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RELIABILITY ANALYSIS OF A COMPLEX REPAIRABLE SYSTEM COMPOSED OF A 2-OUT-OF-3: GSUBSYSTEM AND A SERIES SUBSYSTEM CONNECTED IN PARALLEL

Alka Munjal[1] and S. B. Singh[2]
Department of Mathematics, Statistics and Computer Science
G. B. Pant University of Agriculture and Technology, Pantnagar, INDIA
E Mail: [1]alkamunjal8@gmail.com,[2]drsurajbsingh@yahoo.com

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Abstract

Present study discusses the reliability analysis of a complex system which consists of two repairable subsystems (namely L and M) connected in parallel. Subsystem L is of 2-out-of-3: Gconfiguration which consists of 3 type-A components which are in parallel configuration and subsystem M consists of 5type-B components which are in series configuration. A hot spare of type-A and type-B is connected to the2-out-of-3: G subsystem and the series subsystem respectively.By employing supplementary variable technique, Laplace transforms and Gumbel-Hougaard family of copula various transition state probabilities, reliability, availability, MTTF, cost analysis and sensitivity analysis have been obtained along with the steady state behaviour of the system. At the end some special cases of the system have been taken.

Key Words: System, Reliability, Availability, MTTF, Cost Effectiveness, Sensitivity, *k*-out-of-*m*: G, Gumbel-Hougaard Copula.

1. Introduction

The reliability of a system and its maintenance employs an increasing important issue in modern day systems. As long as man has built things, he has wanted to make them as reliable as possible. In practice, we come across with a number of complex systems where failure of any of the parts results in the reduction of efficiency of whole systems or the complete failure of the system and as a result of it, the reliability of the system reduces. Introducing redundant parts and providing maintenance and repair at the time of need can achieve high degree of reliability. Usually, people use the redundancy design to improve the reliability of the system.In a redundant system, some additional paths are created for the proper functioning of the system. Redundancies can be classified as active, standby and partial. An active redundant system with *n*-units is one which operates with every one unit. A standby redundant system is the one in which one operating unit is followed by spare units called standbys. The redundancy where in two or more redundant units are required to perform function of *k*-out-of-*m* system is called the partial redundancy. *k*-out-of-*m*models are among the most useful models to improve the reliability of electrical and electronic devices/systems.

In the past several studies on reliability analysis of complex system have been done. Yusuf *et al*. [13] analyzed the stochastic modelling of a two unit parallel system under two types of failures. Coit *et al.* [3]have studied the system reliability optimization with*k*-out-of-*n* subsystems and also investigated the reliability analysis of *k*-out-of-*n*: G systems with dependent failures and imperfect coverage.Varma [12]has analyzed the stochastic behaviour of a complex system with standby redundancy.Goel *et al*. [6] have analyzed stochastic behavior of a two unit parallel system with partial and catastrophic failures and preventive maintenance.Bazovsky [1]has discussed reliability theory and practice. Oliveira *et al*. [10]also studied the system by using the supplementary variable technique. Dhillon *et al*. [5]have studied the reliability of an identical unit parallel systemwith common cause failures. Chung [2]has estimated the reliability analysis of a*k*-out-of-*n* redundant system with the presence of chance with multiple critical errors. Zhang [14] dealt with a repairable standby system consisting of (*n*+1) units and a single repair facility, in which unit 1 has preemptive priority both in getting operation and in getting repaired. Nailwal *et al.*[8]have studied performance evaluation and reliability analysis of a complex system with three possibilities in repair with the application of copula.Nailwal *et al*.[9]have applied copula in reliability measures and sensitivity analysis of a complex matrix system including power failure.Goel *et al.*[7]analyzed a 1-out-of-3 warm standby system with two types of spare units: a warm and a cold standby unit and inspection. A lot of literature is available in the field of Markov repairable system, to cite a few,Zheng *et al*.[15] discussed a single-unit Markov repairable system with repair time omission, and Cui *et al*.[4] considered the several indexes including availability for aggregated Markov repairable system with history-dependent up and down states. Ram and Singh [11] have done study on availability, MTTF and cost analysis of complex system under preemptive repeat repair discipline using Gumbel-Hougaard family copula.

In the above mentioned reliability analysis of repairable systems, we have observed that researchers studied the complex system of *k*-out-of-*m*: G (*k*-out-of-*m*: F) with different policies but they have paid no attention to the systems that can have the *k*-out-of-*m*: G (*k*-out-of-*m*: F) system as a subsystem. In the present study we have tried to focus on this issue while modelling a complex repairable system which consists of standby and partial redundancies (*k*-out-of-*m*: G system with spare). In the present study we have considered a parallel system with spares. The considered system composed of two subsystems in which one subsystem L is 2-out-of-3: G and the other M, is in series. The subsystem L consists of 3 type-A components which are in parallel configuration and subsystem M consists of 5 type-B components which are in series configuration. SA and SB denote two different types of spares that can replace only own type components (SA can replace only A, SB can replace only B)in case of their failure. A hot spare or hot standby is used as a failover mechanism to provide reliability and security to the system. The hot spare is active and connected as a part of working system. When a key component fails, the hot spare is switched into operation. Most often hot standby refers to an immediate backup for a critical component, without which the entire system would fail. The switchover may happen manually or automatically. Furthermore, the hot standby component is designed to significantly reduce the time required for a failed system to return to normal operation. In the transition state diagram (see Figure 2) of the system, we denote $A^{x}B^{y}SA^{z}SB^{w}$ by the joint state thatthere x type-A components,y type-B components, $z$type-A spare component and$w$ type-B spare component are functional (x = 2, 3;y = 5;z; w = 0, 1). Each component of the system has two modes- good and failed. Failure rates of component of type-A and type-B are constant.All components of type-A/type-B are repairable and repair rates follow general distribution in all the cases. We have used

Gumbel-Hougaard family of copula to find joint distribution of repairs whenever both the subsystems are being repaired simultaneously with two different repair rates. The repair of the failed component is perfect. After repair each subsystem is as good as new. By the help of Laplace transforms and supplementary variable technique the following reliability characteristics of the system have been analyzed in this model:
(i)Transition state probabilities
(ii)Asymptotic behaviour of system
(iii)Reliability measures such as availability, reliability, mean time to failure,cost effectiveness and sensitivity with respect to different parameter of the system.

At last, some special cases of the complex system are taken to highlight the reliability characteristics of the system. These are as follows:

A. Repairable and non-identical.
B. Repairable and identical.
C. Non-repairable and non-identical.
D. Non-repairable and identical.

The state specification chart of the considered system is given in Table 1 Blockdiagram and transition state diagram of investigated system are shown in Figure 1 and Figure 2 respectively.

2. Assumptions

The following assumptions are associated with the model:

(i) Initially the system is in perfectly good state, i.e. all the components arefunctioning perfectly.
(ii) At t=0 all the components are perfectly well and at t> 0 they start operating.
(iii) The system consists of two subsystems L and M connected in parallel.
(iv) Subsystem L is 2-out-of-3: G system of 3 components of type-A which subsystem M is a series system of 5 components of type-B.
(v) A hot spare of type-A and type-B is connected to the 2-out-of-3: G subsystem and the series subsystem. When a component fails in subsystem, the hot spare is switched into operation.
(vi) Each component is either functional or failed.
(vii) Failure rates of type-A component and type-B component are assumed as constant.
(viii) Each subsystem on complete failure goes for repair.
(ix) The repaired subsystem is as good as new and is immediately reconnected to the system.
(x) Transition from the completely failed state S_{14} to the initial state S_{46} follows two different distributions.
(xi) Joint probability distribution of repair rate from S_{14} to the initial state S_{46} is computed by Gumbel-Hougaard family of copula.
(xii) If both units fail, the system fails completely.

3. State Specification

G = Good state, F = Failed state

States	State of subsystem A	State of subsystem B	State of system
S_{46}	G	G	G
S_{36}	G	G	G
S_{45}	G	G	G
S_{26}	G	G	G
S_{35}	G	G	G
S_{44}	G	F	G
S_{16}	F	G	G
S_{25}	G	G	G
S_{34}	G	F	G
S_{15}	F	G	G
S_{24}	G	F	G
S_{14}	F	F	F

Table 1: State Specification

4. Block and State Transition Diagram

Figure 1 and 2 represent the Block diagram and the state transition diagram of investigated system respectively.

