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A PARALLEL SYSTEM WITH ARRIVAL TIME OF EXPERT SERVER SUBJECT TO MAXIMUM REPAIR AND INSPECTION TIMES OF ORDINARY SERVER

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Abstract

A parallel system of two identical units has been analyzed stochastically considering two types of servers. Initially, the unit is repaired by the ordinary sever who visits the system immediately. And, when ordinary server is unable to do repair of that unit in a given maximum repair time, the unit undergoes for inspection to see the feasibility of its repair by an expert server. If inspection reveals that repair of the unit is also not possible by the expert server, then it is replaced by new unit. The expert server takes some time to arrive at the system. The failure rate of each unit and rate by which unit undergoes for inspection by the ordinary server are taken as constant while the distributions of arrival, inspection, repair and replacement times are assumed as arbitrary with different probability density functions. All random variables are statistically independent. The expressions for several reliability measures are derived using semi-Markov process and regenerative point technique. Graphs are drawn to depict the behavior of MTSF, availability and profit for different values of the parameters.

Key Words: Parallel System, Two Servers, Maximum Repair Time and Reliability Measures.
2000 Mathematics Subject Classification: 90B25 and 60K10

Introduction

Method of redundancy has widely been used in many industrial plants to increase their reliability and safety. Nakagawa (1980) and Singh (1989) analyzed the systems with cold standby redundancy under different sets of assumptions on failure and repair policies. But there exist many systems in which cold standby redundancy is not suggestive and so it is desirable to introduce parallel redundancy. For example, in a power supply system, the transformers having same polarity and voltage ratio are connected in parallel in order to meet the total load requirements as well as to provide continuous power supply for essential services. Kishan and Kumar (2009) and Kumar et al. (2010) investigated stochastic models of parallel systems by taking single server for rectification of faults.

But sometimes complex faults may occur in the system during operation which cannot be repaired by an ordinary server in a given time. In that situation an expert server may be called to get the repair done. Otherwise, the unit may be replaced by new one in order to avoid the unnecessary expenses on repair. Malik and Gitanjali [2012] discussed a parallel system with arrival and maximum repair times of ordinary server.

In view of the above observations and facts, the aim of the present paper is to determine reliability measures of a parallel system of two identical units with repair by two servers – one is ordinary and the other is an expert. Initially, the unit is repaired by the ordinary sever who visits the system immediately. The ordinary server inspects the unit after a maximum repair time to see the feasibility of its repair by an expert server. When inspection reveals that repair of the unit is not also possible by the expert server then it is replaced with new by the ordinary server. The expert server takes some time to arrive at the system. The failure rate of unit and rate by which unit undergoes for inspection are taken as constant while the distributions of arrival time of the expert server, inspection time, repair time and replacement time of the unit are assumed as arbitrary with different probability density functions. All random variables are statistically independent. The switch over is instantaneous and perfect. The unit works as new after repair. The expressions for several reliability measures of vital significance are derived using semi-Markov process and regenerative point technique. The numerical results pertaining to the particular case have been obtained to depict the graphical behavior of MTSF, availability and profit with respect to the replacement rate.

Notations

E	:	Set of regenerative states.
O	:	Unit is operative.
λ	:	Constant failure rate of the unit.
α_0	:	Constant rate by which unit under goes for inspection after a pre-specified time 't' to see the feasibility of repair.
a/b	:	Probability that failed unit is not repairable / repairable by an expert server.
$h(t)/H(t)$	:	pdf/cdf of the inspection time of the unit taken by ordinary server.
$f(t)/F(t)$	:	pdf/cdf of the replacement time of the unit taken by ordinary server.
$w_e(t)/W_e(t)$		pdf / cdf of the waiting time of the expert server for repairing of the failed unit.
$g(t)/G(t)$	:	pdf / cdf of the repair time of the unit taken by ordinary server.
$g_1(t)/G_1(t)$	:	pdf / cdf of the repair time of the unit taken by expert server.
FU_r / FU_R	:	Unit is failed and under repair / under repair continuously from previous state.
FW_r / FW_R	:	Unit is failed and waiting for repair by ordinary server / waiting for repair by ordinary server continuously from previous state.
FW_{re} / FW_{Re}	:	Unit is failed and waiting for repair by expert server / waiting for repair by expert server continuously from previous state.
FU_i / FU_I	:	Unit is failed and under inspection with ordinary server / waiting for inspection by ordinary server continuously from previous state.

