MARKOV MODEL FOR SWITCHING FAILURE OF WARM SPARES IN MACHINE REPAIR SYSTEM

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Abstract

In this investigation, we study the performance characteristics of (m,M) machining systems having warm spares and two heterogeneous servers. The first server is permanent and available full time in the system, whereas the second server takes vacation according to the specific threshold policy. In some real time systems, spares may or may not replace/switch in the system whenever an operating unit failure occurs, as such switching failure has been incorporated. In this paper, we consider a two dimensional continuous time finite state space Markov chain. The steady state queue size distribution for the Markovian machine repair problem, considering switching failure, is obtained computationally using matrix method based on successiveover relaxation. We derive various system characteristics namely, expected number of failed machines in the system, throughput of the system, probability that the server is on vacation, etc. In order to gain maximum net profit, a cost function is constructed in terms of different cost elements to determine the optimal threshold level for the server vacation. For illustration purpose, numerical results are provided. In order to examine the effects of system parameters, the sensitivity analysis has also been facilitated.

Key Words: Machine Repair, Spares, Vacation, Heterogeneous Servers, Switching Failure, Queue Size Distribution.

1. Introduction

Machines are common components of all industries and these are also subject to failure. The operation of any machining system may be stopped due to the failure of the machines and in such a situation the repair facility should be made available so as to restore the functioning of the failed machines. The machine repair/failure problems occur in almost all the areas including the computer networks, communication systems, production systems, transportation systems, flexible manufacturing systems, etc. Due to wide applications, various researchers working in the area of queueing theory devoted their attention on this topic considering various concepts.

In the year 1943, significant milestone for machine interference problem was achieved by Kroning and Mondria when they developed the technique to obtain the

steady state distribution of M/M/1 queue for the machine repair system taking different initial conditions. Later, a lot of works (cf. Pósafalvi and Sztrik, 1987; Jain, 1997; Jiang et al., 2001) was done on the machine interference problems by taking different parameters into consideration. Gray et al. (2002) and Masuyama and Takine (2003) studied different type of queueing models with server breakdowns. Jain el al. (2004) investigated a machine repair system, having the facility of spares and the reneging customers,which follows the *N*-policy. Wu (2010) gave computational algorithm and parameter optimization for the multi-server system with unreliable servers and impatient customers. Jain et al. (2012) analyzed unreliable M/M/K queueing system with controlled rates of failure of machines and arrivals of customers under (N,F) policy having multi–optional phase repair facility. Later, Jain et al. (2013) studied a machine repair problem with an unreliable server in which the arrival of failed machines is governed by F policy.

Maintaining a high or required level of performance efficiency is often an essential requisite for production/manufacturing organizations. If the defective parts are found, the same should be replaced or repaired to get back the operational system. For the uninterrupted service and efficient utilization of installed units in the computer mechanized system these days, it is desirable to provide a spare part support. The machine repair problems including availability of warm spares or other type spares have attracted many researchers (cf. Goel and Gupta, 1983; Berg and Posner, 1990; Wang and Sivazlian, 1992; Gupta, 1994; Gupta and Rao, 1996) in the area of queueing theory to chalk out the better maintenance schedules so as to achieve desired grade of service. Rao and Gupta (2000) established a model of the M/G/1 machine repairman problem with cold, warm and hot standbys and formulated various performance measures. Jain et al. (2007) performed thereliability analysis of k-out-of n: G machining systems with mixed spares and multiple modes of failure. A machine repair problem in production systems with spares and server vacations was investigated by Ke et al. (2009) and Maheshwari et al. (2010). An unreliable machine repair system with heterogeneous servers working under N-policy having warm spares and impatient customers was analysed by Jain et al. (2012). They have also analyzed the performance characteristics of the system. The availability of transient state system with warm standbys was studied using the Runge-Kutta method byJain and Rani (2013).

It is not unusual to assume that the switching device may have a failure probability in real service systems due to error in handling, automation or some natural failures. Thus the switching failure of the spares is also an important factor in the proper functioning of the machining system. Lewis (1996) first introduced the concept of the standby switching failures to facilitate the reliability indices for a standby system. Later this factor was also considered by many other researchers (cf. Wang et al.; 2006; Wang et al., 2007; Ke et al., 2007) in their research. Using the supplementary variable technique, Wang and Chen (2009) developed the explicit expressions for the steady-state availability of three systems with general repair times, reboot delay and switching failures. Ke et al. (2011) formulated reliability measures for a repairable system with standby switching failures and reboot. Jain et al. (2012b) studied an unreliable server machining system by considering switching failure and common failure.

