Generalized Estimator of Population Mean Using Auxiliary Information in Presence of Measurement Errors

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Abstract

It is assumed in survey research that the respondent's reported response is precise. More often, due to prestige bias, the data provided by respondents frequently include estimates that are significantly different from the genuine values. As a consequence, measurement error is present in the sample estimates that may affect the results. Therefore, this study illustrates an improved generalized estimator that utilizes auxiliary data under measurement error. A numerical study to establish its effectiveness is also conducted.

Keywords: Auxiliary variable, bias, mean squared error, efficiency and measurement errors.

1 Introduction

Many survey scientists have focused on the issue of parameter estimation in the face of measurement errors. The characteristics of estimators based

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on data in survey sampling normally presumes that the observations of the attributes being researched is accurate. This premise is not always met in practise, as measurement problems such non-response errors, recording errors, and calculation errors pollute data leading to invalid results. The statistical conclusions drawn from the observed data remain valid if measurement errors are negligibly small and can be disregarded. On the other hand, if they are not comparatively small and inconsequential, the deductions might not only be inaccurate and invalid but frequently have unintended, regrettable, and unfortunate results. For more details one may refer Cochran, W.G. (1968), Lessler Judith. T. and Kalsbeek, William, D. (1992), Paul P. Biemer, Robert M. Groves, Lars E. Lyberg, Nancy A. Mathiowetz and Seymour Sudman (1991), Sukhatme, P. V. and G.R. Seth (1952) etc. Numerous statisticians have addressed the issue of determining population mean when measurement errors are present, including Shalabh (1997), Singh and Karpe (2009), Misra and Yadav (2015), Manisha and Singh R. K (2002).

Let $U = U_1, U_2, \ldots, U_N$ be a finite population of N distinct and identifiable units with Y being the study variable and X being the auxiliary variable taking the value Y_i and X_i for the unit ith of the population U respectively. Additionally, suppose that n paired observations of characteristics X and Y were gathered using a basic random sampling process without replacement. Further, for size n simple random sample, let (x_i, y_i) be the observed values rather than true values (X_i, Y_i) for the ith $(i = 1, 2, \ldots n)$ sampling unit in the sample. Also, $u_i = y_i - Y_i$ and $v_i = x_i - X_i$ where u_i and v_i are related measurement errors that are stochastic in character with mean zero and variance σ_u^2 and σ_v^2 respectively. Additionally, assume that $u_i's$ and $v_i's$ are uncorrelated while $X_i's$ and $Y_i's$ are correlated. Let $\mu_X, \mu_Y, \sigma_X^2, \sigma_Y^2$ and ρ represents the population mean, variances and correlation coefficient between X and Y.

Let $\hat{x} = \frac{1}{n}\sum_{i=1}^{n} x_i \& \hat{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$ denotes the unbiased estimators of populations means μ_X and μ_Y i.e. $E(\hat{x}) = \mu_X$ and $E(\hat{y}) = \mu_Y$.

But in presence of measurement errors, $s_X^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \hat{x})^2$ and $s_Y^2 = \frac{1}{n-1}\sum_{i=1}^{n}(y_i - \hat{y})^2$ are biased estimators of the population variances σ_X^2 and σ_Y^2. $E(s_Y^2) = \sigma_Y^2 + \sigma_u^2$ provides the expected value of s_y^2, in the presence of measurement errors.

In the event where error variances σ_u^2 and σ_v^2 are known in advance, then $\widehat{\sigma}_Y^2 = s_y^2 - \sigma_u^2 > 0$ and $\widehat{\sigma}_X^2 = s_x^2 - \sigma_v^2 > 0$ are unbiased estimators of population variances under measurement errors.

