

Figure 1 Transition state diagram for linear consecutive-*k*-out-of-*n*: G system.

where, $N(m, h, k-1) = \sum_{r=0}^{min(h, \lfloor \frac{m}{k} \rfloor)} (-1)^r \binom{h}{r} \binom{h+m-kr-1}{m-kr}$ and $\lfloor \frac{m}{k} \rfloor$ stands for the greatest integer $\leq \frac{m}{k}$.

Further, the degradation rate, λ_l $(l = 1, 2, \ldots, n-k-1)$ is expressed as

$$\lambda_l = \left[\frac{(l+1)M_{-(l+1)}}{M_{-l}} \right] \lambda.$$

For $l = 0$, we have $\lambda_0 = M_{-1}\lambda$.

The rate of failure of the system, f_l $(l = 1, 2, \ldots, n-k)$ when l units are failed is:

$$f_l = \left[(n-l) - \frac{(l+1)M_{-(l+1)}}{M_{-l}}\right]\lambda.$$

For $l = 0$, we have $f_0 = (n - M_{-1})\lambda$.

The postulations for developing the model are stated as under:

1. The system gets collapses if the consecutive working units in the system are less than *k*.
2. Common cause failure also led to system breakdown.
3. All failures in the system are independent and follow exponential distribution.
4. The rate of repair of each unit is constant and follows exponential distribution.
5. In case of complete failure, the system is repaired by copula method. Gumbel-Hougaard copula is employed to manage two types of repair rates between failed and good states.
6. A repair facility is always available with the system.
7. The repaired system is the same as the new one.

3 Mathematical Analysis of the Model

The mathematical model for linearly ordered consecutive-*k*-out-of-*n*: G system with $k > \frac{n}{2}$ involves following differential equations.

$$\begin{aligned}\left(\frac{d}{dt} + n\lambda + \lambda_{cc}\right) P_0(t) &= \mu P_{-1}(t) + \sum_{l=1}^{n-k+1} \int_0^{\infty} C_{\theta_l}(u(x), \phi_l(x)) P_l(x,t)dx \\ &\quad + \int_0^{\infty} C_{\theta_{cc}}(u(x), \phi_{cc}(x)) P_{cc}(x,t)dx \end{aligned} \tag{1}$$

$$\left(\frac{d}{dt} + (n-1)\lambda + \lambda_{cc} + \mu\right) P_{-1}(t) = \lambda_0 P_0(t) \tag{2}$$

For all, $l = 2, \ldots, n-k$, we have

$$\left(\frac{d}{dt} + (n-l)\lambda + \lambda_{cc} + \mu\right) P_{-l}(t) = \lambda_{l-1} P_{-(l-1)}(t). \tag{3}$$

Corresponding to collapsed states, S_l $(l = 1, 2, \ldots, n-k+1)$ and S_{cc}, we have

$$\left[\frac{\partial}{\partial x} + \frac{\partial}{\partial t} + C_{\theta_l}(u(x), \phi_l(x))\right] P_l(x,t) = 0 \tag{4}$$

and

$$\left[\frac{\partial}{\partial x} + \frac{\partial}{\partial t} + C_{\theta_{cc}}(u(x), \phi_{cc}(x))\right] P_{cc}(x,t) = 0. \tag{5}$$

Initially, system is in a proper working condition means its state is good, hence

$$P_0(0) = 1,$$
$$P_{-l}(0) = 0, \quad \forall\, l = 1, 2, \ldots, n-k$$
$$P_l(0) = P_l(x,0) = 0, \quad \forall\, l = 1, 2, \ldots, n-k+1$$

and

$$P_{cc}(0) = P_{cc}(x,0) = 0.$$

The current system is subjected to the following boundary conditions

$$P_1(0,t) = f_0 P_0(t)$$
$$P_{l+1}(0,t) = f_l P_{-l}(t), \quad \forall\, l = 1, 2, \ldots, n-k$$

and

$$P_{cc}(0,t) = \lambda_{cc}\left(P_0(t) + \sum_{l=1}^{n-k} P_{-l}(t)\right).$$

Taking Laplace transform of Equation (2), we get

$$\overline{P_{-1}}(s) = \frac{\lambda_0}{(s + (n-1)\lambda + \lambda_{cc} + \mu)}\overline{P_0}(s).$$

