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# AVAILABILITY AND RELIABILITY ANALYSIS OF THREE ELEMENTS PARALLEL SYSTEM WITH FUZZY FAILURE AND REPAIR RATES

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## Abstract

The present study proposes an algorithm to evaluate the fuzzy availability, reliability, steady state availability, and mean time to failure of a repairable parallel system which consists of three identical and independent components by using Markov model. This system fails when the three components fail or it goes to the critical case. The failure rate and the repair rate of each component is represented by triangular shaped vague set determined by using statistical data. Two numerical examples are given to illustrate the introduced algoithm and discribe the performance of the model when the life times and the repair times of the system follow exponential or Rayleigh distribution with fuzzy parameters.

**Key Words:** Fuzzy Rates, Availability, Reliability, Markov Model, Vague Sets, Statistical Data.

## 1. Introduction

Reliability is one of the quality characteristics that consumers require from manufacturers and it can be simply defined as the probability of a system to perform a required function under specified working conditions for a specified period of time [1]. Another important reliability related concept is the availability which takes both reliability and maintainability into account and it is defined as the probability that the system performs its required function at a given point of time [2]. The availability and the reliability of the system depend on the availability and the reliability of their components, on the configuration of the system, and on the system failure and repair criteria.

There are many techniques to compute the system availability and reliability. The most widely analytical used technique is Markov model, see [3]. Normally, as in [4] and [5], Markov models were carried out with constant parameters but, in fact, the components' failure and repair rates change during the process so that authers [6-8] introduced Markov models in the presence of time varying failure and repair rates. A lot of studies [9] and [10] proposed the redundant models to calculate the system reliability in the steady state but a few studies use the real time conditions.

In many practical situations, we assume that the failure times of the operating units and the repair times of the failed units are random variables following a known probability distribution except for the values of parameters that are difficult to be determined due to uncertainties and the lack of sufficient data. For this reason, these parameters are vaguely specified in the fuzzy set theory [11] by using membership functions which can be evaluated from collected data or from the opinions of experts. In [12-14], researchers applied this concept 10for analyzing different models and for evaluating the fuzzy reliability and the fuzzy availability.

As a generalization of fuzzy sets, vague sets are used instead because it can descibe the objective world more realistic, practical, and accurate. Vague set was proposed firstly by [15] and it has been widely applied in many situations. In [16], the arithmetic operations between vague sets were presented then they are used for analyzing a lot of fuzzy systems with differnent types of vague sets as [17] and [18] .

In this paper, a new method is developed for analyzing a fuzzy repairable parallel system which consists of three independent and identical components in the presence of common-cause failure by using Markov model. Also, we introduce the steps which are used to evaluate the availability, the reliability, the steady state availability, and the mean time to failure of our system if the life times and the repair times follow exponential distribution or Rayleigh distribution with fuzzy parameters represented by triangular shaped vague sets. Two numerical examples are given to illustrate briefly the introduced method.

## 2. Definitions

In the following, we review some definitions needed for this paper, see [19].

**Definition 2.1:** A vague set $\tilde{V}$ in the universe of discourse X, shown in **Figure 1**, is characterized by a truth membership function $t_{\tilde{V}}$ and a false membership function $f_{\tilde{V}}$ and defined as follows

$$\tilde{V} = \{ ( x, [ t_{\tilde{V}}(x), 1 - f_{\tilde{V}}(x)] ) : x \epsilon X \} \tag{1}$$

The interval $[t_{\tilde{V}}(x), 1 - f_{\tilde{V}}(x)]$ is called the vague value of $x$ in $\tilde{V}$. $t_{\tilde{V}}(x)$, $f_{\tilde{V}}(x)$ associate a value in the interval [0,1] and $t_{\tilde{V}}(x) + f_{\tilde{V}}(x) \leq 1$. $t_{\tilde{V}}(x)$ is represent the lower bound of membership grade of *x* derived from the ‘evidence for x’ and $f_{\tilde{V}}(x)$ is the lower bound on the negation of *x* derived from the ‘evidence against x’.

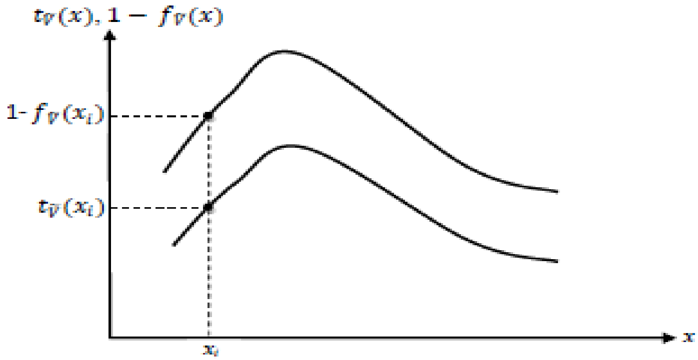



**Figure 1: A vague set $\tilde{V}$**

**Definition 2.2:** Let $\tilde{V}$ be a vague set in a universe $X$ with the true membership function $t_{\tilde{V}}(x)$ and the false membership function $f_{\tilde{V}}(x)$. For $\propto, \beta \in [0,1]$, the ($\propto$, $\beta$)-cut of the vague set $\tilde{V}$ is a crisp subset $\tilde{V}_{(\propto,\beta)}$ of the set $X$ defined as

$$\tilde{V}(\propto, \beta) = \{\, x \in X \mid [\, t_{\tilde{V}}(x),\ 1 - f_{\tilde{V}}(x)] \geq [\propto, \beta] \,\} \quad , \quad \propto \leq \beta \tag{2}$$

**Definition 2.3:** A level-λ triangular shaped membership function is specified by the parameters *a*, *b*, *c*, λ as follows

$$triangle(x; a, b, c) = \begin{cases} \dfrac{\lambda}{b-a}(x-a), & a \leq x < b \\ \dfrac{\lambda}{c-b}(c-x), & b \leq x < c \\ 0 \qquad , & otherwise \end{cases}$$

The interval valued triangular vague set $\tilde{V}$, shown in **Figure 2**, can be specified by

$$\tilde{V} = \langle\, [a_1, b_1, c_1;\ \mu_1],\quad [a_2, b_2, c_2;\ \mu_2]\,\rangle, \quad 0 \leq \mu_1 \leq \mu_2 \leq 1 \tag{3}$$

Where, $\mu_1$, $\mu_2$ are the maximum values for $t_{\tilde{V}}(x)$, $1 - f_{\tilde{V}}(x)$.

