STOCHASTIC ANALYSIS OF A DISCRETE PARAMETRIC MARKOV CHAIN MODEL OF A COMPLEX SYSTEM CONSISTING OF TWO SUB-SYSTEMS

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Abstract

The paper deals with a stochastic model of a complex system composed of two subsystems- A and B. The subsystem-A consists of two identical units in series configuration whereas subsystem-B consists of only a single unit and is capable to keep the system operative when subsystem-A is failed. System failure occurs only when at least one unit of subsystem-A together with subsystem-B is failed. A single repairman is always available with the system to repair a failed unit of subsystem-A and B. Priority in operation as well as in repair is given to the unit of subsystem-A.

1. Introduction

Due to the ever increasing demands of society, the systems are becoming complex day by day with modern technology. Several authors including [1,2,4,7] have analyzed complex system models under different model formulation. Gupta and Ram kishan [3] have analyzed a system model which consists of two subsystems A and B connected in series network. Subsystem-A consists of two identical units. Initially one is online and other is kept into cold standby. Subsystem-B consists of only one unit. The system fails completely due to the failure of any of the subsystem-A or subsystem-B. In analyzing the system models by the above authors, the continuous distributions of all the random variables have been considered. Very few authors namely Gupta and Varshney [5,6] have obtained the reliability characteristics of redundant system models under discrete parametric Markov Chain i.e. the random variables denoting failure and repair times follow discrete distributions.

Keeping the above fact in view, the purpose of the present paper is to analyze a complex system model which consists of two subsystems A and B. Initially subsystem-A is operative and subsystem-B is kept into cold standby. Subsystem-A is further composed of two identical units in series configuration whereas subsystem-B consists of only one unit which is dissimilar from the units of subsystem-A. The main point in analyzing the system model under study is that the random variables denoting time to failure and repair are taken of discrete nature having geometric distributions

with different parameters. The following economic related measures of system effectiveness are obtained by using regenerative point technique:

i) Transition probabilities and mean sojourn times in various states.
ii) Reliability and mean time to system failure.
iii) Point-wise and steady-state availabilities of the system as well as expected up time of the system due to subsystem A and B respectively during time (0, t-1).
iv) Expected busy period of repairman due to subsystem A and B respectively during time (0, t-1).
v) Net expected profit incurred by the system during a finite interval of time and at steady-state.

2. Model Description and Assumptions

i.   The system is consisting of two subsystems A and B. Initially subsystem-A is operative and subsystem-B is kept into cold standby.

ii. The subsystem-A has two identical units in series network whereas the subsystem-B has only one unit alone.

iii. Both the units of subsystem-A are identical whereas the unit of subsystem-B is dissimilar from the unit of subsystem-A.

iv. The system failure occurs when at least one unit of subsystem-A and subsystem-B are failed.

v.   The priority in operation as well as in repair is given to the unit of subsystem-A over the subsystem-B.

vi. A single repairman is always available with the system to repair a filed unit of both the subsystems.

vii. The random variables denoting the failure times and repair times of the units of subsystem-A and subsystem-B are independent of discrete nature and follow geometric distributions with different parameters.

viii. After repair a unit works as good as new.

3. Notations and States of the System
a) Notations

$p_i q_i^t$: p.m.f. of failure time of operating unit of subsystem-A and B respectively for i=a, b; $\left(p_i + q_i = 1 \right)$.

$r_i s_i^t$: p.m.f. of repair time of the unit of subsystem- A and B respectively for i=a, b; $\left(r_i + s_i = 1 \right)$.

$q_{ij}(\bullet), Q_{ij}(\bullet)$: p.m.f. and c.d.f. of one step or direct transition time from state S_i to S_j.

p_{ij} : Steady state transition probability from state S_i to S_j.

$$p_{ij} = Q_{ij}(\infty)$$

$Z_i(t)$: Probability that the system sojourns in state S_i at epochs 0, 1, 2,……, up to (t-1).

ψ_i **:**   Mean sojourn time in state S_i.

$*, h$ **:**   Symbol and dummy variable used in geometric transform

e. g. $GT\left[q_{ij}(t)\right] = q_{ij}^*(h) = \sum_{t=0}^{\infty} h^t q_{ij}(t)$

b) Symbols for the states of the systems

A_O / A_g **:**   Unit of subsystem-A in normal (N) mode and operative/good.

B_O / B_S **:**   Unit of subsystem-B in normal (N) mode and operative/standby.