Further let,

$$C_Y = \frac{\sigma_Y}{\mu_Y} \text{ and } C_X = \frac{\sigma_X}{\mu_X}, \gamma_{2Y} = \beta_{2Y} - 3,$$

$$\gamma_{2X} = \beta_{2X} - 3, \gamma_{2u} = \beta_{2u} - 3, \gamma_{2v} = \beta_{2V} - 3.$$

$$\gamma_{1(X)} = \sqrt{\beta_1(X)}$$

$$\beta_2(X) = \frac{\mu_4(X)}{\mu_2^2(X)},$$

$$\beta_2(u) = \frac{\mu_4(u)}{\mu_2^2(u)},$$

$$\beta_2(v) = \frac{\mu_4(v)}{\mu_2^2(v)},$$

$$\mu_{qrst} = E[(X - \mu_X)^q (Y - \mu_Y)^r v^s u^t]$$

$$\mu_{2000} = \sigma_X^2, \mu_{0200} = \sigma_Y^2, \mu_{0020} = \sigma_v^2 \text{ and } \mu_{0002} = \sigma_u^2.$$

Now suppose that sample values are collected with measurement errors i.e. the observed values (x_i,y_i) as different from the values (X_i,Y_i). It is suggested to use the following generalised estimator to estimate population mean when measurement errors are present.

$$\hat{\bar{y}}_{gME} = g(\bar{y}, b, \bar{x}, \hat{\sigma}_x^2) \tag{1}$$

where 'b' is defined as an estimate of the change in y caused by a unity increase in x and $g(\bar{y}, b, \bar{x}, \hat{\sigma}_x^2)$ is bounded in such a way that at the point $(\mu_Y, \beta, \mu_X, \sigma_X^2)$, we have

$$g(\mu_Y, \beta, \mu_X, \sigma_X^2) = \mu_Y \tag{2}$$

$$g_0 = \frac{\partial}{\partial \bar{y}} g(\bar{y}, b, \bar{x}, \hat{\sigma}_x^2) \Big]_{(\mu_Y, \beta, \mu_X, \sigma_X^2)} = 1 \tag{3}$$

$$g_1 = \frac{\partial}{\partial b} g(\bar{y}, b, \bar{x}, \hat{\sigma}_x^2) \Big]_{(\mu_Y, \beta, \mu_X, \sigma_X^2)} = 0 \tag{4}$$

$$g_2 = \frac{\partial}{\partial \bar{x}} g(\bar{y}, b, \bar{x}, \hat{\sigma}_x^2) \Big]_{(\mu_Y, \beta, \mu_X, \sigma_X^2)} = -\beta \tag{5}$$

$$g_{00} = \frac{\partial}{\partial \bar{y}^2} g(\bar{y}, b, \bar{x}, \hat{\sigma}_x^2)\bigg]_{(\mu_Y,\beta,\mu_X,\sigma_X^2)} = 0 \tag{6}$$

$$g_{22} = \frac{\partial}{\partial \bar{x}^2} g(\bar{y}, b, \bar{x}, \hat{\sigma}_x^2)\bigg]_{(\mu_Y,\beta,\mu_X,\sigma_X^2)} = 0 \tag{7}$$

2 Bias and Mean Squared Error

Here we consider the approximations as

$$\begin{gathered}
\hat{y} = \mu_Y(1+e_0) \\
\hat{x} = \mu_X(1+e_1) \\
\widehat{\sigma}_y^2 = \sigma_Y^2(1+e_2) \\
\widehat{\sigma}_x^2 = \sigma_X^2(1+e_3) \\
\widehat{\sigma}_{xy} = \sigma_{XY}(1+e_4) \\
\text{so that } E(e_i) = 0,\ i = 1,2,3,4
\end{gathered} \tag{8}$$