In many real world repairable systems, it is economical that repairmen may not be available for a random period of time when there are no failed units in the

system at a service completion instance. Sometimes, in many practical multiple server systems, only some servers perform secondary jobs or take vacations when they become idle, and other servers are always available for serving the arriving units. In a queueing system, this type of vacation is called the *partial server vacation*. In the past, a lot of work has been done on the machine interference problem where there is a provision for the server to go on vacation (cf. Gupta, 1997; Zhang and Tian, 2003; Jain et al., 2004; Ke and Wang, 2007). Wang and Chen (2009) studied a machine repair problem with working vacation which means that the server also performs its job at a different pace, even when it is on a vacation. They used Newton's method for the optimal management of the machine repair problem. Jain and Upadhyaya (2009) suggested the threshold N-policy for degraded machining system having many type of spares and multiple vacations. Singh et al. (2012) investigated a queueing model with state dependent arrival of customers and the facility of second optional vacation after availing the first regular vacation by the server.

In this paper, we develop a machine repair model with warm spares incorporating their probabilistic switching failure and facility of vacation for the server. The rest of the paper is organized as follows: In section 2, system description and notations have been given. We formulate the problem mathematically by constructing Chapman Kolmogrove difference equations governing the steady-state model(see figure 1). In section 3, the solution technique is given to obtain steady-state probabilities of the number of failed unit in the system. In section 4, some performance indices are presented in terms of steady-state probabilities. Numerical results are provided in the section 5. Finally, in the last section 6, conclusions are drawn and future scopes are discussed.

2. System Description

For the mathematical modeling of multi-component machining system consisting of M operating machine and S warm spares and two heterogeneous servers, the following assumptions and notations are used (refer Figure1).

- The life time of operating machines and spares follow the exponential distribution with rate of λ and α ($0 < \alpha < \lambda$), respectively. The life times of the operating machine or/and spare are independent of the state of the others.
- For the normal functioning of the system, M operating machines are required. On the failure of the operating machines, the available warm spares are used one by one in the system with negligible switching time. Once a spare machine is put into the system in place of failed operating machine, its characteristics are same as that of the operating machine. The failed machine is immediately sent for repair to the server available.
- If all the spares are exhausted and there are less than M but more than m ($m<M$) operating machines in operation, the system may also function in short mode following the (m,M) policy. Thus, the system breaks down if and only if $L = M+S-m+1$ or more machines fail.
- The switching of spares in place of failed operating machines may or may not be perfect due to mishandling or poor automation. It is assumed that the switching of machine has a failure probability q. If a warm spare fails to replace a failed machine, the next available spare attempts to switch. This process continues until switching is successful or all the warm spares are exhausted.

- The first server *S1* is always available for repairing the failed machines. The second server *S2* leaves for vacation according to the specified threshold policy for random duration, i.e. the server takes vacation on finding less than *N* failed machines in the system. The vacation time is exponentially distributed with rate θ. On returning from a vacation if server *S2* finds more than or equal to *N* failed machines accumulated in the system, the server starts repairing of the failed machines, otherwise goes for another vacation.
- The repair times of server *S1* and server *S2* are exponentially distributed with rates μ_1 and μ_2, respectively.
- Both servers take failed machines for repair in the *First Come First Serve (FCFS)* fashion.
- When the failed machine is repaired, it is as good as a new machine. The repaired machine is used for operation in the system if there are less than *M* operating machines; otherwise the repaired machine joins the spare machine group. The switching time is assumed to be negligible.

Following notations are used for mathematical formulation of birth–death process in continuous time finite state space model:

$P_{n,0}$= The steady-state probability that there are *n* $(0 \leq n \leq L)$ failed machines in the system and server *S2* is in the *vacation* state.

