Journal of Reliability and Statistical Studies; ISSN (Print): 0974-8024, (Online):2229-5666
Vol. 7, Issue Special (2014): 77-88

RELIABILITY MEASURES OF A SINGLE-UNIT SYSTEM UNDER PREVENTIVE MAINTENANCE AND DEGRADATION WITH ARBITRARY DISTRIBUTIONS OF RANDOM VARIABLES

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Received July 29, 2013
Modified January 17, 2014
Accepted March 04, 2014

Abstract

The main objective of this paper is to analyze a single-unit system with arbitrary distributions for all random variables associated with failure time, preventive maintenance (PM), maximum operation time (MOT), inspection and repair times. There is a single server who visits the system immediately whenever needed. The partially failed unit undergoes for PM after a MOT. The unit is considered as degraded after repair while preventive maintenance is perfect. Server inspects the degraded unit at its failure to see the feasibility of repair. If repair of the degraded unit is not feasible, it is replaced immediately by new one. The expressions for some reliability measures are obtained in steady state using regenerative point technique. Giving particular values to various parameters and costs, the numerical results for mean time to system failure (MTSF), availability and profit function are obtained considering exponential and Rayleigh distributions for all random variables.

Key Words: Single-Unit System, Preventive Maintenance, Maximum Operation Time, Degradation, Reliability Measures.

1. Introduction

Now a day's single-unit systems are being preferred over standby systems by the users in every sphere of life because of their affordability and inherent reliability. And, most of these systems have been investigated by the researchers including Nakagawa and Osaki [7], Kuo and Liang [4], Chander [1], Malik [6], Savita et al. [8], Liu and Liu [5], Kaur et al. [3], and Kadyan and Ramniwas [2] under the following assumptions:

i. The unit has a constant failure rate.
ii. The unit can work as good as new after repair.
iii. The unit can work forever without conducting preventive maintenance.
iv. Repair of the unit is always feasible to the system.

But the hazard rates of many components (or systems) such as rotating shafts, valves and cams are of linearly increased nature due to wear out under mechanical stress and so their failure times follow arbitrary distributions like Rayleigh distribution.

Again, the continued operation and ageing of systems gradually reduce their performance, reliability and safety. Therefore, PM of these systems may be conducted after a pre-specific period of time at any stage of their operation to slow the deterioration process as well as to restore the systems in a younger age or state.

Further, sometimes, unit does not work as new after repair. Since the capability of the unit after repair depends on the repair mechanism adopted and unit may have increased failure rate, if it is repaired by an ordinary server. In such a situation unit becomes degraded. Furthermore, repair of a degraded unit is not always feasible to the system may because of its excessive use and high cost of maintenance. In such cases, the failed degraded unit may be replaced by new one and this can be revealed by inspection.

In view of the above, here reliability measures for a single-unit system are obtained considering arbitrary distributions for random variables associated with failure time, PM, MOT, inspection and repair times. The unit may fail completely either directly from normal mode or via partial failure. A single service facility is provided immediately to the system as and when needed. The PM of the unit at its partial failure is conducted after a MOT. However, repair of the unit is done at its complete failure. The unit works as new after PM while it becomes degraded after repair. The degraded unit is inspected by the server to see the feasibility of repair at its failure. If repair of the degraded unit is not feasible, it is replaced immediately by new one in order to avoid the unnecessary expanses on repair. Various reliability measures such as mean sojourn times, MTSF, availability, busy period of the server due to repair activities, expected number of visits by the server to conduct repair activities and profit function are evaluated in steady state using regenerative point technique. Giving particular values to various parameters and costs, the numerical results for MTSF, availability and profit function are obtained considering exponential and Rayleigh distributions for all random variables.

•Methodology

The system has been analyzed using well known semi-Markov process and regenerative point technique which are briefly described as:

Markov Process: If $\{X(t), t\in T\}$ is a stochastic process such that, given the value of X(s), the value of X(t), t>s do not depend on the values of X(u), u<s Then the process $\{X(t), t\in T\}$ is a Markov process.

Semi-Markov Process: A semi-Markov process is a stochastic process in which changes of state occur according to a Markov chain and in which the time interval between two successive transitions is a random variable, whose distribution may depend on the state from which the transition take place as well as on the state to which the next transition take place.