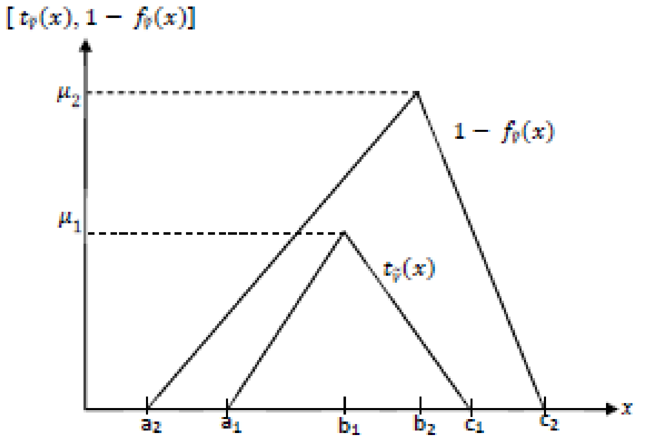



**Figure 2: A triangular vague set $\tilde{V}$**

**Definition 2.4:** Let us consider two interval valued triangular vague sets $\tilde{V}_1$ and $\tilde{V}_2$ :
$\tilde{V}_1 = \langle [a_1, b_1, c_1;\ \mu_1], [\grave{a}_1, \grave{b}_1, \grave{c}_1;\ \mu_2] \rangle, \tilde{V}_2 = \langle [a_2, b_2, c_2;\ \mu_3], [\grave{a}_2, \grave{b}_2, \grave{c}_2;\ \mu_4] \rangle$

The arithmetic operations between $\tilde{V}_1$ and $\tilde{V}_2$ are defined as follow

- $\tilde{V}_1 \oplus \tilde{V}_2 = \langle [a_1 + a_2, b_1 + b_2, c_1 + c_2; Min(\mu_1, \mu_3)], [\grave{a}_1 + \grave{a}_2, \grave{b}_1 + \grave{b}_2, \grave{c}_1 + c2; Min(\mu 2, \mu 4)$
- $\tilde{V}_1 \ominus \tilde{V}_2 = \langle [a_1 - c_2, b_1 - b_2, c_1 - a_2; Min(\mu_1, \mu_3)], [\grave{a}_1 - \grave{c}_2, \grave{b}_1 - \grave{b}_2, \grave{c}_1 - a2; Min(\mu 2, \mu 4)$

- The results of multiplication and division process can be also approximated to have triangular vague sets defined by
  $\tilde{V}_1 \otimes \tilde{V}_2 = \langle [a_1 \times a_2, b_1 \times b_2, c_1 \times c_2; Min(\mu_1, \mu_3)], [\grave{a}_1 \times \grave{a}_2, \grave{b}_1 \times \grave{b}_2, \grave{c}_1 \times c2; Min(\mu 2, \mu 4)$
  $\tilde{V}_1 \oslash \tilde{V}_2 =$
  $\langle [a_1/c_2, b_1/b_2, c_1/a_2; Min(\mu_1, \mu_3)], [\grave{a}_1/\grave{c}_2, \grave{b}_1/\grave{b}_2, \grave{c}_1/\grave{a}_2; Min(\mu_2, \mu_4)] \rangle$

## 3. Model description, availability and reliability

To construct the markov model of our system, a detailed description is given as follows:

- The system is repairable and consists of three independent and similar components work simultaneously and they are connected in parallel.
- At any time *t*, an operating component may fail with a failure rate $\lambda(t)$ and it is repaired with a repair rate $\mu(t)$.
- At any time *t*, the system fails to work if the three components fail or it goes to the critical case due to a common cause failure with failure rate $\lambda_c(t)$ and it is repaired with a repair rate $\mu_c(t)$.

- The life time and the repair time follow arbitrary probability distributions as exponential or Rayleigh distribution with fuzzy rates.

### 3.1. The availability function and the steady state availability

Based on the previous description we can construct a model for our repairable system using non-homogeneous continuous-time Markov chain. Assume that each component has only two binary states, working and failed state, the system has five states as follow

State "0" : All the three system's components are in the working states.
State "1" : One of the three system's components is in the failed state.
State "2" : Two of the three system's components are in the failed state.
State "3" : All the three system's components are in the failed state.
State "*C*" : The system is failed due to a common cause failure.

Let $P_j(t)$ is the probability that the system is in the state $j$ ; $j = 0, 1, 2, 3$ and $P_c(t)$ is the probability that the system is in the critical case. From the state-space diagram of our model shown in **Figure 3**, we can get the Markov's first order differential equations in terms of the failure rates $\lambda(t), \lambda_c(t)$ and the repair rates $\mu(t), \mu_c(t)$ as follow:

$$\dot{P}_0(t) = -\big(3\lambda(t) + \lambda_c(t)\big).P_0(t) + \mu(t).P_1(t) + \mu_c(t).P_c(t) \tag{4.a}$$

$$\dot{P}_1(t) = -(2\lambda(t) + \lambda_c(t) + \mu(t)).P_1(t) + 3\lambda(t).P_0(t) + 2\mu(t).P_2(t) \tag{4.b}$$

$$\dot{P}_2(t) = -(\lambda(t) + \lambda_c(t) + 2\mu(t)).P_2(t) + 2\lambda(t).P_1(t) + 3\mu(t).P_3(t) \tag{4.c}$$

$$\dot{P}_3(t) = -3\mu(t).P_3(t) + \lambda(t).P_2(t) \tag{4.d}$$

$$\dot{P}_c(t) = -\mu_c(t).P_c(t) + \lambda_c(t).[P_0(t) + P_1(t) + P_2(t)] \tag{4.e}$$

If the process is in state "0" at the beginning, the initial conditions for the model are given by:

$$P_0(0) = 1\,, P_1(0) = 0\,, P_2(0) = 0\,, P_3(0) = 0\,, and\ \ P_C(0) = 0 \tag{5}$$

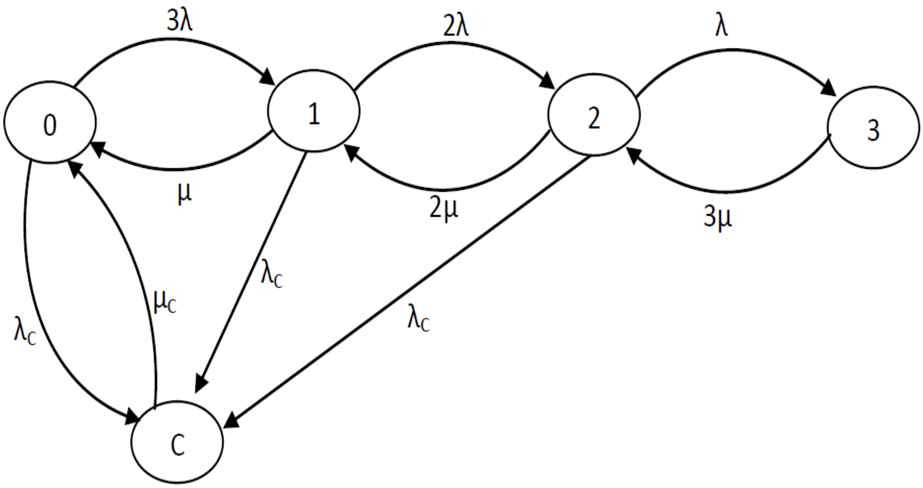



**Figure 3: The state-transition diagram for availability analysis of the three-unite repairable system with common cause failure**

Under the specified initial condition (5), the system of first order differential equations [4.a-4.e] can be solved by any mathematical method to get the transition probabilities $P_j(t);\ j = 0, 1, 2, 3$ and $P_C(t)$.