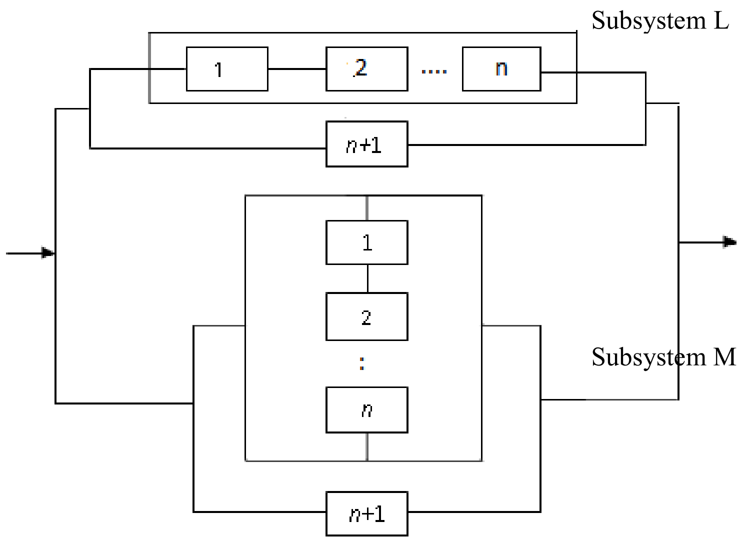

Figure 1: Block diagram of system

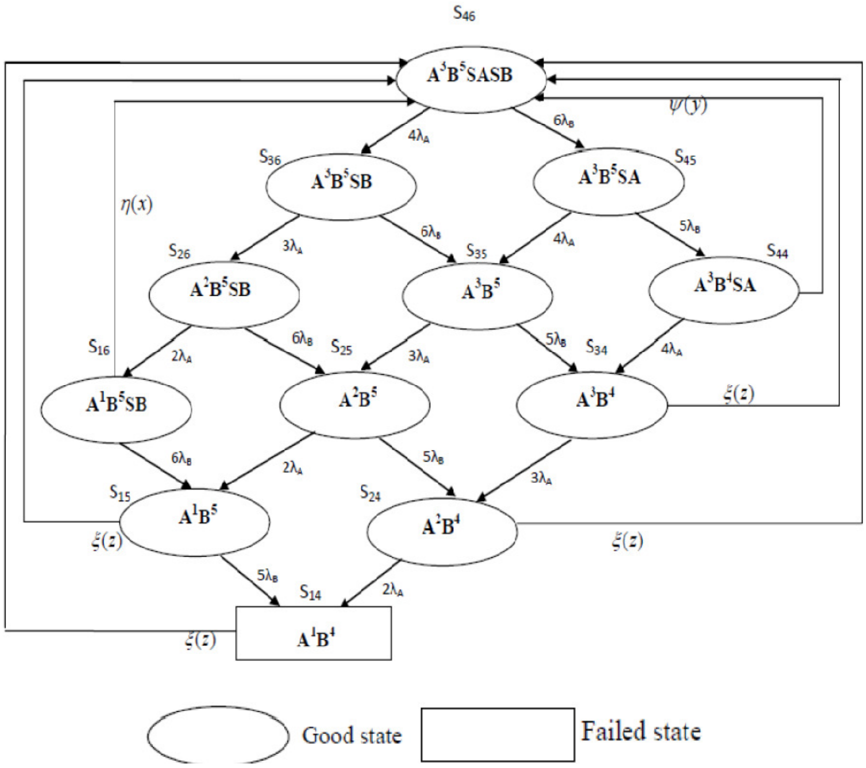

Figure 2. Transition State Diagram

5. Nomenclature

λ_A / λ_B: Failure rate of component of type-A/type-B.

$\eta(x)$: Repair rate of type-A component.

$\psi(y)$: Repair rate of type-B component.

$P_{uv}(t)$: Probability that the system is in S_{uv} state at instant t for u=4 to 1 and v=6 to 4.

$\overline{P_{uv}}(s)$: Laplace transform of $P_{uv}(t)$.

$P_{uv}(j,t)$: The pdf (system is in state S_{uv} and is under repair; elapsed repair time is j, t),where $j= x, y, z$.

$\xi(z)$: Coupled repair rate.

Considering $u_1 = \eta(x)$ and $u_2 = \psi(y)$, the expression for joint probability (failed state S_{14} to good state S_{46}) according to Gumbel-Hougaard family of copula is given by

$$\xi(z) = \exp[(\log(u_1))^{\theta} + (\log(u_2))^{\theta}]^{\frac{1}{\theta}}$$

6. Formation of Mathematical Model

Using the supplementary variable technique, the following set of differential equations associated with the model (as shown in the Figure 2) can be obtained

$$\left[\frac{d}{dt}+4\lambda_A+6\lambda_B\right]P_{46}(t)=\int_0^\infty \psi(y)P_{44}(y,t)dy+\int_0^\infty \eta(x)P_{16}(x,t)dx+\int_0^\infty \xi(z)P_{34}(z,t)dz$$

$$+\int_0^\infty \xi(z)P_{15}(z,t)dz+\int_0^\infty \xi(z)P_{24}(z,t)dz+\int_0^\infty \xi(z)P_{14}(z,t)dz \quad (1)$$

$$\left[\frac{d}{dt}+3\lambda_A+6\lambda_B\right]P_{36}(t)=4\lambda_A P_{46}(t) \quad (2)$$

$$\left[\frac{d}{dt}+4\lambda_A+5\lambda_B\right]P_{45}(t)=6\lambda_B P_{46}(t) \quad (3)$$

$$\left[\frac{d}{dt}+2\lambda_A+6\lambda_B\right]P_{26}(t)=3\lambda_A P_{36}(t) \quad (4)$$

$$\left[\frac{d}{dt}+3\lambda_A+5\lambda_B\right]P_{35}(t)=4\lambda_A P_{45}(t)+4\lambda_B P_{36}(t) \quad (5)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial y}+4\lambda_A+\psi(y)\right]P_{44}(y,t)=0 \quad (6)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+6\lambda_B+\eta(x)\right]P_{16}(x,t)=0 \quad (7)$$

$$\left[\frac{d}{dt}+2\lambda_A+5\lambda_B\right]P_{25}(t)=4\lambda_A P_{45}(t)+6\lambda_B P_{36}(t) \quad (8)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial z}+3\lambda_A+\xi(z)\right]P_{34}(z,t)=0 \quad (9)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial z}+5\lambda_B+\xi(z)\right]P_{15}(z,t)=0 \quad (10)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial z}+2\lambda_A+\xi(z)\right]P_{24}(z,t)=0 \quad (11)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial z}+\xi(z)\right]P_{14}(z,t)=0 \quad (12)$$

Boundary conditions

$$P_{44}(0,t)=5\lambda_B P_{45}(t) \quad (13)$$

$$P_{16}(0,t)=2\lambda_A P_{26}(t) \quad (14)$$

$$P_{34}(0,t)=4\lambda_A P_{44}(t)+5\lambda_B P_{35}(t) \quad (15)$$

$$P_{15}(0,t)=2\lambda_A P_{25}(t)+6\lambda_B P_{16}(t) \quad (16)$$

$$P_{24}(0,t)=3\lambda_A P_{34}(t)+5\lambda_B P_{25}(t) \quad (17)$$

$$P_{14}(0,t)=2\lambda_A P_{24}(t)+5\lambda_B P_{15}(t) \quad (18)$$

Initial condition

$P_{46}(0)=1$ and other probabilities are zero at t=0. (19)