Regenerative Process: Let X(t) be the state of the system of epoch. If t_1, t_2,… are the epochs at which the process probabilistically restarts, then these epochs are called regenerative epochs and the process $\{X(t), t = t_1, t_2\ldots\ldots\}$ is called regenerative process. The state in which regenerative points occur is known as regenerative state.

2. System Description and Assumptions

1) The system has a single unit may fail totally either directly from normal mode or via. partial failure.

2) There is a single server who visits the system immediately whenever needed.
3) The partially failed unit undergoes for PM after a MOT.
4) The unit is considered as degraded after repair while preventive maintenance is perfect.
5) Server inspects the degraded unit at its failure to see the feasibility of repair.
6) If repair of the degraded unit is not feasible, it is replaced immediately by new unit.
7) Distributions for all random variables associated with failure time, PM, MOT, inspection and repair times are taken as arbitrary.
8) All random variables are uncorrelated and mutually independent.

3. Notations

E	Set of regenerative states.
O	The unit is new and operative
Do	The unit is degraded and operative.
PFO	The unit is partially failed and operative.
DFUi	Degraded unit is failed and under inspection.
DFUr	Degraded unit is failed and under repair.
PFP_m	The unit is partially failed and under PM.
Fur	The unit is completely failed and under repair.
$f(t)/F(t)$, $f_1(t)/F_1(t)$, $f_2(t)/F_2(t)$	Probability density function (pdf)/cumulative distribution function (cdf) of failure time of the unit from normal mode to complete failure/Normal mode to partial failure/ partial failure to complete failure.
$f_3(t)/F_3(t)$	pdf/cdf of failure time of degraded unit.
$z(t)/Z(t)$	pdf/cdf of the MOT after partial failure.
$g(t)/G(t)$	pdf /cdf of PM time of the unit.
$g_1(t)/G_1(t)$	pdf/cdf of repair time of the failed new unit.
$g_2(t)/G_2(t)$	pdf/cdf of repair time of the failed degraded unit.
$h(t)/H(t)$	pdf/cdf of inspection time of degraded unit.
p/q	Probability that repair of degraded system is feasible/not feasible.
$q_{ij}(t)/Q_{ij}(t)$	pdf/cdf of first passage time from regenerative state *i* to a regenerative state *j* or to a failed state *j* without visiting any other regenerative state in (0,t].
$M_i(t)$	Probability that system is up initially in state $S_i \in E$ is up at time *t* without visiting to any other regenerative sate
$W_i(t)$	Probability that server is busy in the state S_i up to time *t* without making any transition to any other regenerative state or returning to the same via one or more non-regenerative states

4. State Specification

S_0	:	The unit is new and operative.
S_1	:	The unit is partially failed.
S_2	:	The unit is completely failed and under repair.
S_3	:	The unit is partially failed and under PM
S_4	:	The unit is degraded and operative
S_5	:	Degraded unit is failed and under inspection

S_6 : Degraded unit is failed and under repair.

The following are the transition states of the system model:

Up states: S_0= (O), S_1= (PFO), S_4= (Do).

Down states: S_2= (FUr), S_3= (PFP_m), S_5= (DFUi), S_6= (DFUr).

All the transition states are regenerative. Thus E = {S_i ; 0≤i ≤6}. The state transition diagram has been shown in Figure 1.

5. Transition Probabilities and Mean Sojourn Times

The transition probability matrix (t.p.m.) of embedded Markov-chain is

$$p = (p_{ij}) = (Q_{ij}(\infty) = Q(\infty))$$

By probabilistic arguments, the non-zero elements p_{ij} are

$$p_{01} = \int_0^\infty f_1(t)\overline{F}(t)dt \qquad (**)$$

Where p_{01} means that probability that the complete failure of new unit does not occur until time t and the new unit is partially failed at time t.

All other transition probabilities can be explained in the same manner.

$$p_{02} = \int_0^\infty f(t)\overline{F_1}(t)dt, \quad p_{12} = \int_0^\infty f_2(t)\overline{Z}(t)dt, \quad p_{13} = \int_0^\infty z(t)\overline{F_2}(t)dt,$$

$$p_{24} = \int_0^\infty g_1(t)dt, \quad p_{30} = \int_0^\infty g(t)dt, \quad p_{45} = \int_0^\infty f_3(t)dt,$$

$$p_{50} = \int_0^\infty qh(t)dt, \quad p_{56} = \int_0^\infty ph(t)dt \quad p_{64} = \int_0^\infty g_2(t)dt. \qquad (1)$$