Our parallel system will stop working when a common cause failure occurs or all the system's components fail which mean that both states "3" and "$C$" are the only down states of the system so the availability function can be expressed as follow:

$$A(t) = P_0(t) + P_1(t) + P_2(t) = 1 - P_3(t) - P_C(t)\,, \quad t \geq 0 \tag{6}$$

Then the steady state availability can be calculated from

$$A_{ss} = \lim_{t\to\infty} A(t) \tag{7}$$

## 3.2. The reliability function and the mean time to failure

To obtain the system reliability function $R$(t) and the mean time to failure *MTTF*, repairs that return our system from unacceptable state "3", "$C$" should be forbidden and treated as absorbing states. The initial model should be transformed as shown in **Figure 4** and the system of first order differential equations (4) are changed to be

$$\dot{P}_0^*(t) = -(3\lambda(t) + \lambda_c(t)).P_0^*(t) + \mu(t).P_1^*(t) \tag{8.a}$$

$$\dot{P}_1^*(t) = -(2\lambda(t) + \lambda_c(t) + \mu(t)).P_1^*(t) + +3\lambda(t).P_0^*(t) + 2\mu(t).P_2^*(t) \tag{8.b}$$

$$\dot{P}_2^*(t) = -(\lambda(t) + \lambda_c(t) + 2\mu(t)).P_2^*(t) + 2\lambda(t).P_1^*(t) \tag{8.c}$$

$$\dot{P}_3^*(t) = \lambda(t).\, P_2^*(t) \quad (8.d)$$

$$\dot{P}_C^*(t) = \lambda_c(t).\, [P_0^*(t) + P_1^*(t) + P_2^*(t)] \quad (8.e)$$

After solving the above equations [8.a-8.e] with the same initial condition (5), the reliability function of the system can be obtained by

$$R(t) = P_0^*(t) + P_1^*(t) + P_2^*(t) = 1 - P_3^*(t) - P_C^*(t), \quad t \geq 0 \quad (9)$$

Then the mean time to failure of the system is

$$MTTF = \int_0^\infty R(t)\, dt \quad (10)$$

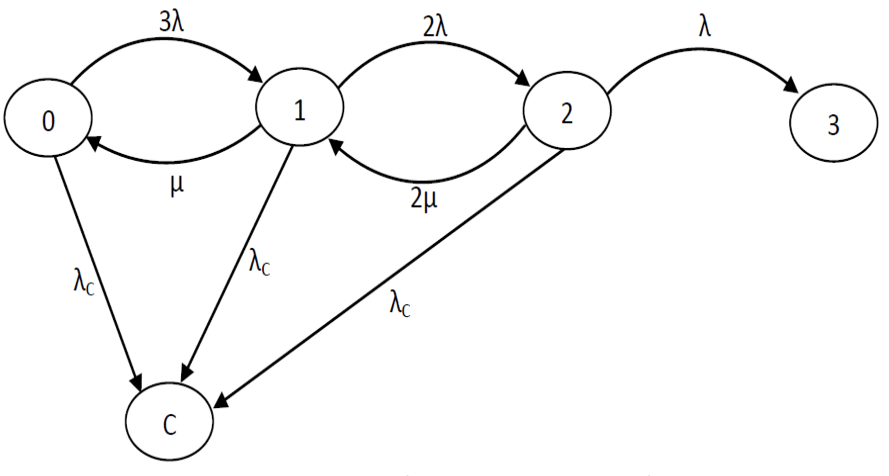



**Figure 4: The state-transition diagram for reliability analysis of the three-unite repairable system with common cause failure**

## 4. System availability and reliability under Fuzzy Failure and repair rates

To extend the applicability of our system, we assume that the failure rates $\lambda(t)$ and $\lambda_c(t)$ are random variables following the same known probability distribution $h_i(t) = f(t;\, \theta_i), i = 1, 2$ with different parameters $\theta_i, i = 1, 2$ and the repair rates $\mu(t)$ and $\mu_c(t)$ are random variables following the same known probability distribution $\grave{h}_i(t) = f(t;\, \theta_i), i = 3, 4$ with different parameters $\theta_i, i = 3, 4$. Due to uncertainty and lack of sufficient information, the values of the four parameters $\theta_i$ , $i = 1, 2, 3, 4$ are difficult to be determined so they are represented by vague sets with triangular shaped truth and fault membership functions, as equation (3) estimated from statistical data which are taken from random samples as follow:

$$\widetilde{\theta}_i = \langle\ [L_i, M_i, U_i\,;\, \eta_{1i}]\ ,\ [L_i', M_i', U_i';\, \eta_{2i}]\ \rangle \quad ;\ i = 1, 2, 3, 4 \quad (11)$$

Where, $M_i, \grave{M}_i$ are the point estimation, $L_i, \grave{L}_i, U_i, \grave{U}_i$ are the lower and the upper limits of the triangular truth and fault membership functions of the parameter $\widetilde{\theta_i}$ , respectively. The values of $L_i, \grave{L}_i, M_i, \grave{M}_i$ , $U_i$, and $\grave{U}_i$ can be estimated for each parameter $\widetilde{\theta_i}$ by using the random samples with $(1-\gamma_i)100\%$ and $(1-\gamma_i{}')100\%$ confidence intervals. Also, $\eta_{1i}$ , $\eta_{2i}$ are the maximum values for the truth membership funtion $t_{\widetilde{\theta_i}}(x_i)$ and the fault membership function $(1- f_{\widetilde{\theta_i}}(x_i))$ of the parameter $\widetilde{\theta}_i$.

Then we can evaluate the fuzzy availability function, reliability function, steady state availability, and mean time to failure of our model by using the $(\propto, \beta)-cut$ technique with the following procedures:

**Step 1:** Depending on the probability distribution of the failure and repair rates $f(t;\ \theta_i)$, we can determine $L_i,\ M_i,\ U_i$ , the point estimation, the lower, and the upper limits of the triangular truth membership function for each parameter $\widetilde{\theta}_i\ ; i = 1, 2, 3, 4$ with $(1-\gamma_i)100\%$ confidence intervals by using a sample data $(X_1,\ X_2, \ldots, X_{m_i})$ of size $m_i$ .

**Step 2:** By using another sample data $(\grave{X}_1{}',\ X_2{}', \ldots,\ X'_{r_i})$ of size $r_i$ , we can determine $L_i{}',\ M_i{}',\ U_i{}'$, the point estimation, the lower, and the upper limits of the triangular fault membership function for each parameter $\widetilde{\theta}_i\ ; i = 1, 2, 3, 4$ with $(1-\gamma_i^{'})100\%$ confidence interval.

**Step 3:** Finding $\propto_i = Max\{t_{\widetilde{\theta_i}}(x_i)\}$ and $\beta_i = Max\{1- f_{\widetilde{\theta_i}}(x_i)\}$ corresponding to each parameter $\widetilde{\theta}_i\ ; i = 1, 2, 3, 4$.

**Step 4:** Finding the value of $\propto = Min\{\propto_i\}$ and $\beta = Min\{\beta_i\}\ ; i = 1, 2, 3, 4$.

**Step 5:** For certain values of $t_{\widetilde{\theta_i}}(x_i)$ lying in the interval $[0, \propto]$, such that $\propto -cut =$ *tθixi, 0≤∝−cut≤∝* , the corresponding intervals for *θi,* will be determined from the following relation:

$$\left[\widetilde{\theta}_i{}^L,\ \widetilde{\theta}_i{}^U\right]_{\propto -cut} = \left[L_i + \frac{\propto -cut}{\propto_i}(M_i - L_i)\ , U_i - \frac{\propto -cut}{\propto_i}(U_i - M_i)\right]$$

**Step 6:** Also, for certain values of $1 - f_{\widetilde{\theta_i}}(x_i)$ lying in the interval $[0, \beta]$, such that $\beta - cut = 1 - f_{\widetilde{\theta_i}}(x_i), 0 \le \beta - cut \le \beta$ , the corresponding intervals for $\widetilde{\theta}_i$, will be determined from the following relation:

$$\left[\widetilde{\theta}_i{}^L,\ \widetilde{\theta}_i{}^U\right]_{\beta -cut} = \left[L'_i + \frac{\beta -cut}{\beta_i}(M'_i - L'_i)\ , U'_i - \frac{\beta -cut}{\beta_i}(U'_i - M'_i)\right]$$

**Step 7:** Finding the intervals for the failure and repair rates $h_i(t) = f(t;\ \theta_i)$ corresponding to the $\propto -cuts$ and $\beta - Cuts$ of the parameters $\widetilde{\theta}_i\ ; i = 1, 2, 3, 4$.