Similarly, from Equation (3), we obtain

$$\overline{P_{-l}}(s) = \frac{\lambda_{l-1}}{(s + (n-l)\lambda + \lambda_{cc} + \mu)}\overline{P_{-(l-1)}}(s), \quad \forall\, l = 2, \ldots, n-k. \tag{6}$$

Solving Equation (4), we have

$$\overline{P_1}(s) = f_0\left(\frac{1 - \overline{q_1}(s)}{s}\right)\overline{P_0}(s)$$

and

$$\overline{P_l}(s) = f_{l-1}\left(\frac{1-\overline{q_l}(s)}{s}\right)\overline{P_{-(l-1)}}(s), \quad \forall\, l = 2, \ldots, n-k+1. \tag{7}$$

Using Equation (6) in Equation (7), $\forall\, l = 3, \ldots, n-k+1$, we get,

$$\overline{P_l}(s) = f_{l-1}\left(\frac{1-\overline{q_l}(s)}{s}\right)\left[\frac{\lambda_{l-2}}{(s+(n-l+1)\lambda+\lambda_{cc}+\mu)}\right]\overline{P_{-(l-2)}}(s).$$

The Laplace transform of Equation (5) gives

$$\overline{P_{cc}}(s) = \lambda_{cc}\left(\frac{1-\overline{q_{cc}}(s)}{s}\right)\left(\overline{P_0}(s) + \sum_{l=1}^{n-k}\overline{P_{-l}}(s)\right).$$

4 Specific Case of Linear Consecutive-(n-1)-out-of-n: G System

The model of consecutive-(n-1)-out-of-n: G system, as shown in the transition state diagram in Figure 2, has five states. All the assumptions in this model are the same as discussed in Section 2.

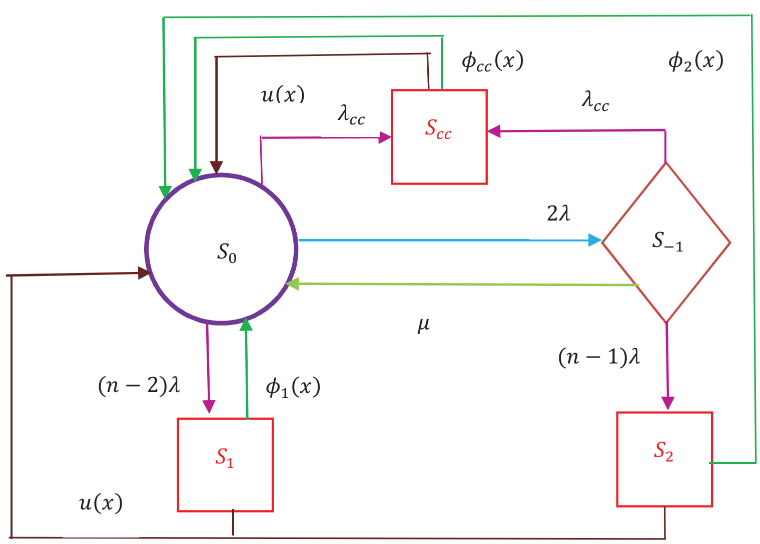

Figure 2 Transition state diagram for consecutive-(n-1)-out-of-n: G system.

For this special case, Equation (1) reduces to

$$\begin{aligned}\left(\frac{d}{dt} + n\lambda + \lambda_{cc}\right) P_0(t) &= \mu P_{-1}(t) + \int_0^\infty C_{\theta_1}(u(x), \phi_1(x)) P_1(x,t)dx \\ &\quad + \int_0^\infty C_{\theta_2}(u(x), \phi_2(x)) P_2(x,t)dx \\ &\quad + \int_0^\infty C_{\theta_{cc}}(u(x), \phi_{cc}(x)) P_{cc}(x,t)dx.\end{aligned}$$

There are two cases in the state S_{-1} and degradation rate, λ_0 from state S_0 to S_{-1} is 2λ. The failures rates for states S_1 and S_2 are $(n-2)\lambda$ and $(n-1)\lambda$, respectively. Thus, Equation (2) reduces to

$$\left(\frac{d}{dt} + (n-1)\lambda + \lambda_{cc} + \mu\right) P_{-1}(t) = 2\lambda P_0(t). \tag{8}$$