For these transition probabilities, it can be verified that

$p_{01}+p_{02}=p_{12}+p_{13}=p_{24}=p_{30}=p_{45}=p_{50}+p_{56}=p_{64}=1$

5.1 Mean Sojourn Times

The mean sojourn times (μ_i) in state S_i are given by

$\mu_i = E(T) = \int_0^\infty P(T > t)dt$; where T denotes the time to system failure.

$$\mu_0 = \int_0^\infty \overline{F}(t)\overline{F_1}(t)dt, \quad \mu_1 = \int_0^\infty \overline{Z}(t)\overline{F_2}(t)dt, \quad \mu_2 = \int_0^\infty \overline{G_1}(t)dt,$$

$$\mu_3 = \int_0^\infty \overline{G}(t)dt, \quad \mu_4 = \int_0^\infty \overline{F_3}(t)dt, \quad \mu_5 = \int_0^\infty \overline{H}(t)dt,$$

$$\mu_6 = \int_0^\infty \overline{G_2}(t)dt \qquad (2)$$

The unconditional mean (m_{ij}) time taken by the system to transit from any regenerative state S_i when time is counted from epoch of entrance into state S_j is given by

$$m_{ij} = \int t dQ_{ij}(t) = -\left[\frac{d}{ds}\left(Q_{ij}^{**}(s)\right)\right]_{s=0} \tag{3}$$

$$\mu_i = \sum_j m_{ij} \tag{4}$$

5.2 Relationship between Unconditional Mean and Mean Sojourn Times

$m_{01} + m_{02} = \mu_0$, $m_{12} + m_{13} = \mu_1$ $m_{24} = \mu_2$

$m_{34} = \mu_3$, $m_{45} = \mu_4$ $m_{50} + m_{56} = \mu_5$,

$m_{64} = \mu_6$

6. Reliability Measures

6.1 Mean Time to System Failure (MTSF)

Let $\phi_i(t)$ be the cdf of the first passage time from regenerative state *i* to a failed state. Regarding the failed state as absorbing state, we have the following recursive relations for $\phi_i(t)$:

$$\phi_i(t) = \sum_j Q_{i,j}(t)Ⓢ\phi_j(t) + \sum_k Q_{i,k}(t) \tag{5}$$

where *j* is an operative regenerative state to which the given regenerative state *i* can transit and *k* is a failed state to which the state *i* can transit directly.

Taking Laplace Stieltjes Transform of relations (5) and solving for $\tilde{\phi}_0(s)$.

We have

$$R^*(s) = (1 - \tilde{\phi}_0(s))/s \tag{6}$$

The reliability R(t) can be obtained by taking Laplace inverse transform of (6).

$MTSF = \mu_0 + p_{01}\mu_1$

6.2 Availability Analysis

Let $A_i(t)$ be the probability that the system is in up-state at instant *t* given that the system entered regenerative state *i* at *t=0*. The recursive relations for $A_i(t)$ are given by

$$A_i(t) = M_i(t) + \sum_j q_{i,j}(t)©A_j(t) \tag{7}$$

where *j* is any successive regenerative state to which the regenerative state *i* can transit. We have

$M_0(t) = \int_0^\infty \overline{F}(t)\overline{F_1}(t)dt$ $M_1(t) = \int_0^\infty \overline{Z}(t)\overline{F_2}(t)dt$ $M_4(t) = \int_0^\infty \overline{F_3}(t)dt$

Taking Laplace Transform of relations (7) and solving for $A_0^*(s)$.

The steady-state availability of the system can be given by

$$A_0(\infty) = \lim_{s\to 0} sA_0^*(s) = \frac{N_2}{D_2} \tag{8}$$

Where

$N_2 = p_{50}(\mu_0 + p_{01}\mu_1) + \mu_4[1 - p_{01}p_{13}]$

$D_2 = p_{50}[\mu_0 + p_{01}(\mu_1 + p_{13}\mu_3)] + (1 - p_{01}p_{13})[p_{50}\mu_2 + \mu_4 + \mu_5 + p_{56}\mu_6]$

6.3 Busy Period Analysis

6.3.1 Busy Period Analysis Due to Repair

Let $BR_i(t)$ be the probability that the server is busy in repairing the unit at an instant 't' given that the system entered regenerative state i at t = 0. The recursive relations for $BR_i(t)$ are given as

$$BR_i(t) = W_i(t) + \sum_j q_{i,j}(t) © BR_j(t) \tag{9}$$

where j is any successive regenerative state to which the regenerative state i can transit. We have

$$W_2(t) = \int_0^\infty \overline{G_1}(t)dt \ , \qquad W_6(t) = \int_0^\infty \overline{G_2}(t)dt$$