**Step 8:** Substituting the rates $h_i(t)\ ; i = 1, 2, 3, 4$ in equations (4) and then by MAPLE program, we solve these under the initial condition (5) to obtain the intervals for the fuzzy availability function $\widetilde{A}(t)$ and steady state availability $\tilde{A}_{ss}$ corresponding to the *α-cuts* and *β-cuts* by using relations (6) and (7).

**Step 9:** Substituting the rates $h_i(t)\,; i = 1, 2, 3, 4$ in equations (8) and then by MAPLE program, we solve these with the same initial condition to obtain the intervals for the system fuzzy availability function $\widetilde{R}(t)$ and mean time to failure $\widetilde{MTTF}$ corresponding to the *α-cuts* and *β-cuts* by using relations (9) and (10).

## 5. Numerical examples

These examples to illustrate the performance of our model by applying the previous algorithm to evaluate the fuzzy availability and reliability function when life time and the repair time follow arbitrary probability distributions with fuzzy failure and repair rates. We will focus on two cases of the life and repair times' distributions which are exponential and Rayleigh distributions, as follows:

### 5.1. The life and repair times with fuzzy exponential distribution [20]

In this case, our system will be modeled by homogenous Markov chain with constant failure and repair rates $h_i(t) = \theta_i, i = 1, 2, 3, 4$ but not fixed (triangular vague set) so we substitute in the set of equations (4) and (8) by

$$\lambda(t) = \tilde{\lambda}\,, \lambda_C(t) = \widetilde{\lambda_C}\,, \mu(t) = \tilde{\mu}\,, \mu_C(t) = \widetilde{\mu_C}$$

For each rate, we can calculate the point estimation, the lower, and the upper limits of the true and the fault membership functions of each fuzzy parameter $\widetilde{\theta}_i, i = 1, 2, 3, 4$ by using Chi-square distribution with $(1\text{-}\gamma_i)100\%$ , $(1-\gamma_i{}')100\%$ as follow:

$$\begin{aligned} M_i &= \frac{m_i}{\sum_{j=1}^{m_i} X_j}\,, \quad L_i = \frac{\chi^2_{2m_i,(1-\gamma_i/2)}}{2\sum_{j=1}^{m_i} X_j}\,, \quad U_i = \frac{\chi^2_{2m_i,(\gamma_i/2)}}{2\sum_{j=1}^{m_i} X_j}\,, \\ M'_i &= \frac{r_i}{\sum_{j=1}^{r_i} X_j{}'}\,, \quad L'_i = \frac{\chi^2_{2r_i,(1-\gamma_i{}'/2)}}{2\sum_{j=1}^{r_i} X_j{}'}\,, \quad U'_i = \frac{\chi^2_{2r_i,(\gamma_i{}'/2)}}{2\sum_{j=1}^{r_i} X_j{}'}\,. \\ & \qquad i=1,2,3,4 \end{aligned} \tag{12}$$

Where we take two random samples to get each fuzzy parameter $\widetilde{\theta}_i$ with number of observations $m_i$ , $r_i$ and total test times $\sum_{j=1}^{m_i} X_j$ , $\sum_{j=1}^{r_i} X_j{}'$, respectively.

**Table 1** shows the samples' statistical data used to estimate the point estimation, lower, and upper limits of the truth and the fault membership functions of each fuzzy parameter $\widetilde{\theta}_i\,, i = 1, 2, 3, 4$ at $(1\text{–}\gamma_i)100\%$ , $(1-\gamma_i{}')100\%$ confidence interval by using relation (12). Then the triangular vague sets of the failure and repair rates $\tilde{\lambda}$, $\tilde{\lambda}_C$, $\tilde{\mu}$, and $\tilde{\mu}_C$ can be written as follow:

$$\tilde{\lambda} = \langle\, [0.01, 0.024, 0.045; 0.2], [0.014, 0.027, 0.042; 0.5]\,\rangle,$$

$$\tilde{\lambda}_C = \langle\, [0.02, 0.043, 0.07; 0.4], [0.022, 0.056, 0.11; 0.6]\,\rangle$$

$\tilde{\mu} \;\; = \langle\, [0.038, 0.068, 0.127; 0.3], [0.036, 0.073, 0.12; 0.4] \,\rangle\,,$

$$\tilde{\mu}_C \;= \langle\, [0.041, 0.074, 0.11; 0.4], [0.039, 0.075, 0.123; 0.5] \,\rangle$$

So,

$$\propto = Min\{\propto_i\} = Min\{0.2, 0.4, 0.3, 0.4\} = 0.2 \quad , \quad \beta = Min\{\beta_i\} = Min\{0.5, 0.6, 0.4, 0.5\} = 0.4$$

| $i$ | $m_i$ | $\sum_{j=1}^{m_i}$ | $\gamma_i$ | $r_i$ | $\sum_{j=1}^{r_i}$ | $\gamma_i{}'$ | $M_i$ , $L_i$ , $U_i$ | $M'_i$ , $L'_i$ , $U'_i$ | $\propto_i$ | $\beta_i$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 10 | 420 | 0.02 | 13 | 480 | 0.05 | 0.024, 0.0098, 0.0447 | 0.027, 0.0144, 0.042 | 0.2 | 0.5 |
| 2 | 11 | 255 | 0.05 | 9 | 160 | 0.02 | 0.043, 0.02, 0.0696 | 0.056, 0.022, 0.11 | 0.4 | 0.6 |
| 3 | 10 | 148 | 0.05 | 8 | 110 | 0.1 | 0.027, 0.0375, 0.127 | 0.0727, 0.036, 0.12 | 0.3 | 0.4 |
| 4 | 7 | 95 | 0.05 | 12 | 160 | 0.05 | 0.0737, 0.036, 0.12 | 0.075, 0.039, 0.123 | 0.4 | 0.5 |

**Table 1: The information used to calculate the truth and false membership functions of the four rates**

Hence, the crisp intervals for $\tilde{\lambda}$, $\tilde{\lambda}_C$, $\tilde{\mu}$, and $\tilde{\mu}_C$ corresponding to the $(\propto, \beta) - cuts$ can be calculated at specific values of $\propto - cut \in [0, 0.2]$ and $\beta - cut \in [0, 0.4]$ as shown in **Table 2** and **Table 3**.

