COST- BENEFIT ANALYSIS OF A SINGLE-UNIT SYSTEM SUBJECT TO RANDOM SHOCKS

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Abstract

The purpose of the present study is to carry out cost-benefit analysis of a single-unit system subject to random shocks. The operative unit may be affected by the impact of random shocks with some probabilities. The unit may fail completely due to the reasons other than shocks. There is a single server who visits the system immediately. The maintenance of the unit is carried out by the server if it is affected by the impact of a shock. However, repair of the unit is done at its failure. Maintenance and repair are perfect. All the random variables are statistically independent. The shock and failure times of the unit are exponentially distributed whereas distributions of maintenance and repair times are taken as arbitrary. The expressions for various reliability measures are evaluated in steady state using semi-Markov process and regenerative point technique. The values of MTSF, availability and profit functions are obtained for a particular case to depict their graphical behavior with respect to shock rate.

1. Introduction

No doubt that the technique of redundancy has been proved as one of the effective strategy for performance improvement of a system. But there are many systems in which a unit cannot be kept as spare due to its high cost. And, so single-unit systems are being preferred by the users due to their affordability and inherent reliability. Several authors including Chander and Bansal (2005) and Malik (2008) have analyzed single-unit reliability models considering different failure and repair policies. But most of these models have been probed under the common assumptions that failures occur in the system due to reasons other than shocks. In fact, shocks are the events which can be one of the causes of the system failure and deterioration. Murari and Al-Ali (1988) developed a reliability model of a single unit system with the impact of random shocks. Gupta and Chaudhary (1992) analyzed a two-unit priority standby system subject to random shocks and Rayleigh failure time distribution. Wu and Wu (2011) obtained reliability of a two-unit cold standby repairable system under Poison shocks.

On the other hand, maintenance can be conducted to stop further damage of the system affected by random shocks. However, not much work related to the reliability modeling of single-unit systems subject to random shocks has been reported

so far in the literature of reliability using the concepts of maintenance and repair simultaneously. Malik and Chhillar (2012) tried to establish reliability model of a cold standby system under maintenance and repair subject to random shocks.

While considering the above observations and facts in mind, here a shock model for a single unit system is developed under maintenance and repair. The operative unit suffers damage with the impact of random shocks with some probabilities. The unit may fail completely due to the reasons other than shocks such as wear out. There is a single server who visits the system immediately. The maintenance of the unit is carried out by the server if the unit is affected by the impact of a shock. Server repairs the unit at its failure. Repair, maintenance and switch devices are perfect. All random variables are statistically independent. The shock and failure times of the unit are exponentially distributed whereas distributions of maintenance and repair times are taken as arbitrary. To carry out cost-benefit analysis, the expressions for various reliability measures such as transition probabilities, mean sojourn times, mean time to system failure (MTSF), availability, busy period of the server due to repair and maintenance, expected number of maintenances, expected number of repairs and profit function are evaluated in steady state using semi-Markov process and regenerative point technique. The values of MTSF, availability and profit functions are obtained for a particular case to depict their graphical behavior with respect to shock rate.

2. Notations

O   :   The unit is operative and in normal mode.
p_0 :   The probability that shock is effective.
q_0 :   The probability that shock is not effective.
μ :   Constant rate of the occurrence of a shock.
λ :   Constant failure rate of the unit.
m(t)/M(t)   :   pdf / cdf of maintenance time of the unit after the effect of a shock.
FUr   :   The Unit is completely failed and under repair
SUm   :   Shocked unit under maintenance
g (t) / G (t)   :   pdf / cdf of repair time of the completely failed unit
$q_{ij}(t) / Q_{ij}(t)$:   pdf and cdf of direct transition time from a regenerative state i to a
                                    regenerative state j without visiting any other regenerative state
$M_i(t)$:   Probability that the system is up initially in state $S_i \in E$ is up at
                                    time t without visiting to any other regenerative state.
$W_i(t)$: Probability that the server is busy in state S_i up to time t without
                                    making transition to any other regenerative state or returning to
                                    the same via one or more non regenerative states.
m_{ij} :   Contribution to mean sojourn time (μ_i) in state S_i when system
                                    transit directly to state S_j so that $\mu_i = \sum_j m_{ij}$ and

$$m_{ij} = \int t dQ_{ij}(t) = -q_{ij}^{*}{}'(0)$$

(s) / ©   :   Symbol for Laplace Stieltjes convolution / Laplace convolution.
~ / *   :   Symbol for Laplace Stieltjes Transform (LST) / Laplace
                                    Transform (LT).
All the transitions states S_0, S_1 and S_2 are regenerative as shown in Figure 1.

