Effect of Load on Sequential Imperfect Preventive Maintenance and Replacement Schedules of Mechanically Repairable Machines

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Abstract

This paper examines the impact of load on the operational time and maintenance cost of mechanically repairable machines. Three different levels of load with multiplicative impact on the hazard rate of the failure distribution were applied to the working of a cassava grinding machine using a two-parameter Weibull distribution with respective hazard and cumulative hazard functions. Their effect on the preventive maintenance (PM) and replacement schedules revealed that at above maximum load level, the length of the machine's operational time decreased drastically compared to the decrease at maximum load level and relative decrease at the below maximum load level when compared to the machine's operational time at the minimum load level. The application of load also results in frequent preventive maintenance actions and an increase in machine downtime for a given cost ratio. This implies that the influence of load on the PM and replacement maintenance schedule

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of mechanically repairable machines is essential to the design and operation of such machines. The results also provide maintenance engineers with an operational guide for PM and replacement maintenance actions in order to prevent failure maintenance and increase the machine's availability for enhanced productivity.

Keywords: Load, mechanically repairable systems, preventive maintenance, preventive replacement, Weibull distribution.

1 Introduction

Load in this context, is simply the additional weight on a machine. In other words, it is a stress-induced factor. Each machine has a moderated load capacity. When the machine's load level exceeds its threshold limit, the hazard rate increases. A machine's load can have either a multiplicative or additive effect. It is assumed that machines degrade continuously as a result of either additive or cumulative load impact (Liu, 2016). This causes frequent machine breakdowns, resulting in downtime, unavailability, and increased machine maintenance costs. Consequent upon the above reasons, the effect of load on the preventive maintenance (PM) and machine replacement schedule is of interest in this work because of the implications for maintenance and machine lifespan.

Preventive maintenance (PM) is a proactive approach to keeping machines in good working order. It saves money over time by avoiding repairs and minimizing other expenses such as lost production, higher costs for spare parts and shipping, downtime, and customer goodwill; (see, Udoh and Ekpenyong, 2019). One of the previous contributors to PM, Moghaddam and Usher (2010) presented a mathematical formulation to find the best PM and replacement schedule for a given system under three options: one can either continue to use the system as it is; maintain the system; or replace it. These decisions have cost effect, and how those costs are distributed can affect how frequently a given system fails. Later, Basri, Razak, and Absmart (2017) examined the effectiveness of the various maintenance policies from the standpoint of four major topics. This includes the overall maintenance policy, preventive maintenance planning, preventive maintenance planning concept, and preventive maintenance planning based on developing an optimal plan for carrying out the PM actions.

The goal of this paper is therefore to improve the effectiveness of maintenance by generating PM and replacement schedules for repairable systems using imperfect PM and replacement models. Earlier studies on imperfect PM replacement models are prominent in Nakagawa (1986) models for both periodic and sequential PM; Nakagawa (1988) models on age and hazard; Lin et al. (2000) hybridized generalized sequential maintenance model that combined the age and hazard models of Nakagawa (1988); Udoh and Ekpenyong (2019) sequential imperfect PM and replacement model for cassava grinding machine; Udoh and Effanga (2023) geometric imperfect PM and replacement model and Udoh and Uko (2023) PM and replacement models for mechanically repairable systems with linearly increasing hazard rate. Nonetheless, these authors did not account for the impact of load on the amount of time it takes to operate the system and the cost of maintenance in their models. This study seeks to investigate the consequences of taking these factors into consideration.

Here unto, this paper considers a situation in which a machine is subjected to levels of load as additional stress. Mohammad et al. (2013) had employed Cox's proportional hazard model, which incorporates load as a multiplicative effect on the hazard rate. Load has a multiplicative effect that can be either constant or cumulative, Sergey et al. (2014). They demonstrated that when load imposed on a machine exceeds its specific carrying limit, the hazard rate of the machine increases, resulting in an increase in the deterioration rate and a reduction in the reliability of such machines; Whereas Gao et al. (2019) demonstrated that for a non-linear PM model with an environmental factor based on the Weibull distribution, the PM action affects the system's hazard rate, which can be restored to any of the following conditions following PM: "as good as new" state: $h_k(t) = h(t)$, "as bad as old" state: $h_k(t) = h_{k-1}(t_{k-1} + t)$, "better than new" state: $h_k(t) < h(t)$ almost everywhere and for "better than old" state: $h_k(t) < h_{k-1}(t)$ almost everywhere. This means that any stress action, such as load, will have an additive or multiplicative effect on the system's hazard rate, is affecting the PM schedule.