Taking Laplace Transform of relations (9) and solving for $BR_0^*(s)$ and using this, we can obtain the fraction of time for which the repairman is busy in steady state

$$BR_0 = \lim_{s\to 0} s.BR_0(s) = \frac{N_3}{D_2} \tag{10}$$

Where

$N_3 = (1 - p_{01}p_{13})(p_{56}\mu_6 + p_{50}\mu_2)$,

D_2 is already mentioned.

6.3.2 Busy Period Analysis Due to PM

Let $BP_i(t)$ be the probability that the server is busy for PM at an instant't' given that the system entered regenerative state i at t = 0. The recursive relations for $BP_i(t)$ are given as

$$BP_i(t) = W_i(t) + \sum_j q_{i,j}(t) © BP_j(t) \tag{11}$$

where j is any successive regenerative state to which the regenerative state i can transit.

$$W_3(t) = \int_0^\infty \overline{G}(t)dt \ ,$$

Taking Laplace Transform of above relations (11) and solving for $BP_0^*(s)$ we get in the long run time for which the system is under preventive maintenance as

$$BP_0 = \lim_{s\to 0} s.BP_0(s) = \frac{N_4}{D_2} \tag{12}$$

where

$N_4 = p_{01}p_{13}p_{50}\mu_3$,

D_2 is already mentioned.

6.3.3 Busy Period Analysis Due to Inspection

Let $BI_i(t)$ be the probability that the server is busy for inspection at an instant 't' given that the system entered regenerative state i at t = 0. The recursive relations for $BI_i(t)$ are given as

$$BI_i(t) = W_i(t) + \sum_j q_{i,j}(t) © BI_j(t) \tag{13}$$

Where *j* is any successive regenerative state to which the regenerative state *i* can transit.
We have

$$W_5(t) = \int_0^\infty \overline{H}(t)dt,$$

Taking Laplace Transform of above relations (13) and solving for $BI_0^*(s)$, we get in the long run time for which the system is under preventive maintenance as

$$BI_0 = \lim_{s\to 0} s.BI_0(s) = \frac{N_5}{D_2} \tag{14}$$

Where
$N_5 = (1 - p_{01}p_{13})\mu_5$,
D_2 is already mentioned.

6.4 Expected Number of Visit by Server

6.4.1 Expected Number of Visit by Server Due to Repair

Let $NR_i(t)$ be the expected number of visits by the server due to repair in (0, t] given that the system entered the regenerative state i at t = 0. The recursive relations for $NR_i(t)$ are given as

$$NR_i(t) = \sum_j Q_{i,j}(t) ® \left[\delta_j + NR_j(t)\right] \tag{15}$$

Where j is any regenerative state to which the given regenerative state *i* transits and $\delta_i = 1$, if *j* is the regenerative state where the server does job afresh, otherwise $\delta_i = 0$.

Taking Laplace Stieltjes Transform of relations (15) and solving for $\widetilde{NR}_0(s)$.
We get the expected number of visits by server for repair per unit time as

$$NR_0 = \lim_{s\to 0} NR_0(s) = \frac{N_6}{D_2},$$

$N_6 = p_{02}p_{50}$,
D_2 is already mentioned.

6.4.2 Expected Number of Visit Due to Inspection

Let $NI_i(t)$ be the expected number of visits by the server due to inspection in (0, t] given that the system entered the regenerative state i at t = 0. The recursive relations for $NI_i(t)$ are given as

$$NI_i(t) = \sum_j Q_{i,j}(t) ® \left[\delta_j + NI_j(t)\right] \tag{16}$$

Where j is any regenerative state to which the given regenerative state *i* transits and $\delta_i = 1$, if *j* is the regenerative state where the server does job afresh, otherwise $\delta_i = 0$.

Taking Laplace Stieltjes Transform of relations (16) and solving for $\widetilde{NI}_0(s)$.
We get the expected number of visits by server for inspection per unit time as

$$NI_0 = \lim_{s\to 0} s.NI_0(s) = \frac{N_7}{D_2}$$

$N_7 = (1 - p_{01}p_{13})$,
D_2 is already mentioned.