FU_{re}/FU_{Re}	:	Unit is failed and under repair with expert server / under repair continuously from previous state with expert server.
$FU_{rp}/\ FU_{RP}$	:	Unit is failed and under replacement / under replacement continuously from previous state.
m_{ij}	:	Contribution to mean sojourn time in state $S_i \in E$ and non-regenerative state if occurs before transition to $S_j \in E$. Mathematically, it can be written as $m_{ij} = \int_0^\infty t\, d\left(Q_{ij}(t)\right) = -q_{ij}^{*'}(0)$.
μ_i	:	The mean sojourn time in state S_i which is given by $\mu_i = E(T) = \int_0^\infty P(T>t)\,dt = \sum_j m_{ij}$, where T denotes the time to system failure.
$\sim/*$	:	Symbol for Laplace Stieltjes transform / Laplace transform.
$\boxed{S}$ /©	:	Symbols for Stieltjes convolution / Laplace convolution.

The possible transitions between states along with transitions rates for the system model are shown in figure 1.The states S_0, S_1, S_2, S_3, S_4 and S_5 are regenerative while the other states are non-regenerative.

Transition Probabilities and Mean Sojourn Times

Simple probabilistic considerations yield the following expressions for the non-zero elements $p_{ij} = Q_{ij}(\infty) = \int q_{ij}(t)\,dt$ as

$p_{01} = 1$, $p_{10} = g^*(\lambda+\alpha_0)$, $p_{12} = \frac{\alpha_0}{\alpha_0+\lambda}[1-g^*(\lambda+\alpha_0)]$,
$p_{17} = \frac{\lambda}{\alpha_0+\lambda}[1-g^*(\lambda+\alpha_0)]$, $p_{23} = ah^*(\lambda)$, $p_{24} = bh^*(\lambda)$, $p_{2,11} = \left(1-h^*(\lambda)\right)$,
$p_{30} = f^*(\lambda)$, $p_{36} = \left(1-f^*(\lambda)\right)$, $p_{45} = w_e{}^*(\lambda)$, $p_{4,13} = \left(1-w_e{}^*(\lambda)\right)$,
$p_{50} = g_1{}^*(\lambda)$, $p_{5,14} = (1-g_1{}^*(\lambda))$, $p_{61} = f^*(0)$, $p_{71} = g^*(\alpha_0)$,
$p_{78} = 1-g^*(\alpha_0)$, $p_{89} = a$, $p_{8,10} = b$, $p_{91} = f^*(0)$ $p_{10,12} = w_e{}^*(0)$,
$p_{11,9} = a$, $p_{11,10} = b$, $p_{12,1} = g_1{}^*(0)$, $p_{13,12} = w_e{}^*(0)$,
$p_{14,1} = g_1{}^*(0)$, $p_{11.7} = \frac{\lambda}{\alpha_0+\lambda}[1-g^*(\lambda+\alpha_0)]g^*(\alpha_0)$,
$p_{11.7,8,9} = \frac{a\lambda}{\alpha_0+\lambda}[1-g^*(\lambda+\alpha_0)](1-g^*(\alpha_0))$, $p_{11.7,8,10,12} = \frac{b\lambda}{\alpha_0+\lambda}[1-g^*(\lambda+\alpha_0)](1-g^*(\alpha_0))$,
$p_{21.11,9} = a\left(1-h^*(\lambda)\right)$, $p_{21.11,10,12} = b\left(1-h^*(\lambda)\right)$, $p_{31.6} = \left(1-f^*(\lambda)\right)$,
$p_{41.13,12} = \left(1-w_e{}^*(\lambda)\right)$, $p_{51.14} = \left(1-g_1{}^*(\lambda)\right)$ (1)

It can easily be verified that

$p_{01} = p_{10} + p_{12} + p_{17} = p_{10} + p_{12} + p_{11.7} + p_{11.7,8,9} + p_{11.7,8,10,12} = p_{23} + p_{24} + p_{2,11} = p_{23} + p_{24} + p_{21.11,9} + p_{21.11,10,12} = p_{30} + p_{36} = p_{30} + p_{31.6} = p_{45} + p_{41.13,12} = p_{45} + p_{4,13} = p_{50} + p_{5,14} = p_{50} + p_{51.14} = p_{61} = p_{71} + p_{78} = p_{89} + p_{8,10} = p_{91} = p_{10,12} = p_{11,9} + p_{11,10} = p_{12,1} = p_{13,12} = p_{14,1} = 1$ (2)