A Novel strategy for modelling the reliability of systems under shared load could be seen in Liu (2016). The paper made a significant contribution to our understanding of load and its consequences for machines by modelling the reliability of load-sharing systems with ageing parts and proposing maintenance strategies to keep them running smoothly. Xiao et al. (2016) also reported on optional element loading for a linear sliding window system. While Wang (2018) divides load rate into three states to assess its impact on distribution network reliability: light load, heavy load, and overload. These data were used to derive the failure rate-load relationship. Also, Strunk et al. 2021 investigated the influence of external loads on surfaces and sub surfaces, while the effect of loads on materials were studied by Dixon and Kajtaz (2021), Xue et al. (2021) and Sun et al. (2023).

2 Methodology

2.1 Definition of Load

A load may be constant or cumulative. In this paper, we examine the cumulative load application with multiplicative effect on the PM model. According to Liu (2016), the definition of load can be expressed mathematically as:

$$Z_k = \begin{cases} \sum_{k=1}^{N} L_k; & \textit{if } N > 0 \\ 0; & \textit{if } N = 0 \end{cases}$$

$$\therefore\ Z_k = \sum_{k=1}^{N} L_k, \quad \textit{for } N = 1, 2, 3, \ldots \tag{1}$$

N is the number of loads arriving the machine
L_k is the respective load at Nth point
Z_k is the cumulative load which is the sum of respective L_k.

2.2 Choice of Weibull Failure Distribution

Several parametric models have been successfully used as population model for failure times distribution of both repairable and nonrepairable systems associated with a wide range of products. These distributions are exhibited by systems according to their mode of failure and the failure mechanism. Therefore, choosing appropriate model for failure times distribution can either be based on probabilistic views of the physics of the failure mode or the success in fitting empirical data. Hence, the choice of Weibull distribution in this work is based on the concept of increasing hazard rate of mechanically repairable systems and its empirical success.

2.3 Goodness-of-Fit Test for Weibull Distribution

This paper uses a cassava grinding machine as its case study. The chi-squared goodness-of-fit test was used to investigate whether the empirical

Table 1 Observed and expected frequencies

Interval of Time, t	O_i	E_i
t < 1600	11	10.95
1600 < t < 4800	9	9.1
4800 < t < 8000	10	10.0
Total	30	30

data of failure times of the cassava grinding machine follows the expected two parameters Weibull distribution. The expected frequencies of failure times of the machine at a given interval of time is given by;

$$E_{ij} = N \int_0^t \left(\frac{\beta}{\alpha}\right)\left(\frac{t}{\alpha}\right)^{\beta-1} e^{-\left(\frac{t}{\alpha}\right)^{\beta}} dt$$

Table 1 shows the observed, O_i and pooled expected, E_i frequencies ($i = 1, 2{,}3$) of failure times intervals of the machine.

The test, $\chi^2 < \chi^2{}_{(0.05),2}$ shows that the distribution of failure times of the machine follows a Weibull distribution. This result was validated by the use of Easyfit (5.6) software with best-fit rank of 1. This software was also used in the estimation of the shape and scale parameters of the Weibull distribution; $\alpha = 1.3$ and $\beta = 1386$ respectively. The respective hazard and cumulative hazard functions of the distribution are;

$$h(y_k) = \alpha\beta^{\alpha} y_k^{\alpha-1} \quad \text{and} \quad H(y_k) = \beta^{\alpha} y_k^{\alpha}$$

where α and β are the shape and scale parameters of the Weibull distribution and y_k is the effective age of the system after kth PM.

By incorporating the cumulative load factor, Z_k into the hazard and cumulative hazard functions with multiplicative impact, we obtain (2);

$$h(y_k Z_k) = \alpha\beta^{\alpha}(y_k Z_k)^{\alpha-1} \quad \text{and} \quad H(y_k Z_k) = \beta^{\alpha}(y_k Z_k)^{\alpha} \tag{2}$$

2.4 Imperfect Preventive and Replacement Maintenance Model for Repairable Systems

A hybrid model of Lin et al. (2000) which is a combination of the hazard rate adjustment model and the age reduction model of Nakagawa (1988) is given by (3);

$$g(t_1 + x) = ahg(bt_1 + x) \tag{3}$$

where $a \geq 1; 0 \leq b \leq 1; x \in (0, t_2 - t_1)$ and $h(t)$ is the failure rate function for $t \in (0, t_1)$.