7. Profit Analysis

Any manufacturing industry is basically a profit making organization and no organization can survive for long without minimum financial returns for its investment. There must be an optimal balance between the reliability aspect of a product and its cost. The major factors contributing to the total cost are availability, busy period of server and expected number of visits by the server. The cost of these individual items varies with reliability or mean time to system failure. In order to increase the reliability of the products, we would require a correspondingly high investment in the research and development activities. The production cost also would increase with the requirement of greater reliability.

The revenue and cost function lead to the profit function of a firm, as the profit is excess of revenue over the cost of production. The profit function in time t is given by:-

P(t) = Expected revenue in (0, t] – Expected total cost in (0, t]

In general, the optimal policies can more easily be derived for an infinite time span or compared to a finite time span. The profit per unit time, in infinite time span is expressed as:

$$\lim_{t \to \infty} \frac{P(t)}{t}$$

i.e. profit per unit time = total revenue per unit time – total cost per unit time. Considering the various costs, the profit equation is given as
Profit incurred to the system model in steady state is given by

$$P = K_1A_0 - K_2BR_0 - K_3BP_0 - K_4BI_0 - K_5NR_0 - K_6NI_0$$

Where
K_1: Revenue per unit up-time of the system.
K_2: Cost per unit time for which server is busy in repair.
K_3: Cost per unit time for which server is busy in preventive maintenance.
K_4: Cost per unit time for which server is busy in inspection.
K_5: Cost per unit visit by the server for repair.
K_6: Cost per unit visit by the server for inspection.

8. Results and Discussion

To show the importance of results and characterize the behavior of MTSF, availability and profit of the system, here we assume that failure, MOT, PM, inspection and repair times as Weibull distributed with two parameters. Probability density function of Weibull distribution with two parameters is given by

$$f(t) = \lambda t^b \exp\left[-\lambda t^{b+1} \big/ b+1\right] \qquad t \geq 0$$

Where b and λ are positive constants and are known as shape and scale parameters respectively. From the properties of Weibull distribution, If b = 0, it become the exponential distribution and when b = 1, it become the Rayleigh distribution.
Let

$$F_1(t)=1-\exp\left[\frac{-\lambda_1 t^{b+1}}{b+1}\right],\qquad F_2(t)=1-\exp\left[\frac{-\lambda_2 t^{b+1}}{b+1}\right],$$

$$F_3(t)=1-\exp\left[\frac{-\lambda_3 t^{b+1}}{b+1}\right],\qquad G(t)=1-\exp\left[\frac{-\theta t^{b+1}}{b+1}\right],$$

$$G_1(t)=1-\exp\left[\frac{-\theta_1 t^{b+1}}{b+1}\right],\qquad G_2(t)=1-\exp\left[\frac{-\theta_2 t^{b+1}}{b+1}\right],$$

$$Z(t)=1-\exp\left[\frac{-\alpha t^{b+1}}{b+1}\right],\qquad H(t)=1-\exp\left[\frac{-\alpha_1 t^{b+1}}{b+1}\right],$$

$$F(t)=1-\exp\left[\frac{-\lambda t^{b+1}}{b+1}\right].$$

$$\text{MTSF}=\frac{\Gamma\left(\frac{1}{b+1}\right)}{(b+1)^{b/(b+1)}}\left[\frac{1}{(\lambda+\lambda_1)^{1/(b+1)}}+\frac{\lambda_1}{(\lambda+\lambda_1)(\alpha+\lambda_2)^{1/(b+1)}}\right]$$

$$N_2=\frac{\Gamma\left(\frac{1}{b+1}\right)}{(b+1)^{b/(b+1)}}\left[\begin{array}{l} q\left(\frac{1}{(\lambda+\lambda_1)^{1/(b+1)}}+\frac{\lambda_1}{(\lambda+\lambda_1)(\alpha+\lambda_2)^{1/(b+1)}}\right)+\left(\frac{1}{(\lambda_3)^{1/(b+1)}}\right)\\ \left(\frac{(\lambda+\lambda_1)(\alpha+\lambda_2)-\alpha\lambda_1}{(\lambda+\lambda_1)(\alpha+\lambda_2)}\right)\end{array}\right]$$