The mean sojourn times μ_i in state S_i is given by

$\mu_0 = \int_0^\infty P(T>t)\,dt = m_{01} = \frac{1}{2\lambda}$, $\mu_1 = m_{10} + m_{12} + m_{17} = \frac{1}{\alpha_0+\lambda}[1-$
$g*\lambda+\alpha 0$, $\mu 2$=m23+m24+m2,11=$1\lambda 1-h*\lambda$,

$\mu_3 = \mathrm{m}_{30} + \mathrm{m}_{36} = \frac{1}{\lambda}\left(1 - f^*(\lambda)\right)$, $\mu_4 = \mathrm{m}_{45} + \mathrm{m}_{4,13} = \frac{1}{\lambda}\left(1 - w_e{}^*(\lambda)\right)$,
$\mu_5 = \mathrm{m}_{50} + \mathrm{m}_{5,14} = \frac{1}{\lambda}\left(1 - g_1{}^*(\lambda)\right)$,
$\mu_1^{'} = \mathrm{m}_{10} + \mathrm{m}_{12} + \mathrm{m}_{11.7} + \mathrm{m}_{11.7,8,9} + \mathrm{m}_{11.7,8,10,12} = \frac{[1-\mathrm{g}^*(\alpha_0+\lambda)]}{\alpha_0+\lambda}[1 + \lambda\big(1 - g^*(\alpha_0)\big)\left(\frac{1}{\alpha_0} - bg_1{}^{*'}(0) - af^{*'}(0) - h^{*'(0)} - \mathrm{b}w_e{}^{*'}(0)\right)]$
$\mu_2^{'} = \mathrm{m}_{23} + \mathrm{m}_{24} + \mathrm{m}_{21.11,9} + \mathrm{m}_{21.11,10,12} = \left(1 - h^*(\lambda)\right)\left[\left(\frac{1}{\lambda} - \left(bg_1{}^{*'}(0) + af^{*'}(0) + h^{*'(0)} + \mathrm{b}w_e{}^{*'}(0)\right)\right)\right]$,
$\mu_3^{'} = \mathrm{m}_{30} + \mathrm{m}_{31.6} = \left(1 - f^*(\lambda)\right)\left(\frac{1}{\lambda} - f^{*'}(0)\right)$,
$\mu_4^{'} = \mathrm{m}_{45} + \mathrm{m}_{41.13,12} = \left(1 - w_e{}^*(\lambda)\right) = \left(\frac{1}{\lambda} - \left(g_1{}^{*'}(0) + w_e{}^{*'}(0)\right)\right)$,
$\mu_5^{'} = \mathrm{m}_{50} + \mathrm{m}_{51.1.4} = \left(1 - g_1{}^*(\lambda)\right)\left(\frac{1}{\lambda} - g_1{}^{*'}(0)\right)$. (3)

Mean Time to System Failure (MTSF)

Let $\varphi_i(t)$be the cdf of first passage time from the regenerative state to a failed state S_i. Regarding the failed state as absorbing state, we have the following recursive relations for $\varphi_i(t)$:
$\varphi_i(t) = \sum_j Q_{i,j}(t)\boxed{S}\,\varphi_j(t) + \sum_k Q_{i,j}(t)$ (4)
where S_j is an un-failed regenerative state to which the given regenerative state S_i can transit and k is a failed state to which the state i can transit directly. Taking LST of above relation (4) and solving for $\tilde{\boldsymbol{\varphi}}_0(s)$, we get
$MSTF(T_0) = \lim_{s\to 0}\frac{1-\tilde{\varphi}_0(s)}{s} = \frac{N_1}{D_1}$ (5)
Where $N_1 = \mu_0 + \mu_1 + p_{12}\mu_2 + p_{12}p_{23}(\mu_3 + p_{24}\mu_4 + p_{24}p_{45}\mu_5)$ and
$D_1 = 1 - p_{10} - p_{12}\left(p_{23}p_{30} + p_{24}p_{45}p_{5,14}\right)$.