Using the results from Singh and Karpe (2009), we have

$$E(e_0^2) = \frac{C_Y^2}{n\theta_Y} \quad \text{and} \quad E(e_1^2) = \frac{C_X^2}{n\theta_X} \tag{9}$$

where, $\theta_Y = \frac{\sigma_Y^2}{\sigma_Y^2+\sigma_u^2}$ and $\theta_X = \frac{\sigma_X^2}{\sigma_X^2+\sigma_v^2}$

$$E(e_3^2) = \frac{A_X}{n}, \quad \text{where } A_X = \gamma_{2X} + \gamma_{2\nu}\frac{\sigma_\nu^4}{\sigma_X^4} + 2\left(1+\frac{\sigma_\nu^2}{\sigma_X^2}\right)^2 \tag{10}$$

$$\mathrm{E}(\mathrm{e}_2^2) = \frac{\mathrm{A}_Y}{\mathrm{n}}, \quad \text{where } \mathrm{A}_\mathrm{Y} = \gamma_{2\mathrm{Y}} + \gamma_{2\mathrm{u}}\frac{\sigma_u^4}{\sigma_\mathrm{Y}^4} + 2\left(1+\frac{\sigma_\mathrm{u}^2}{\sigma_\mathrm{Y}^2}\right)^2 \tag{11}$$

$$\mathrm{E}(\mathrm{e}_1\mathrm{e}_3) = \frac{\mu_{3000}}{\mathrm{n}\sigma_\mathrm{X}^2\mu_\mathrm{X}} \tag{12}$$

$$\mathrm{E}(\mathrm{e}_0\mathrm{e}_2) = \frac{\mu_{0300}}{\mathrm{n}\sigma_\mathrm{Y}^2\mu_\mathrm{Y}} \tag{13}$$

$$E(e_0e_3) = \frac{\mu_{2100}}{n\sigma_X^2\mu_Y} \tag{14}$$

$$\mathrm{E}(\mathrm{e}_1\mathrm{e}_2) = \frac{\mu_{1200}}{\mathrm{n}\sigma_{\mathrm{Y}}^2\mu_{\mathrm{X}}} \tag{15}$$

$$\mathrm{E}(\mathrm{e}_0\mathrm{e}_1) = \frac{\sigma_{\mathrm{XY}}}{\mathrm{n}\mu_{\mathrm{X}}\mu_{\mathrm{Y}}} = \frac{\rho\mathrm{C}_{\mathrm{X}}\mathrm{C}_{\mathrm{Y}}}{\mathrm{n}} \tag{16}$$

$$\mathrm{E}(\mathrm{e}_1\mathrm{e}_4) = \frac{\mu_{2100}}{\mathrm{n}\sigma_{\mathrm{XY}}\mu_{\mathrm{X}}} \tag{17}$$

$$E(e_3e_4) = \frac{\mu_{3100}}{n\sigma_X^2\sigma_{XY}} \tag{18}$$

$$E(e_2e_3) = \frac{\delta - 1}{n}, \quad \text{where } \delta = \frac{\mu_{2200}}{\sigma_X^2\sigma_Y^2} \tag{19}$$

$$E(e_2e_4) = \frac{\mu_{1200}}{n\sigma_Y^2\sigma_{XY}} \tag{20}$$

Using Taylor's Series expansion, we now expand $g(\bar{y}, b, \bar{x}, \hat{\sigma}_x^2)$ as

$$\begin{aligned}
\hat{\bar{y}}_{g_{ME}} &= g(\mu_Y, \beta, \mu_X, \sigma_X^2) + (\bar{y} - \mu_Y)g_0 + (b - \beta)g_1 + (\bar{x} - \mu_X)g_2 \\
&+ (\hat{\sigma}_x^2 - \sigma_X^2)g_3 + \frac{1}{2}\{(\bar{y} - \mu_Y)^2 g_{00} + (b - \beta)^2 g_{11} + (\bar{x} - \mu_X)^2 g_{22} \\
&+ (\hat{\sigma}_x^2 - \sigma_X^2)^2 g_{33} + 2(\bar{y} - \mu_Y)(b - \beta)g_{01} \\
&+ 2(\bar{y} - \mu_Y)(\bar{x} - \mu_X)g_{02} + 2(\bar{y} - \mu_Y)(\hat{\sigma}_x^2 - \sigma_X^2)g_{03} \\
&+ 2(b - \beta)(\bar{x} - \mu_X)g_{12} + 2(b - \beta)(\hat{\sigma}_x^2 - \sigma_X^2)g_{13} \\
&+ 2(\bar{X} - \mu_X)(\hat{\sigma}_x^2 - \sigma_X^2)g_{23}\} \\
&+ \frac{1}{3!}\left\{(\bar{y} - \mu_Y)\frac{\partial}{\partial\bar{y}} + (b - \beta)\frac{\partial}{\partial b} + (\bar{x} - \mu_X)\frac{\partial}{\partial\bar{x}}\right. \\
&\left. + (\hat{\sigma}_x^2 - \sigma_X^2)\frac{\partial}{\partial\hat{\sigma}_x^2}\right\}^3 g(\bar{y}^*, b^*, \bar{x}^*, \hat{\sigma}_x^{2*})
\end{aligned} \tag{21}$$