Corresponding to the failed states, S_1, S_2 and S_{cc}, we have

$$\left[\frac{\partial}{\partial x} + \frac{\partial}{\partial t} + C_{\theta_1}(u(x), \phi_1(x))\right] P_1(x,t) = 0 \tag{9}$$

$$\left[\frac{\partial}{\partial x} + \frac{\partial}{\partial t} + C_{\theta_2}(u(x), \phi_2(x))\right] P_2(x,t) = 0 \tag{10}$$

and

$$\left[\frac{\partial}{\partial x} + \frac{\partial}{\partial t} + C_{\theta_{cc}}(u(x), \phi_{cc}(x))\right] P_{cc}(x,t) = 0. \tag{11}$$

The initial, as well as the boundary conditions to govern the system are given as under:

$$P_0(0) = 1,$$

$$P_{-1}(0) = P_1(0) = P_2(0) = P_1(x,0) = P_2(x,0) = P_{cc}(0) = P_{cc}(x,0) = 0,$$

$$P_1(0,t) = (n-2)\lambda\ P_0(t),$$

$$P_2(0,t) = (n-1)\lambda\ P_{-1}(t)$$

and

$$P_{cc}(0,t) = \lambda_{cc}(P_0(t) + P_{-1}(t)).$$

Solving Equations (8)–(11), we get

$$\overline{P_0}(s) = \frac{1}{A(s)} \tag{12}$$

$$\overline{P_{-1}}(s) = \frac{2\lambda}{(s + (n-1)\lambda + \lambda_{cc} + \mu)}\overline{P_0}(s) \tag{13}$$

$$\overline{P_1}(s) = (n-2)\lambda \left(\frac{1 - \overline{q_1}(s)}{s}\right)\overline{P_0}(s)$$

$$\overline{P_2}(s) = (n-1)\lambda \left(\frac{1 - \overline{q_2}(s)}{s}\right)\overline{P_{-1}}(s)$$

and

$$\overline{P_{cc}}(s) = \lambda_{cc}\left(\frac{1 - \overline{q_{cc}}(s)}{s}\right)(\overline{P_0}(s) + \overline{P_{-1}}(s)).$$

where,

$$\begin{aligned} A(s) = s + &\frac{2\lambda}{(s + \lambda(n-1) + \lambda_{cc} + \mu)}[s + \lambda(n-1)(1 - \overline{q_2}(s)) \\ &+ \lambda_{cc}(1 - \overline{q_{cc}}(s))] + \lambda(n-2)(1 - \overline{q_1}(s)) + \lambda_{cc}(1 - \overline{q_{cc}}(s)). \end{aligned} \tag{14}$$

The Laplace transform of the probability of the system being in upstate is given by –

$$\overline{P_{up}}(s) = \overline{P_0}(s) + \overline{P_{-1}}(s). \tag{15}$$

In numerical calculation, for $l = 1, 2$, and cc we have assumed $\phi_l(x) = \theta = x = 1$, and consequently, we get, $\overline{q_l}(s) = \frac{e}{s+e}$. Further, considering $\mu = 1$ and using Equations (12)–(15), the obtained expression for $\overline{P_{up}}(s)$ is as under-

$$\overline{P_{up}}(s) = \frac{(s+e)(s + (n+1)\lambda + \lambda_{cc} + 1)}{B(s)} \tag{16}$$

where,

$$\begin{aligned} B(s) = &(s + \lambda(n-1) + \lambda_{cc} + 1)(s+e)(s + \lambda(n-2) + \lambda_{cc}) \\ &+ 2\lambda(s+e)(s + \lambda(n-1) + \lambda_{cc}) \\ &- e((n-2)\lambda + \lambda_{cc})(s + \lambda(n-1) + \lambda_{cc} + 1) \\ &- 2\lambda e(\lambda(n-1) + \lambda_{cc}). \end{aligned}$$

For failure rates, $\lambda = 0.15$ and $\lambda_{cc} = 0.20$, we have attained the following expressions for the availability as well as reliability of the linear consecutive-9-out-of-10: G system-

$$\begin{aligned} A(t) = 0.344250236133439e^{-4.081748393349267t} \\ - 0.001781669384188e^{-2.886533435109778\,t} \\ + 0.657531433250748 \end{aligned} \tag{17}$$

and

$$\begin{aligned} R(t) = -1.000000000000031e^{1.699999999999998\,t} \\ + 2.000000000000031e^{-1.550000000000001\,t}. \end{aligned} \tag{18}$$