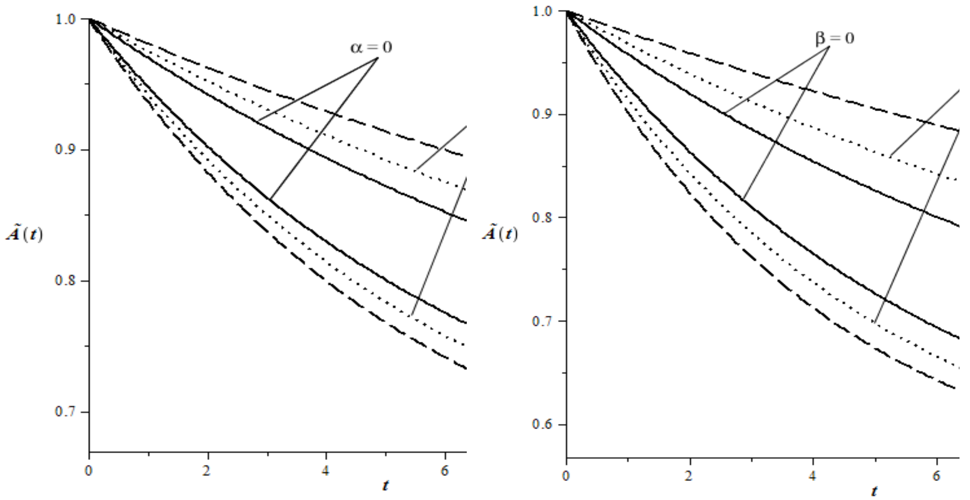
| $\propto$ | [ | [ | [ | [ |
|---|---|---|---|---|
| 0 | [0.010, 0.045] | [0.020, 0.070] | [0.038, 0.127] | [0.041, 0.110] |
| 0.1 | [0.017, 0.034] | [0.026, 0.063] | [0.048, 0.107] | [0.049, 0.100] |
| 0.2 | [0.024, 0.024] | [0.032, 0.056] | [0.058, 0.087] | [0.057, 0.092] |

**Table 2: The intervals for $\tilde{\lambda}$, $\tilde{\lambda}_C$, $\tilde{\mu}$, and $\tilde{\mu}_C$ corresponding to $\propto$-*cut* = 0, 0.1, 0.2**

| ***β* –cut** | [ | [ | [ | [ |
|---|---|---|---|---|
| 0 | [0.014, 0.042] | [0.022, 0.110] | [0.036, 0.120] | [0.039, 0.123] |
| 0.1 | [0.022, 0.039] | [0.028, 0.103] | [0.045, 0.108] | [0.046, 0.113] |
| 0.2 | [0.029, 0.036] | [0.033, 0.095] | [0.054, 0.096] | [0.053, 0.104] |
| 0.3 | [0.036, 0.033] | [0.039, 0.088] | [0.0635,0.085] | [0.061, 0.094] |
| 0.4 | [0.043, 0.030] | [0.045, 0.080] | [0.073, 0.073] | [0.068, 0.068] |

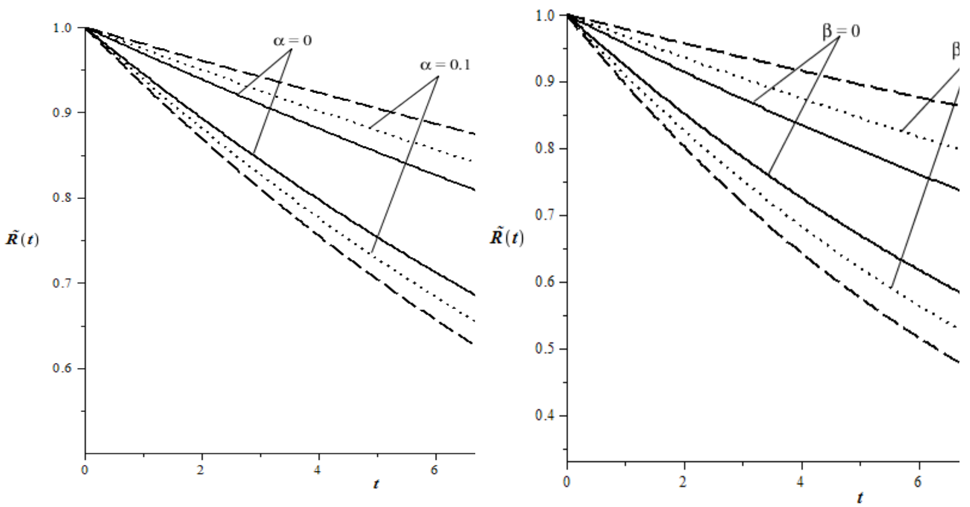
**Table 3: The intervals for $\tilde{\lambda}$, $\tilde{\lambda}_C$, $\tilde{\mu}$, and $\tilde{\mu}_C$ corresponding to *β*-*cut* = 0, 0.1, 0.2, 0.3, 0.4**

It is difficult to solve our model equations analytically and obtain a closed form for the system fuzzy availability and reliability functions so by using Maple program, we can approximate these system characteristics instead by collecting numerical solutions of our model equations (4) and (8) at arbitrary values $(\propto, \beta) - cuts$ ; $\alpha - cut \in [0, 0.2]$ and $\beta - cut \in [0, 0.4]$. As shown in **Figure 5** and **Figure 6**, we can represent the system availability and the reliability functions $\tilde{A}(t)$ and $\tilde{R}(t)$ versus the time at $\propto -cut = 0, 0.2$ and at $\beta - cut = 0, 0.2, 0.4$. At any instant value of time, the system availability and reliability are not crisp values but they are represented by vague sets as shown in **Figure 7**.



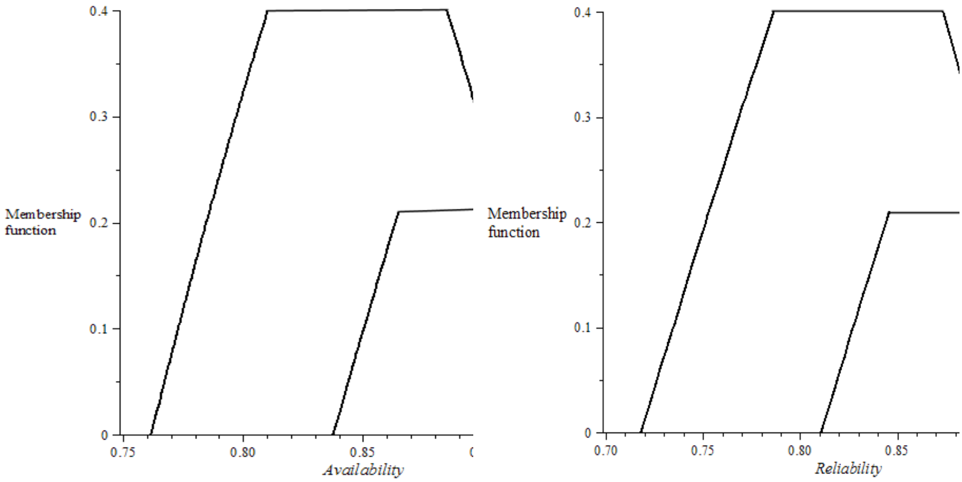

**(a)** $\tilde{A}(t)$ at ∝-*cut* = 0, 0.1, 0.2 **(b)** $\tilde{A}(t)$ at *β-cut* = 0, 0.2, 0.4

**Figure 5: The system fuzzy availability function versus the time $\tilde{A}(t)$ (Case 1)**




**(a)** $\tilde{R}(t)$ at ∝-*cut* = 0, 0.1, 0.2 **(b)** $\tilde{R}(t)$ at *β-cut* = 0, 0.2, 0.4

**Figure 6: The system fuzzy reliability function versus the time $\tilde{R}(t)$ (Case 1)**




**(a)** $\tilde{A}(t)$ at $t = 3$ **(b)** $\tilde{R}(t)$ at $t = 3$

**Figure 7: The system fuzzy availability and reliability at time $t = 3$ (Case 1)**

## 5.2. The life and repair times with fuzzy Rayleigh distribution [21]

In this case, our system will be modeled with time varying failure and repair rates given by the following relations:

$$\lambda(t) = \frac{t}{\widetilde{\vartheta_1}^2}, \lambda_C(t) = \frac{t}{\widetilde{\vartheta_2}^2}, \mu(t) = \frac{t}{\widetilde{\vartheta_3}^2}, \mu_C(t) = \frac{t}{\widetilde{\vartheta_4}^2}$$