where $g_0, g_1, g_2, g_{00} \& g_{22}$ are already defined and

$$g_3 = \frac{\partial}{\partial\hat{\sigma}_x^2} g(\bar{y}, b, \bar{x}, \hat{\sigma}_x^2)\bigg]_{(\mu_Y, \beta, \mu_X, \sigma_X^2)}$$

$$g_{02} = \frac{\partial}{\partial\bar{y}\partial\bar{x}} g(\bar{y}, b, \bar{x}, \hat{\sigma}_x^2)\bigg]_{(\mu_Y, \beta, \mu_X, \sigma_X^2)}$$

$$g_{12} = \frac{\partial}{\partial b \partial \bar{x}} g(\bar{y}, b, \bar{x}, \hat{\sigma}_x^2)\Big]_{(\mu_Y, \beta, \mu_X, \sigma_X^2)}$$

$$g_{23} = \frac{\partial}{\partial \bar{x} \partial \hat{\sigma}_x^2} g(\bar{y}, b, \bar{x}, \hat{\sigma}_x^2)\Big]_{(\mu_Y, \beta, \mu_X, \sigma_X^2)}$$

$$g_{01} = \frac{\partial}{\partial \bar{y} \partial b} g(\bar{y}, b, \bar{x}, \hat{\sigma}_x^2)\Big]_{(\mu_Y, \beta, \mu_X, \sigma_X^2)}$$

$$g_{03} = \frac{\partial}{\partial \bar{y} \partial \sigma_x^2} g(\bar{y}, b, \bar{x}, \hat{\sigma}_x^2)\Big]_{(\mu_Y, \beta, \mu_X, \sigma_X^2)}$$

$$g_{13} = \frac{\partial}{\partial b \partial \hat{\sigma}_x^2} g(\bar{y}, b, \bar{x}, \hat{\sigma}_x^2)\Big]_{(\mu_Y, \beta, \mu_X, \sigma_X^2)}$$

$$\bar{y}* = \mu_Y + h(\bar{y} - \mu_Y)$$

$$b* = \beta + h(b - \beta)$$

$$\bar{x}* = \mu_X + h(\bar{x} - \mu_X)$$

$$\hat{\sigma}_x^2* = \sigma_X^2 + h(\hat{\sigma}_x^2 - \sigma_X^2) \quad \text{for } 0 < \text{h} < 1.$$

Under the above mentioned conditions given from (2) to (7), the expression (21) in terms of e_i's reduces to,

$$\begin{aligned}\hat{\bar{y}}_{g_{ME}} = \mu_Y &+ \mu_Y e_0 + \mu_X e_1(-\beta) + \sigma_X^2 e_3 g_3 \\ &+ \frac{1}{2}\{\sigma_X^4 e_3^2 g_{33} + 2\mu_Y e_0 \sigma_X^2 e_3 g_{03} \\ &+ 2\beta(e_4 - e_3 + e_3^2 - e_3 e_4)\mu_X e_1 g_{12} \\ &+ 2\beta(e_4 - e_3 + e_3^2 - e_3 e_4)\sigma_X^2 e_3 g_{13} + 2\mu_X e_1 \sigma_X^2 e_3 g_{23}\}\end{aligned}$$

or

$$\begin{aligned}\hat{\bar{y}}_{g_{ME}} - \mu_Y = \mu_Y e_0 &- \beta\mu_X e_1 + \sigma_X^2 e_3 g_3 \\ &+ \frac{1}{2}\{\sigma_X^4 e_3^2 g_{33} + 2\mu_Y \sigma_X^2 e_0 e_3 g_{03} + 2\beta\mu_X(e_1 e_4 - e_1 e_3) g_{12} \\ &+ 2\beta\sigma_X^2(e_3 e_4 - e_3^2) g_{13} + 2\mu_X \sigma_X^2 e_1 e_3 g_{23}\} \qquad (22)\end{aligned}$$