The above-mentioned Equation (17) has been obtained using Equation (16) and the inverse Laplace Transform tool. The Equation (18) has been attained by the inverse Laplace transform of Equation (15), wherein all repair rates are assumed to be zero and it is the system reliability. The limiting behaviour of $\overline{P_{up}}(s)$ (Equation (16)), with nil repair rates as s tends to zero results in Equation (19) of system MTTF.

$$\text{MTTF} = \frac{\lambda(n+1) + \lambda_{cc}}{n(n-1)\lambda^2 + (2n-1)\lambda\,\lambda_{cc} + {\lambda_{cc}}^2} \tag{19}$$

The primary objective of any industry is to earn a reasonable profit with time. Reliable systems are less prone to failure, so their operation involves less repair cost. Thus, the system earned profit can be improved by boosting its reliability. The system expected profit is evaluated with the help of the below-mentioned formula:

$$E_p(t) = r_c \int_0^t P_{up}(t) - s_c t$$

where, r_c and s_c are revenue and service cost per unit time, respectively.

Using the above basic formula, corresponding to parameters $\lambda = 0.15$, $\lambda_{cc} = 0.20$ and revenue cost as 1, the under mentioned expression is obtained for the expected profit of consecutive-9-out-of-10: G system:

$$\begin{aligned} E_p(t) = -0.084338916307128e^{-4.081748393349267\,t} \\ + 0.000617234972066e^{-0.001781669384188\,t} \\ + 0.657531433250748\,t + 0.083721681335062 - s_c t. \end{aligned}$$

5 Result and Discussion

The availability of the linear consecutive-9-out-of-10: G system given in Table 2 initially declines sharply, and after a certain moment ($t = 8$), it attains a constant value of 0.6575314333. The reliability of the system also reduces with the lapse of time.

The early decrease in reliability is steep, whereas, after $t = 4$, it decreases gradually. Due to corrective maintenance, at any moment *t* system availability consistently exceeds its reliability. These two metrics have been investigated in depth by varying rate of common cause failure and the number of components.

Figures 3 and 4 reveal that at any instant, *t* and for $\lambda_{cc} = 0$, both the availability as well as reliability attained the highest levels concerning linear consecutive-9-out-of-10: G system. These figures further depict that the increased common cause failure has led to reduced system availability and reliability.

The availability and the reliability of system are influenced by the value of *n* and that is shown in Figures 5 and 6. It is obvious that the performance of system is improved with decreasing the value of *n*. The observed differences in system availabilities and reliabilities at $t = 1$ are 0.2910 and 0.3866, respectively, when *n* is varied from 6 to 20.

MTTF of the consecutive-(*n*-1)-out-of-*n*: G system has been analyzed by varying one parameter at a time in Equation (19). Figures 7, 8, and 9 indicate that the system MTTF follows a decreasing trend with increasing the number of components as well as the failure rates. The constant parameters assumed in Figures 7, 8, and 9 are $\lambda_{cc} = 0.20$, $\lambda = 0.15$ and $n = 10$, respectively.

Table 2 Availability and reliability of system

Time	Availability	Reliability
0	1.0000000000	1.0000000000
1	0.6632423026	0.2418124236
2	0.6576239566	0.0567251348
3	0.6575327793	0.0130264573
4	0.6575314440	0.0029450861
5	0.6575314328	0.0006580167
6	0.6575314332	0.0001456781
7	0.6575314332	0.0000320188
8	0.6575314333	0.0000069967
9	0.6575314333	0.0000015217
10	0.6575314333	0.0000003297

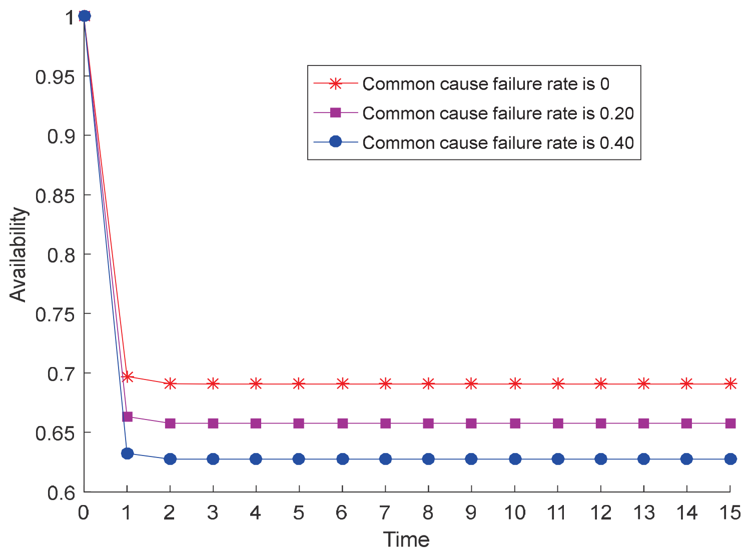

Figure 3 Influence of common cause failure on system availability.