$$D_2=\frac{\Gamma\left(\frac{1}{b+1}\right)}{(b+1)^{b/(b+1)}}\left[\begin{array}{l} q\left(\frac{1}{(\lambda+\lambda_1)^{1/(b+1)}}+\left(\frac{\lambda_1}{(\lambda+\lambda_1)}\right)\left(\frac{1}{(\alpha+\lambda_2)^{1/(b+1)}}+\frac{\alpha}{(\alpha+\lambda_2)(\theta)^{1/(b+1)}}\right)\right)\\ +\left(\frac{(\lambda+\lambda_1)(\alpha+\lambda_2)-\alpha\lambda_1}{(\lambda+\lambda_1)(\alpha+\lambda_2)}\right)\left(\begin{array}{l}\frac{q}{(\theta_1)^{1/(b+1)}}+\frac{1}{(\lambda_3)^{1/(b+1)}}\\ +\frac{1}{(\alpha_1)^{1/(b+1)}}+\frac{p}{(\theta_2)^{1/(b+1)}}\end{array}\right)\end{array}\right]$$

$$N_3=\frac{\Gamma\left(\frac{1}{b+1}\right)}{(b+1)^{b/(b+1)}}\left[\left(\frac{(\lambda+\lambda_1)(\alpha+\lambda_2)-\alpha\lambda_1}{(\lambda+\lambda_1)(\alpha+\lambda_2)}\right)\left(\frac{q}{(\theta_1)^{1/(b+1)}}+\frac{p}{(\theta_2)^{1/(b+1)}}\right)\right]$$

$$N_4=\frac{\Gamma\left(\frac{1}{b+1}\right)}{(b+1)^{b/(b+1)}}\left[\left(\frac{(\lambda_1)(\alpha)q}{(\lambda+\lambda_1)(\alpha+\lambda_2)(\theta)^{1/(b+1)}}\right)\right]$$

$$N_5=\frac{\Gamma\left(\frac{1}{b+1}\right)}{(b+1)^{b/(b+1)}}\left[\left(\frac{(\lambda+\lambda_1)(\alpha+\lambda_2)-\alpha\lambda_1}{(\lambda+\lambda_1)(\alpha+\lambda_2)(\alpha_1)^{1/(b+1)}}\right)\right]\qquad N_6=\frac{q\lambda}{(\lambda+\lambda_1)}$$

Giving particular values to various parameters and costs, the numerical results for MTSF, availability and profit function are obtained by considering exponential and Rayleigh distributions for all random variables associated with failure, preventive maintenance, inspection, maximum operation and repair times as shown in tables 1,2 and 3.

Table 1 to table 3 reflect the behavior of MTSF, availability and profit of the system model with respect to maximum rate of operation time when the distribution of failure, maximum rate of operation, PM, inspection and repair times of the unit are taken as exponential and Rayleigh distribution.

Tables 1 indicates that MTSF keeps on decreasing with the increase of maximum rate of operation (α). The same trends are found for MTSF in respect of failure rates (λ and λ_1) for fixed values of other parameters. It is also observed that the value of MTSF is more for exponential distribution in comparision to Rayleigh distribution.

The behavior of availability and profit of the system with respect to maximum rate of operation time is shown in table 2 and table 3 respectively. These tables show that availability and profit of the system decrease with the increase of maximum rate of operation (α) and failure rates (λ and λ_1) for fixed values of other parameters K_1=5000, K_2=500, K_3=100, K_4=75, K_5=50, K_6=25. Further, we found that availability and profit of the system increase with the increase of repair rate of new unit (θ_1) for fixed values of other parameters including K_1=5000, K_2=500, K_3=100, K_4=75, K_5=50, K_6=25. It can also be seen that availability and profit of the system increase by interchange the values of probabilities (p and q) of feasibility of repair or replacement of degraded failed unit

From tables 1 to table 3 we examined that the behavior for MTSF, availability and profit of the system is same for both the distributions.

9. Conclusion

The measures of performance (MTSF, availability, busy period of the server due to repair, busy period of the server due to PM, expected number of visits by the server due to repair, busy period of the server due to inspection, expected number of visits by the server due to inspection and profit of the system) obtained in this paper by using arbitrary distributions will help the system analyst and reliability engineers to improve the system reliability in their respective fields. From the tables 1, 2 and 3, it is concluded that exponential distribution has more values for MTSF, availability and profit of the system as compare to the Rayleigh distribution under stated conditions.

Acknowledgement

The authors are thankful to the reviewers for their valuable comments that led to an improved presentation of the paper.