A_r / A_{wr} **:**   Unit of subsystem-A in total failure (F) mode and under repair/waiting for repair.

B_r / B_{wr} **:**   Unit of subsystem-B in total failure (F) mode and under repair/waiting for repair.

With the help of above symbols the possible states of the system along with failure and repair rates are shown in the transition diagram (fig.1):

TRANSITION DIAGRAM
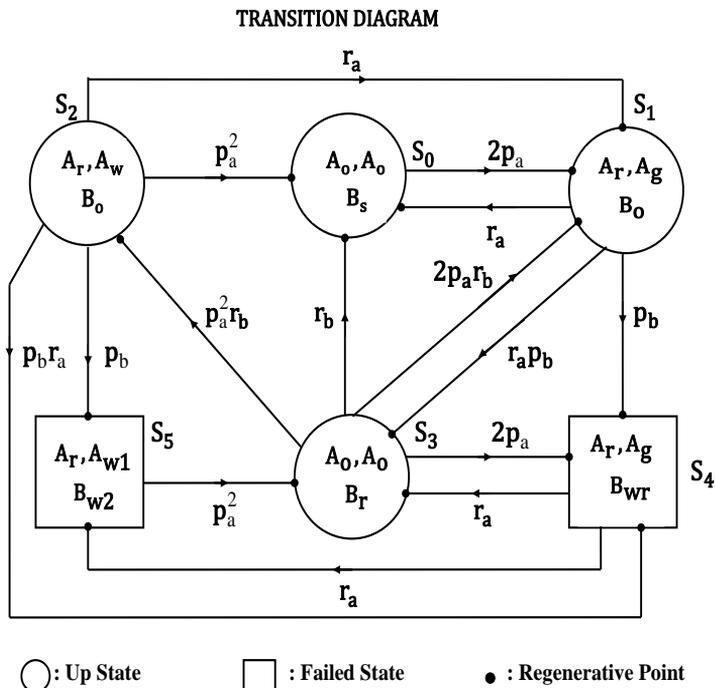
Fig.1

4. Transition Probabilities

Let $Q_{ij}(t)$ be the probability that the system transits from state S_i to S_j during time interval (0, t) i.e., if T_{ij} is the transition time from state S_i to S_j then

$$Q_{ij}(t) = P\left[T_{ij} \leq t\right]$$

By using simple probabilistic arguments, we have

$$Q_{01}(t) = \frac{2p_a q_a}{\left(1-q_a^2\right)}\left\{1-\left(q_a^2\right)^{t+1}\right\},$$

$$Q_{02}(t) = \frac{p_a^2}{\left(1-q_a^2\right)}\left\{1-\left(q_a^2\right)^{t+1}\right\}$$

$$Q_{10}(t) = Q_{21}(t) = \frac{r_a q_b}{\left(1-s_a q_b\right)}\left\{1-\left(s_a q_b\right)^{t+1}\right\},$$

$$Q_{13}(t) = Q_{24}(t) = \frac{r_a p_b}{\left(1-s_a q_b\right)}\left\{1-\left(s_a q_b\right)^{t+1}\right\}$$

$$Q_{14}(t) = Q_{25}(t) = \frac{s_a p_b}{\left(1-s_a q_b\right)}\left\{1-\left(s_a q_b\right)^{t+1}\right\},$$

$$Q_{30}(t) = \frac{q_a^2 r_b}{\left(1-q_a^2 s_b\right)}\left\{1-\left(q_a^2 s_b\right)^{t+1}\right\}$$

$$Q_{31}(t) = \frac{2p_a q_a r_b}{\left(1-q_a^2 s_b\right)}\left\{1-\left(q_a^2 s_b\right)^{t+1}\right\},$$

$$Q_{32}(t) = \frac{p_a^2 r_b}{\left(1-q_a^2 s_b\right)}\left\{1-\left(q_a^2 s_b\right)^{t+1}\right\}$$

$$Q_{34}(t) = \frac{2p_a q_a s_b}{\left(1-q_a^2 s_b\right)}\left\{1-\left(q_a^2 s_b\right)^{t+1}\right\},$$

$$Q_{35}(t) = \frac{p_a^2 s_b}{\left(1-q_a^2 s_b\right)}\left\{1-\left(q_a^2 s_b\right)^{t+1}\right\}$$

$$Q_{43}(t) = Q_{54}(t) = 1-s_a^{t+1}$$

$$(1-11)$$

The steady state transition probabilities from state S_i to S_j can be obtained from $(1-11)$ by taking $t \rightarrow \infty$, as follows:

$$p_{01} = \frac{2p_a q_a}{\left(1 - q_a^2\right)}, \qquad\qquad p_{02} = \frac{p_a^2}{\left(1 - q_a^2\right)},$$

$$p_{10} = p_{21} = \frac{r_a q_b}{\left(1 - s_a q_b\right)}$$

$$p_{13} = p_{24} = \frac{r_a p_b}{\left(1 - s_a q_b\right)}, \qquad p_{14} = p_{25} = \frac{s_a p_b}{\left(1 - s_a q_b\right)},$$

$$p_{30} = \frac{q_a^2 r_b}{\left(1 - q_a^2 s_b\right)}$$

$$p_{31} = \frac{2p_a q_a r_b}{\left(1 - q_a^2 s_b\right)}, \qquad\qquad p_{32} = \frac{p_a^2 r_b}{\left(1 - q_a^2 s_b\right)},$$

$$p_{34} = \frac{2p_a q_a s_b}{\left(1 - q_a^2 s_b\right)}$$

$$p_{35} = \frac{p_a^2 s_b}{\left(1 - q_a^2 s_b\right)}, \qquad\qquad p_{43} = p_{54} = 1$$

We observe that the following relations hold-

$$p_{01} + p_{02} = 1, \qquad\qquad p_{10} + p_{13} + p_{14} = 1,$$

$$p_{21} + p_{24} + p_{25} = 1$$

$$p_{30} + p_{31} + p_{32} + p_{34} + p_{35} = 1, \qquad p_{43} = p_{54} = 1$$

$$\left(12 - 16\right)$$

5. Mean Sojourn Times

ψ_i be the sojourn time in state S_i (i = 0, 1, 2, 3, 4, 5) then mean sojourn time in state S_i is given by

$$\psi_i = \sum_{t=1}^{\infty} P[T \geq t]$$

In particular,

$$\psi_0 = \frac{q_a^2}{p_a^2}, \qquad\qquad \psi_1 = \psi_2 = \frac{s_a q_b}{\left(1 - s_a q_b\right)} = \psi,$$

$$\psi_3 = \frac{q_a^2 s_b}{\left(1 - q_a^2 s_b\right)}$$

$$\psi_4 = \psi_5 = \frac{s_a}{r_a} = \psi_A$$

$$\left(17 - 20\right)$$

6. Methodology for Developing Equations

In order to obtain various interesting measures of system effectiveness we develop the recurrence relations for reliability, availability, busy period of repairman as follows :

a) Reliability of the system

Here we define $R_i(t)$ as the probability that the system does not fail up to epochs 0, 1, 2,.., (t-1) when it is initially started from up state S_i. To determine it, we regard the failed states S_4 and S_5 as absorbing states. Now, the expressions for $R_i(t)$; i=0, 1, 2, 3; we have the following set of convolution equations.

$$R_0(t) = q_A^{2t} + \sum_{u=0}^{t-1} q_{01}(u)R_1(t-1-u) + \sum_{u=0}^{t-1} q_{02}(u)R_2(t-1-u)$$
$$= Z_0(t) + q_{01}(t-1)\mathbb{C}R_1(t-1) + q_{02}(t-1)\mathbb{C}R_2(t-1)$$

Similarly,

$$R_1(t) = Z(t) + q_{10}(t-1)\mathbb{C}R_0(t-1) + q_{12}(t-1)\mathbb{C}R_2(t-1) + q_{13}(t-1)\mathbb{C}R_3(t-1)$$
$$R_2(t) = Z(t) + q_{21}(t-1)\mathbb{C}R_1(t-1)$$
$$R_3(t) = Z_3(t) + q_{10}(t-1)\mathbb{C}R_0(t-1) + q_{31}(t-1)\mathbb{C}R_1(t-1) + q_{12}(t-1)\mathbb{C}R_2(t-1)$$
$$(21-24)$$

Where,

$$Z(t) = s_a^t q_b^t, \qquad Z_3(t) = q_b^t s_b^t$$

b) Availability of the system-

Let $A_i^a(t)$ and $A_i^b(t)$ be the respective probabilities that the system is up at epoch t-1 due to operation of unit of subsystem A and B, when it initially started from state S_i. Then, by using simple probabilistic arguments as in case of reliability, the following recurrence relations can be easily developed for $A_i^a(t)$ and $A_i^b(t)$; i=0 to 5.