Availability Analysis

Let $A_i(t)$ be the probability that the system is in up-state at instant 't' given that the system entered regenerative state S_i at t = 0. The recursive relations for $A_i(t)$ are given as
$A_i(t) = M_i(t) + \sum_j q_{i,j}^{(n)}(t) © A_j(t)$ (6)
where $M_i(t)$ is the probability that the system is up initially in regenerative state $S_i \in E$ at time $'t`$ without visiting to any other regenerative state and
$M_0(t) = e^{-2\lambda t}$, $M_1(t) = e^{-(\lambda+\alpha_0)t}\,\bar{G}(t)$, $M_2(t) = e^{-\lambda t}(t)\,\overline{H}(t)$,
$M_3(t) = e^{-\lambda t}(t)\,\bar{F}(t)$ $M_4(t) = e^{-\lambda t}(t)\,\overline{W_e}(t)$, *and* $M_5(t) = e^{-\lambda t}(t)\,\overline{G_1}(t)$
Taking $L.T.$ of relation (6) and solving for $A_0^*(s)$, we get steady-state availability as
$A_0(s) = \lim_{s\to 0} A_0{}^*(s) = \frac{N_2}{D_2}$ (7)
Where $N_2 = \left(p_{10} + p_{12}(p_{23}p_{30} + p_{24}p_{45}p_{50})\right)\mu_0 + \mu_1 + p_{12}\mu_2 + p_{12}(p_{23}\mu_3 + p_{24}\mu_4 + p_{24}p_{45}\mu_5)$ and $D_2 = \left(p_{10} + p_{12}(p_{23}p_{30} + p_{24}p_{45}p_{50})\right)\mu_0 + \mu_1^{'} + p_{12}\,\mu_2^{'} + p_{12}(p_{23}\,\mu_3^{'} + p_{24}\,\mu_4{}' + p_{24}p_{45}\,\mu_5{}')$.

Busy Period Analysis of Ordinary Server Due to Repair

Let $B_i^R(t)$ be the probability that the ordinary server is busy in repairing the unit at an instant 't'given that the system entered regenerative state S_i at $t = 0$. The recursive relations for $B_i^R(t)$ are given as

$$B_i^R(t) = W_i(t) + \sum_j q_{i,j}^{(n)}(t) © B_j^R(t) \quad (8)$$

where $W_1(t) = e^{-(\lambda+\alpha_0)t}\bar{G}(t) + \left(\lambda e^{-\lambda t} © \mathbf{1} © e^{-\alpha_0 t}\right)\bar{G}(t)$ (9)

Taking $L.T.$ of relation (8) and solving for$B_0^{R^*}(s)$, we get in the long run the time for which the system is under repair is given by

$$B_0^R = \lim_{s\to 0} s\, B_0^{R^*}(s) = \frac{N_3}{D_2} \quad (10)$$

Where $N_3 = W_1^*(0)$ and D_2 is already specified.

Busy Period Analysis of Ordinary Server Due To Replacement

Let $B_i^{Rp}(t)$ be the probability that the ordinary server is busy in repairing the unit at an instant 't'given that the system entered regenerative state S_i at $t = 0$.The recursive relations for $B_i^{Rp}(t)$ are given as

$$B_i^{Rp}(t) = W_i(t) + \sum_j q_{i,j}^{(n)}(t) © B_j^{Rp}(t) \quad (11)$$

Where $W_3(t) = e^{-\lambda t}\bar{F}(t) + \left(\lambda e^{-\lambda t} © \mathbf{1} ©\right)\bar{F}(t)$ (12)

Taking $L.T.$ of relation (11) and solving for $B_0^{Rp^*}(s)$, we get in the long run the time for which the system is under replacement is given by

$$B_0^{Rp} = \lim_{s\to 0} s\, B_0^{Rp^*}(s) = \frac{N_4}{D_2} \quad (13)$$

where $N_4 = p_{12}p_{23}W_3^*(0)$ and D_2 is already specified.

Busy Period Analysis of Ordinary Server Due To Inspection

Let $B_i^i(t)$ be the probability that the ordinary server is busy in inspection of the unit at an instant 't'given that the system entered regenerative state S_i at $t = 0$. The recursive relations for $B_i^i(t)$ are given as

$$B_i^i(t) = W_i(t) + \sum_j q_{i,j}^{(n)}(t) © B_j^i(t) \quad (14)$$

Where $W_2(t) = e^{-\lambda t}\bar{H}(t) + \left(\lambda e^{-\lambda t} © \mathbf{1}\right)\bar{H}(t)$ (15)

Taking $L.T.$ of relation (14) and solving for$B_0^{i^*}(s)$, we get in the long run the time for which the system is under inspection is given by

$$B_0^i = \lim_{s\to 0} s\, B_0^{i^*}(s) = \frac{N_5}{D_2} \quad (16)$$

Where $N_5 = p_{12}W_2^*(0)$ and D_2 is already specified.