The PM activity at time, t_1 produces a new failure rate function g(t) for $t \in (t_2, t_1)$ with $ah(x)$ as the failure rate function in the next PM interval which depends only on $h(x)$ and the corresponding PM activity. In other word, $g(t)$ depends on both $h(t); t \in (0, t_1)$ and b, the extent of the PM activity in time, t_1.

2.5 The Mean Cost Per Unit Time of Operating the System

The associated cost model to any maintenance and replacement function is often used to evaluate the performance of the system that is repairable and also to determine the expected amount of time needed for maintenance that is both safe and appropriate. As a result, the associated expected cost rate model for (3) is given in (4) as follows:

$$\begin{aligned} C &= C(y_1, y_2, \dots y_N) \\ &= \frac{C_r + (N-1)C_p + C_m \sum_{k=1}^{N} P_k[H(y_k) - H(r_{k-1}y_{k-1})]}{\sum_{k=1}^{N-1}(1-r_k)y_k + y_N} \end{aligned} \tag{4}$$

where C_r, C_p and C_m are respectively the costs of replacement maintenance, preventive maintenance and minimal repair of the machine, $P_k = \prod_{i=1}^{k-1} \rho_i$, $0 = r_0 < r_1 < r_2 < \cdots < 1$, where P_k and $H(y_k)$ are the product of the hazard rate adjustment factor and the cumulative hazard function occurring within the interval (t_{k-1}, t_k), which is between the time of $(k-1)^{th}$ PM and the kth PM respectively and $r_{k-1}y_{k-1}$ is the effective age of the system right after $(k-1)^{th}$ PM.

2.6 Optimization of the Expected Cost Per Unit Time of Operation

To generate optimal PM and replacement schedule for the cassava grinding machine, we shall determine optimal PM intervals by finding the optimal values of y_k; $k = 1, 2, \dots N-1$ and at replacement point, N as decision variables to minimize the expected cost rate in (4), Nakagawa (1986), (1988), Lin et al. (2000) and Udoh and Ekpenyong (2019). Let $C(y_1, y_2, \dots y_N) = C$, so that taking the partial derivative of (4) with respect to y_k and equating the obtained derivative to zero, we have;

$$\frac{\partial C}{\partial y_k} = \frac{\begin{matrix}\left[y_N + \sum_{k=1}^{N-1}(1-r_k)y_k\right] C_m[P_k h(y_k) - r_k P_{k+1} h(r_k y_k)] \\ -\Big[C_r + (N-1)C_p + C_m \sum_{k=1}^{N} \\ P_k[H(y_k) - H(r_{k-1}y_{k-1})]\Big](1-r_k)\end{matrix}}{\left[y_N + \sum_{k=1}^{N-1}(1-r_k)y_k\right]^2} = 0$$

$$= \frac{\begin{matrix}C_m[P_k h(y_k) - r_k P_{k+1} h(r_k y_k)] \\ -\frac{\left[C_r+(N-1)C_p+C_m\sum_{k=1}^{N} P_k[H(y_k)-H(r_{k-1}y_{k-1})]\right](1-r_k)}{\left[y_N+\sum_{k=1}^{N-1}(1-r_k)y_k\right]}\end{matrix}}{\left[y_N + \sum_{k=1}^{N-1}(1-r_k)y_k\right]} = 0 \tag{5}$$

And substituting (4) into (5) we obtain (6);

$$\frac{\partial C}{\partial y_k} = \frac{C_m[P_k h(y_k) - r_k P_{k+1} h(r_k y_k)] - C(1-r_k)}{\left[y_N + \sum_{k=1}^{N-1}(1-r_k)y_k\right]} = 0$$

$$\Rightarrow C_m[P_k h(y_k) - r_k P_{k+1} h(r_k y_k)] = C[(1-r_k)];$$
$$k = 1, 2, 3, \ldots, N-1 \tag{6}$$

where $h(r_k y_k)$ is the adjusted hazard function of the machine after k^{th} PM; $k = 1, 2, 3, \ldots, N-1$.