3. Transition Probabilities and Mean Sojourn Times

Simple probabilistic considerations yield the following expressions for the non-zero elements

$$p_{ij} = Q_{ij}(\infty) = \int_0^\infty q_{ij}(t)dt \quad \text{as}$$

$$p_{00} = \frac{q_0\mu}{\lambda+\mu}, \quad p_{01} = \frac{p_0\mu}{\lambda+\mu}, \quad p_{02} = \frac{\lambda}{\lambda+\mu}, \quad p_{10} = m*(0) \text{ and } p_{20} = g*(0) \qquad (1)$$
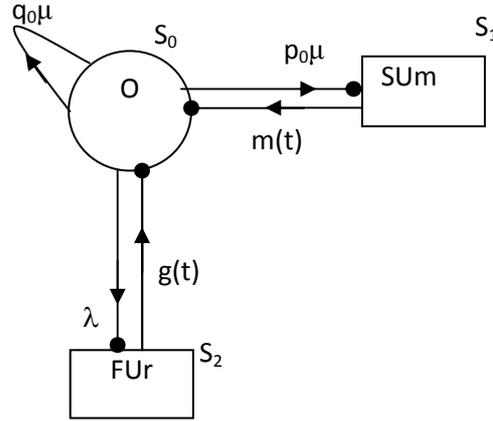
Fig. 1: State Transition Diagram

It can be easily verified that

$$p_{00} + p_{01} + p_{02} = p_{10} = p_{20} = 1 \qquad (2)$$

The mean sojourn times (μ_i) in the state S_i are

$$m_{00} + m_{01} + m_{02} = \mu_0, \quad m_{10} = \mu_1, \quad m_{20} = \mu_2$$

4. Reliability and Mean Time to System Failure (MTSF)

Let $\phi_i(t)$ be the cdf of first passage time from the regenerative state S_i to a failed state. Regarding the failed state as absorbing state, we have the following recursive relations for $\phi_i(t)$:

$$\varphi_i(t) = \sum_j Q_{i,j}(t) \circledR \varphi_j(t) + \sum_k Q_{i,k}(t) \qquad (3)$$

where S_j is an un-failed regenerative state to which the given regenerative state S_i can transit and k is a failed state to which the state S_i can transit directly. Taking LST of above relation (3) and solving for $\tilde{\phi}_0(s)$, we have

$$R*(s) = \frac{1 - \tilde{\varphi}_0(s)}{s} \qquad (4)$$

The reliability of the system model can be obtained by taking inverse Laplace transform of (4).

The mean time to system failure (MTSF) is given by

$$\text{MTSF} = \lim_{s \to o} \frac{1 - \tilde{\phi}_0(s)}{s} = \frac{N_1}{D_1} \tag{5}$$

Where $N_1 = \mu_0$ and $D_1 = 1 - p_{00}$

5. Steady State Availability

Let $A_i(t)$ be the probability that the system is in up-state at instant 't' given that the system entered regenerative state S_i at t = 0. The recursive relations for $A_i(t)$ are given as

$$A_i(t) = M_i(t) + \sum_j q_{i,j}(t) © A_j(t) \tag{6}$$

where S_j is any successive regenerative state to which the regenerative state S_i can transit. $M_i(t)$ is the probability that the system is up initially in state $S_i \in E$ is up at time t without visiting to any other regenerative state, we have

$$M_0(t) = e^{-(\lambda + \mu)t} \tag{7}$$

Taking LT of above relations (6) and solving for $A_0^*(s)$, the steady state availability is given by

$$A_0(\infty) = \lim_{s \to 0} s A_0^*(s) = \frac{N_2}{D_2} \text{, Where } N_2 = \mu_0 \text{ and } D_2 = 1\text{-}p_{00}\text{-}p_{10}p_{01}\text{-}p_{02}p_{20}$$