Busy Period Analysis of Expert Server Due To Repair

Let $B_i^e(t)$ be the probability that the expert sever is busy in repairing the unit at an instant 't'given that the system entered regenerative state S_i at $t = 0$. The recursive relation for $B_i^e(t)$ are given by:

$$B_i^e(t) = W_i(t) + \sum_j q_{i,j}^{(n)}(t) © B_j^e(t) \quad (17)$$

Where $W_5(t) = e^{-\lambda t}\overline{G_1}(t) + \left(\lambda e^{-\lambda t} © \mathbf{1}\right)\overline{G_1}(t)$ (18)

Taking $L.T.$ of relation (17) and solving for $B_0^{e*}(s)$, we get the time for which the system is under repair done by expert server is given by

$$B_0^e = \lim_{s\to 0} s\, B_0^{e*}(s) = \frac{N_6}{D_2} \quad (19)$$

where $N_6 = p_{12}p_{24}p_{45}W_5^*(0)$ and D_2 is already specified.

Expected Number of Visits by the Ordinary Server

Let $N_i(t)$ be the expected number of visits by the ordinary server in $(0, t]$ given that the system entered the regenerative state S_i at $t = 0$. The recursive relation for $N_i(t)$ are given by

$$N_i(t) = \sum_j Q_{i,j}^{(n)}(t)\boxed{S}\left[\delta_j + N_i(t)\right] \tag{20}$$

Where δ_j=1, if S_j is the regenerative state where the ordinary server does job afresh, otherwise δ_j= 0. Taking $L.S.T.$ of relation (20) and solving for $\widetilde{N_0}(s)$, we get the expected number of visits by ordinary server per unit time as

$$N_0 = \lim_{s\to 0} s\,\widetilde{N_0}(s) = \frac{N_7}{D_2} \tag{21}$$

Where $N_7 = p_{10} + p_{12}(p_{23}p_{30} + p_{24}p_{45}p_{50})$ and D_2 is already specified.

Expected Number of Visits by Expert Server

Let $N_i^e(t)$ be the expected number of visits by expert server $(0, t]$ given that the system entered the regenerative state S_i at $t = 0$. The recursive relation for $N_i^e(t)$ are given by:

$$N_i^e(t) = \sum_j Q_{i,j}^{(n)}(t)\boxed{S}\left[\delta_j + N_i^e(t)\right] \tag{22}$$

Where δ_j=1, if S_j is the regenerative state where the expert server does job afresh, otherwise δ_j= 0. Taking $L.S.T.$ of relation (22) and solving for $\widetilde{N_0^e}(s)$, we get the expected number of visits by expert server per unit time as

$$N_0^e = \lim_{s\to 0} s\,\widetilde{N_0^e}(s) = \frac{N_8}{D_2} \tag{23}$$

Where $N_8 = p_{12}p_{24} + p_{11.7,8,10,12} + p_{12}p_{21.11,10,12}$ and D_2 is already specified.

Expected Number of Replacements of the Unit

Let $R_i(t)$ be the expected number of replacement of unit in $(0, t]$ given that the system entered the regenerative state S_i at $t = 0$. The recursive relation for $R_i(t)$ are given by

$$R_i(t) = \sum_j Q_{i,j}^{(n)}(t)\boxed{S}\left[\delta_j + R_i(t)\right] \tag{24}$$

δ_j=1, if S_j is the regenerative state where the ordinary server does job afresh, otherwise δ_j= 0.

Taking $L.S.T.$ of relation (24) and solving for $\widetilde{R_0}(s)$, we get the expected number of replacements per unit time as

$$R_0 = \lim_{s\to 0} s\,\widetilde{R_0}(s) = \frac{N_9}{D_2} \tag{25}$$

Where N_9=$p_{12}p_{23} + p_{11.7,8,9} + p_{12}p_{21.11,9}$ and D_2 is already specified.