By employing Laplace transforms in the equation (1-18) and using the initial conditions given in (19), we get

$$[s+4\lambda_A+6\lambda_B]\overline{P}_{46}(s)=1+\int_0^\infty \psi(y)\overline{P}_{44}(y,s)dy+\int_0^\infty \eta(x)\overline{P}_{16}(x,s)dx+\int_0^\infty \xi(z)\overline{P}_{34}(z,s)dz$$
$$+\int_0^\infty \xi(z)\overline{P}_{15}(z,s)dz+\int_0^\infty \xi(z)\overline{P}_{24}(z,s)dz+\int_0^\infty \xi(z)\overline{P}_{14}(z,s)dz \quad (20)$$

$$[s+3\lambda_A+6\lambda_B]\overline{P}_{36}(s)=4\lambda_A\overline{P}_{46}(s) \quad (21)$$

$$[s+4\lambda_A+5\lambda_B]\overline{P}_{45}(s)=6\lambda_B\overline{P}_{46}(s) \quad (22)$$

$$[s+2\lambda_A+6\lambda_B]\overline{P}_{26}(s)=3\lambda_A\overline{P}_{36}(s) \quad (23)$$

$$[s+3\lambda_A+5\lambda_B]\overline{P}_{35}(s)=4\lambda_A\overline{P}_{45}(s)+4\lambda_B\overline{P}_{36}(s) \quad (24)$$

$$\left[s+\frac{\partial}{\partial y}+4\lambda_A+\psi(y)\right]\overline{P}_{44}(y,s)=0 \quad (25)$$

$$\left[s+\frac{\partial}{\partial x}+6\lambda_B+\eta(x)\right]\overline{P}_{16}(x,s)=0 \quad (26)$$

$$[s+2\lambda_A+5\lambda_B]\overline{P}_{25}(s)=4\lambda_A\overline{P}_{45}(s)+6\lambda_B\overline{P}_{36}(s) \quad (27)$$

$$\left[s+\frac{\partial}{\partial z}+3\lambda_A+\xi(z)\right]\overline{P}_{34}(z,s)=0 \quad (28)$$

$$\left[s+\frac{\partial}{\partial z}+5\lambda_B+\xi(z)\right]\overline{P}_{15}(z,s)=0 \quad (29)$$

$$\left[s+\frac{\partial}{\partial z}+2\lambda_A+\xi(z)\right]\overline{P}_{24}(z,s)=0 \quad (30)$$

$$\left[s+\frac{\partial}{\partial z}+\xi(z)\right]\overline{P}_{14}(z,s)=0 \quad (31)$$

$$\overline{P}_{44}(0,s)=5\lambda_B\overline{P}_{45}(s) \quad (32)$$

$$\overline{P}_{16}(0,s)=2\lambda_A\overline{P}_{26}(s) \quad (33)$$

$$\overline{P}_{34}(0,s)=4\lambda_A\overline{P}_{44}(s)+5\lambda_B\overline{P}_{35}(s) \quad (34)$$

$$\overline{P}_{15}(0,s)=2\lambda_A\overline{P}_{25}(s)+6\lambda_B\overline{P}_{16}(s) \quad (35)$$

$$\overline{P}_{24}(0,s)=3\lambda_A\overline{P}_{34}(s)+5\lambda_B\overline{P}_{25}(s) \quad (36)$$

$$\overline{P}_{14}(0,s)=2\lambda_A\overline{P}_{24}(s)+5\lambda_B\overline{P}_{15}(s) \quad (37)$$

The transition state probabilities for the system can be obtained as a result of solving the set of equations (20-31) with the help of (32-37)

$$\overline{P}_{46}(s)=\frac{1}{D(s)} \quad (38)$$

$$\overline{P}_{36}(s)=\frac{4\lambda_A}{(s+3\lambda_A+6\lambda_B)D(s)} \quad (39)$$

$$\overline{P}_{45}(s)=\frac{6\lambda_B}{(s+4\lambda_A+5\lambda_B)D(s)} \quad (40)$$

$$\overline{P}_{26}(s) = \frac{3.4\lambda_A^{\ 2}}{(s+3\lambda_A+6\lambda_B)(s+2\lambda_A+6\lambda_B)D(s)} \qquad (41)$$

$$\overline{P}_{35}(s) = \frac{4.6\lambda_A\lambda_B}{A(s)D(s)} \qquad (42)$$

$$\overline{P}_{44}(s) = \frac{5.6\lambda_B^{\ 2}\left[1-\overline{S}_{\psi}(s+4\lambda_A)\right]}{(s+4\lambda_A+5\lambda_B)(s+4\lambda_A)D(s)} \qquad (43)$$

$$\overline{P}_{16}(s) = \frac{2.3.4\lambda_A^{\ 3}\left[1-\overline{S}_{\eta}(s+6\lambda_B)\right]}{(s+3\lambda_A+6\lambda_B)(s+2\lambda_A+6\lambda_B)(s+6\lambda_B)D(s)} \qquad (44)$$

$$\overline{P}_{25}(s) = \frac{3.4.6\lambda_A^{\ 2}\lambda_B}{B(s)D(s)} \qquad (45)$$

$$\overline{P}_{34}(s) = \frac{4.5.6\lambda_A\lambda_B^{\ 2}\left[1-\overline{S}_{\xi}(s+3\lambda_A)\right]}{(s+3\lambda_A)D(s)}\left[\frac{1}{A(s)}+\frac{1-\overline{S}_{\psi}(s+4\lambda_A)}{(s+4\lambda_A)(s+4\lambda_A+5\lambda_B)}\right] \qquad (46)$$

$$\overline{P}_{15}(s) = \frac{2.3.4.6\lambda_A^{\ 3}\lambda_B\left[1-\overline{S}_{\xi}(s+5\lambda_B)\right]}{(s+5\lambda_B)D(s)}\left[\frac{1}{B(s)}+\frac{1-\overline{S}_{\eta}(s+6\lambda_B)}{(s+6\lambda_B)(s+3\lambda_A+6\lambda_B)(s+2\lambda_A+6\lambda_B)}\right] \qquad (47)$$

$$\overline{P}_{24}(s) = \frac{3.4.5.6\lambda_A^{\ 2}\lambda_B^{\ 2}}{C(s)D(s)} \qquad (48)$$

$$\overline{P}_{14}(s) = \frac{2.3.4.5.6\lambda_A^{\ 3}\lambda_B^{\ 2}\left[1-\overline{S}_{\xi}(s)\right]}{sD(s)}\left\{\frac{1-\overline{S}_{\xi}(s+5\lambda_B)}{(s+5\lambda_B)}\left[\frac{1}{B(s)}+\frac{1-\overline{S}_{\eta}(s+6\lambda_B)}{(s+6\lambda_B)(s+3\lambda_A+6\lambda_B)(s+2\lambda_A+6\lambda_B)}\right]+\frac{1}{C(s)}\right\} \qquad (49)$$

where

$$D(s) = (s+4\lambda_A+6\lambda_B) - \left[5.6\lambda_B^{\ 2}\overline{S}_{\psi}(s+4\lambda_A)+2.3.4\lambda_A^{\ 3}\overline{S}_{\eta}(s+5\lambda_B)+4.5.6\lambda_B^{\ 2}\lambda_A\overline{S}_{\xi}(s+3\lambda_A)\right.$$
$$\left.+3.4.5.6\lambda_B^{\ 2}\lambda_A^{\ 2}\overline{S}_{\xi}(s+2\lambda_A)+2.3.4\lambda_A^{\ 3}\overline{S}_{\xi}(s+5\lambda_B)+2.3.4.5.6\lambda_B^{\ 2}\lambda_A^{\ 3}\overline{S}_{\xi}(s)\right] \qquad (50)$$

$$\frac{1}{A(s)} = \frac{1}{(s+3\lambda_A+6\lambda_B)(s+3\lambda_A+5\lambda_B)}+\frac{1}{(s+4\lambda_A+5\lambda_B)(s+3\lambda_A+5\lambda_B)} \qquad (51)$$

$$\frac{1}{B(s)} = \frac{1}{(s+3\lambda_A+6\lambda_B)(s+2\lambda_A+6\lambda_B)(s+2\lambda_A+5\lambda_B)}+\frac{1}{(s+3\lambda_A+6\lambda_B)(s+3\lambda_A+5\lambda_B)(s+2\lambda_A+5\lambda_B)}$$
$$+\frac{1}{(s+4\lambda_A+5\lambda_B)(s+3\lambda_A+5\lambda_B)(s+2\lambda_A+5\lambda_B)} \qquad (52)$$

$$\frac{1}{C(s)} = \frac{\left[1-\overline{S}_{\xi}(s+2\lambda_A)\right]}{(s+2\lambda_A)}\left[\frac{1}{B(s)}+\frac{1-\overline{S}_{\xi}(s+3\lambda_A)}{(s+3\lambda_A)}\left(\frac{1}{A(s)}+\frac{1-\overline{S}_{\psi}(s+4\lambda_A)}{(s+4\lambda_A)(s+4\lambda_A+5\lambda_B)}\right)\right] \qquad (53)$$