$$A_0^j(t) = (1-\delta)Z_0(t) + q_{01}(t-1)\mathbb{C}A_1^j(t-1) + q_{02}(t-1)\mathbb{C}A_2^j(t-1)$$
$$A_1^j(t) = \delta Z(t) + q_{10}(t-1)\mathbb{C}A_0^j(t-1) + q_{12}(t-1)\mathbb{C}A_2^j(t-1) + q_{13}(t-1)\mathbb{C}A_3^j(t-1)$$

$$A_2^j(t) = \delta Z(t) + q_{21}(t-1)\mathbb{C}A_1^j(t-1) + q_{24}(t-1)\mathbb{C}A_4^j(t-1) + q_{25}(t-1)\mathbb{C}A_5^j(t-1)$$

$$A_3^j(t) = (1-\delta)Z_3 + q_{30}(t-1)©A_0^j(t-1) + q_{31}(t-1)©A_1^j(t-1) + q_{32}(t-1)©A_2^j(t-1)$$

$$+q_{34}(t-1)©A_4^j(t-1) + q_{35}(t-1)©A_5^j(t-1)$$

$$A_4^j(t) = q_{43}(t-1)©A_3^j(t-1)$$

$$A_5^j(t) = q_{54}(t-1)©A_4^j(t-1)$$

$$(25-30)$$

Where,

$\delta = 0$ and 1 respectively for j=a and b. The values of $Z(t)$ and $Z_i(t)$; i=0 and 3 are same as given in section 6(a).

c) Busy period of repairman

Let $B_i^a(t)$ and $B_i^b(t)$ be the respective probabilities that the repairman is busy at epoch (t-1) in the repair of failed unit of subsystem A and B when system initially starts from state S_i. Using simple probabilistic arguments in case of reliability and availability analysis, the relations for $B_i^j(t)$; i=0 to 5 can be easily developed as below.

$$B_0^j(t) = q_{01}(t-1)©B_1^j(t-1) + q_{01}(t-1)©B_1^j(t-1)$$

$$B_1^j(t) = (1-\delta)Z(t) + q_{10}(t-1)©B_0^j(t-1) + q_{13}(t-1)©B_3^j(t-1) + q_{14}(t-1)©B_4^j(t-1)$$

$$B_2^j(t) = (1-\delta)Z + q_{21}(t-1)©B_1^j(t-1) + q_{24}(t-1)©B_4^j(t-1) + q_{25}(t-1)©B_5^j(t-1)$$

$$B_3^j(t) = \delta Z_3(t) + q_{30}(t-1)©B_0^j(t-1) + q_{31}(t-1)©B_1^j(t-1) + q_{32}(t-1)©B_2^j(t-1)$$

$$+q_{34}(t-1)©B_4^j(t-1) + q_{35}(t-1)©B_5^j(t-1)$$

$$B_4^j(t) = Z'(t) + q_{43}(t-1)©B_3^j(t-1)$$

$$B_5^j(t) = Z'(t) + q_{54}(t-1)©B_4^j(t-1)$$

$$(31-36)$$

Where, $\delta = 0$ and 1 respectively for j=a and b. The values of $Z(t)$ and $Z_i(t)$; i=0, 3 are same as given in section 6(a) and $Z'(t) = s_a^t$.

7. Analysis of Reliability and MTSF

Taking geometric transforms of (21-24) and simplifying the resulting set of algebraic equations for $R_0^*(h)$ we get

$$R_0^*(h) = \frac{N_1(h)}{D_1(h)}$$

$$(37)$$

Where,

$$N_1(h) = \left[1 - hq_{13}^*\left(hq_{31}^* + h^2q_{32}^*q_{21}^*\right)\right]Z_0^* + \left[hq_{01}^*\left(1 + h^2q_{13}^*q_{32}^*\right) + hq_{02}^*\left(1 + hq_{21}^* - h^2q_{13}^*q_{31}^*\right)\right]Z^*$$

$$+ hq_{13}^*\left[hq_{31}^* + h^2q_{32}^*q_{21}^*\right]Z_3^*$$

$$D_1(h) = 1 - hq_{13}^*\left(hq_{31}^* + h^2q_{32}^*q_{21}^*\right) - \left(hq_{10}^* + h^2q_{13}^*q_{30}^*\right)\left(hq_{01}^* + h^2q_{02}^*q_{21}^*\right)$$