$P_{n,1}$= The steady-state probability that there are *n* $(0 \leq n \leq L)$ failed machines in the system and server *S2* is in the *working* state.
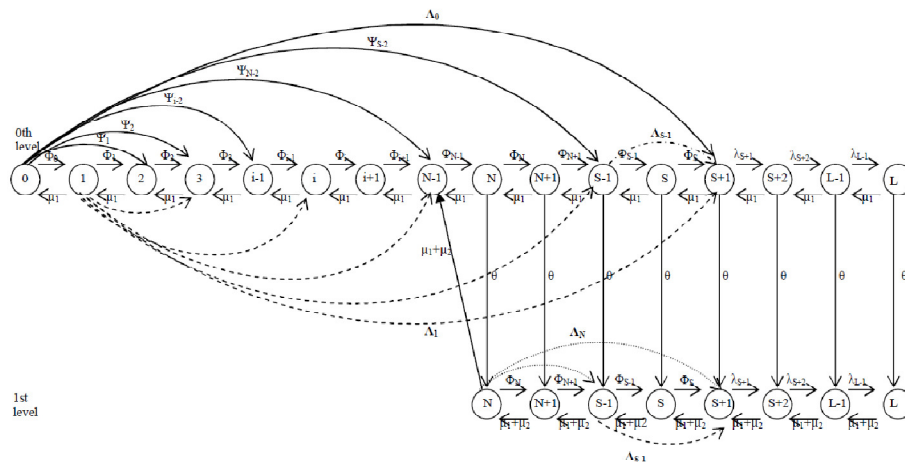

Figure 1: State transition diagram with rates $\Lambda_n = M\lambda q^{S-n}$; $\Psi_n = M\lambda q^{n-1}(1-q)$;
$\Phi_n = M\lambda + (S-n)\alpha$; $\lambda_n = (M+S-n)\lambda$

Using the quasi birth-death process and the above defined assumptions and notations, the Chapman-Kolmogroveequations governing the present machine repair model are constructed as follows:

$$- (M\lambda + S\alpha)P_{0,0} + \mu_1 P_{1,0} = 0 \qquad (1)$$

$$[M\lambda(1-q) + S\alpha]P_{0,0} - [M\lambda + (S-1)\alpha + \mu_1]P_{1,0} + \mu_1 P_{2,0} = 0 \qquad (2)$$

$$\left[M\lambda(1-q)+(S-i+1)\alpha\right]P_{i-1,0}-\left[M\lambda+(S-i)\alpha+\mu_1\right]P_{i,0}+\mu_1 P_{i+1,0}$$

$$+\sum_{n=0}^{i-2}M\lambda q^{i-1-n}(1-q)P_{n,0}=0\ ; \qquad 2\le i\le N-2 \tag{3}$$

$$\left[M\lambda(1-q)+(S-N+2)\alpha\right]P_{N-2,0}-\left[M\lambda+(S-N+1)\alpha+\mu_1\right]P_{N-1,0}$$

$$+\mu_1 P_{N,0}+(\mu_1+\mu_2)P_{N,1}+\sum_{n=0}^{N-3}M\lambda q^{N-2-n}(1-q)P_{n,0}=0 \tag{4}$$

$$\left[M\lambda(1-q)+(S-N+1)\alpha\right]P_{N-1,0}-\left[M\lambda+(S-N)\alpha+\mu_1+\theta\right]P_{N,0}$$

$$+\mu_1 P_{N+1,0}+\sum_{n=0}^{N-2}M\lambda q^{N-1-n}(1-q)P_{n,0}=0 \tag{5}$$

$$\left[M\lambda(1-q)+(S-i+1)\alpha\right]P_{i-1,0}-\left[M\lambda+(S-i)\alpha+\mu_1+\theta\right]P_{i,0}$$

$$+\mu_1 P_{i+1,0}+\sum_{n=0}^{i-2}M\lambda q^{i-1-n}(1-q)P_{n,0}=0\ ; \qquad N+1\le i\le S \tag{6}$$

$$-\left[(M-1)\lambda+\mu_1+\theta\right]P_{S+1,0}+\mu_1 P_{S+2,0}+\sum_{n=0}^{S}M\lambda q^{S-n}P_{n,0}=0 \tag{7}$$

$$(M+S-i+1)\lambda P_{i-1,0}-\left[(M+S-i)\lambda+\mu_1+\theta\right]P_{i,0}$$

$$+\mu_1 P_{i+1,0}=0\ ;\ S+2\le i\le L-1 \tag{8}$$

$$m\lambda P_{L-1,0}-(\mu_1+\theta)P_{L,0}=0 \tag{9}$$

$$\theta P_{N,0}-\left[M\lambda+(S-N)\alpha+\mu_1+\mu_2\right]P_{N,1}+(\mu_1+\mu_2)P_{N+1,1}=0 \tag{10}$$