Transition state probability that the system is in up and down states are obtained as

$$\overline{P}_{up}(s) = \overline{P}_{46}(s)+\overline{P}_{36}(s)+\overline{P}_{45}(s)+\overline{P}_{26}(s)+\overline{P}_{35}(s)+\overline{P}_{44}(s)+\overline{P}_{16}(s)+\overline{P}_{25}(s)+\overline{P}_{34}(s)+\overline{P}_{15}(s)+\overline{P}_{24}(s) \qquad (54)$$

$$\overline{P}_{up}(s) = \frac{1}{D(s)}\left\{\left[1+\frac{4\lambda_A}{(s+3\lambda_A+6\lambda_B)}+\frac{6\lambda_B}{(s+4\lambda_A+5\lambda_B)}+\frac{3.4\lambda_A^{\ 2}}{(s+3\lambda_A+6\lambda_B)(s+2\lambda_A+6\lambda_B)}+\frac{4.6\lambda_A\lambda_B}{A(s)}\right]\right.$$
$$+\frac{5.6\lambda_B^{\ 2}\left[1-\overline{S}_{\psi}(s+4\lambda_A)\right]}{(s+4\lambda_A+5\lambda_B)(s+4\lambda_A)}+\frac{2.3.4\lambda_A^{\ 3}\left[1-\overline{S}_{\eta}(s+6\lambda_B)\right]}{(s+3\lambda_A+6\lambda_B)(s+2\lambda_A+6\lambda_B)(s+6\lambda_B)D(s)}+\frac{3.4.6\lambda_A^{\ 2}\lambda_B}{B(s)}$$

$$+\frac{4.5.6\lambda_A\lambda_B^{2}\left[1-\bar{S}_{\xi}(s+3\lambda_A)\right]}{(s+3\lambda_A)}\left[\frac{1}{A(s)}+\frac{1-\bar{S}_{\psi}(s+4\lambda_A)}{(s+4\lambda_A)(s+4\lambda_A+5\lambda_B)}\right]+\frac{3.4.5.6\lambda_A^{2}\lambda_B^{2}}{C(s)}$$

$$+\frac{2.3.4.6\lambda_A^{3}\lambda_B\left[1-\bar{S}_{\xi}(s+5\lambda_B)\right]}{(s+5\lambda_B)D(s)}\left[\frac{1}{B(s)}+\frac{1-\bar{S}_{\eta}(s+6\lambda_B)}{(s+6\lambda_B)(s+3\lambda_A+6\lambda_B)(s+2\lambda_A+6\lambda_B)}\right]\Bigg\} \tag{55}$$

$$\bar{P}_{down}(s)=\frac{2.3.4.5.6\lambda_A^{3}\lambda_B^{2}\left[1-\bar{S}_{\xi}(s)\right]}{sD(s)}\left\{\frac{1-\bar{S}_{\xi}(s+5\lambda_B)}{(s+5\lambda_B)}\left[\frac{1}{B(s)}+\frac{1-\bar{S}_{\eta}(s+6\lambda_B)}{(s+6\lambda_B)(s+3\lambda_A+6\lambda_B)(s+2\lambda_A+6\lambda_B)}\right]+\frac{1}{C(s)}\right\} \tag{56}$$

It is worth mentioning that

$$\overline{P_{up}}(s)+\overline{P_{down}}(s)=\frac{1}{s}$$

7. Asymptotic Behaviour of the System

Using Abel's lemma in Laplace transforms,

$$\lim_{s\to 0} s\overline{F}(s)=\lim_{t\to\infty} \mathrm{A(t)}=\mathrm{F} \tag{57}$$

provided the limit on the right hand side exits, the time independent operational probabilities are obtained as follows:

$$P_{46}=\frac{1}{D(0)} \tag{58}$$

$$P_{36}=\frac{4\lambda_A}{(3\lambda_A+6\lambda_B)D(0)} \tag{59}$$

$$P_{45}=\frac{6\lambda_B}{(4\lambda_A+5\lambda_B)D(0)} \tag{60}$$

$$P_{26}=\frac{3.4\lambda_A^{2}}{(3\lambda_A+6\lambda_B)(2\lambda_A+6\lambda_B)D(0)} \tag{61}$$

$$P_{35}=\frac{4.6\lambda_A\lambda_B}{A(0)D(0)} \tag{62}$$

$$P_{44}=\frac{5.6\lambda_B^{2}}{(4\lambda_A+5\lambda_B)(4\lambda_A+\psi(y))D(0)} \tag{63}$$

$$P_{16}=\frac{2.3.4\lambda_A^{3}}{(3\lambda_A+6\lambda_B)(2\lambda_A+6\lambda_B)(6\lambda_B+\eta(x))D(0)} \tag{64}$$

$$P_{25}=\frac{3.4.6\lambda_A^{2}\lambda_B}{B(0)D(0)} \tag{65}$$

$$P_{34}=\frac{4.5.6\lambda_A\lambda_B^{2}}{(3\lambda_A+\xi(z))D(0)}\left[\frac{1}{A(0)}+\frac{1}{(4\lambda_A+\psi(y))(4\lambda_A+5\lambda_B)}\right] \tag{66}$$

$$P_{15}=\frac{2.3.4.6\lambda_A^{3}\lambda_B}{(5\lambda_B+\xi(z))D(0)}\left[\frac{1}{B(0)}+\frac{1}{(6\lambda_B+\eta(x))(3\lambda_A+6\lambda_B)(2\lambda_A+6\lambda_B)}\right] \tag{67}$$

$$P_{24}=\frac{3.4.5.6\lambda_A^{2}\lambda_B^{2}}{C(0)D(0)} \tag{68}$$

$$P_{14} = \frac{2.3.4.5.6\lambda_A^{\,3}\lambda_B^{\,2}}{D(0)\xi(z)}\left\{\frac{1}{(5\lambda_B+\xi(z))}\left[\frac{1}{B(0)}+\frac{1}{(6\lambda_B+\eta(x))(3\lambda_A+6\lambda_B)(2\lambda_A+6\lambda_B)}\right]+\frac{1}{C(0)}\right\} \tag{69}$$

where

$$D(0)=(4\lambda_A+6\lambda_B)-[5.6\lambda_B^{\,2}\overline{S}_\psi(4\lambda_A)+2.3.4\lambda_A^{\,3}\overline{S}_\eta(5\lambda_B)+4.5.6\lambda_B^{\,2}\lambda_A\overline{S}_\xi(3\lambda_A)+3.4.5.6\lambda_B^{\,2}\lambda_A^{\,2}\overline{S}_\xi(2\lambda_A)$$
$$+\,2.3.4\lambda_A^{\,3}\overline{S}_\xi(5\lambda_B)+2.3.4.5.6\lambda_B^{\,2}\lambda_A^{\,3}\overline{S}_\xi(0)] \tag{70}$$

$$\frac{1}{A(0)}=\frac{1}{(3\lambda_A+6\lambda_B)(3\lambda_A+5\lambda_B)}+\frac{1}{(4\lambda_A+5\lambda_B)(3\lambda_A+5\lambda_B)} \tag{71}$$

$$\frac{1}{B(0)}=\frac{1}{(3\lambda_A+6\lambda_B)(2\lambda_A+6\lambda_B)(2\lambda_A+5\lambda_B)}+\frac{1}{(3\lambda_A+6\lambda_B)(3\lambda_A+5\lambda_B)(2\lambda_A+5\lambda_B)}\frac{1}{(4\lambda_A+5\lambda_B)(3\lambda_A+5\lambda_B)(2\lambda_A+5\lambda_B)} \tag{72}$$

$$\frac{1}{C(0)}=\frac{1}{(2\lambda_A+\xi(z))}\left[\frac{1}{B(0)}+\frac{1}{(3\lambda_A+\xi(z))}\left(\frac{1}{A(0)}+\frac{1}{(4\lambda_A+\psi(y))(4\lambda_A+5\lambda_B)}\right)\right] \tag{73}$$

8. Special Cases

When repair follows exponential distribution. In this case the result can be derived by putting

$$\overline{S}_\mu(s)=\frac{\eta(x)}{s+\eta(x)},\ \overline{S}_\psi(s)=\frac{\psi(y)}{s+\psi(y)},$$
$$\overline{S}_\xi(s)=\frac{\exp\left\{(\log\eta(x))^\theta+(\log\psi(x))^\theta\right\}^{\frac{1}{\theta}}}{s+\exp\left\{(\log\eta(x))^\theta+(\log\psi(x))^\theta\right\}^{\frac{1}{\theta}}} \tag{74}$$