Where, $\widetilde{\vartheta_i}\,; i = 1,2,3,4$ are fuzzy parameters defined by triangular vague sets. The point estimation, the lower, and the upper limits of the (1-$\gamma_i$)100% , $(1 - \gamma_i{}')$100% confidence interval of each parameter $\widetilde{\vartheta_i}\,; i = 1,2,3,4$ can be calculated as follow

$$M_i = \sqrt{\frac{\sum_{j=1}^{m_i}(X_j)^2}{2m_i}} \quad , \quad [L_i\,, U_i] = M_i \mp Z_{\gamma_i/2}\sqrt{var(M_i)} \quad , \quad var(M_i) = \frac{(M_i)^2}{4m_i} \quad ,$$

$$M_i{}' = \sqrt{\frac{\sum_{j=1}^{r_i}(X_j{}')^2}{2r_i}} \quad , \quad [L_i{}', U_i{}'] = M_i' \mp Z_{\gamma_i{}'/2}\sqrt{var(M_i{}')} \quad , \quad var(M_i{}') = \frac{(M_i{}')^2}{4r_i} \quad .$$

$$i=1,2,3,4 \qquad (13)$$

We can use this relation, containing the normal distribution, if the sizes of two random samples taken to estimate these parameters are large $m_i\,,\ r_i\ \geq 30$ .

**Table 4** shows the samples' statistical data used to estimate the point estimation, lower, and upper limits of the truth and the fault membership functions of each fuzzy parameter $\widetilde{\vartheta_i}, i = 1,2,3,4$ at $(1-\gamma_i)100\%$ , $(1-\gamma_i{}')100\%$ confidence interval by using relation (13). Then the triangular vague sets of the parameters $\widetilde{\vartheta_i}, i = 1,2,3,4$ can be written as follow:

$\widetilde{\vartheta_1} = \langle\, [2.6, 2.95, 3.296\,; 0.2], [2.48, 2.887, 3.29\,; 0.5]\,\rangle$,

$$\widetilde{\vartheta_2} = \langle\, [2.83, 3.25, 3.67\,; 0.3], [2.85, 3.32, 3.79\,; 0.7]\,\rangle$$

$\widetilde{\vartheta_3} = \langle\, [2.95, 3.47, 3.99\,; 0.4], [2.93, 3.61, 4.287\,; 0.6]\,\rangle$, $\widetilde{\vartheta_4} = \langle\, [2.7, 3.16, 3.6\,; 0.2], [2.67, 3.22, 3.766\,; 0.8]\,\rangle$

So, $\propto = Min\{\propto_i\} = Min\{0.2, 0.3, 0.4, 0.2\} = 0.2$ , $\beta = Min\{\beta_i\} = Min\{0.5, 0.7, 0.6, 0.8\} = 0.5$

| $i$ | $m_i$ | $\sum_{j=1}^{m_i}(X$ | $\gamma_i$ | $r_i$ | $\sum_{j=1}^{r_i}(X_j{}'$ | $\gamma_i{}'$ | $M_i, L_i, U_i$ | $M'_i, L'_i, U$ | $\alpha_i$ | $\beta_i$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 70 | 1220 | 0.05 | 60 | 1000 | 0.03 | 2.95, 2.6, 3.296 | 2.887, 2.48, 3.29 | 0.2 | 0.5 |
| 2 | 45 | 255 | 0.08 | 50 | 160 | 0.045 | 3.25, 2.83, 3.67 | 3.32, 2.85, 3.79 | 0.3 | 0.7 |
| 3 | 36 | 865 | 0.075 | 30 | 780 | 0.04 | 3.47, 2.95, 3.99 | 3.61, 2.93, 4.287 | 0.4 | 0.6 |
| 4 | 40 | 800 | 0.065 | 35 | 725 | 0.05 | 3.16, 2.7, 3.6 | 3.22, 2.67, 3.766 | 0.2 | 0.8 |

**Table 4: The information used to calculate the truth and false membership functions of the four parameters $\widetilde{\vartheta_1}, \widetilde{\vartheta_2}, \widetilde{\vartheta_3}, \widetilde{\vartheta_4}$**

Hence, the crisp intervals for the parameters $\widetilde{\vartheta_i}, i = 1,2,3,4$ and the failure and repair rates $\tilde{\lambda}(t)$, $\tilde{\lambda}_C(t)$, $\tilde{\mu}(t)$, and $\tilde{\mu}_C(t)$ corresponding to the $(\propto,\beta) - cuts$ can be calculated at specific values of $\propto -cut \in [0, 0.2]$ and $\beta - cut \in [0, 0.5]$ as shown in **Table 5** and **Table 6**.

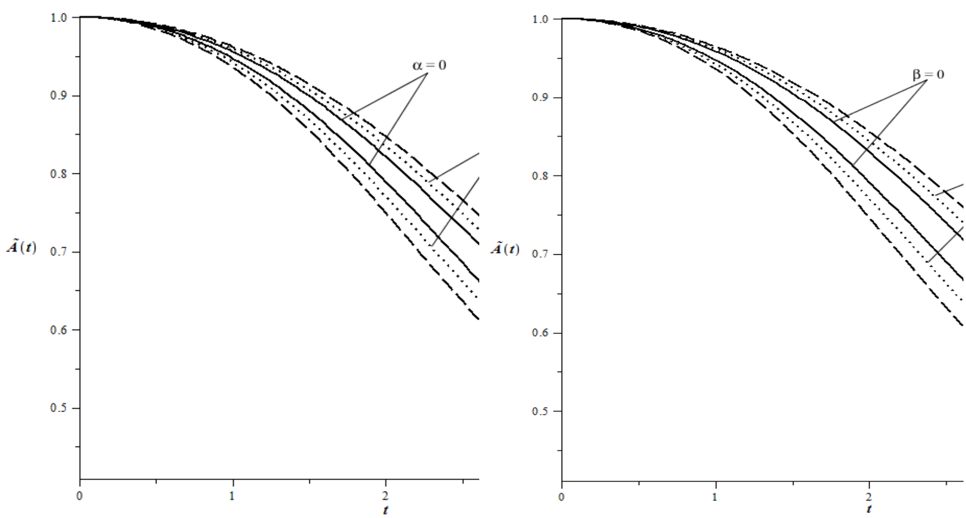
| $\propto$-*cut* | $[\tilde{\vartheta}_1^L, \tilde{\vartheta}_1^U]$ | $[\tilde{\lambda}^L(t), \tilde{\lambda}^U(t)]$ | $[\tilde{\vartheta}_2^L, \tilde{\vartheta}_2^U]$ | $[\tilde{\lambda}_C^L(t), \tilde{\lambda}_C^U(t)]$ |
|---|---|---|---|---|
| 0 | [2.600, 3.296] | [0.092t, 0.148t] | [2.83, 3.67] | [0.074t, 0.125t] |
| 0.1 | [2.775, 3.123] | [0.103t, 0.130t] | [2.97, 3.53] | [0.080t, 0.113t] |
| 0.2 | [2.950, 2.950] | [0.115t, 0.115t] | [3.11, 3.39] | [0.103t, 0.087t] |
| $\propto$-*cut* | $[\tilde{\vartheta}_3^L, \tilde{\vartheta}_3^U]$ | $[\square^L(t), \square^U(t)]$ | $[\tilde{\vartheta}_4^L, \tilde{\vartheta}_4^U]$ | $[\square_C^L(t), \square_C^U(t)]$ |
| 0 | [2.95, 3.99] | [0.063t, 0.115t] | [2.70, 3.60] | [0.077t, 0.137t] |
| 0.1 | [3.08, 3.86] | [0.067t, 0.110t] | [2.93, 3.38] | [0.088t, 0.116t] |
| 0.2 | [3.21, 3.73] | [0.072t, 0.097t] | [3.16, 3.16] | [0.100t, 0.100t] |