Collecting the coefficient of h^t from expression (37), we can get the reliability of the system $R_0(t)$. The MTSF is given by-

$$E(T) = \lim_{h \to 1}\sum_{t=1}^{\infty} h^t R(t) = \frac{N_1(1)}{D_1(1)} - 1$$

$$\tag{38}$$

$$N_1(1) = \left[1 - p_{13}\left(p_{32}p_{21} + p_{31}\right)\right]\psi_0 + \left[1 + p_{01}p_{13}p_{32} + p_{02}\left(p_{21} - p_{13}p_{31}\right)\right]\psi$$

$$+ \left[p_{13}\left(p_{01} + p_{02}p_{21}\right)\right]\psi_2$$

$$D_1(1) = 1 - p_{13}\left(p_{31} + p_{32}p_{21}\right) - \left(p_{10} + p_{13}p_{30}\right)\left(p_{01} + p_{02}p_{21}\right)$$

8. Availability Analysis

On taking geometric transforms of (25-30) and simplifying the resulting equations for $A_i^a(t)$ and $A_i^b(t)$ we get

$$A_0^{a*}(h) = \frac{N_2(h)}{D_2(h)} \qquad A_0^{b*}(h) = \frac{N_3(h)}{D_2(h)}$$

and

$$\tag{39-40}$$

Where,

$$N_2(h) = \left[1 - hq_{43}^*\left(hq_{34}^* + h^2q_{35}^*q_{54}^*\right) - h^2q_{43}^*q_{32}^*\left(hq_{24}^* + h^2q_{25}^*q_{54}^*\right) - \left(h^2q_{32}^*q_{21}^* + hq_{31}^*\right)\right.$$

$$\left.\left(hq_{13}^* + h^2q_{14}^*q_{43}^*\right)\right]Z_0^* + \left[hq_{01}^*\left(hq_{13}^* + h^2q_{14}^*q_{43}^*\right) + hq_{02}^*\left\{hq_{43}^*\left(hq_{24}^* + h^2q_{25}^*q_{54}^*\right)\right.\right.$$

$$\left.\left. + hq_{21}^*\left(hq_{13}^* + h^2q_{14}^*q_{43}^*\right)\right\}\right]Z_3^*$$

$$N_3(h) = \left[hq_{01}^*\left\{1 - hq_{43}^*\left(hq_{34}^* + h^2q_{35}^*q_{54}^*\right) - h^2q_{43}^*q_{32}^*\left(hq_{24}^* + h^2q_{25}^*q_{54}^*\right) + hq_{32}^*\left(hq_{13}^*\right.\right.\right.$$

$$\left.\left.+ h^2q_{14}^*q_{43}^*\right)\right\} + hq_{02}^*\left\{\left(1 + hq_{21}^*\right)\left(1 - hq_{43}^*\left(hq_{34}^* + h^2q_{35}^*q_{54}^*\right)\right)\right.$$

$$\left.\left. + hq_{31}^*\left(hq_{43}^*\left(hq_{24}^* + h^2q_{25}^*q_{54}^*\right) - \left(hq_{13}^* + h^2q_{14}^*q_{43}^*\right)\right)\right\}\right]Z^*$$

$$D_2(h) = 1 - hq_{43}^*\left(hq_{34}^* + h^2q_{35}^*q_{54}^*\right) - h^2q_{43}^*q_{32}^*\left(hq_{24}^* + h^2q_{25}^*q_{54}^*\right) - \left(h^2q_{32}^*q_{21}^* + hq_{31}^*\right)\left(hq_{13}^*\right.$$

$$\left.+ h^2q_{14}^*q_{43}^*\right) - hq_{10}^*\left[hq_{01}^*\left\{1 - hq_{43}^*\left(hq_{34}^* + h^2q_{35}^*q_{54}^*\right) - h^2q_{43}^*q_{32}^*\left(hq_{24}^* + h^2q_{25}^*q_{54}^*\right)\right\}\right.$$

$$\left.+ hq_{02}^*\left\{hq_{21}^*\left(1 - hq_{43}^*\left(hq_{34}^* + h^2q_{35}^*q_{54}^*\right)\right) + h^2q_{43}^*q_{31}^*\left(hq_{24}^* + hq_{30}^{*2}q_{25}^*q_{54}^*\right)\right\}\right]$$

$$- hq_{30}^*\left[hq_{01}^*\left(hq_{13}^* + h^2q_{14}^*q_{43}^*\right) + hq_{02}^*\left\{hq_{43}^*\left(hq_{24}^* + h^2q_{25}^*q_{54}^*\right)\right.\right.$$

$$\left.\left.+ hq_{21}^*\left(hq_{13}^* + h^2q_{14}^*q_{43}^*\right)\right\}\right]$$