A. Repairable and Non Identical

When the considered system is assumed to be repairable and units are non-identical then the transition state probabilities corresponding to present system are given by

$$\overline{P}_{46}(s)=\frac{1}{D(s)} \tag{75}$$

$$\overline{P}_{36}(s)=\frac{4\lambda_A}{(s+3\lambda_A+6\lambda_B)}\overline{P}_{46}(s) \tag{76}$$

$$\overline{P}_{45}(s)=\frac{6\lambda_B}{(s+4\lambda_A+5\lambda_B)}\overline{P}_{46}(s) \tag{77}$$

$$\overline{P}_{26}(s)=\frac{3.4\lambda_A^{\,2}}{(s+3\lambda_A+6\lambda_B)(s+2\lambda_A+6\lambda_B)}\overline{P}_{46}(s) \tag{78}$$

$$\overline{P}_{35}(s)=\frac{4.6\lambda_A\lambda_B}{A(s)}\overline{P}_{46}(s) \tag{79}$$

$$\overline{P}_{44}(s)=\frac{5.6\lambda_B^{\,2}}{(s+4\lambda_A+5\lambda_B)(s+4\lambda_A+\psi(y))}\overline{P}_{46}(s) \tag{80}$$

$$\overline{P}_{16}(s)=\frac{2.3.4\lambda_A^{\,3}}{(s+3\lambda_A+6\lambda_B)(s+2\lambda_A+6\lambda_B)(s+6\lambda_B+\eta(x))}\overline{P}_{46}(s) \tag{81}$$

$$\overline{P}_{25}(s)=\frac{3.4.6\lambda_A^{\,2}\lambda_B}{B(s)}\overline{P}_{46}(s) \tag{82}$$

$$\overline{P}_{34}(s)=\frac{4.5.6\lambda_A\lambda_B^{\,2}\overline{P}_{46}(s)}{(s+3\lambda_A+\xi(z))}\left[\frac{1}{A(s)}+\frac{1}{(s+4\lambda_A+\psi(y))(s+4\lambda_A+5\lambda_B)}\right] \tag{83}$$

$$\overline{P_{15}}(s)=\frac{2.3.4.6\lambda_A{}^3\lambda_B\overline{P_{46}}(s)}{(s+5\lambda_B+\xi(z))}\left[\frac{1}{B(s)}+\frac{1}{(s+6\lambda_B+\eta(x))(s+3\lambda_A+6\lambda_B)(s+2\lambda_A+6\lambda_B)}\right] \quad (84)$$

$$\overline{P_{24}}(s)=\frac{3.4.5.6\lambda_A{}^2\lambda_B{}^2}{C(s)}\overline{P_{46}}(s) \quad (85)$$

$$\overline{P_{14}}(s)=\frac{2.3.4.5.6\lambda_A{}^3\lambda_B{}^2\overline{P_{46}}(s)}{(s+\xi(z))}\left\{\frac{1}{(s+5\lambda_B+\xi(z))}\left[\frac{1}{(s+6\lambda_B+\eta(x))(s+3\lambda_A+6\lambda_B)(s+2\lambda_A+6\lambda_B)}\right.\right.$$

$$\left.\left.+\frac{1}{B(s)}\right]+\frac{1}{C(s)}\right\} \quad (86)$$

B. Repairable and Identical

When the considered system is taken to be repairable and units are identical then the transition state probabilitiesof the present system are given by

$$\overline{P_{46}}(s)=\frac{1}{D(s)} \quad (87)$$

$$\overline{P_{36}}(s)=\frac{4\lambda}{(s+9\lambda)}\overline{P_{46}}(s) \quad (88)$$

$$\overline{P_{45}}(s)=\frac{6\lambda}{(s+9\lambda)}\overline{P_{46}}(s) \quad (89)$$

$$\overline{P_{26}}(s)=\frac{3.4\lambda^2}{(s+9\lambda)(s+8\lambda)}\overline{P_{46}}(s) \quad (90)$$

$$\overline{P_{35}}(s)=\frac{4.6\lambda^2}{A(s)}\overline{P_{46}}(s) \quad (91)$$

$$\overline{P_{44}}(s)=\frac{5.6\lambda^2}{(s+9\lambda)(s+4\lambda+\psi(y))}\overline{P_{46}}(s) \quad (92)$$

$$\overline{P_{16}}(s)=\frac{2.3.4\lambda^3}{(s+9\lambda)(s+8\lambda)(s+6\lambda+\eta(x))}\overline{P_{46}}(s) \quad (93)$$

$$\overline{P_{25}}(s)=\frac{3.4.6\lambda^3}{B(s)}\overline{P_{46}}(s) \quad (94)$$

$$\overline{P_{34}}(s)=\frac{4.5.6\lambda^3\overline{P_{46}}(s)}{(s+3\lambda+\xi(z))}\left[\frac{1}{A(s)}+\frac{1}{(s+4\lambda+\psi(y))(s+9\lambda)}\right] \quad (95)$$

$$\overline{P_{15}}(s)=\frac{2.3.4.6\lambda^4\overline{P_{46}}(s)}{(s+5\lambda+\xi(z))}\left[\frac{1}{B(s)}+\frac{1}{(s+6\lambda+\eta(x))(s+9\lambda)(s+8\lambda)}\right] \quad (96)$$

$$\overline{P_{24}}(s)=\frac{3.4.5.6\lambda^4}{C(s)}\overline{P_{46}}(s) \quad (97)$$

$$\overline{P}_{14}(s)=\frac{2.3.4.5.6\lambda^5\overline{P}_{46}(s)}{(s+\xi(z))}\left\{\frac{1}{(s+5\lambda+\xi(z))}\left[\frac{1}{B(s)}+\frac{1}{(s+6\lambda+\eta(x))(s+9\lambda)(s+8\lambda)}\right]+\frac{1}{C(s)}\right\} \quad (98)$$

C. Non Repairable and Non Identical

Had the considered system be non-repairable and units are non-identical then the transition state probabilities corresponding to present system are given by

$$\overline{P}_{46}(s)=\frac{1}{D(s)} \quad (99)$$

$$\overline{P}_{36}(s)=\frac{4\lambda_A}{(s+3\lambda_A+6\lambda_B)}\overline{P}_{46}(s) \quad (100)$$

$$\overline{P}_{45}(s)=\frac{6\lambda_B}{(s+4\lambda_A+5\lambda_B)}\overline{P}_{46}(s) \quad (101)$$

$$\overline{P}_{26}(s)=\frac{3.4\lambda_A^2}{(s+3\lambda_A+6\lambda_B)(s+2\lambda_A+6\lambda_B)}\overline{P}_{46}(s) \quad (102)$$

$$\overline{P}_{35}(s)=\frac{4.6\lambda_A\lambda_B}{A(s)}\overline{P}_{46}(s) \quad (103)$$

$$\overline{P}_{44}(s)=\frac{5.6\lambda_B^2}{(s+4\lambda_A+5\lambda_B)(s+4\lambda_A)}\overline{P}_{46}(s) \quad (104)$$

$$\overline{P}_{16}(s)=\frac{2.3.4\lambda_A^3}{(s+3\lambda_A+6\lambda_B)(s+2\lambda_A+6\lambda_B)(s+6\lambda_B)}\overline{P}_{46}(s) \quad (105)$$

$$\overline{P}_{25}(s)=\frac{3.4.6\lambda_A^2\lambda_B}{B(s)}\overline{P}_{46}(s) \quad (106)$$

$$\overline{P}_{34}(s)=\frac{4.5.6\lambda_A\lambda_B^2\overline{P}_{46}(s)}{(s+3\lambda_A)}\left[\frac{1}{A(s)}+\frac{1}{(s+4\lambda_A)(s+4\lambda_A+5\lambda_B)}\right] \quad (107)$$

$$\overline{P}_{15}(s)=\frac{2.3.4.6\lambda_A^3\lambda_B\overline{P}_{46}(s)}{(s+5\lambda_B)}\left[\frac{1}{B(s)}+\frac{1}{(s+6\lambda_B)(s+3\lambda_A+6\lambda_B)(s+2\lambda_A+6\lambda_B)}\right] \quad (108)$$

$$\overline{P}_{24}(s)=\frac{3.4.5.6\lambda_A^2\lambda_B^2}{C(s)}\overline{P}_{46}(s) \quad (109)$$

$$\overline{P}_{14}(s)=\frac{2.3.4.5.6\lambda_A^3\lambda_B^2\overline{P}_{46}(s)}{s}\left\{\frac{1}{(s+5\lambda_B)}\left[\frac{1}{B(s)}+\frac{1}{(s+6\lambda_B)(s+3\lambda_A+6\lambda_B)(s+2\lambda_A+6\lambda_B)}\right]+\frac{1}{C(s)}\right\} \quad (110)$$