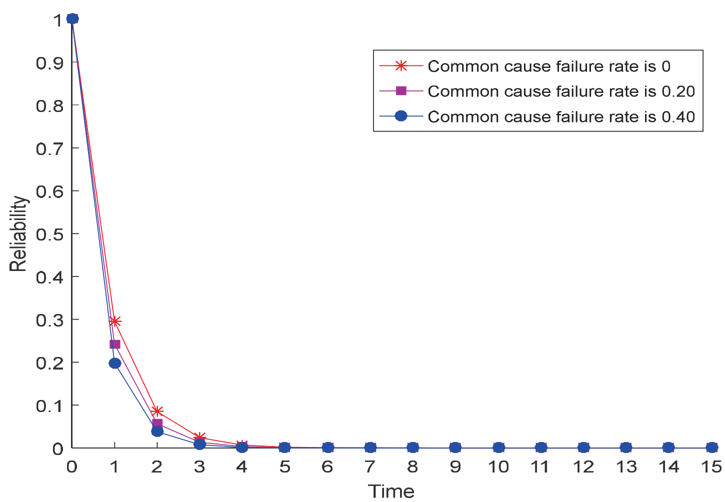

Figure 4 Influence of common cause failure on system reliability.

Figure 7 reveals that for the higher failure rates, λ as 0.40, 0.50, and 0.60 MTTF is nearly the same for all the linear consecutive (n-1)-out-of-n: G systems having more than 12 units. Similarly, Figure 8 shows that for the increased common cause failure rate, λ_{cc} as 0.30, 0.40, 0.50, and 0.60, all

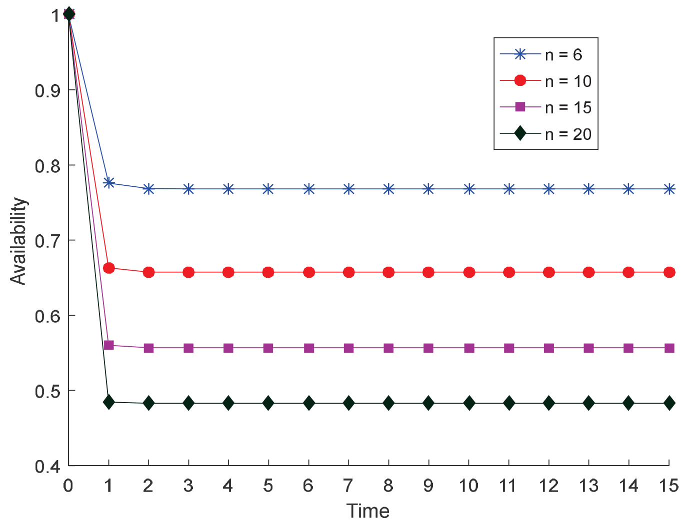

Figure 5 Effect of n on availability of the system.

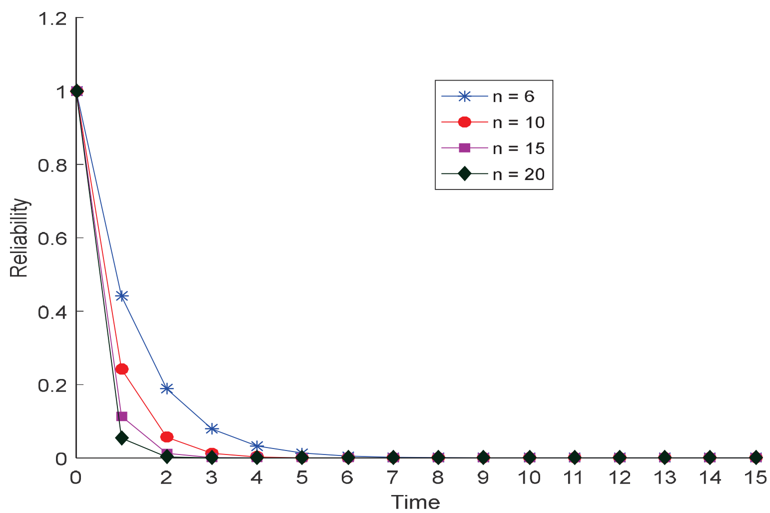

Figure 6 Effect of n on reliability of the system.

the linear consecutive (n-1)-out-of-n: G systems with more than 8 units have almost alike MTTF.