The steady state availabilities of the system due to the operation of unit of subsystem A and unit B of the system are given by-

$$A_0^a = \lim_{t \to \infty} A_0^a(t) = \lim_{h \to 1} (1-h) \frac{N_2(h)}{D_2(h)}$$

$$A_0^b = \lim_{t \to \infty} A_0^b(t) = \lim_{h \to 1} (1-h) \frac{N_2(h)}{D_2(h)}$$

But $D_2(h)$ at h=1 is zero, therefore by applying L. Hospital rule, we get

$$A_0^a = -\frac{N_2(1)}{D_2'(1)} \qquad \text{and} \qquad A_0^b = -\frac{N_3(1)}{D_2'(1)}$$

(41-42)

Where,

$$N_2(1) = \left[P_{10}(P_{31} + P_{32}P_{21}) + P_{24} \right] \psi_0 + \left[1 - P_{10}(P_{01} + P_{02}P_{21}) \right] \psi_3$$

$$N_3(1) = \left[1 - P_{34} - P_{35} - P_{10}(P_{01}P_{32} - P_{02}P_{31}) + P_{21}(P_{32} + P_{30}P_{02}) \right] \psi$$

and

$$D_2'(1) = -\left[\left\{ P_{10}(P_{31} + P_{32}P_{21}) + P_{24} \right\} \psi_0 + \left\{ 1 - P_{34} - P_{35} - P_{10}(P_{01}P_{32} - P_{02}P_{31}) + P_{21}(P_{32} \right. \right.$$
$$+ P_{30}P_{02}) \right\} \psi + \left\{ 1 - P_{10}(P_{01} + P_{02}P_{21}) \right\} \psi_3 + \left\{ 1 - P_{10}P_{01} - P_{13}P_{32}P_{21} \right.$$
$$- P_{02}P_{21}(P_{10} + P_{13}P_{30}) - P_{13}(P_{31} + P_{30}P_{01}) + (1 - P_{10}P_{01})(P_{35} + P_{32}P_{25})$$
$$\left. \left. + P_{02}(P_{25}(P_{30} + P_{31}P_{10}) - P_{21}P_{10}P_{35}) \right\} \psi_A \right]$$

Now the expected uptime of the system due to the working of subsystem A and B respectively upto epoch (t-1) are given by

$$\mu_{up}^a(t) = \sum_{x=0}^{t-1} A_0^a(x) \quad \text{and} \qquad \mu_{up}^a(t) = \sum_{x=0}^{t-1} A_0^a(x)$$

so that

$$\mu_{up}^{a*}(h) = \frac{A_0^{a*}(h)}{(1-h)} \qquad \text{and} \qquad \mu_{up}^{b*}(h) = \frac{A_0^{b*}(h)}{(1-h)}$$

(43-44)

9. Busy Period Analysis

On taking geometric transforms of (31-36) and simplifying the resulting equations for j=a and b we get

$$B_0^{a*}(h) = \frac{N_4(h)}{D_2(h)} \qquad \text{and} \qquad B_0^{b*}(h) = \frac{N_5(h)}{D_2(h)}$$

(45-46)

Where,

$$N_4(h) = \left[hq_{01}^* \left\{ 1 - hq_{43}^*(hq_{34}^* + h^2 q_{35}^* q_{54}^*) - h^2 q_{43}^* q_{32}^*(hq_{24}^* + h^2 q_{25}^* q_{54}^*) + hq_{32}^*(hq_{13}^* \right. \right.$$
$$+ h^2 q_{14}^* q_{43}^*) \right\} + hq_{02}^* \left\{ (1 + hq_{21}^*)(1 - hq_{43}^*(hq_{34}^* + h^2 q_{35}^* q_{54}^*)) \right.$$
$$\left. \left. + hq_{31}^* \left(hq_{43}^*(hq_{24}^* + h^2 q_{25}^* q_{54}^*) - (hq_{13}^* + h^2 q_{14}^* q_{43}^*) \right) \right\} \right] Z^*$$

$$N_5(h) = \left[hq_{01}^* \left(hq_{13}^* + h^2 q_{14}^* q_{43}^* \right) + hq_{02}^* \left\{ hq_{43}^* \left(hq_{24}^* + h^2 q_{25}^* q_{54}^* \right) + hq_{21}^* \left(hq_{13}^* + h^2 q_{14}^* q_{43}^* \right) \right\} \right] Z_3^*$$

and $D_2(h)$ is same as in availability analysis.