D. Non Repairable and Identical

When the considered system is assumed to be non-repairable and units are identical then the transition state probabilities corresponding to present system are given by

$$\overline{P}_{46}(s)=\frac{1}{D(s)} \quad (111)$$

$$\overline{P}_{36}(s)=\frac{4\lambda}{(s+9\lambda)}\overline{P}_{46}(s) \quad (112)$$

$$\overline{P}_{45}(s)=\frac{6\lambda}{(s+9\lambda)}\overline{P}_{46}(s) \quad (113)$$

$$\overline{P}_{26}(s)=\frac{3.4\lambda^2}{(s+9\lambda)(s+8\lambda)}\overline{P}_{46}(s) \quad (114)$$

$$\overline{P}_{35}(s) = \frac{4.6\lambda^2}{A(s)}\overline{P}_{46}(s) \quad (115)$$

$$\overline{P}_{44}(s) = \frac{5.6\lambda^2}{(s+9\lambda)(s+4\lambda)}\overline{P}_{46}(s) \quad (116)$$

$$\overline{P}_{16}(s) = \frac{2.3.4\lambda^3}{(s+9\lambda)(s+8\lambda)(s+6\lambda)}\overline{P}_{46}(s) \quad (117)$$

$$\overline{P}_{25}(s) = \frac{3.4.6\lambda^3}{B(s)}\overline{P}_{46}(s) \quad (118)$$

$$\overline{P}_{34}(s) = \frac{4.5.6\lambda^3\overline{P}_{46}(s)}{(s+3\lambda)}\left[\frac{1}{A(s)} + \frac{1}{(s+4\lambda)(s+9\lambda)}\right] \quad (119)$$

$$\overline{P}_{15}(s) = \frac{2.3.4.6\lambda^4\overline{P}_{46}(s)}{(s+5\lambda)}\left[\frac{1}{B(s)} + \frac{1}{(s+6\lambda)(s+9\lambda)(s+8\lambda)}\right] \quad (120)$$

$$\overline{P}_{24}(s) = \frac{3.4.5.6\lambda^4}{C(s)}\overline{P}_{46}(s) \quad (121)$$

$$\overline{P}_{14}(s) = \frac{2.3.4.5.6\lambda^5\overline{P}_{46}(s)}{s}\left\{\frac{1}{(s+5\lambda)}\left[\frac{1}{B(s)} + \frac{1}{(s+6\lambda)(s+9\lambda)(s+8\lambda)}\right] + \frac{1}{C(s)}\right\} \quad (122)$$

9. Numerical Computation

The Maple software has been used to analyze reliability, availability, MTTF, cost effectiveness and sensitivity of the system.

(I) Reliability Analysis

Let us fix failure rates as λ_A=0.2 and λ_B=0.1, repair rates $\eta(x) = \psi(y) = \xi(z) = 0$, θ = 1, and x = y = z = 1. Also assume that the repair follows exponential distribution, i.e. equation (74) holds. Now by putting all these values in equation (55), using equation (74) and setting t = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, one can obtain Table 2 and Figure 3 which represent how reliability varies as the time increases.

(II) Availability Analysis

Let the failure rates λ_B=0.2, λ_B=0.1, repair rates $\eta(x) = \psi(y) = \xi(z) = 1$, θ = 1 and$x$= y = z =1. Putting all values in equation and taking inverse Laplace transformation, we get

$P_{up}(t)$ = 6.785162075exp(-0.3982589672t)-0.413918134exp(-1.72273319t)-28.5681895exp(0.9t)-0.4178526776exp(-1.55997341t)-0.153028 exp (-1.420949617t)-2.417188899exp(1.3t) +0.2820005597 exp(-2.563587411t) -12.25114855exp(-1.2t) +34.225844exp(-1.1t) +3.928319264exp(-1.034497403t) (123)

Now setting$t$=0, 10, 20, 30, 40, 50, 60, 70, 80, 90, 100, one can obtain Table 3 Figure 4 shows the variation of availability with respect to time.

(III) MTTF Analysis

Let us suppose that repair follows exponential distribution then using equation (74) and fromthe following equation, MTTF can be obtained

$$\text{MTTF} = \lim_{s\to 0} \text{P}_{\text{up}}(s)$$

We have the following three cases when repair rates $\eta(x) = \psi(y) = \xi(z) = 0$, $\theta = 1$ and $x = y = z = 1$:

(a) Let us set λ_A=0.06 and varying the value of λ_B as 0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.07, 0.08, 0.09, 0.10, one can obtain variation of MTTF with respect to λ_B.

(b) Fixing λ_B=0.05 and varying λ_A as 0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.07, 0.08, 0.09, 0.10, one can obtain changes of MTTF with respect to λ_A.

(c) Increasing the value of λ_A and λ_B from 0.01 to 0.10, we obtain the manner in which MTTF varies with respect to λ_A and λ_B simultaneously. Table 4 and Figure 5 show how MTTF varies with respect to different failure rate.

(IV) Cost Analysis

Setting λ_A=0.2, λ_B=0.1, repair rates $\eta(x) = \psi(y) = \xi(z) = 0$, $\theta = 1$ and $x = y = z = 1$. Putting all these values and taking inverse Laplace transforms, one can obtain equation (125). If the repair facility is always available, then expected profit during the interval (0, 100] is given by

$$E_P(t) = c_1 \int_0^t P_{up}(t)dt - c_2 t \tag{124}$$

where c_1 and c_2 are revenue rate per unit time and service cost per unit time respectively.

$$\begin{aligned} E_P(t) = {} & c_1(6.78516207\ 5\exp(-0.39\ 82589672\ t) - 0.41391813\ 4\exp(-1.72\ 273319\ t) \\ & - 28.5681895\ \exp(0.9\ t) - 0.41785267\ 76\exp(-1.5\ 5997\ t) - 0.15303\exp(-1.420949\ 67t) \\ & - 2.41718889\ 9\exp(1.3\ t) + 0.28200\exp(-2.563587\ 411t) - 12.2511485\ 5\exp(-1.2\ t) \\ & + 34.225844e\ xp(-1.1\ t) + 3.92831926\ 4\exp(-1.03\ 4497403\ t)) - tc_2 \end{aligned} \tag{125}$$

Taking $c_1 = 1$ and $c_2 = 0.1, 0.2, 0.3, 0.4, 0.5$ and using equation (74), variation of $E_P(t)$ with respect to time can be obtained. The computational values obtained are given in Table 5 and depicted in Figure 6.

(V) Sensitivity Analysis

Performing sensitivity analysis for changes in R(t) resulting from changes in system parametersλ_A and λ_B yield

$$\begin{aligned} \frac{\partial R(t)}{\partial \lambda_A} = {} & 4(2\sinh(1/2xt)\exp((-7/2x-6y)t) + (-15xt+1)\exp(-(4x+6y)t) \\ & + (30xt-1)\exp(-(3x+6y)t))/x - 12t\exp(-2xt) + 24t\exp(-3xt) - 12t\exp(-4xt) \\ & + 72t\exp(-(2x+5y)t) - 60t\exp(-(2x+6y)t) + 72t\exp(-(4x+5y)t) - 144t\exp(-(3x+5y)t) \end{aligned} \tag{126}$$

$$\begin{aligned} \frac{\partial R(t)}{\partial \lambda_B} = {} & 6((1-15yt)\exp(-(4x+6y)t) + 2\sinh(1/2yt)\exp((-4x-11/2y)t) \\ & + (+15yt-1)\exp(-(4x+5y)t))/y - 30t\exp(-5yt) + 180t\exp(-(2x+5y)t) \\ & - 180t\exp(-(2x+6y)t) + 30t\exp(-6yt) - 240t\exp(-(3x+5y)t) + 240t\exp(-(3x+6y)t) \end{aligned} \tag{127}$$

Numerical results of the sensitivity analysis for the system reliability with respect to change in λ_A and λ_B are given in Tables 6 and 7. Corresponding behaviour of sensitivity has been shown in Figures 7 and 8.