6. Busy Period Analysis of the Server

a. Due to repair

Let $B_i^R(t)$ be the probabilities that the server is busy in repair at an instant 't' given that the system entered state S_i at t = 0. The recursive relations for $B_i^R(t)$ are as follows

$$B_i^R(t) = W_i(t) + \sum_j q_{i,j}(t) © B_j^R(t) \tag{8}$$

where S_j is any successive regenerative state to which the regenerative state S_i can transit. Let $W_i(t)$ be the probability that the server is busy in state S_i due to repair up to time t without making any transition to any other regenerative state or returning to the same via one or more non-regenerative states. We have $W_2(t) = \overline{G(t)}$

Taking LT of above relations (8) and, solving for $B_i^{*R}(s)$ the time for which server is busy due to repair is given by

$$B_0^R = \lim_{s \to 0} s B_0^{*R}(s) = \frac{N_3^R}{D_2} \tag{9}$$

where

$N_3 = p_{02} w_2^*(s)$ and D_2 is already defined.

b. Due to Corrective Maintenance

Let $B_i^M(t)$ be the probabilities that the server is busy in Corrective maintenance at an instant 't' given that the system entered state S_i at $t = 0$. The recursive relations for $B_i^M(t)$ are as follows

$$B_i^M(t) = W_i(t) + \sum_j q_{i,j}(t) © B_j^M(t) \tag{10}$$

where S_j is any successive regenerative state to which the regenerative state S_i can transit. Let $W_i(t)$ be the probability that the server is busy in state S_i due to corrective maintenance up to time t without making any transition to any other regenerative state or returning to the same via one or more non-regenerative states. We have

$$W_1(t) = \begin{aligned} & e^{-(\lambda+\mu)t}\,\overline{M(t)} + (\lambda e^{-(\lambda+\mu)t} © 1)\overline{M(t)} + (p_0 \mu e^{-(\lambda+\mu)t} © 1)\overline{M(t)} \\ & + (q_0 \mu e^{-(\lambda+\mu)t} © 1)\overline{M(t)} \end{aligned}$$

Taking LT of above relations (10) and, solving for $B_i^{*M}(s)$ the time for which server is busy due to corrective maintenance is given by

$$B_0^M = \lim_{s \to 0} s B_0^{*M}(s) = \frac{N_3^M}{D2} \tag{11}$$

where

$N_4 = p_{01}\, w_1^*(s)$ and D_2 is already defined.

7. Expected Number of Corrective Maintenance

Let $N_i^M(t)$ be the probabilities that the server is busy in corrective maintenance at an instant 't' given that the system entered state S_i at $t = 0$. The recursive relations for $N_i^M(t)$ are as follows

$$N_i^M(t) = \sum_j Q_{i,j}(t) ® \left[\delta_j + N_j^M(t) \right] \tag{12}$$

where S_j is any successive regenerative state to which the regenerative state S_i can transit. Taking LST of above relations (12) and, solving for $N_i^{*M}(s)$ the time for which server is busy due to corrective maintenance is given by

$$N_0^M = \lim_{s \to 0} \frac{N_0^{M*}(s)}{s} = N_4/D_2 \text{ where } N_4 = p_{10}\, p_{01} \text{ and } D_2 \text{ is already defined.}$$

8. Expected Number of Repairs

Let $N_i^R(t)$ be the probabilities that the server is busy in repair at an instant 't' given that the system entered state S_i at $t = 0$. The recursive relations for $N_i^R(t)$ are as follows

$$N_i^R(t) = \sum_j Q_{i,j}(t) ® \left[\delta_j + N_j^R(t) \right] \tag{13}$$

Where S_j is any successive regenerative state to which the regenerative state S_i can transit. Taking LST of above relations (13) and, solving for $N_i^{*R}(s)$ the time for which server is busy due to repair is given by

$$N_0^R = \lim_{s \to 0} \frac{N_0^{R*}(s)}{s} = N_5/D_2$$

Where $N_5 = p_{20}p_{02}$ and D_2 is already defined.