**Table 5: The intervals for $\widetilde{\vartheta_1}$, $\tilde{\lambda}(t)$, $\widetilde{\vartheta_2}$, $\tilde{\lambda}_C(t)$, $\widetilde{\vartheta_3}$, $\tilde{\mu}(t)$, $\widetilde{\vartheta_4}$, $\tilde{\mu}_C(t)$ corresponding to $\propto$-*cut* = 0, 0.1, 0.2**

| $\beta$ –cut | $[\tilde{\vartheta}_1^L, \tilde{\vartheta}_1^U]$ | $[\tilde{\lambda}^L(t), \tilde{\lambda}^U(t)]$ | $[\tilde{\vartheta}_2^L, \tilde{\vartheta}_2^U]$ | $[\tilde{\lambda}_C^L(t), \tilde{\lambda}_C^U(t)]$ |
|---|---|---|---|---|
| 0 | [2.48, 3.29] | [0.092t, 0.163t] | [2.85, 3.79] | [0.070t, 0.123t] |
| 0.1 | [2.56, 3.21] | [0.097t, 0.152t] | [2.92, 3.72] | [0.072t, 0.118t] |
| 0.2 | [2.64, 3.13] | [0.102t, 0.143t] | [2.98, 3.66] | [0.075t, 0.112t] |
| 0.3 | [2.72, 3.05] | [0.108t, 0.135t] | [3.05, 3.59] | [0.078t, 0.107t] |
| 0.4 | [2.81, 2.97] | [0.113t, 0.127t] | [3.12, 3.52] | [0.081t, 0.103t] |
| 0.5 | [2.89, 2.89] | [0.120t, 0.120t] | [3.19, 3.45] | [0.084t, 0.099t] |
| **$\beta$ –cut** | $[\tilde{\vartheta}_3^L, \tilde{\vartheta}_3^U]$ | $[\tilde{\mu}^L(t), \tilde{\mu}^U(t)]$ | $[\tilde{\vartheta}_4^L, \tilde{\vartheta}_4^U]$ | $[\tilde{\mu}_C^L(t), \tilde{\mu}_C^U(t)]$ |
| 0 | [2.93, 4.29] | [0.054t, 0.116t] | [2.67, 3.77] | [0.071t, 0.140t]] |
| 0.1 | [3.04, 4.17] | [0.057t, 0.108t] | [2.74, 3.70] | [0.073t, 0.133t]] |
| 0.2 | [3.16, 4.06] | [0.060t, 0.100t] | [2.81, 3.63] | [0.076t, 0.127t] |
| 0.3 | [3.27, 3.95] | [0.064t, 0.093t] | [2.88, 3.56] | [0.079t, 0.121t] |
| 0.4 | [3.38, 3.83] | [0.068t, 0.087t] | [2.95, 3.49] | [0.082t, 0.115t] |
| 0.5 | [3.50, 3.72] | [0.072t, 0.082t] | [3.01, 3.42] | [0.085t, 0.110t] |

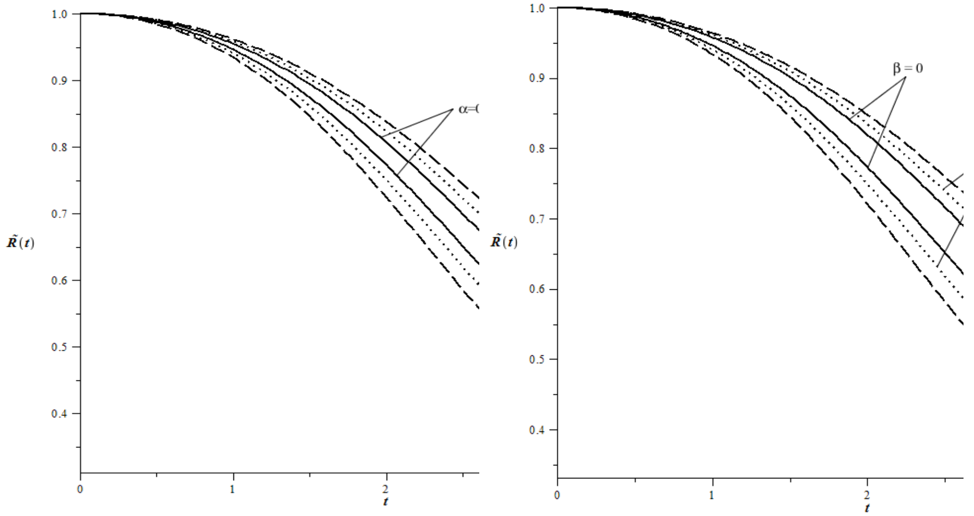
**Table 6: The intervals for $\tilde{\vartheta}_1, \tilde{\lambda}(t)$, $\tilde{\vartheta}_2, \tilde{\lambda}_C(t)$, $\tilde{\vartheta}_3, \tilde{\mu}(t)$, $\tilde{\vartheta}_4$, $\tilde{\mu}_C(t)$ corresponding to $\beta$-*cut* = 0, 0.1, 0.2, 0.3, 0.4, 0.5**

We substitute in our model equations (4) and (8) by $\tilde{\lambda}(t)$, $\tilde{\lambda}_C(t)$, $\tilde{\mu}(t)$, $\tilde{\mu}_C(t)$ and then use Maple program to collect numerical solutions of these equations at arbitrary values $(\propto, \beta) - cuts$ ; $\propto -cut \in [0, 0.2]$ and $\beta - cut \in [0, 0.5]$. The fuzzy system availability and reliability functions $\tilde{A}(t)$ and $\tilde{R}(t)$, as shown in **Figure 8** and **Figure 9**, can represented versus the time at $\propto -cut = 0, 0.2$ and at $\beta - cut = 0, 0.2, 0.4$. At any instant value of time, the system availability and reliability are not crisp values but they are represented by vague sets as shown in **Figure 10**.



