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FORECASTING AVAILABILITY OF A STANDBY SYSTEM USING FUZZY TIME SERIES

Ritu Chandna[1] and Mangey Ram[2]
Department of Mathematics,
Graphic Era University, Dehradun 248002, India
E Mail: [1]rituchandna15@gmail.com; [2]mangeyram@gmail.com

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Abstract

Inthis paper, the fuzzy time series is applied to forecast the availability of a standby system incorporating waiting time to repair. In doingso, a fuzzy time series model is developed using historical data. Fuzzy time series is an effective tool to deal with problems when historical data are linguistic values. A complete procedure isproposed which includes: fuzzifying the historical data, developing a fuzzy time series model, and calculating and interpreting theoutputs. A numerical example is presented to illustrate the utility of the model.

Key Words: Forecasting, Fuzzy Time Series, Availability, Linguistic Values, Historical Data, Fuzzification

1. Introduction

Wang et al. (2012) described the reliability of a wireless sensor network with tree topology and analyzed the infrastructure communication reliability of a wireless sensor network. Azaron et al. (2005) discussed reliability evaluation and optimization of dissimilar components cold standby redundant systems. Gupta and Aggarwal (1984) have considered the reliability mean time to failure (MTTF) of a complex system, with different types of failures and one type of repair. They described the reliability of a parallel redundant complex system with two types of failure under preemptive- repeat repair discipline. Levitin and Amari (2007) analyzed the fault tolerant system with multi fault coverage and suggested a modification of the generalized reliability block diagram method for evaluating reliability indices of systems with multi fault coverage.
Ram et al. (2013) investigated the reliability of a standby system incorporating waiting time to repair. The considered system consists of two units, namely, the main unit and the standby unit. Chandna and Ram (2013) applied the fuzzy reliability evaluation approach to merit the input failure rates of the system. The fuzzy reliability index is evaluated with the help of the linguistic variables assessed by experts in the form of performance rating and importance weights of different parameters and multi-criteria decision making technique to measure the reliability of a system. Ram and Chandna (2013) incorporated the concepts of fuzzy logic [Zadeh (1965)], fuzzy inference system and linguistic variables [Zadeh (1976)] to calculate availability of the system.

The forecasting problem of time series data, consisting of time-dependent sequences of continuous values have been applied to reliability analysis. Yadav et al. (2012) proposed a procedure to forecast times-between-failures of software during its testing phase by employing the fuzzy time series approach, where time-between-failures of software is represented by a fuzzy set having trapezoidal membership function.Biswas(2007) proposed an application of Atanassov's intuitionistic fuzzyset theory in reliability engineering. The proposedmethod reduces to a method of fuzzy computing ofsystem reliability as a special case. Aliev and Kara (2004) used the concept ofγ-cut (intervalof confidence) and time dependent fuzzy set theory to propose ina general procedure to construct the membership function ofthefuzzy reliability, when the failure rate is fuzzy.

Lee et al.(2012) compared the performance of forecasting between classical methods (Box-Jenkins methods Seasonal Auto-Regressive Integrated Moving Average (SARIMA), Holt Winters andtime series regression) and modern methods (fuzzy time series) by using data oftourist arrivals to Bali and Soekarno-Hatta gate in Indonesia as a case study.Chen (2003) presented a method for analyzing fuzzy system reliability using vague set theory is demonstrated, where the reliabilities of the components of a system are represented by vague sets defined in the universe of discourse [0, 1].

Radmehr and Gharneh(2012)dealt with a new forecasting model based on the simulated annealing (SA) heuristic and fuzzy time series (FTS) to forecast the Alabama University's enrollment dataset. Li and Cheng (2007) proposed a deterministic forecasting model to deal with the forecasting problem of fuzzytime series. The proposed model is provoked by the need for controlling the uncertainty which exists in the fuzzyrelationships groups and removing the inconsistency of partitioning intervals in the area of forecasting the University of Alabama's enrollment.

2. Description of system

As per previous work of Ram et al. (2013), here the authors have extended that work under fuzzy time series. In that work, the authors have analyzed a mathematical model of a system having standby unit incorporating waiting time to repair and human error. They found various reliability measures in different situations. In this work, we have analyzed the comprehensive state availability in fuzzy time series environment.

3. Some Concepts of Fuzzy Time Series

Song and Chissom (1993a,1993b) presented the definition of fuzzy time series. General definitions of fuzzy time series are given as follows:
Let U be the universe of discourse, where $U = \{u_1, u_2,\ldots,u_b\}$. A fuzzy set A_i of U is defined as $A_i = f_{Ai}(u_1)/ u_1 + f_{Ai}(u_2)/ u_2 + \ldots + f_{Ai}(u_b)/u_b$, where f_{Ai} is the membership function of the fuzzy set A_i; f_{Ai}: $U\rightarrow[0,1]$. $u_a$is a generic element of fuzzy set A_i; $f_{Ai}(u_a)$ is the degree of belongingness of u_a to A_i; $f_{Ai}(u_a)\in[0,1]$ and $1\leq a\leq b$.