$$\theta P_{N+1,0}+\left[M\lambda(1-q)+(S-N)\alpha\right]P_{N,1}$$

$$-\left[M\lambda+(S-N-1)\alpha+\mu_1+\mu_2\right]P_{N+1,1}+(\mu_1+\mu_2)P_{N+2,1}=0 \tag{11}$$

$$\theta P_{i,0}+\left[M\lambda(1-q)+(S-i+1)\alpha\right]P_{i-1,1}-$$

$$\left[M\lambda+(S-i)\alpha+\mu_1+\mu_2\right]P_{i,1}+(\mu_1+\mu_2)P_{i+1,1}+$$

$$\sum_{n=N}^{i-2}M\lambda q^{i-n-1}(1-q)P_{n,1}=0\ ; \qquad N+2\le i\le S \tag{12}$$

$$\theta P_{S+1,0}-\left[(M-1)\lambda+\mu_1+\mu_2\right]P_{S+1,1}+(\mu_1+\mu_2)P_{S+2,1}$$

$$+\sum_{n=N}^{S}M\lambda q^{S-n}P_{n,1}=0 \tag{13}$$

$$\theta P_{i,0} + (M+S-i+1)\lambda P_{i-1,1} - [(M+S-i)\lambda + \mu_1 + \mu_2]P_{i,1}$$
$$+(\mu_1 + \mu_2)P_{i+1,1} = 0; \qquad S+2 \le i \le L-1 \tag{14}$$

$$\theta P_{L,0} + m\lambda P_{L-1,1} - (\mu_1 + \mu_2)P_{L,1} = 0 \tag{15}$$

3. The Solution Technique

The governing difference equations (1)-(15) of the present model can be expressed in the matrix form

$$\mathbf{AX} = \mathbf{0} \tag{16}$$

where $\mathbf{A}$ is the coefficient matrix of an order $2L$-N+2, $\mathbf{X}$ is the column vector having

elements $\left(P_{00}, P_{10}, ..., P_{N-1,0}, P_{N0}, P_{N1}, ..., P_{L,0}, P_{L,1} \right)^T$ and $\mathbf{0}$ is the null column vector

of order $2L$-N+2. Using the normalizing condition

$$\sum_{i=0}^{L} P_{i,0} + \sum_{i=N}^{L} P_{i,1} = 1 \tag{17}$$

the system of linear equations in (16) can be expressed as

$$\mathbf{A}^*\mathbf{X} = \mathbf{B} \tag{18}$$

where $\mathbf{A}^*$ is the matrix $\mathbf{A}$ replacing the last row with a row vector having all unit elements and $\mathbf{B}$ is the column vector of the form $[0,0,...,0,1]^T$ of order $2L$-N+2.

Equation (18) has been solved using the numerical technique *'Successive Over Relaxation (SOR) method'* in MATLAB 7.1. This technique is an extrapolation to Gauss-Seidal method which accelerate the convergence rate by taking the relaxation parameter w>1 (w=1.25) which is unity in case of the Gauss-Seidal method.

4. Performance Characteristics

Using the steady-state probabilities derived in the previous section, we compute some performance measures for queuing model under consideration in present study as follows:

- The expected number of failed machines in the system is as follows

$$E(n) = \sum_{n=0}^{L} nP_{n,0} + \sum_{n=N}^{L} nP_{n,1} \tag{19}$$

- The throughput of the system is derived as given below

$$\tau = \sum_{n=1}^{L} \mu_1 P_{n,0} + \sum_{n=N}^{L} (\mu_1 + \mu_2)P_{n,1} \tag{20}$$

- The expected number of spare units in the system is given by

$$E(S) = \sum_{n=0}^{S} (S-n)P_{n,0} + \sum_{n=N}^{S} (S-n)P_{n,1} \tag{21}$$

- The probability that the second server *S2* in on vacation is obtained using

$$P_2(V) = \sum_{n=0}^{L} P_{n,0} \qquad (22)$$

- The probability that server *S2* is in busy state is given by

$$P_2(B) = \sum_{n=N}^{L} P_{n,1} \qquad (23)$$

- The steady state probability that at least m operating units are in operation to function properly (system availability) is

$$A = 1 - P_{L,0} - P_{L,1} \qquad (24)$$

- The steady-state failure frequency is determined as follows

$$SF = m\lambda[P_{L-1,0} + P_{L-1,1}] \qquad (25)$$

- The expected number of operating machines in the system is given by

$$E(O) = M - \left[\sum_{n=1}^{L-S} nP_{S+n,0} + \sum_{n=N}^{L-S} nP_{S+n,1} \right] \qquad (26)$$