Similarly, at replacement point, N;

$$\frac{\partial C}{\partial y_N} = \frac{\begin{matrix}\left[y_N + \sum_{k=1}^{N-1}(1-r_k)y_k\right] \\ C_m[P_N h(y_N) - r_N P_{k+1} h(r_N y_N)] \\ -\Big[C_r + (N-1)C_p + C_m \sum_{k=1}^{N} \\ P_k[H(y_k) - H(r_{k-1}y_{k-1})]\Big]\end{matrix}}{\left[y_N + \sum_{k=1}^{N-1}(1-r_k)y_k\right]^2} = 0$$

$$= \frac{\begin{matrix}C_m[P_N h(y_N) - r_N P_{k+1} h(r_N y_N)] \\ -\frac{\left[C_r+(N-1)C_p+C_m\sum_{k=1}^{N} P_k[H(y_k)-H(r_{k-1}y_{k-1})]\right]}{\left[y_N+\sum_{k=1}^{N-1}(1-r_k)y_k\right]}\end{matrix}}{\left[y_N + \sum_{k=1}^{N-1}(1-r_k)y_k\right]} = 0 \tag{7}$$

Substituting (4) into (7), we obtain (8) as follows;

$$\frac{\partial C}{\partial y_N} = \frac{C_m[P_N h(y_N) - r_N P_{k+1} h(r_N y_N)] - C}{\left[y_N + \sum_{k=1}^{N-1}(1-r_k)y_k\right]} = 0$$

$$\Rightarrow C_m[P_N h(y_N) - r_N P_{k+1} h(r_N y_N)] = C; \quad k = 1, 2, 3, \ldots, N-1$$

Noting that the k^{th} PM is the replacement point, therefore $P_{k+1} = 0$, and

$$C = C_m[P_N h(y_N)] \tag{8}$$

Now substituting (8) into (6) we have (9) as follows;

$$C_m[P_k h(y_k) - r_k P_{k+1} h(r_{k-1} y_{k-1})] = C_m[P_N h(y_N)][(1-r_k)]$$

$$[P_k h(y_k) - r_k P_{k+1} h(r_{k-1} y_{k-1})]$$
$$= [P_N h(y_N)][(1-r_k)] \forall k = 1, 2, 3, \ldots, N-1 \tag{9}$$

In order to test for convexity of the function, we obtain the second partial derivative as follows;

$$\frac{\partial^2 C}{\partial {y^2}_k} = \frac{\begin{array}{l} \left[y_N + \sum_{k=1}^{N-1}(1-r_k)y_k\right]^2 \{C_m[P_k h'(y_k) - r_k P_{k+1} h'(r_k y_k)] \\ \quad -r_k P_{k+1} h(r_k y_k)(1-r_k)\} \\ \quad -2(1-r_k)\left[y_N + \sum_{k=1}^{N-1}(1-r_k)y_k\right] \\ C_m[P_k h(y_k) - P_{k+1} r_k h(r_k y_k)] - \left[C_r + (N-1)C_p + C_m \sum_{k=1}^{N} \right. \\ \left. P_k[H(y_k) - H(r_{k-1} y_{k-1})]\right](1-r_k) \end{array}}{\left[y_N + \sum_{k=1}^{N-1}(1-r_k)y_k\right]^3}$$

$$= \left[y_N + \sum_{k=1}^{N-1}(1-r_k)y_k\right]^2 \{C_m[P_k h'(y_k) - r_k P_{k+1} h'(r_k y_k)]$$
$$- r_k P_{k+1} h(r_k y_k)(1-r_k)\} - 2(1-r_k)\left[y_N + \sum_{k=1}^{N-1}(1-r_k)y_k\right]$$
$$C_m[P_k h(y_k) - P_{k+1} r_k h(r_k y_k)]$$
$$- \left[C_r + (N-1)C_p + C_m \sum_{k=1}^{N} P_k[H(y_k) - H(r_{k-1} y_{k-1})]\right]$$
$$(1-r_k) > 0 \tag{10}$$

It follows from (10) that (6) is convex with a global minimum solution and has a unique solution for y_k since $1 - \rho_k r_k > 0, k = 1, 2, 3, \ldots, N-1$, $h(y_N)$ is continuous and differentiable and $0 < y_N < \infty$ is fixed. $\rho_k = \frac{6k+1}{2k+1}$ is the hazard rate adjustment factor and $r_k = \frac{k}{2k+1}$ is the age improvement factor.