Time	Reliability
0	1
1	0.997841
2	0.967774
3	0.881451
4	0.748451
5	0.598083
6	0.455709
7	0.335008
8	0.239835
9	0.168389
10	0.116551

Table 2: Time vs. Reliability

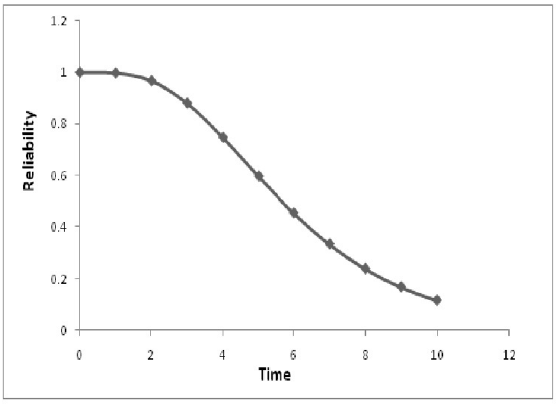

Figure 3: Time vs. Reliability

Time	Availability
0	1
10	0.123549
20	2.36E-03
30	4.39E-05
40	8.19E-07
50	1.53E-08
60	2.84E-10
70	5.30E-12
80	9.88E-14
90	1.84E-15
100	3.43E-17

Table 3: Time vs. Availability

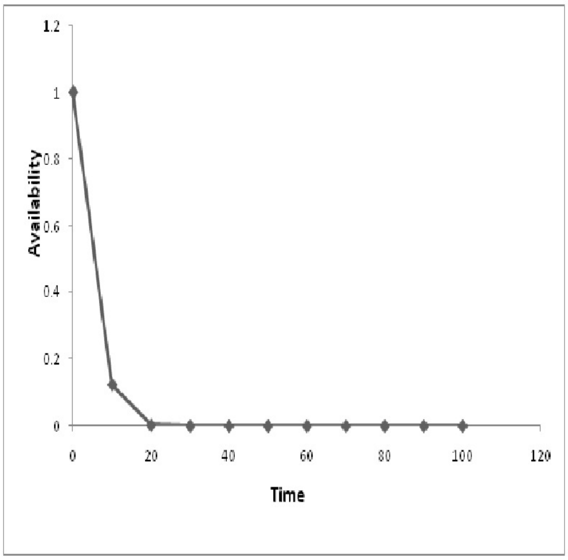

Figure 4: Time vs. Availability

λ_A	MTTF	λ_B	MTTF	λ_A and λ_B	MTTF
0.01	108.3485	0.01	39.5842	0.01	111.2698
0.02	54.25319	0.02	24.57325	0.02	55.63492
0.03	36.32675	0.03	20.84832	0.03	37.08995
0.04	27.47102	0.04	19.47416	0.04	27.81746
0.05	22.25397	0.05	18.85782	0.05	22.25397
0.06	18.85782	0.06	18.54497	0.06	18.54497
0.07	16.49991	0.07	18.37163	0.07	15.89569
0.08	14.78722	0.08	18.26901	0.08	13.90873
0.09	13.50077	0.09	18.20501	0.09	12.36332
0.1	12.50899	0.1	18.16338	0.1	11.12698

Table 4: Failure rates vs. MTTF

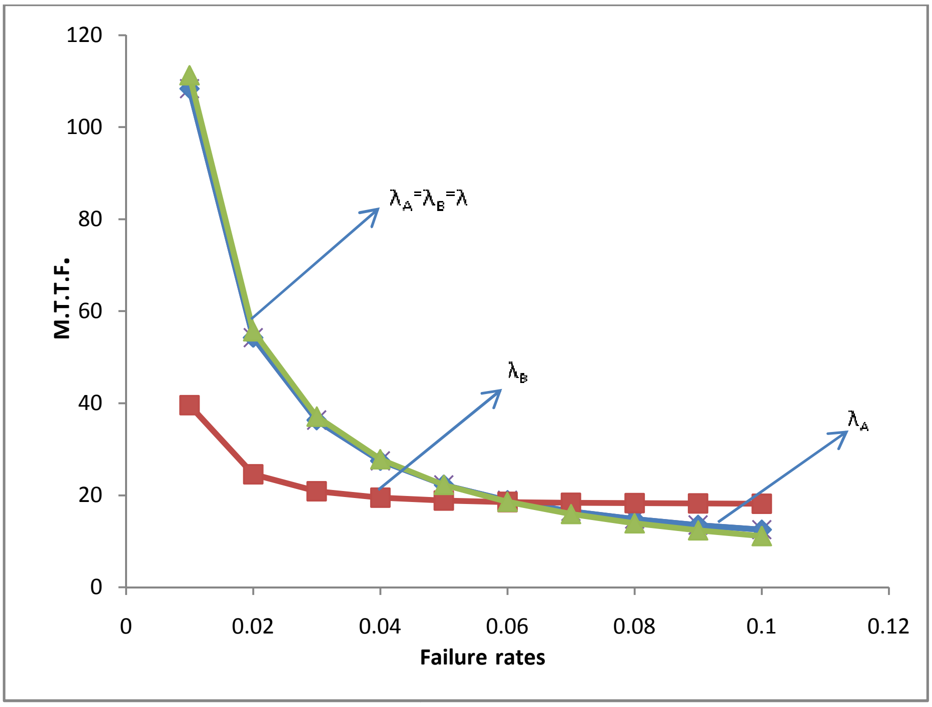

Figure 5: Failure rates vs. MTTF

Time	Ep(t)				
	$C_2 = 0.1$	$C_2 = 0.2$	$C_2 = 0.3$	$C_2 = 0.4$	$C_2 = 0.5$
0	0	0	0	0	0
10	0.988968	0.888968	0.788968	0.688968	0.588968
20	2.16E+00	1.959071	1.759071	1.559071	1.359071
30	3.31E+00	3.010643	2.710643	2.410643	2.110643
40	4.29E+00	3.888245	3.488245	3.088245	2.688245
50	5.04E+00	4.538272	4.038272	3.538272	3.038272
60	5.57E+00	4.971894	4.371894	3.771894	3.171894
70	5.93E+00	5.227133	4.527133	3.827133	3.127133
80	6.15E+00	5.346451	4.546451	3.746451	2.946451
90	6.27E+00	5.366954	4.466954	3.566954	2.666954
100	6.32E+00	5.317685	4.317685	3.317685	2.317685

Table 5: Time vs. expected profit

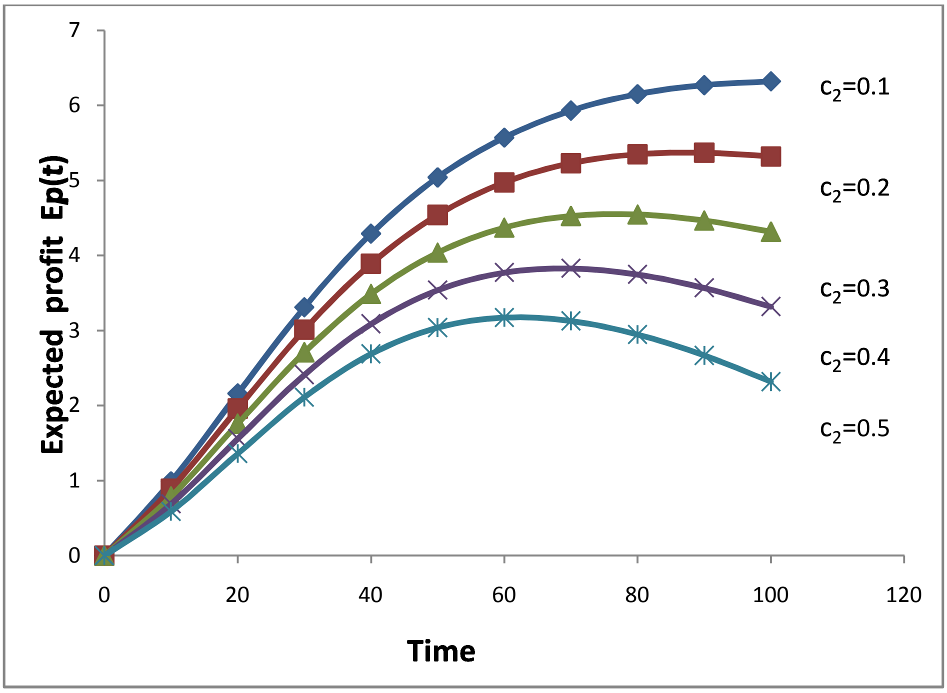

Figure 6: Time vs. expected profit

Time	Value of $\partial R(t)/\partial \lambda_A$		
0	0	0	0
10	-1.28907	-2.85822	-2.15764
20	-9.82089	-12.3116	-12.33
30	-21.1004	-15.2452	-20.0326
40	-29.0596	-12.324	-21.0272
50	-32.3584	-8.19396	-17.98
60	-31.8758	-4.89597	-13.698
70	-28.9898	-2.73951	-9.69874
80	-24.9248	-1.46766	-6.53191
90	-20.56	-0.76285	-4.24426
100	-16.4323	-0.38794	-2.68569