- Since any one of the *S* spares machines in the queue may have switching failure, the average switching failure rate is given by

$$SR = \sum_{n=1}^{S} M\lambda q P_{n-1,0} + \sum_{n=N+1}^{S} M\lambda q P_{n-1,1} \qquad (27)$$

- The probability that the first server *S1* is in idle state is

$$P_1(I) = P_{0,0} \qquad (28)$$

- The probability that the first server *S1* is busy is

$$P_1(B) = 1 - P_1(I) \qquad (29)$$

5. Numerical Results

To validate the model, we compute various performance measures established in the previous sections, and display in tables and graphs. In tables 1-4, numerical results for *E(O), E(S), P₂(V), P₂(B), P₁(I), P₁(B), SR* and *SF* are summarized for varying values of various parameters.

In table 1, performance measures are summarized for different values of *M, S* and *λ* by fixing *m=12, α=0.1, μ₁=2, μ₂=3, q=0.8, N=4 and θ=0.6*. It is clear that the *E(O), E(S), P₂(V), P₁(I)* and *SR* are decreasing with respect to failure rate (*λ*) but *P₂(B), P₁(B)* and *SF* are increasing with respect to *λ*.

M	S	Λ	E(O)	E(S)	P₂(V)	P₂(B)	P₁(I)	P₁(B)	SR	SF
20	5	0.2	17.72	0.1885	0.3561	0.6440	0.0019	0.9981	0.3389	0.0743
		0.4	14.21	0.0105	0.0350	0.9650	0.0000	1.0000	0.0457	0.8122
		0.6	12.46	0.0005	0.0021	0.9979	0.0000	1.0000	0.0035	1.8562
	15	0.2	19.04	0.5322	0.0038	0.9962	0.0000	1.0000	0.6700	0.0211
		0.4	14.35	0.0155	0.0000	1.0000	0.0000	1.0000	0.0584	0.7903
		0.6	12.46	0.0006	0.0000	1.0000	0.0000	1.0000	0.0039	1.8554

Table 1 Performance measures by varying the number of operating machine (*M*), warm spares (*S*) and failure rate of operating machine (*λ*)

In table 2 for given value $\lambda = 0.3$ and taking other parameters same as in table 1, the value of $E(O)$ is almost constant with respect to the failure rate of warm spares (α), which proves that the expected number of operating units in the system $E(O)$ is not affected. The gradual increasing trends are observed for $P_2(B)$ and $P_1(B)$ with respect to the failure rate of warm spares (α) and gradual decrease is observed for $E(S)$, $P_2(B)$, $P_1(I)$, SR and SF. These trends reveal that the failure rate of warm spares is not a sensitive parameter for the given model.

M	S	A	E(O)	E(S)	$P_2(V)$	$P_2(B)$	$P_1(I)$	$P_1(B)$	SR	SF
20	5	0.05	15.96	0.0496	0.1333	0.8667	0.0002	0.9998	0.1498	0.3222
		0.15	15.96	0.0487	0.1327	0.8673	0.0002	0.9998	0.1486	0.3220
		0.30	15.96	0.0474	0.1318	0.8683	0.0001	0.9999	0.1469	0.3216
	15	0.05	16.59	0.0978	0.0002	0.9998	0.0000	1.0000	0.2373	0.2548
		0.15	16.59	0.0950	0.0001	0.9999	0.0000	1.0000	0.2346	0.2549
		0.30	16.59	0.0911	0.0001	0.9999	0.0000	1.0000	0.2309	0.2550

Table 2 Performance measures by varying the number of operating machine (*M*), warm spares (*S*) and failure rate of spare (*α*)

By fixing $\lambda = 0.8$ and $\alpha = 0.5$ in table 3 and keeping other parameters same as in table 1, we note the great differences in all performance measures with respect to the threshold value (m) which depicts that it is a very important factor of our study. $P_2(B)$ is decreasing with respect to m whereas $E(O)$, SR, $P_2(B)$ and SF are increasing with respect to same parameters.