The expression of bias is now derived as

$$\begin{aligned}Bias(\hat{\hat{y}}_{g_{ME}}) &= E(\hat{\hat{y}}_{g_{ME}} - \mu_Y)\\ &= \frac{1}{2n}\left\{\sigma_X^4 A_X g_{33} + 2\mu_Y \sigma_X^2 \frac{\mu_{2100}}{\sigma_X^2 \mu_Y} g_{03}\right.\\ &\quad + 2\beta\mu_X\left(\frac{\mu_{2100}}{\sigma_{XY}\mu_X} - \frac{\mu_{3000}}{\sigma_X^2\mu_X}\right) g_{12}\\ &\quad \left.+ 2\beta\sigma_X^2\left(\frac{\mu_{31}}{\sigma_X^2\sigma_{XY}} - A_X\right) g_{13} + 2\mu_X\sigma_X^2\frac{\mu_{3000}}{\sigma_X^2\mu_X} g_{23}\right\} \end{aligned} \tag{23}$$

Now on squaring (22) on both the sides and approximating to the first degree, we have

$$\begin{aligned}MSE(\hat{\hat{y}}_{g_{ME}}) &= E(\hat{\hat{y}}_{g_{ME}} - \mu_Y)^2\\ &= E(\mu_Y e_0 - \beta\mu_X e_1 + \sigma_X^2 e_3 g_3)^2\\ &= E\left(\mu_Y^2 e_0^2 + \frac{\sigma_{XY}^2}{\sigma_X^4}\mu_X^2 e_1^2 + \sigma_X^4 e_3^2 g_3^2\right.\\ &\quad - 2\mu_Y\frac{\sigma_{XY}}{\sigma_X^2}\mu_X e_0 e_1 + 2\mu_Y\sigma_X^2 e_0 e_3 g_3\\ &\quad \left.- 2\frac{\sigma_{XY}}{\sigma_X^2}\mu_X\sigma_X^2 e_1 e_3 g_3\right)\end{aligned}$$

on using values of the expectations given from (8) to (20), the above expression becomes

$$\begin{aligned}MSE(\hat{\hat{y}}_{g_{ME}}) &= (1-\rho^2)\frac{\sigma_Y^2}{n} + \frac{1}{n}\left(\sigma_u^2 + \rho^2\sigma_Y^2\frac{\sigma_v^2}{\sigma_X^2}\right)\\ &\quad + \left(\sigma_X^4\frac{A_X}{n}g_3^2 + 2\frac{\mu_{2100}}{n}g_3 - 2\frac{\sigma_{XY}}{\sigma_X^2 n}\mu_{3000}g_3\right)\end{aligned} \tag{24}$$

Now for optimizing (24) w.r.t g_3, the optimum value of g_3 is given by

$$g_3 = \frac{\left(\frac{\sigma_{XY}}{\sigma_X^2}\mu_{3000} - \mu_{2100}\right)}{\sigma_X^4 A_X}. \tag{25}$$

Therefore, using optimum value of g_3 from (25), we get

$$MSE(\hat{\bar{y}}_{g_{ME}})_{\min} = (1-\rho^2)\frac{\sigma_Y^2}{n} + \frac{1}{n}\left(\sigma_u^2 + \rho^2\sigma_Y^2\frac{\sigma_v^2}{\sigma_X^2}\right) - \frac{\left(\frac{\sigma_{XY}}{\sigma_X^2}\mu_{3000} - \mu_{2100}\right)^2}{n\sigma_X^4 A_X} \tag{26}$$