Table 6: Sensitivity analysis of the system MTTF w. r. t. λ_A

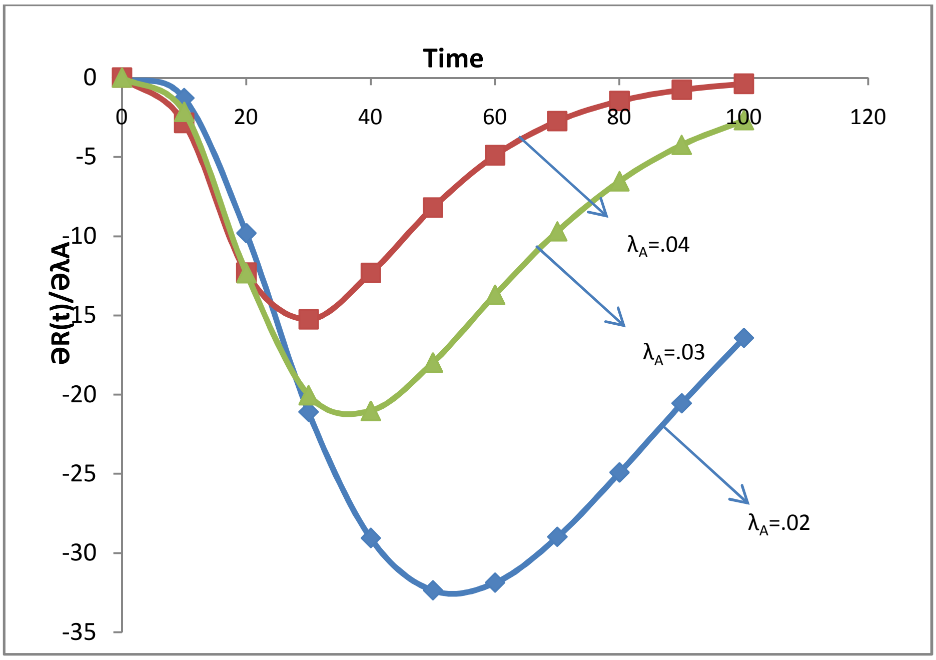

Figure 7: Sensitivity of system MTTF with respect to different values of λ_A

Time	**Value of $\partial R(t)/\partial\lambda_B$**		
0	**0**	**0**	**0**
10	-1.12241	-0.75097	-0.97338
20	-6.50713	-1.47096	-3.27613
30	-9.37826	-0.72316	-2.75228
40	-7.86207	-0.2087	-1.35024
50	-5.00059	-4.61E-02	-0.50447
60	-2.71242	-8.75E-03	-0.1613
70	-1.33E+00	-1.52E-03	-4.69E-02
80	-6.15E-01	-2.48E-04	-1.28E-02
90	-2.71E-01	-3.90E-05	-3.37E-03
100	-1.16E-01	-5.99E-06	-9.74E-01

Table 7: Sensitivity analysis of the system MTTF w. r. t. λ_B

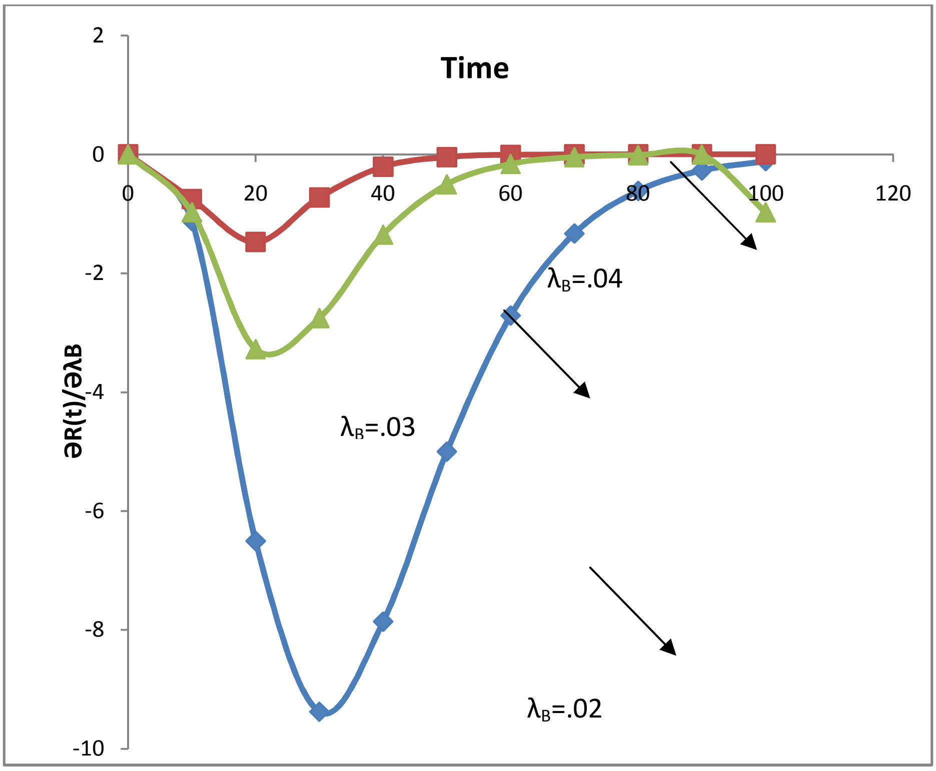

Figure 8: Sensitivity of system MTTF with respect to different values of λ_B

10. Interpretation of the Result and Conclusion

In the present study different reliability measures of the complex system such as transition state probabilities, asymptotic behaviour, reliability, availability, MTTF, expected profit and sensitivity with respect to different parameters have been obtained.

The Table 2 gives the variation of reliability with respect to the time and the Figure 3 shows the graph of “Reliability vs. Time”. At time $t = 0$ the reliability of the system is obtained to be 1 and it decreases with the increment in time.

Figure 4 shows the graph of “Availability vs. Time” and its value has been given in Table 3. Critical observation of Figure 4 concludes that availability decreases fast in the beginning but thereafter it decreases approximately in a constant manner.

Figure 5 is the graph of “MTTF vs. λ_A”, “MTTF vs. λ_B” and “MTTF vs. λ ($\lambda_A = \lambda_B$)”. The corresponding values of MTTF have been given in Table 4. Observation of the figure reveals thatbehaviour of MTTF is approximately same with respect to λ_A and λ but it is different with respect to λ_B. However in all three cases they decrease as failure rates increase.One of the interesting facts is that at failure rate 0.05, MTTF with respect to λ_A and λ are same. But prior to failure rate 0.05, MTTF is higher with respect to λ than λ_A and after wards situation got reversed. We also observed that prior to failure rate 0.06, value of the MTTF is higher with respect to λ than λ_B and after this the value of MTTF got reversed. It is worth mentioning that the value of MTTF with respect to λ_B and λ are the same at the failure rate 0.06.

From the Table 5 one can observe the variation of effective profit with respect to time. The corresponding Figure 6 has been drawn by keeping the revenue cost per unit time C_1 set at 1.0, service cost C_2 is varied as 0.1, 0.2, 0.3, 0.4 and 0.5 and failure rates are kept at constant value as $\lambda_A = 0.2$ and $\lambda_B = 0.1$. By observation of the figure, one can draw the conclusion that expected profit decreases as service cost increases with respect to time.

The sensitivities of the system reliability with respect to the system failure rates λ_A and λ_B are depicted in Figures 7 and 8 respectively. In the Figure 7, along the time coordinate, we show the sensitivity of reliability with respect to λ_A by varying λ_A from 0.02, 0.03 and 0.04 when the λ_B is fixed at λ_B=0.03. In the Figure 8, along the time coordinate, we show the sensitivity of reliability with respect to λ_B by varying λ_B from 0.02, 0.03 and 0.04 when the λ_A is fixed at λ_A=0.03. We observe that influence of λ_A and λ_B on system reliability increases as λ_A and λ_B decreases and the time with maximum sensitivity delays. We observe that sensitivity of the reliability with respect to λ_B is more than sensitivity with respect to λ_A when other failure rate is fixed at 0.02 whereas when we decrease the value of fixed failure rate then sensitivity with respect to λ_B is less than sensitivity with respect to λ_A. We can see that sensitivity of the system reliability decreases with the increases in the value of λ_A and λ_B. It reveals that the system reliability is more sensitive with respect to λ_B.

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