9. Profit Analysis
        The profit incurred to the system model in steady state can be obtained as

$$P = K_0 A_0 - K_1 B_0^M - K_2 B_0^R - K_3 N_0^M - K_4 N_0^R - K_5 \qquad (14)$$

where

K_0 = Revenue per unit up-time of the system

K_1 = Cost per unit time for which server is busy due to maintenance

K_2 = Cost per unit time for which server is busy due to repair

K_3 = Maintenance cost per unit

K_4 = Repair cost per unit

K_5 = fixed cost of the server and $A_0, B_0^M, B_0^R, N_0^M, N_0^R$ are already defined.
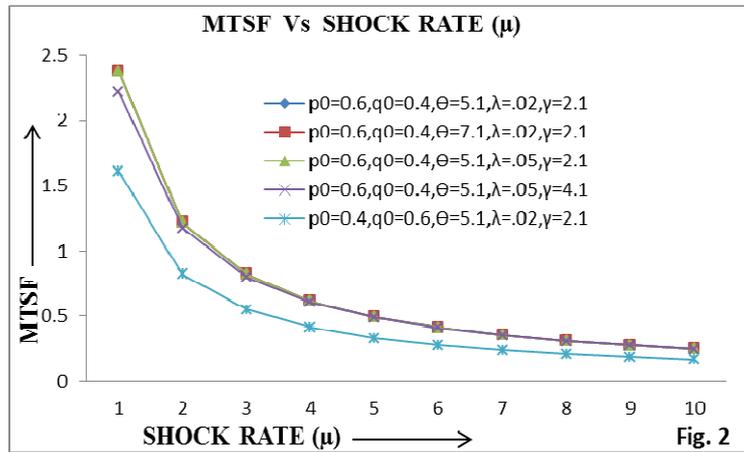
Fig. 2: MTSF Vs. Shock Rate
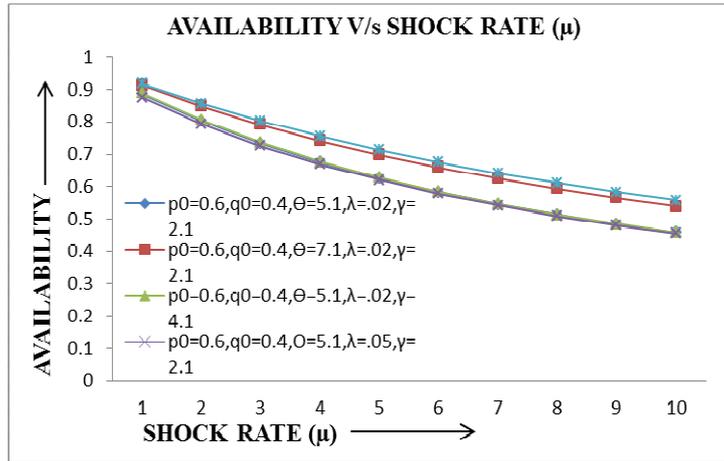
Fig. 3: Availability Vs. Shock Rate
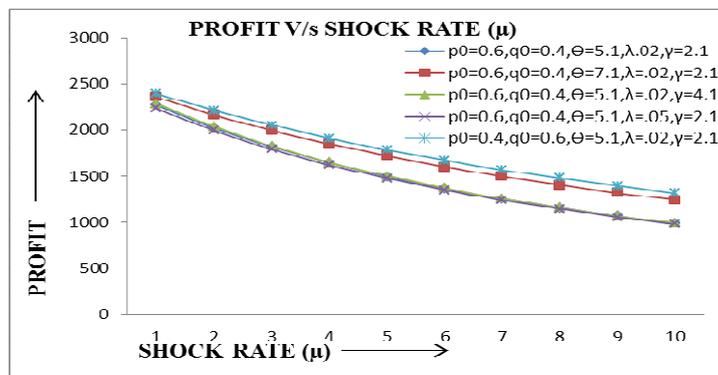
Fig. 4: Profit Vs. Shock Rate

10. Conclusion

The results obtained for a particular case $g(t) = \gamma e^{-\gamma t}$, $m(t) = \theta e^{-\theta t}$ indicate that mean time to system failure (MTSF), availability and profit go on decreasing with the increase of shock rate (μ) and failure rate (λ) for fixed values of other parameters and costs as shown respectively in figures 2, 3 and 4. However, their values keep on increasing with the increase of maintenance rate and repair rate of the unit. It is interesting to note that MTSF decreases by interchanging the values of p_0 and q_0, i.e., $p_0=0.4$ and $q_0= 0.6$ while availability and profit increase. Hence, the study reveals that a single- unit system subjected to random shocks can be made more profitable and reliable to use by increasing the maintenance rate of the shocked unit.

11. References

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