Definition 1: Fuzzy time series. Let Y(t) (t=…,0,1,2,…), a subset of real numbers R, be the universe of discourse by which fuzzy sets f_j(t) are defined. If F(t) is a collection of f_1(t), f_2(t), … then F(t) is called a fuzzy time series defined on Y(t).

Definition 2: If there exists a fuzzy relationship R(t-1, t), such that F(t)= F(t-1) $\circ$ R(t-1, t), where $\circ$ is an arithmetic operator, then F(t) is said to be caused by F(t-1). The relationship between$F$(t) and F(t-1) can be denoted by F(t-1)$\rightarrow$ F(t).

Definition 3: Suppose F(t) is calculated by F(t-1) only, and F(t)= F(t-1) ° R(t-1, t). For any t, if R(t-1, t) is independent of t, then F(t) is considered as a time-invariant fuzzy time series, otherwise F(t) is time-variant.

Definition 4: Suppose F(t-1)= A_i and F(t)= A_j, a fuzzy logical relationship can be defined as

$$A_i \rightarrow A_j$$

whereA_i and A_j are called the left-hand side (LHS) and right-hand side (RHS) of the fuzzy logical relationship, respectively.

Definition 5:Fuzzy Relationship Group(FLRG). Relationships with the same fuzzy set on the left hand side can be further grouped into a relationship group. Relationship groups are also referred to as fuzzy logical relationship groups (FLRG's). Suppose there are relationships such that

$$A_i \rightarrow A_{j1}$$
$$A_i \rightarrow A_{j2}$$
$$\ldots$$
$$A_i \rightarrow A_{jn},$$

then they can be grouped into a relationship group as follows:$A_i \rightarrow A_{j1}, A_{j2}, \ldots, A_{jn}$

There are six main steps in FTS:
Step 1: Define and partition the universe of discourse.
Step 2: Define fuzzy sets for the observations.
Step 3: Partition the intervals.
Step 4: Fuzzify the observations.
Step 5: Establish the fuzzy relationship (FLRs) and forecast.
Step 6: Defuzzify the forecasting results.

Step 1:*Define the universe of discourse and partition it into equally lengthy intervals*

The universe of discourse U is defined as [$D_{min} - D_1$, $D_{max+}D_2$] where D_{min} and D_{max} are minimum and maximum availability of the system in the comprehensive staterespectively. From Table 1, we get D_{min}=0.38372 and D_{max}=1.00000. The variables D_1 and D_2 are just two positive numbers, properly chosen by the user. If we let D_1=0.00372 and D_2 =0.22000 we get U =[0.38000, 1.22000]. Chen used seven intervals which are the same number used in most cases observed in the literature. Dividing U into seven evenly lengthy intervals u_1, u_2, u_3, u_4, u_5, $u_6$and u_7, we get u_1=[0.38000-0.50000], u_2=[0.50000-0.62000], u_3=[0.62000-0.74000], u_6=[0.98000-1.10000] and u_7=[1.10000-1.22000].

Time (t)	Availability $P_{up}(t)$
	System in Comprehensive state
0	1.00000
1	0.99663
2	0.98348
3	0.95586
4	0.91695
5	0.87031
6	0.81892
7	0.76514

8	0.71074
9	0.65702
10	0.60491
11	0.55503
12	0.50779
13	0.46344
14	0.42207
15	0.38372

Table 1. Availability of the system with respect to time

Step 2:*Define fuzzy sets on the universe of discourse*

Assume A_1, A_2,……..,A_k to be fuzzy sets which are linguistic values of the linguistic variable 'availability'. Then the fuzzy sets A_1, A_2,……..,A_k are defined on the universe of discourse as

$A_1 = a_{11}/u_1 + a_{12}/u_2 + \ldots + a_{1m}/u_m$,
$A_2 = a_{21}/u_1 + a_{22}/u_2 + \ldots + a_{2m}/u_m$,
.
.
.
$A_k = a_{k1}/u_1 + a_{k2}/u_2 + \ldots + a_{km}/u_m$,

where$a_{ij} \in [0,1]$, $1 \leq i \leq k$, and $1 \leq j \leq m$. The variables a_{ij} represents the membership degree of the crisp interval u_j in the fuzzy set A_i. Linguistic values should be assigned to each fuzzy set before defining fuzzy sets on U. Chen uses the linguistic values A_1= (not many), A_2 =(not too many), A_3 = (many), A_4 = (many many), A_5= (very many), A_6= (too many) and A_7=(too many many).