Graphical presentation of the results shown in Figure 9 exhibits a sharp decrease in the system MTTF owing to increase in failure rate λ. For a

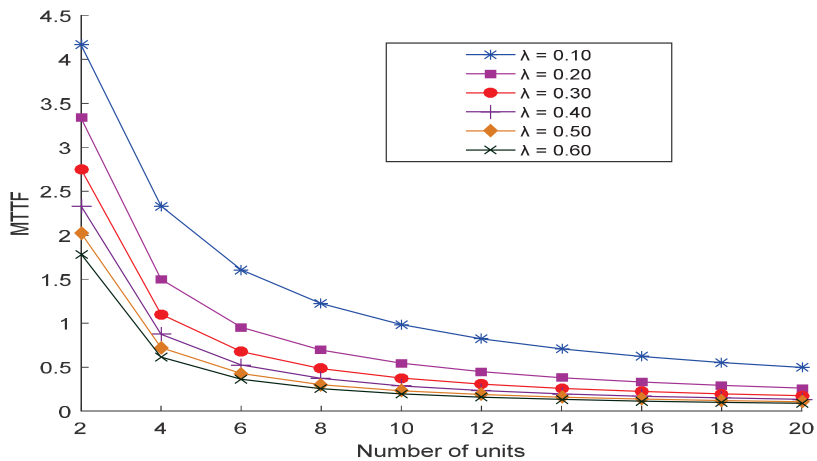

Figure 7 Effect of n and λ on system MTTF.

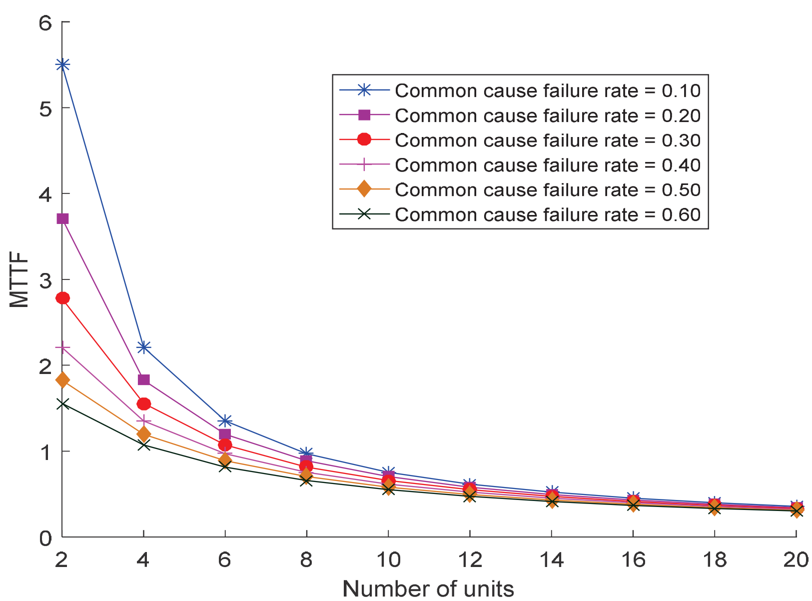

Figure 8 Effect of n and λ_{cc} on system MTTF.

fixed assumed common cause failure rate, $\lambda_{cc} = 0.20$, the MTTF of linear consecutive-9-out-of-10: G system decreases from 0.9848 to 0.1194 as, λ varies from 0.1 to 1.0 while this reduction is 0.3032 for a fixed value of $\lambda = 0.15$ when the common cause failure rate increases from 0.1 to 1.0.