M	S	M	E(O)	E(S)	$P_2(V)$	$P_2(B)$	$P_1(I)$	$P_1(B)$	SR	SF
20	5	1	6.25	0.0000	0.0000	1.0000	0.0000	1.0000	0.0000	0.0097
		6	7.28	0.0000	0.0000	1.0000	0.0000	1.0000	0.0000	1.0270
		12	11.88	0.0000	0.0002	0.9998	0.0000	1.0000	0.0004	2.5536
	15	1	6.25	0.0000	0.0000	1.0000	0.0000	1.0000	0.0000	0.0097
		6	7.28	0.0000	0.0000	1.0000	0.0000	1.0000	0.0000	1.0270
		12	11.88	0.0000	0.0000	1.0000	0.0000	1.0000	0.0004	2.5540

Table 3 Performance measures by varying the number of operating machine (*M*), warm spares (*S*) and threshold number of operating units (*m*)

In table 4, a gradual decrease is observed in $E(O)$, $E(S)$, $P_2(B)$ and $P_1(I)$ with respect to the probability of switching failure of warm spares (q). Also a gradual increase is seen in $P_2(B)$, $P_1(B)$, SR and SF with respect to same parameters , for the same data as chosen for table 3 and $m=12$.

M	S	Q	E(O)	E(S)	$P_2(V)$	$P_2(B)$	$P_1(I)$	$P_1(B)$	SR	SF
20	5	0.1	16.10	0.0959	0.1699	0.8301	0.0005	0.9995	0.0354	0.3126
		0.5	16.00	0.0646	0.1462	0.8538	0.0003	0.9997	0.1216	0.3189
		0.9	15.94	0.0450	0.1293	0.8708	0.0002	0.9998	0.1540	0.3229
	10	0.1	16.96	0.4021	0.0377	0.9623	0.0001	0.9999	0.0897	0.2305
		0.5	16.68	0.1638	0.0118	0.9882	0.0000	1.0000	0.2237	0.2496
		0.9	16.54	0.0798	0.0042	0.9958	0.0000	1.0000	0.2304	0.2589

Table 4 Performance measures by varying the number of operating machine (*M*), warm spares (*S*) and switching failure rate (*q*)

All tables reveal the increasing trends in *E(O), E(S), P₂(B), P₁(B)* and *SR* with respect to number of warm spares (*S*) but *P₂(V), P₁(I)* and *SF* are decreasing. On the other hand, the values of performance measures namely, *E(S), P₂(B), P₁(I), SR* and *SF* decrease with respect to the number of operating units (*M*) and rest of the performance measures show the increasing trends. From all tables, we conclude that there are major changes in the performance measures for failure rate of operating machine, service rate of both servers and the minimum number of operating units required for the system operation.

Figure 2 depicts the relation between expected number of failed units in the system *E(n)* with respect to various input parameters for other default parameters fixed as in tables 1-4. In figure 2(i) we see that *E(n)* increases with high rate for lower values of *λ* but increases gradually for higher values of *λ*. Figure 2(ii), 2(iii), and 2(iv) respectively, show that there is not much significance changes in *E(n)* with respect to *α, q* and *θ*.
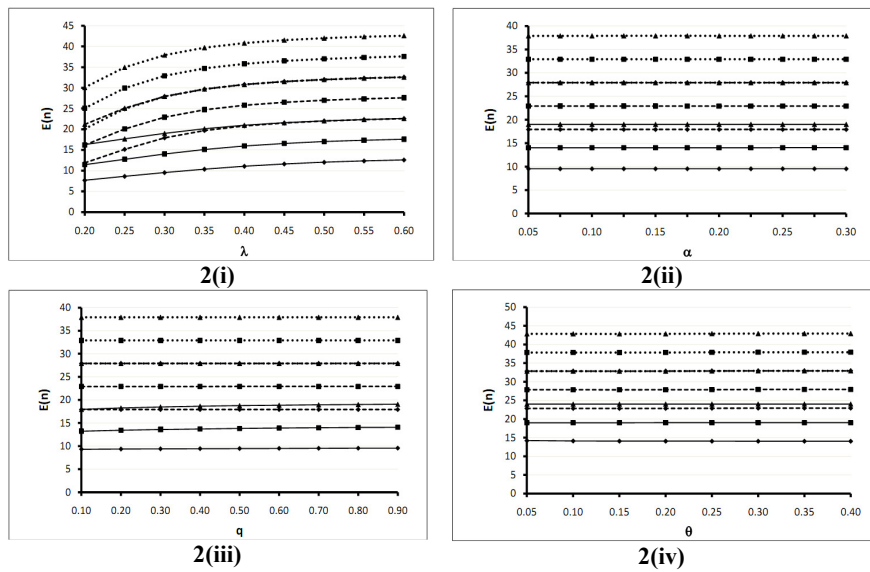