Fuzzy sets can be defined on the universe of discourse as follows:

$A_1 = 1/u_1 + 0.5/u_2 + 0/u_3 + 0/u_4 + 0/u_5 + 0/u_6 + 0/u_7$
$A_2 = 0.5/u_1 + 1/u_2 + 0.5/u_3 + 0/u_4 + 0/u_5 + 0/u_6 + 0/u_7$
$A_3 = 0/u_1 + 0.5/u_2 + 1/u_3 + 0.5/u_4 + 0/u_5 + 0/u_6 + 0/u_7$
$A_4 = 0/u_1 + 0/u_2 + 0.5/u_3 + 1/u_4 + 0.5/u_5 + 0/u_6 + 0/u_7$
$A_5 = 0/u_1 + 0/u_2 + 0/u_3 + 0.5/u_4 + 1/u_5 + 0.5/u_6 + 0/u_7$
$A_6 = 0/u_1 + 0/u_2 + 0/u_3 + 0/u_4 + 0.5/u_5 + 1/u_6 + 0.5/u_{7t}$
$A_7 = 0/u_1 + 0/u_2 + 0/u_3 + 0/u_4 + 0/u_5 + 0.5/u_6 + 1/u_7$

Step 3:*Fuzzify historical data*

In this context, fuzzification is the process of identifying associations between the historical values in the datasetand the fuzzy sets defined in the previous step. Each historical value is fuzzified according to its highest degree of membership. If the highest degree of belongingness of a certain historical time variable, say $F(t\text{-}1)$, occur at fuzzy set A_k, then $F(t\text{-}1)$ is fuzzified as A_k. To exemplify this, let us fuzzify time t =0. According to table 1, the availability for time t=0 is 1.00000 which lies within the boundaries of the interval u_6. Since the highest membership degree of $u_6$occurs at A_6, the historical time variable $F(0)$ is fuzzified as A_6. Actual availability for time t =3 is 0.95586 which lies within the boundaries of interval u_5. Hence $F(3)$ is fuzzified as A_5. A complete overview of the fuzzified availabilities is shown in Table 2.

Time (t)	Availability	Interval	Fuzzified Availability
0	1.00000	[0.98000-1.10000]	A_6
1	0.99663	[0.98000-1.10000]	A_6
2	0.98348	[0.98000-1.10000]	A_6
3	0.95586	[0.86000-0.98000]	A_5
4	0.91695	[0.86000-0.98000]	A_5
5	0.87031	[0.86000-0.98000]	A_5
6	0.81892	[0.74000-0.86000]	A_4
7	0.76514	[0.74000-0.86000]	A_4
8	0.71074	[0.62000-0.74000]	A_3
9	0.65702	[0.62000-0.74000]	A_3
10	0.60491	[0.50000-0.62000]	A_2
11	0.55503	[0.50000-0.62000]	A_2
12	0.50779	[0.50000-0.62000]	A_2
13	0.46344	[0.38000-0.50000]	A_1
14	0.42207	[0.38000-0.50000]	A_1
15	0.38372	[0.38000-0.50000]	A_1

Table 2.Fuzzified historical availabilities

Step 4:*Identify fuzzy relationships*

Relationships are identified from the fuzzified historical data. If the time series variableF(t-1) is fuzzified as A_k and F(t) as A_m, then $A_k$is related to A_m. We denote this relationship as $A_k \rightarrow A_{m,}$, where A_k is the current state of enrollment and A_m is the next state of availability. From table 2, we see that time t=0 and t = 1 both are fuzzified as A_6, which provides the following relationship; $A_6 \rightarrow A_6$. The complete sets of relationships identified from Table 2 are presented in Table 3.

$A_6 \rightarrow A_6$	$A_6 \rightarrow A_5$	$A_5 \rightarrow A_5$	$A_5 \rightarrow A_4$
$A_4 \rightarrow A_4$	$A_4 \rightarrow A_3$	$A_3 \rightarrow A_3$	$A_3 \rightarrow A_2$
$A_2 \rightarrow A_2$	$A_2 \rightarrow A_1$	$A_1 \rightarrow A_1$	

Table 3. Fuzzy set relationships

Step 5:*Establish fuzzy relationship groups (FLRG)*

If the same fuzzy set is related to more than one set, the right hand sides are merged. This process is referred as the establishment of FLRG. For example, from table 3, A_6 is related to itself and to A_5. This provides the following FLRG: $A_6 \rightarrow A_6$, A_5. A complete overview of the relationship groups obtained from Table 3 is shown in Table 4.