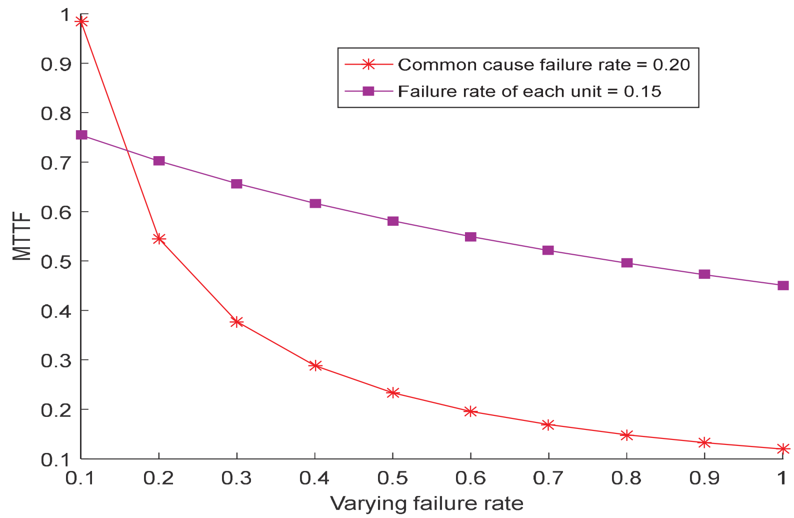

Figure 9 Effect of λ and λ_{cc} on system MTTF.

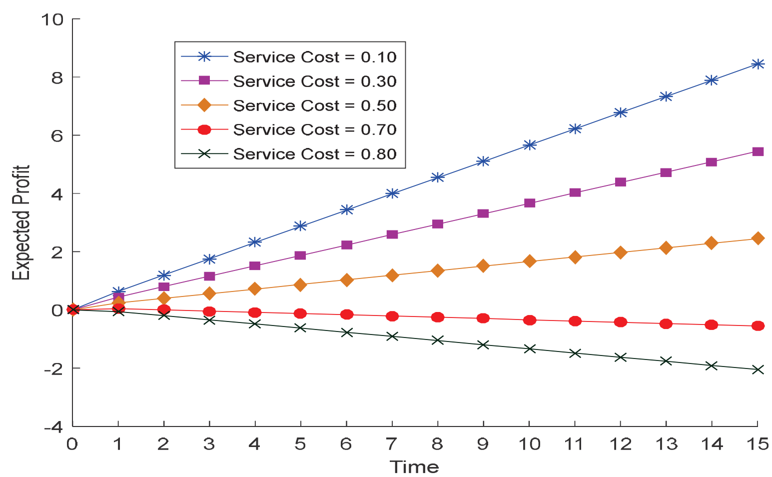

Figure 10 Effect of service cost on expected profit.

Figure 10 indicates that for the service costs of 0.10, 0.30, and 0.50 linear consecutive-9-out-of-10: G system is profitable, and the profit also increases with time. However, for $s_c = 0.60$ and $s_c = 0.80$, the expected profit becomes negative, and the system goes into a loss. The study suggests that controlled lower failure rates and an optimum number of units may result in a highly efficient and profitable linear consecutive-(n-1)-out-of-n: G system.

6 Conclusion

A reliability model for linear consecutive-*k*-out-of-*n*: G system is developed using SVT by incorporating common cause failure and copula repair. In the present study a specific case of consecutive-(*n*-1)-out-of-*n*: G system has also been worked out. The reliability metrics of such system ($n = 10$) have been evaluated and explored in depth by plotting different graphs which confirms that availability, reliability, and MTTF of linear consecutive-(*n*-1)-out-of-*n*: G system decrease on account of rise in common cause failure, hardware failure rate, and the number of units. In the proposed model, MTTF of consecutive-9-out-of-10: G system is strongly influenced by increased hardware failure rate, λ. It has been observed that the MTTF of linear consecutive-9-out-of-10: G system declines more rapidly with rise in the hardware failure rate, λ, as compared to the common cause failure rate, λ_{cc}. For the assumed parameters, corresponding to revenue cost one and service costs of 0.10, 0.30, 0.50, 0.70, and 0.80, the considered linear consecutive-9-out-of-10: G system is earning reasonable profit only if the involved service cost is 0.10, 0.30 and 0.50. The linear consecutive-(*n*-1)-out-of-*n*: G system with a lower hardware failure rate, no common cause failure, and an optimum number of units need to be designed, as otherwise, due to the high involved service cost, it may not give significant profit. The findings of this study are expected to be very beneficial for the industries because they may assist them in developing highly reliable and cost-efficient linear consecutive-*k*-out-of-*n*: G systems. Managers or the concerned persons can easily adopt the developed model for evaluating reliability measures of linear consecutive-(*n*-1)-out-of-*n*: G system. The developed stochastic model can be made more applicable by incorporating the notion of multi-state units and imperfect repair. The proposed model can be further improved by combining inspection and maintenance policies.

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