Also, from (4);

$$C\left[\sum_{k=1}^{N-1}(1-r_k)y_k + y_N\right] = C_r + (N-1)C_p + C_m\sum_{k=1}^{N} P_k[H(y_k) - H(r_{k-1}y_{k-1})]$$

$$C\left[\sum_{k=1}^{N-1}(1-r_k)y_k + y_N\right] - C_m\sum_{k=1}^{N} P_k[H(y_k) - H(r_{k-1}y_{k-1})] = C_r + (N-1)C_p \tag{11}$$

Substituting (8) into (11) we obtain (12) as follows;

$$C_m[P_N h(y_N)]\left[\sum_{k=1}^{N-1}(1-r_k)y_k + y_N\right] - C_m\sum_{k=1}^{N} P_k[H(y_k) - H(r_{k-1}y_{k-1})] = C_r + (N-1)C_p \tag{12}$$

Dividing through (12) by C_m we have;

$$[P_N h(y_N)]\left[\sum_{k=1}^{N-1}(1-r_k)y_k + y_N\right] - \sum_{k=1}^{N} P_k[H(y_k) - H(r_{k-1}y_{k-1})] = \frac{C_r + (N-1)C_p}{C_m} \tag{13}$$

The effect of load with multiplicative impact on the hazard and cumulative hazard functions from (2) into (13) yields (14);

$$[P_N h(y_N Z_N)]\left[\sum_{k=1}^{N-1}(1-r_k)y_k Z_k + y_N Z_N\right]$$

$$-\sum_{k=1}^{N} P_k[H(y_k Z_k) - H(r_{k-1}y_{k-1}Z_{k-1})]$$
$$= \frac{C_r + (N-1)C_p}{C_m} \quad (14)$$

2.7 Algorithm for Generating Sequential PM and Replacement Schedule with Load Effect

The following computational algorithm is used in accordance with the analytical results:

Step 1: Solve for $y_k Z_k$ as a function of $y_N Z_N$ in (9) to obtain (15)

Step 2: Substitute $y_k Z_k$ into (15) to obtain $y_N Z_N$

Step 3: Choose N to minimize $P_N h(y_N Z_N)$ in order to obtain optimal number, N^* of PM

Step 4: Obtain y_k from the expression in step 1 following from step2

Step 5: Obtain the optimal length of operating time; $x_k = y_k - r_{k-1}y_{k-1}$, $k = 1, 2, 3, \ldots, N$

The input parameters are the costs; C_r, C_p and C_m with ratios $\frac{C_r}{C_p}$ and $\frac{C_m}{C_p}$, the Weibull parameters are α and β and the adjustment factors are ρ_k and r_k

3 Implementation of the Optimal PM and Replacement Algorithm to Obtain Sequential PM and Replacement Schedule for 8hp-PML Gold Engine Cassava Grinding Machine with Load Effect

According to Udoh and Ekpenyong (2019), a sequential imperfect PM and replacement schedule for an 8hp-PML gold cassava grinding machine was obtained, the failure distribution of the device was shown to follow a conventional two-parameter Weibull distribution with hazard and cumulative hazard functions given as: $h(t) = \alpha\beta^{\alpha}(t)^{\alpha-1}$, $\alpha > 1$, $\beta > 0$ and $H(t) = \beta^{\alpha}t^{\alpha}$ with estimated parameters; $\alpha = 1.3$ and $\beta = 1386$.

On the application of load, (9) becomes;

$$[P_k h(y_k Z_k) - r_k P_{k+1} h(r_k y_k Z_k)] = [P_N h(y_N Z_N)][(1 - r_k)] \quad (15)$$

Step 1: Substitute the Weibull hazard function with load effect from (2) into (15), we have (16) as follows;

$$[P_k \alpha \beta^{\alpha} (y_k Z_k)^{\alpha-1} - r_k P_{k+1} \alpha \beta^{\alpha} (r_k y_k Z_k)^{\alpha-1}]$$
$$= [P_N \alpha \beta^{\alpha} (y_N Z_N)^{\alpha-1}][(1 - r_k)]$$
$$(y_k Z_k) = \left[\frac{P_N(1 - r_k)}{[P_k - P_{k+1} r_k^{\alpha-1+1}]} \right]^{1/\alpha-1} (y_N Z_N) \qquad (16)$$

Step 2: By substituting (16) into (14) we obtain (17) as follows;

$$[P_N h(y_N Z_N)] \left[\sum_{k=1}^{N-1} (1 - r_k) \left[\frac{P_N(1 - r_k)}{[P_k - P_{k+1} r_k^{\alpha}]} \right]^{1/\alpha-1} (y_N Z_N) + y_N Z_N \right]$$
$$- \sum_{k=1}^{N} P_k [H(y_k Z_k) - H(r_{k-1} y_{k-1} Z_{k-1})] = \frac{C_r + (N-1)C_p}{C_m}$$