Cost-Benefit Analysis

Profit incurred to the system model in steady state is given by

$$P = K_1A_0 - K_2B_0^R - K_3B_0^{Rp} - K_4R_0 - K_5B_0^i - K_6B_0^e - K_7N_0 - K_8N_0^e$$

Where

K_1 = Revenue per unit uptime of the system

K_2 = Cost per unit time for which ordinary server is busy due to repair
K_3 = Cost per unit time for which ordinary server is busy due to replacement
K_4 = Cost per unit time replacement of the unit
K_5 = Cost per unit time for which ordinary server is busy due to inspection
K_6 = Cost per unit time for which expert server is busy due to repair
K_7 = Cost per unit visits by the ordinary server
K_8 = Cost per unit visits by the expert server

Conclusion

To make the study more concrete and informative, numerical results for the particular case $(t) = \theta e^{-\theta t}$, $g_1(t) = \theta_0 e^{-\theta_0 t}$, $f(t) = \beta e^{-\beta t}$, $w(t) = \gamma e^{-\gamma t}$, $w_e(t) = \gamma_0 e^{-\gamma_0 t}$ and $h(t) = \eta e^{-\eta t}$ are obtained to depict the graphical behavior of MTSF, availability and profit function with respect to replacement rate (β) keeping fixed values of other parameters including $K_1 = 5000$, $K_2 = 600$, $K_3 = 100$, $K_4 = 450$, $K_5 = 50$, $K_6 = 900$, $K_7 = 150$, $K_8 = 200$ with $a = 0.6$ and $b = 0.4$ as shown in figures 2 to 4 respectively. It is observed that MTSF, availability and profit of the system model go on increasing with the increase of replacement rate (β), repair rate (θ) of ordinary server, repair rate (θ_0) of expert server, inspection rate (η) of the unit and arrival rate (γ_0) of the expert server while their values decrease as the rate ($\boldsymbol{\alpha}_0$) by which unit undergoes for inspection and failure rate (λ) increases. Furthermore, system becomes less profitable for $K_2 < K_4$. Thus, a parallel system of two identical units in which expert server takes some arrival time at the completion of maximum repair time of the ordinary server can be made more profitable to use by increasing the repair rate of ordinary server in case replacement cost is high.

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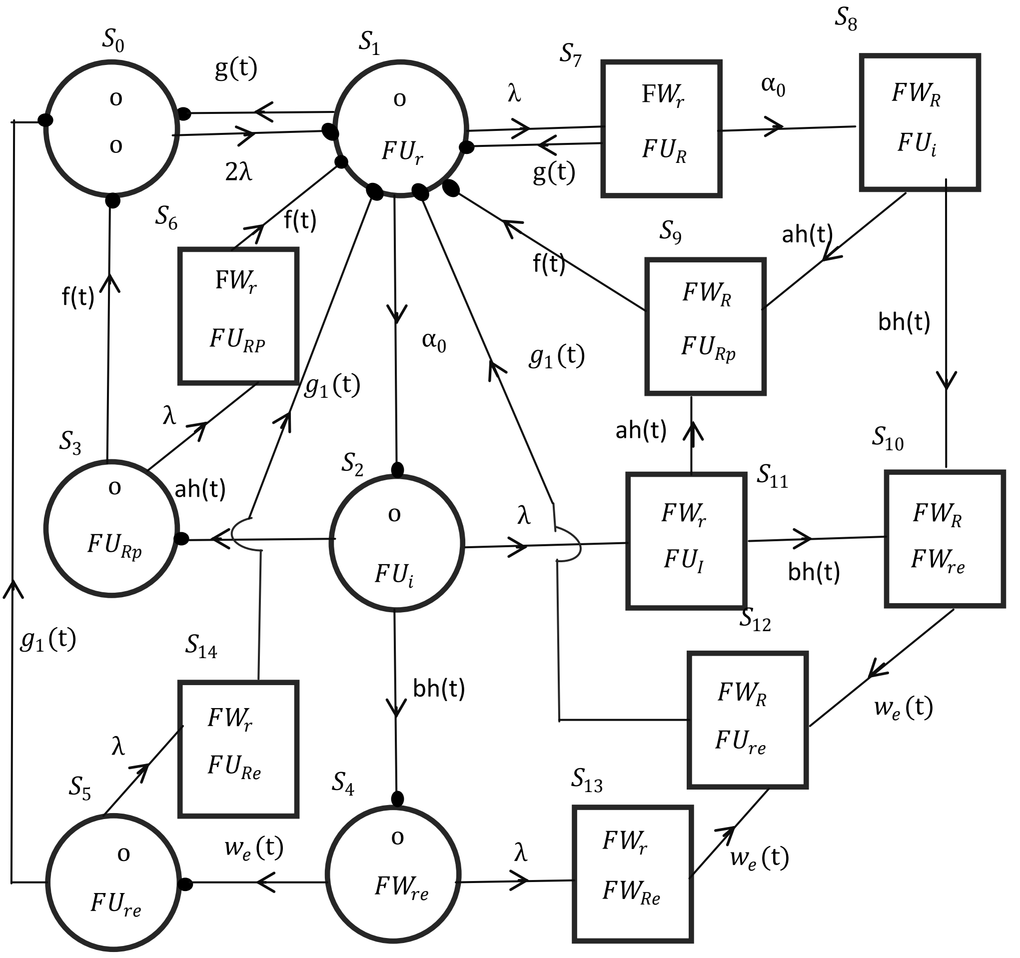
State Transition Diagram