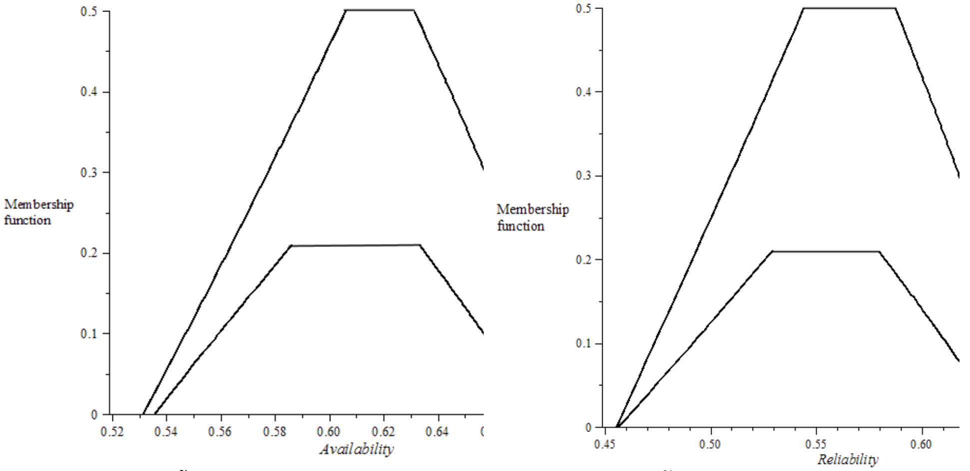

**(a)** $\tilde{A}(t)$ at ∝-*cut* = 0, 0.1, 0.2 **(b)** $\tilde{A}(t)$ at $\beta$-*cut* = 0, 0.2, 0.4

**Figure 8: The system fuzzy availability function versus the time $\tilde{A}(t)$ (Case 2)**




**(a)** $\tilde{R}(t)$ at ∝-*cut* = 0, 0.1, 0.2 **(b)** $\tilde{R}(t)$ at *β-cut* = 0, 0.2, 0.4

**Figure 9: The system fuzzy reliability function versus the time $\widetilde{\boldsymbol{R}}(\boldsymbol{t})$ (Case 2)**




**(a)** $\tilde{A}(t)$ at $t = 3$ **(b)** $\tilde{R}(t)$ at $t = 3$

**Figure 10: The system fuzzy availability and reliability at $\boldsymbol{t = 3}$ (Case 2)**

## 6. Conclusion

In this paper, Markov model was used to analyze a repairable parallel system with three similar components in the presence of common cause failure and we introduced the procedures to determine the fuzzy availability and the fuzzy reliability of the system when the time to failure and the time to repair of each component followed exponential or Rayleigh distribution with unknown parameters. Due to lack of data, these parameters were represented by triangular vague sets estimated by using statistical data taken from random samples. Finally, illustrative examples were presented to illustrate the performance of our model. This model provides more effective, realistic and flexible measures and we can apply it to wide variety of industrial problems.

As an extension to this work, we can develop other complex repairable systems as parallel-series systems, series-parallel systems, k-out of n systems or standby systems which could be studied with the vague set concepts.

## References


1. Høyland, Arnljot, and Marvin Rausand (2009). System Reliability Theory: Models and Statistical Methods, Vol. 420. Wiley. com.
2. Stapelberg, Rudolph Frederick (2009). Handbook of Reliability, Availability, Maintainability and Safety in Engineering Design, Springer, ISBN: 1848001746.
3. Carrasco, Juan A (2003). Markovian Dependability/Performability Modeling of Fault-Tolerant Systems- Handbook of Reliability Engineering, Springer London, p. 613-642.
4. Singh, C. J. and Jain, M. (2000). Reliability of repairable multicomponent redundant system, International Journal of Engineering, Vol. 13(4), p. 17-22.
5. Sharifi, M., M. Ganjian, and P. Shafiee (2006). Reliability of a System with n Parallel and Not Identical Elements with Constant Failure Rates, Computational Intelligence for Modelling, Control and Automation, 2006 and International Conference on Intelligent Agents, Web Technologies and Internet Commerce, International Conference on IEEE.
6. Zhang, Tieling, and Horigome, Michio (2001). Availability and reliability of system with dependent components and time-varying failure and repair rates, IEEE Transactions on Reliability, 50(2), p. 151-158.
7. Hassett, Thomas F., Duane L. Dietrich and Ferenc, Szidarovszky (1995). Time-varying failure rates in the availability and reliability analysis of repairable systems, IEEE Transactions on Reliability, 44(1), p. 155-160.
8. Tang, Shengdao and Fengquan, Wang (2005). Reliability analysis for a repairable parallel system with time-varying failure rates, Applied Mathematics-A Journal of Chinese Universities, 20(1), p. 85-90.
9. Jain, M. and Singh, C.J. (2000). Reliability of repairable multi-component redundant system, Int. J. Eng., Vol. 3, p. 107-114.
10. Srinivasan, S. K., and Subramanian, R. (2006). Reliability analysis of a three unit warm standby redundant system with repair, Annals of Operations Research, 143(1), p. 227-235.
11. Zadeh, L. A. (1965). Fuzzy sets, Information and control, 8(3), p. 338-353.
12. Buckley, James J., and Eslami, Esfandiar (2008). Fuzzy Markov chains: uncertain probabilities, Mathware & soft computing, 9(1), p. 33-41.
13. Sharifi, M., Ganjian, M. and Ghajar, H.R. (2005). Expansion of Reliability Models based on Markov Chain with Consideration of Fuzzy Failure Rates: System with two Parallel and Identical Elements with Constant Failure Rates, Computational Intelligence for Modelling, Control and Automation, 2005 and International Conference on Intelligent Agents, Web Technologies and Internet Commerce, International Conference on. Vol. 2. IEEE.

14. Hashemzadeh, Gholam Reza (2010). Set a Productive Mathematical Reliability Model for a System with Fuzzy Constant Failure Rates, Int. J. Contemp. Math. Sciences, 5(32), p. 1583-1590.
15. Gau, W. L. and Buehrer, D. J. (1993). Interval valued vague sets, IEEE Transactions on Systems Maa and Cybernetics, 23(2), p. 610- 614.
16. Chen, S. M. (1995). Arithmetic operations between interval valued vague sets, Proc. Int. Joint Conference CFSA/IFIS/SOFT on Fuzzy Theory and Applications, p. 206-211.
17. Chen, S. M. (2003). Analyzing fuzzy system reliability using interval valued vague set theory, International Journal of Applied Science and Engineering, l(1), p. 82-88.
18. Chang, J. R., Chang K. H., Liao, S. H. and Cheng, C. H. (2006). The reliability of general vague fault-tree analysis on weapon systems fault diagnosis, Soft Computing, 10(7), p. 531-542.
19. Kumar, Amit, Yadav, S.P. and Kumar, S. (2008). Fuzzy system reliability using different types of vague sets, International Journal of Applied Science and Engineering, 6(1), p. 71-83.
20. Chang, J. R., Chang K. H., Liao, S. H. and Cheng, C. H. (2006). The reliability of Fuzzy System Reliability Using Different Types of Vague Sets, Int. J. Appl. Sci. Eng., 2008. 6, 1 83 general vague fault-tree analysis of weapon systems fault diagnosis, Soft Computing, 10, p. 531-542.
21. Kumar, A., Yadav, S. P., and Kumar, S. (2006). A new approach for analyzing the fuzzy reliability of networks having three-state devices, Proc. Second Int Conf. on Reliability and Safety Engineering, p. 14-20.
22. Romeu, J. L. (2012). Fuzzy estimation of the exponential life time, Rac start,10(7).
23. Kundu, D. and Raqab, M. Z. (2005). Generalized Rayleigh distribution: different methods of estimations, Computational Statistics and Data Analysis, 49(1), p. 187-200.