2(i)   **2(ii)**

2(iii)   **2(iv)**

Figure 2 Expected number of failed machines v/s (i) *λ*, (ii) *α*, (iii) *q*, (iv) *θ*

—♦— S=10, M=20   —■— S=15, M=20   —▲— S=20, M=20   --♦-- S=10, M=30   --■-- S=15, M=30

--▲-- S=20, M=30   ···♦·· S=10, M=40   ··■·· S=15, M=40   ···▲·· S=20, M=40

Figures 3 and 4 exhibit the behavior of throughput *τ* and availability *A* respectively with respect to various parameters. The throughput increases with the increment in *λ* but is almost constant with respect to *α*.. System availability *A* is decreasing with respect to *λ*but its values are almost constant for *α*.
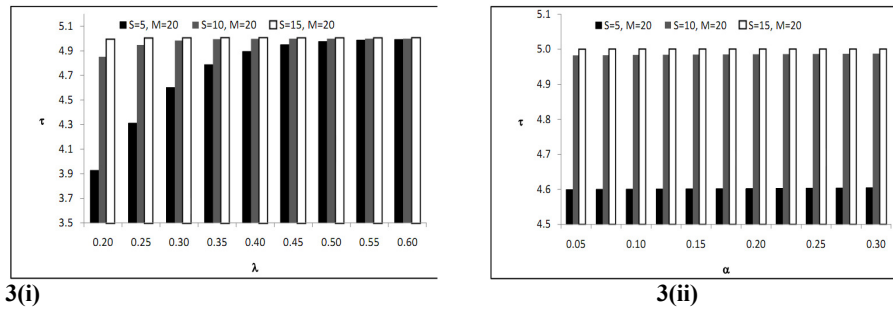
3(i)   **3(ii)**
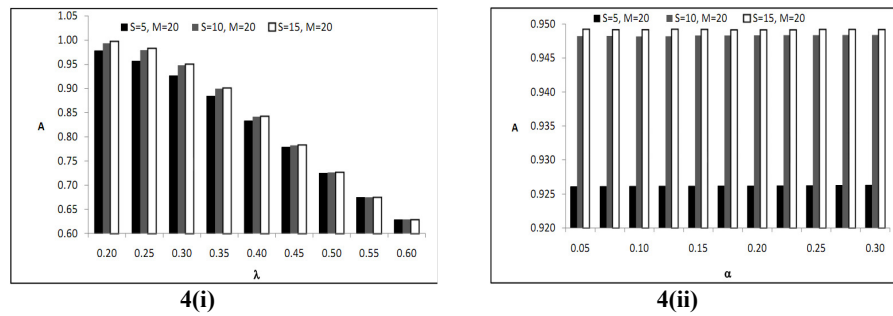
Figure 3: Throughput of the system v/s (i) λ, (ii) α

4(i)   **4(ii)**

Figure 4: Steady-state availability of the system v/s (i) λ, (ii) α

From all tables and figures, we conclude that the major changes in various performance measures are observed for failure rate of operating machine and spre machines servers. Our recommendations from present study are that we have to evaluate various performance measures of the system concerned to examine the sensitivity of different input parameters. It is evident that just by increasing the service rate nominally after some extent we will not get the reduction in the workload of failed machines. We also have to determine the minimum number (m) of operating machines as well as the number of warm spares (S) required so that the system may run successfully without interruption.

6. Conclusion and Future Scope

In the present investigation, various performance indices of (m,M) machine repair problem are obtained to provide an insight into the performance and availability of such system. The machine repair problems are likely to be more complex in future, and as such it is natural to pay attention towards new technologies and congestion problems arising out of machine schedules which are prone to failure. Our study may be helpful to the system designers and decision makers to determine the optimal policy to achieve required efficiency and availability of the system under unavoidable techno-economic constraints of spare provisioning and repair facility.

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