Let $\pi_k = \left[\frac{(1-r_k)^{\alpha}}{[P_k - P_{k+1} r_k^{\alpha}]} \right]^{1/\alpha-1}$

$$[P_N h(y_N Z_N)] \left[\sum_{k=1}^{N-1} \pi_k P_N^{1/\alpha-1} (y_N Z_N) + y_N Z_N \right]$$
$$- \sum_{k=1}^{N} P_k [H(y_k Z_k)] = \frac{C_r + (N-1)C_p}{C_m}$$

Substituting the hazard and cumulative hazard functions of the two parameter Weibull function, we have;

$$P_N \beta^{\alpha} (y_N Z_N)^{\alpha} \left[\alpha \left(P_N^{1/\alpha-1} \sum_{k=1}^{N-1} \pi_k + 1 \right) - P_N^{1/\alpha-1} \sum_{k=1}^{N-1} \pi_k - 1 \right]$$
$$= \frac{C_r + (N-1)C_p}{C_m}$$

$$\therefore\ y_N Z_N = \frac{[C_r + C_p(N-1)]^{\frac{1}{\alpha}}}{\left[C_m(\alpha - 1) \left(P_N + P_N^{\alpha/\alpha-1} \sum_{k=1}^{N-1} \pi_k \right) \right]^{\frac{1}{\alpha}} \beta};\ \alpha > 1 \qquad (17)$$

Step 3: To obtain optimal *N*, we seek optimal number N^* which minimizes (17) by substituting $y_N z_N$ into (8) as follows:

Let

$$B(N) = P_N h(y_N Z_N) = P_N \alpha\beta^\alpha (y_N Z_N)^{\alpha-1}$$

$$= \frac{\alpha\beta^\alpha P_N [C_r + C_p(N-1)]^{\frac{1}{\alpha}}}{[C_m(\alpha-1)]^{\alpha-1}\left[P_N + P_N^{\alpha/\alpha-1}\sum_{k=1}^{N-1}\pi_k\right]^{\frac{1}{\alpha}}\beta^{\alpha-1}}$$

$$\therefore B(N) = \theta\frac{[C_r + C_p(N-1)]}{\left[P_N^{-1/\alpha-1} + \sum_{k=1}^{N-1}\pi_k\right]}; \quad \text{where } \theta = \frac{(\alpha\beta)^{\frac{\alpha}{\alpha-1}}}{[C_m(\alpha-1)]^\alpha}$$

A necessary condition for the existence of a finite N^* which minimizes B(N) is that N^* satisfies the inequalities; $B(N+1) \geq B(N)$ and $B(N) < B(N-1)$, (see Nakagawa,1988 and Lin et al., 2000). That is,

$$\frac{\theta[C_r + (N)C_p]}{\left[P_{N+1}^{-1/\alpha-1} + \sum_{k=1}^{N}\pi_k\right]} \geq \frac{\theta[C_r + (N-1)C_p]}{\left[P_N^{-1/\alpha-1} + \sum_{k=1}^{N-1}\pi_k\right]}$$

$$\therefore B(N) = \frac{P_N^{-1/\alpha-1} + \sum_{k=1}^{N-1}\pi_k}{[P_{N+1}^{-1/\alpha-1} - P_N^{-1/\alpha-1} + \pi_N]} - (N-1) \geq \frac{C_r}{C_p} \tag{18}$$

where $\pi_k = \left[\frac{(1-r_k)^\alpha}{[P_k - P_{k+1}r_k^\alpha]}\right]^{1/\alpha-1}$ and $P_k = \prod_{i=1}^{k-1}\rho_i, \forall k = 1, 2, 3, \ldots, N$

Similarly,

$$B(N-1) < \frac{C_r}{C_p} \tag{19}$$

From (17):

$$y_N = \left[\frac{[C_r + (N-1)C_p]}{C_m(\alpha-1)\left\{\left[P_N^{\alpha/\alpha-1}\sum_{k=1}^{N-1}\pi_k + P_N\right]\right\}}\right]^{1/\alpha}\frac{1}{Z_N\beta} \tag{20}$$

Substituting (20) into (16), we have;

$$(y_k Z_k) = \left[\frac{P_N(1-r_k)}{[P_k - P_{k+1}r_k^{\alpha-1+1}]}\right]^{1/\alpha-1}$$

$$\left[\left[\frac{[C_r + (N-1)C_p]}{C_m(\alpha-1)\left\{\left[P_N^{\alpha/\alpha-1}\sum_{k=1}^{N-1}\pi_k + P_N\right]\right\}}\right]^{1/\alpha}\frac{Z_N}{Z_N\beta}\right]$$

$$\therefore y_k = \left[\frac{P_N(1-r_k)}{[P_k - P_{k+1}r_k^{\alpha-1+1}]}\right]^{1/\alpha-1}\frac{A_N}{Z_k} \tag{21}$$

where

$$A_N = \left[\frac{[C_r + (N-1)C_p]}{C_m(\alpha-1)\left\{\left[P_N^{\alpha/\alpha-1}\sum_{k=1}^{N-1}\pi_k + P_N\right]\right\}}\right]^{1/\alpha}\frac{1}{\beta}$$

The cost ratios $\frac{C_r}{C_p}$ are obtained with the corresponding values of N in (18) as shown in Table 2.