In the long run the respective probabilities that the repairman is busy in the repair of failed unit of subsystem A and B are given by-

$$B_0^a = \lim_{t \to \infty} B_o^a(t) = \lim_{h \to 1} (1-h) \frac{N_4(h)}{D_2(h)}$$

$$B_0^b = \lim_{t \to \infty} B_o^b(t) = \lim_{h \to 1} (1-h) \frac{N_5(h)}{D_2(h)}$$

But $D_2(h)$ at h=1 is zero, therefore by applying L. Hospital rule, we get

$$B_0^a = -\frac{N_4(1)}{D_2'(1)} \text{ and } B_0^b = -\frac{N_5(1)}{D_2'(1)} \qquad (47\text{-}48)$$

Where,

$$N_4(1) = \left[1 - p_{34} - p_{35} - p_{10}(p_{01}p_{32} - p_{02}p_{31}) + p_{21}(p_{32} + p_{30}p_{02}) \right]\psi + \left[1 - p_{10}p_{01} - p_{13}p_{32}p_{21} \right.$$
$$\left. - p_{02}p_{21}(p_{10} + p_{13}p_{30}) - p_{13}(p_{31} + p_{30}p_{01}) + (1 - p_{10}p_{01})(p_{35} + p_{32}p_{25}) \right.$$
$$\left. + p_{02}(p_{25}(p_{30} + p_{31}p_{10}) - p_{21}p_{10}p_{35}) \right]\psi_A$$

$$N_5(1) = \left[1 - p_{10}(p_{01} + p_{02}p_{21}) \right]\psi_3$$

and $D_2'(1)$ is same as in availability analysis.

Now the expected busy period of the repairman in repair of failed unit of subsystem-A and B up to epoch (t-1) are respectively given by-

$$\mu_b^a(t) = \sum_{x=0}^{t-1} B_0^a(x) \qquad \qquad \mu_b^b(t) = \sum_{x=0}^{t-1} B_0^b(x)$$
,

So that,

$$\mu_b^{a*}(h) = \frac{B_0^{a*}(h)}{(1-h)} \qquad \qquad \mu_b^{b*}(h) = \frac{B_0^{b*}(h)}{(1-h)}$$
,
$$\qquad (49\text{-}50)$$

10. Profit Function Analysis

We are now in the position to obtain the net expected profit incurred up to epoch (t-1) by considering the characteristics obtained in earlier section.

Let us consider,

K_0 = revenue per-unit time by the system when it is operative due to unit of subsystem-A.

K_1 = revenue per-unit time by the system when it is operative due to unit of subsystem-B.

K_2 = cost per-unit time when repairman is busy in the repairing failed unit of subsystem-A.

K_3 = cost per-unit time when repairman is busy in the repairing failed unit of subsystem-B.

Then, the net expected profit incurred up to epoch (t-1) given by

$$P(t) = K_0 \mu_{up}^a(t) + K_1 \mu_{up}^b(t) - K_2 \mu_b^a(t) - K_3 \mu_b^b(t) \tag{51}$$

The expected profit per unit time in steady state is given by-

$$P = \lim_{t \to \infty} \frac{P(t)}{t} = \lim_{h \to 1} (1-h)^2 P^*(h)$$

$$= K_0 \lim_{h \to 1} (1-h)^2 \frac{A_0^{a*}(h)}{(1-h)} + K_1 \lim_{h \to 1} (1-h)^2 \frac{A_0^{b*}(h)}{(1-h)} - K_2 \lim_{h \to 1} (1-h)^2 \frac{B_0^{a*}(h)}{(1-h)}$$

$$- K_3 \lim_{h \to 1} (1-h)^2 \frac{B_0^{b*}(h)}{(1-h)}$$

$$= K_0 A_0^a + K_1 A_0^b - K_2 B_0^a - K_3 B_0^b \tag{52}$$

11. Graphical Representation

The curves for MTSF and profit function have been drawn for different values of parameters. Fig. 2 depicts the variations in MTSF with respect to failure rate (p_a) of operative unit of subsystem-A for different values of repair rate (r_a) of failed unit of subsystem-A and failure rate (p_b) of operative unit of subsystem-B when repair rate (r_b) of unit of subsystem-B is kept fixed as $r_b = 0.35$. The smooth curves shows the trends for three different values 0.4, 0.6 and 0.8 of r_a when p_b is taken as 0.17 where as dotted curves shows the trend for same three values of r_a as above when p_b is taken as 0.30. From the curves we observed that MTSF decreases uniformly as the value of p_a and p_b increase and increase with the increase in r_a.