3 Theoretical Comparison

To establish the effectiveness and superiority of the recommended estimator we now contrast it to the usual estimator of mean when measurement errors are present.

$$\bar{y}_m = \frac{1}{n}\sum_{E_i}^{n} y_i \tag{27}$$

Expressing above in terms of e_i's, we have

$$\bar{y}_m = \mu_Y(1+e_0)$$

$$\bar{y}_m - \mu_Y = \mu_Y e_0$$

Therefore

$$Bias(\bar{y}_m) = 0 \tag{28}$$

& From Salabh (1997), we have

$$MSE(\bar{y}_m) = \frac{\sigma_Y^2}{n}\left(1 + \frac{\sigma_u^2}{\sigma_Y^2}\right) \tag{29}$$

If there exists measurement error, the proposed estimator $\hat{\bar{y}}_{g_{ME}}$ will now be more effective than the conventional estimate of mean if

$$MSE(\bar{y}_m) - MSE(\hat{\bar{y}}_{g_{ME}})_{\min} > 0$$

i.e.

$$\frac{\sigma_Y^2}{n}\left(1 + \frac{\sigma_u^2}{\sigma_Y^2}\right) - (1-\rho^2)\frac{\sigma_Y^2}{n} - \frac{1}{n}\left(\sigma_u^2 + \rho^2\sigma_Y^2\frac{\sigma_v^2}{\sigma_X^2}\right) + \frac{\left(\frac{\sigma_{XY}}{\sigma_X^2}\mu_{3000} - \mu_{2100}\right)^2}{n\sigma_X^4 A_X} > 0$$

$$\text{or } h > \rho^2 \frac{\sigma_Y^2}{n}\left(\frac{\sigma_v^2}{\sigma_X^2} - 1\right) \quad \text{where } h = \frac{\left(\frac{\sigma_{XY}}{\sigma_X^2}\mu_{3000} - \mu_{2100}\right)^2}{n\sigma_X^4 A_X}. \tag{30}$$

Therefore, if condition (30) is met, the suggested estimator $\hat{\hat{y}}_{g_{ME}}$ will be more effective than the already accepted typical estimator of mean in the presence of measurement errors.

4 Empirical Study

Using a known population data set, we compare the effectiveness of the proposed estimator to that of the conventional mean estimator in this section. The population set is described as follows. We have taken the data set for empirical study from Gujrati, Porter and Sangeetha (2012) as

$\mathrm{Y_i}$ = "True Consumption Expenditure"
$\mathrm{X_i}$ = "True Income"
$\mathrm{y_i}$ = "Measured consumption expenditure"
$\mathrm{x_i}$ = "Measure Income"

The population characteristics obtained using above data is as follows

$$n = 10, \mu_X = 170, \mu_Y = 127, \sigma_X^2 = 3300, \sigma_Y^2 = 1278, \sigma_u^2 = 32.4001,$$

$$\sigma_v^2 = 32.3998, C_Y = 0.2815, C_X = 0.3379, \rho_{XY} = 0.9641, \beta_{2Y} = 1.9026,$$

$$\beta_{2X} = 1.7758, \beta_{2u} = 1.7186, \beta_{2v} = 1.8409$$

Substituting the above parameters in Equations (26) and (29), the MSE's of the usual and recommended estimator with measurement errors is given by

$$MSE(\hat{y}_m) = 131.033$$

$$MSE(\widehat{\hat{y}}_{ME})_{min} = 13.372$$

The above results justify the efficiency of the recommended estimator over the usual counterparts.

5 Conclusion

Mean squared error criterion has been used to assess the effectiveness of the estimators in both theoretical and empirical studies. The proposed estimator is contrasted with the standard mean estimator and it is discovered that the proposed estimator is more effective in terms of MSE. Based on the aforementioned MSEs, the suggested estimator's percent relative efficiency (PRE) over the standard estimate of mean under measurement error is 979, demonstrating its improved efficiency.

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Biography

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