Table 2 Computed values of $\frac{C_r}{C_p}$ and corresponding values of N^*

N^*	1	3	5	7	9	11	13
C_r/C_p	8	80	800	1600	8000	112000	128000

3.1 Computation of Optimal PM and Replacement Schedule Without Load

Udoh and Ekpenyong (2019) determined the optimal PM schedule for the 8pml gold engine cassava grinding machine without the application of load. This was accomplished while maintaining the same cost ratios in Table 2 and Weibull parameters in Section 2.3. It is presented in this work as a reproduction in Table 3 so that it can be compared to the results of this work.

3.2 Computation of Optimal PM and Replacement Schedule with Load Effect

The optimal PM intervals would be computed in step 5 resulting from step 4 of the algorithm previously stated in Section 2.7 using: $x_k = y_k - r_{k-1}y_{k-1}, k = 1, 2, 3, \ldots, N$ where r_{k-1} is analogous to the step length in gradient search algorithm. The levels of load to be considered in this work was calibrated as: below maximum = 30 kg, maximum = 50 kg and above maximum = 70 kg. By using the cost ratios in Table 2 and taking $C_m/C_p = 4$, y_k was calculated using the expression in (21) which takes into

Table 3 Optimal PM and replacement schedule without load

N^*	1	3	5	7	9	11	13
C_r/C_p	8	80	800	1600	8000	112000	128000
x_1	0.0031	0.0128	0.0743	0.1242	0.4274	2.5259	3.5991
x_2		0.0037	0.0214	0.0358	0.1231	0.7266	1.0368
x_3		0.0025	0.0162	0.0146	0.0468	0.2968	0.4229
x_4			0.0007	0.0065	0.0241	0.1336	0.1904
x_5			0.00354	0.0032	0.0111	0.0658	0.0937
x_6				0.0017	0.0057	0.0337	0.048
x_7				0.0015	0.0027	0.0158	0.0225
x_8					0.0015	0.0086	0.0123
x_9					0.0012	0.0044	0.0065
x_{10}						0.0022	0.0032
x_{11}						0.0024	0.0009
x_{12}							0.0005
x_{13}							0.1801

Table 4 Optimal PM and Replacement schedule with below maximum load level at 30 kg

N^*	1	3	5	7	9	11	13
C_r/C_p	8	80	800	1600	8000	112000	128000
x_1	0.0000937	0.000552	0.003247	0.005537	0.019117	0.145873	0.161727
x_2		0.000297	0.001727	0.002941	0.010155	0.077454	0.08795
x_3		0.000212	0.00121	0.002072	0.007149	0.054541	0.059592
x_4			0.000786	0.001335	0.004598	0.035075	0.038878
x_5			0.000507	0.000861	0.002966	0.022625	0.025074
x_6				0.000502	0.00173	0.013186	0.014616
x_7				0.000267	0.000918	0.007004	0.007763
x_8					0.00181	0.003502	0.003882
x_9					0.000448	0.001721	0.001906
x_{10}						0.000854	0.000944
x_{11}						0.00043	0.000478
x_{12}							0.000247
x_{13}							0.00013

consideration the multiplicative impact of the cumulative load. The resulting PM intervals for the machine with load at below maximum, maximum and above maximum levels are presented in Tables 4, 5 and 6.

4 Discussion

The result obtained for the PM schedule (in 00,000 hours, say) for the 8HP-PML gold engine cassava grinding machine without load is shown in Table 3

Table 5 Optimal PM and Replacement schedule with maximum load level at 50 kg

N^*	1	3	5	7	9	11	13
C_r/C_p	8	80	800	1600	8000	112000	128000
x_1	0.0000562	0.00031	0.001948	0.003322	0.01147	0.087524	0.097036
x_2		0.000178	0.001036	0.001765	0.006093	0.046472	0.05277
x_3		0.000127	0.000726	0.001243	0.00429	0.032724	0.035755
x_4			0.000472	0.000801	0.002759	0.021045	0.023327
x_5			0.000304	0.000517	0.00178	0.013575	0.015044
x_6				0.000301	0.001038	0.007912	0.008769
x_7				0.00016	0.000551	0.004203	0.004658
x_8					0.000366	0.002101	0.002329
x_9					0.000269	0.001033	0.001144
x_{10}						0.000512	0.000566
x_{11}						0.000258	0.000287
x_{12}							0.000148
x_{13}							0.000078