Similarly, Fig. 3 reveals the variation in profit (P) with respect to p_a for varying values of r_a and p_b as in case of MTSF, when the values of other parameters are kept fixed as $r_b = 0.35$, $K_0 = 120$, $K_1 = 80$, $K_2 = 420$ and $K_3 = 300$. From this figure same trends in respect of p_a, r_a and p_a have been observed as in MTSF. Further it is also revealed smooth curves that system is profitable only if p_a is less than 0.221, 0.290 and 0.387 respectively for $r_a = 0.4$, 0.6 and 0.8 for fixed $p_b = 0.17$. From dotted curves that system is profitable only if p_a is less than 0.215, 0.280 and 0.365 respectively for $r_a = 0.4$, 0.6 and 0.8 for fixed $p_b = 0.30$.

12. References

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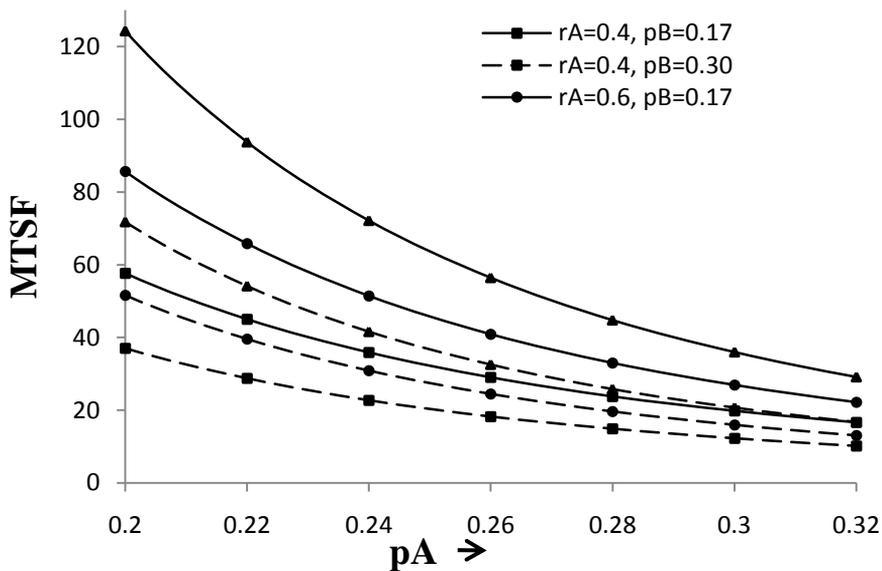
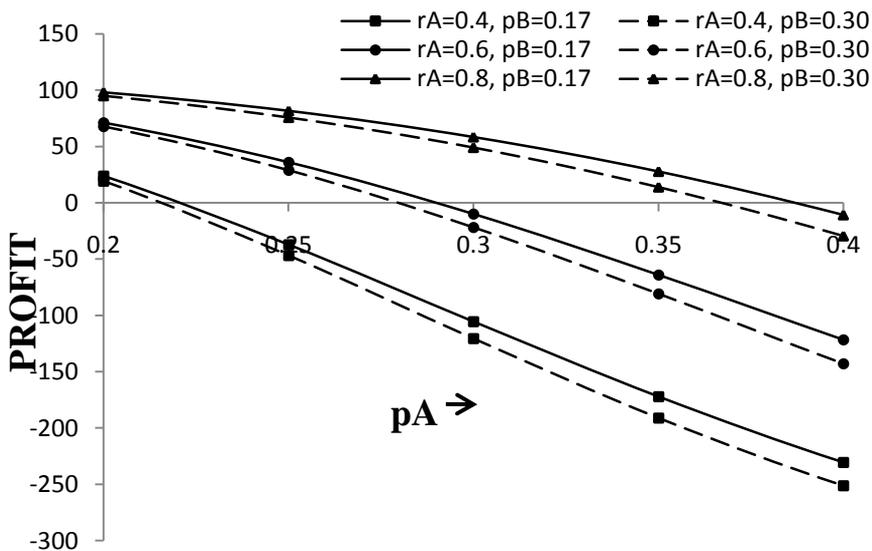**Fig. 2: Behavior of MTSF with respect to** p_A **,** r_A **and** p_B

Fig. 3: Behavior of Profit (P) with respect to p_A **,** r_A **and** p_B