Group 1	$A_6 \rightarrow A_6$	$A_6 \rightarrow A_5$
Group 2	$A_5 \rightarrow A_5$	$A_5 \rightarrow A_4$
Group 3	$A_4 \rightarrow A_4 A_4 \rightarrow A_3$	
Group 4	$A_3 \rightarrow A_3 A_3 \rightarrow A_2$	

Group 5	$A_2 \rightarrow A_2 A_2 \rightarrow A_1$
Group 6	$A_1 \rightarrow A_1$

Table 4. FLRG's

Step 6:*Defuzzify the forecasted output*

Assume the fuzzified availability of F(t-1) is A_j, then the forecasted output of the F (t) is determined according to the following principles:

(i) If there exists a one-to-one relationship in the relationship group of A_j, say $A_j \rightarrow A_k$, and the highest degree of belongingness ofA_k occurs at interval u_k, then the forecasted output of F(t) equals the midpoint of u_k.

(ii) If A_j is empty, i.e. $A_j \rightarrow \emptyset$, and the interval where A_j has the highest degree of belongingness is u_j, then the forecasted output equals the midpoint of u_j.

(iii) If there exists a one-to-many relationship in the relationship group of A_j, say $A_j \rightarrow A_1, A_2, \ldots\ldots, A_n$, and the highest degrees of belongingness occurs at set $u_1, u_2, \ldots, u_n$, then the forecasted output is computed as the average of the midpoints $m_1, m_2, \ldots, m_n$ of $u_1, u_2, \ldots, u_n$. This equation can be expressed as; $(m_1 + m_2 + \ldots + m_n)/n$

For example, the availability for time t = 5, is forecasted using the fuzzified availability of time t = 4. According to Table 2, fuzzified availability of the time t = 4 is A_5. From Table 4, it can be seen that A_5 is related to A_5 and A_4. The highest degrees of belongingness of A_5 andA_4 are the sets u_5 and u_4, where u_4=[0.74000-0.86000] and u_5=[0.86000-0.98000]. The midpoints of the intervals u_4 and u_5 are 0.80000 and 0.92000 respectively.Using rule 3, the forecasted availability of time t = 5 is computed as (0.80000 + 0.92000)/2 = 0.86000.

Time	Actual Availability	Forecasted Availability	FLRG	Interval midpoints
0	1.00000		$A_6 \rightarrow A_6, A_5$	0.92000, 1.04000
1	0.99663	0.98000	$A_6 \rightarrow A_6, A_5$	0.92000, 1.04000
2	0.98348	0.98000	$A_6 \rightarrow A_6, A_5$	0.92000, 1.04000
3	0.95586	0.98000	$A_5 \rightarrow A_5, A_4$	0.92000, 0.80000
4	0.91695	0.86000	$A_5 \rightarrow A_5, A_4$	0.92000, 0.80000
5	0.87031	0.86000	$A_5 \rightarrow A_5, A_4$	0.92000, 0.80000
6	0.81892	0.86000	$A_4 \rightarrow A_4, A_3$	0.80000,0.68000
7	0.76514	0.74000	$A_4 \rightarrow A_4, A_3$	0.80000,0.68000
8	0.71074	0.74000	$A_3 \rightarrow A_3, A_2$	0.68000,0.56000
9	0.65702	0.62000	$A_3 \rightarrow A_3, A_2$	0.68000,0.56000
10	0.60491	0.62000	$A_2 \rightarrow A_2, A_1$	0.56000,0.44000
11	0.55503	0.50000	$A_2 \rightarrow A_2, A_1$	0.56000,0.44000
12	0.50779	0.50000	$A_2 \rightarrow A_2, A_1$	0.56000,0.44000
13	0.46344	0.50000	$A_1 \rightarrow A_1$	0.44000
14	0.42207	0.44000	$A_1 \rightarrow A_1$	0.44000
15	0.38372	0.44000	$A_1 \rightarrow A_1$	0.44000

Table 5. Forecasted Availability for the time t= 0 to t= 16

4. Conclusion

In this paper, we have proposed a forecasting model to forecast availability of a system. The proposed model is useful due to the need for controlling the uncertainty which exists in the measurement of availability.Especially two factors highly influence forecast accuracy and are of primary focus to FTS. The first is the selection of interval partitions (i.e. the length and number of intervals). The second is the formulation of fuzzy relationships.

There are also some limitations to this approach:
(1) there is a lack of consistency between forecast rules and the data they represent,
(2) forecast accuracy is sensitive to selected interval partitions,
(3) data becomes underutilized as model’s order increases.

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