Table 6 Optimal PM and Replacement schedule with above maximum load level at 70 kg

N^*	1	3	5	7	9	11	13
C_r/C_p	8	80	800	1600	8000	112000	128000
x_1	0.0000401	0.000236	0.001392	0.002373	0.008193	0.06252	0.06931
x_2		0.000127	0.00074	0.001261	0.004352	0.03319	0.03769
x_3		0.0000908	0.000518	0.000888	0.003064	0.02338	0.02554
x_4			0.000337	0.000572	0.001971	0.01503	0.01667
x_5			0.000217	0.000369	0.001271	0.009697	0.01075
x_6				0.000215	0.000741	0.005651	0.006264
x_7				0.000114	0.000393	0.003002	0.003327
x_8					0.000776	0.001501	0.001664
x_9					0.000192	0.000738	0.000817
x_{10}						0.000366	0.000405
x_{11}						0.000184	0.000205
x_{12}							0.000106
x_{13}							0.0000557

and at different levels of load effect in Tables 4, 5 and 6. The values of N^* in row 1 in Tables 2 are the optimal number of PM before replacement at the N^* time for the given cost ratios. Also, row 2 in Table 2 contains specified cost ratios generated from (18). Thus, the values under N^* with the associated cost ratio in each column in Tables 3, 4, 5 and 6 represent the expected length of operational time, $x_1, x_2, \ldots, x_{N-1}$ of the machine for PM actions and replacement at x_N. For instance in Table 4, under $N^* = 3$ and $c_r/c_p = 80$: $x_1 = 0.000552$, $x_2 = 0.000297$ and $x_3 = 0.000212$ implies that PM should be performed a total of 2 times at $x_1 = 55.2$ hours and $x_2 = 29.7$ hours and the 3^{rd} time being replacement after $x_3 = 21.2$ hours, for $t = 0$ at the beginning of each maintenance cycle. It is observed that the PM and replacement schedules for the respective cost ratios and respective load effects in Tables 4, 5 and 6, decrease for all values of x_k. This calls for frequent PM in line with Zhang (2002) and Wang and Zhang (2009). In addition, the decreasing trend of the operational time of the machine at succeeding values of $x_{i's}$ and N^* is similar to the results obtained by Nakagawa (1988), Lin et al. (2000), Udoh and Ekpenyong (2019) and Udoh and Effanga (2023). Also observed is the compensatory relationship between cost ratios and expected operational time of the machine. This shows that the higher the cost ratio, the longer the expected length of operational time of the machine, perhaps to compensate for the increase cost implication.

Furthermore, a comparison of the PM and replacement schedules of the machine without load and with load at different levels explain the following:

(a) The PM and replacement schedule without load by Udoh and Ekpenyong (2019) in Table 3 has longer length of operational time; $x_1, x_2, \ldots, x_{13}$ compared to the schedules of the machine with load in Tables 4, 5 and 6 for corresponding cost ratios.
(b) The PM and replacement schedule with below maximum load level of 30 kg of cassava in Table 4 has longer length of operational time compared to the schedule with maximum load level of 50 kg of Table 5 and 70 kg of Table 6.
(c) Similarly, the PM and replacement schedule with maximum load level of 50 kg in Table 5 has longer length of operational times compared to the schedules with above maximum load level of 70 kg in Tables 6 for corresponding cost ratios.

5 Conclusion

The PM and replacement maintenance schedule for mechanically repairable machines has been obtained for the cassava grinding machine in this paper. This was done subject to varying levels of cumulative load, each of which has a multiplicative effect on the hazard function of the machine. When the expected length of the machine's operational time before each PM was compared to the impact of various levels of load, it was found that the application of load shortens the length of the machine's operational time before each PM, and that the higher the level of load, the shorter the operating time of the machine due to an increase in stress that resulted in deterioration. Hence, the need for more frequent PM and replacement maintenance actions, which came at a higher cost to users. Practically, the implication of increasing levels of load is the decrease in the length of operational time of the machine before next PM in addition to frequent PM to avoid failure maintenance with higher costs and to preserve the useful life of the machine for better performance.

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