● : Regenerative Point
O : Upstate
□ : Failed State

Figure. 1

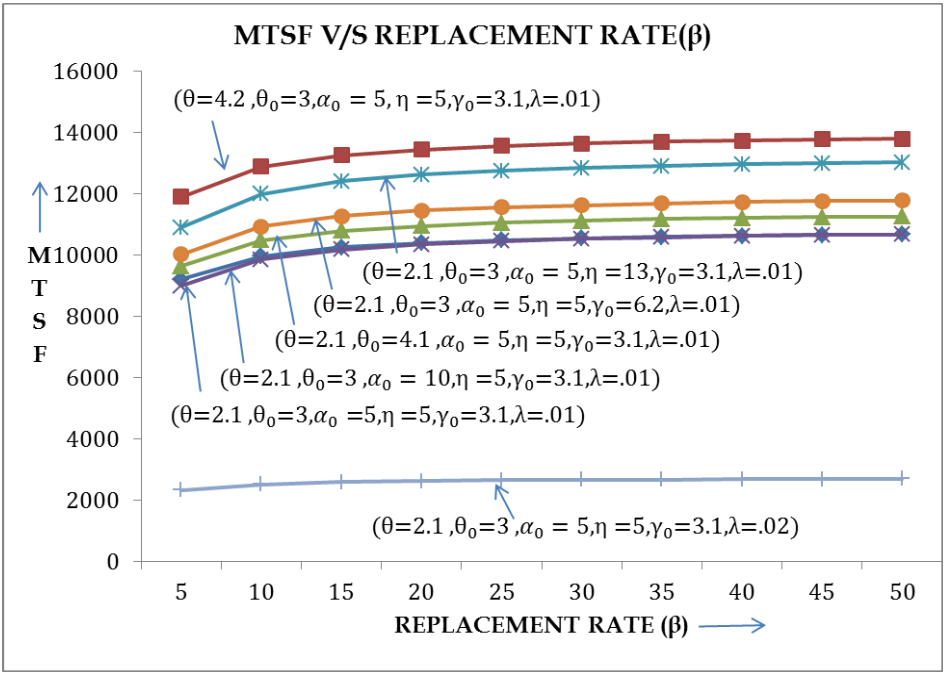


Figure 2

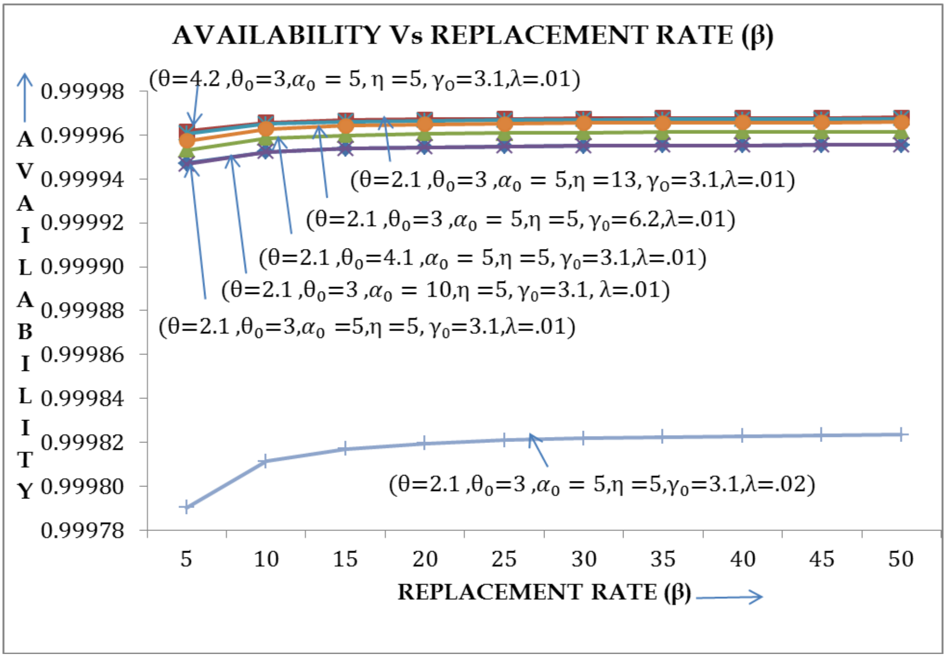


Figure 3

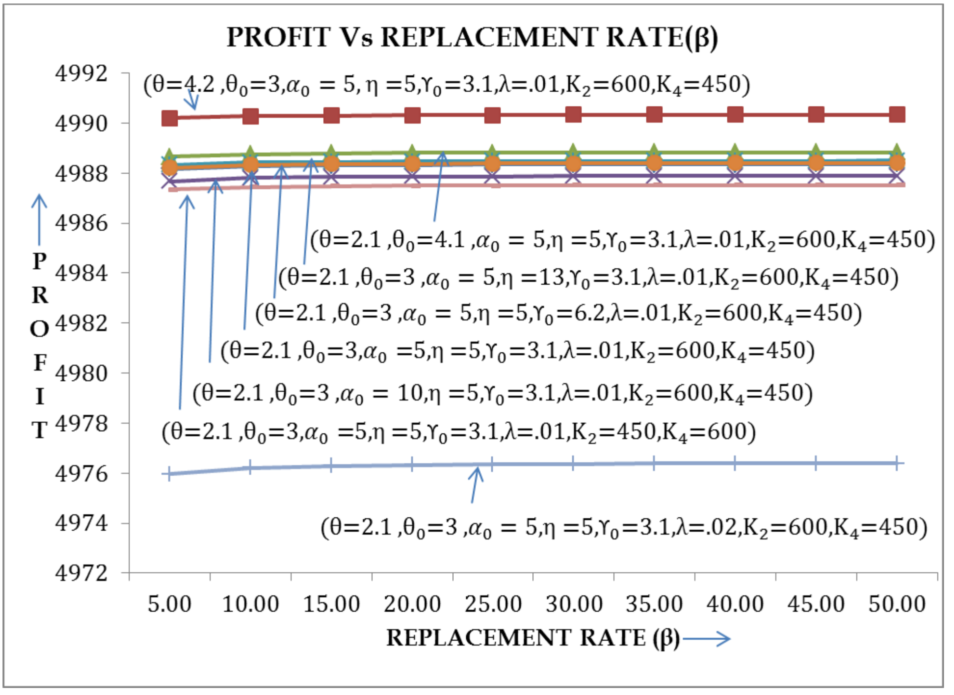

PROFIT Vs REPLACEMENT RATE(β)
PROFIT
REPLACEMENT RATE (β)
(θ=4.2 ,θ0=3,α0 = 5, η =5,ϒ0=3.1,λ=.01,K2=600,K4=450)
(θ=2.1 ,θ0=4.1 ,α0 = 5,η =5,ϒ0=3.1,λ=.01,K2=600,K4=450)
(θ=2.1 ,θ0=3 ,α0 = 5,η =13,ϒ0=3.1,λ=.01,K2=600,K4=450)
(θ=2.1 ,θ0=3 ,α0 = 5,η =5,ϒ0=6.2,λ=.01,K2=600,K4=450)
(θ=2.1 ,θ0=3,α0 =5,η =5,ϒ0=3.1,λ=.01,K2=600,K4=450)
(θ=2.1 ,θ0=3 ,α0 = 10,η =5,ϒ0=3.1,λ=.01,K2=600,K4=450)
(θ=2.1 ,θ0=3,α0 =5,η =5,ϒ0=3.1,λ=.01,K2=450,K4=600)
(θ=2.1 ,θ0=3 ,α0 = 5,η =5,ϒ0=3.1,λ=.02,K2=600,K4=450)
4992
4990
4988
4986
4984
4982
4980
4978
4976
4974
4972
5.00
10.00
15.00
20.00
25.00
30.00
35.00
40.00
45.00
50.00

Figure 4