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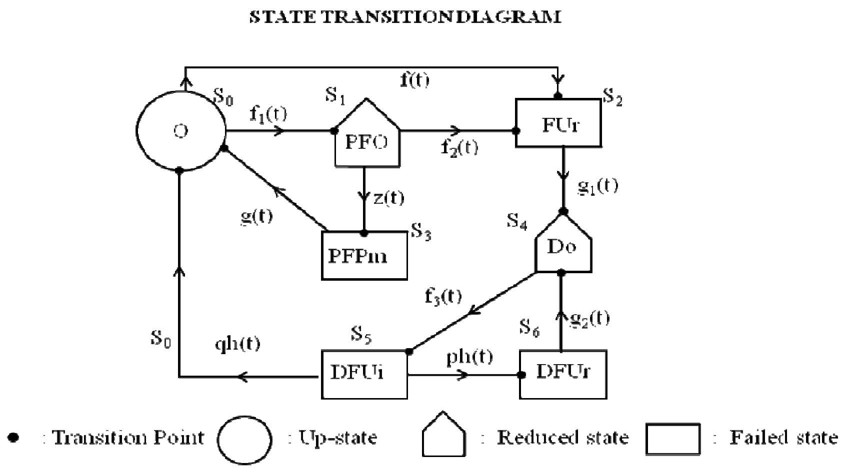

Figure 1. State transition diagram

Mean Time to System Failure (MTSF)						
α ↓	Exponential Distribution			Rayleigh distribution		
	λ =0.13	λ =0.15	λ_1=0.20	λ =0.13	λ =0.15	λ_1=0.20
5	3.4421	3.2270	3.1466	2.5994	2.5073	2.5145
10	3.3888	3.1770	3.0897	2.3437	2.2676	2.2411
15	3.3706	3.1559	3.0701	2.3255	2.2505	2.2216
20	3.3614	3.1513	3.0603	2.3163	2.2419	2.2117
25	3.3558	3.1461	3.0543	2.3107	2.2366	2.2058
30	3.3521	3.1426	3.0504	2.3070	2.2332	2.2018
35	3.3494	3.1401	3.0475	2.3043	2.2307	2.1990
40	3.3474	3.1382	3.0454	2.3023	2.2288	2.1968
45	3.3459	3.1268	3.0437	2.3008	2.2273	2.1951

Table-1: Comparison between the effects of the exponential and Rayleigh distributions with respect to maximum rate of operation (α) and other parameters (λ =0.13, λ_1=0.17, λ_2=0.21, λ_3=0.27, θ=3.7, θ_1=2.1, θ_2=2.7, p=0.7, q=0.3, α=10) on the Mean Time to System Failure (MTSF)

Availability										
α	Exponential Distribution					Rayleigh distribution				
↓	λ =0.13	λ =0.15	λ_1=0.20	θ_1=2.5	p=0.3, q=0.7	λ =0.13	λ =0.15	λ_1=0.20	θ_1=2.5	p=0.3, q=0.7
5	0.9089	0.9065	0.9066	0.9121	0.9211	0.7438	0.7394	0.7430	0.7467	0.7832
10	0.9085	0.9061	0.9061	0.9117	0.9205	0.7360	0.7322	0.7340	0.7389	0.7722
15	0.9084	0.9060	0.9059	0.9116	0.9203	0.7354	0.7317	0.7334	0.7384	0.7714
20	0.9083	0.9059	0.9058	0.9115	0.9202	0.7352	0.7315	0.7331	0.7381	0.7710
25	0.9083	0.9058	0.9058	0.9114	0.9201	0.7350	0.7313	0.7329	0.7380	0.7708
30	0.9083	0.9058	0.9058	0.9114	0.9201	0.7349	0.7312	0.7328	0.7379	0.7706
35	0.9083	0.9058	0.9057	0.9114	0.9201	0.7348	0.7312	0.7327	0.7378	0.7705
40	0.9082	0.9058	0.9057	0.9114	0.9200	0.7348	0.7311	0.7326	0.7377	0.7704
45	0.9082	0.9058	0.9057	0.9114	0.9200	0.7347	0.7311	0.7325	0.7377	0.7703

Table 2: Comparison between the effects of the exponential and Rayleigh distributions with respect to maximum rate of operation (α) and other parameters (λ =0.13, λ_1=0.17, λ_2=0.21, λ_3=0.27, θ=3.7, θ_1=2.1, θ_2=2.7, p=0.7, q=0.3, α=10) on the availability

Profit of the System										
α	Exponential Distribution					Rayleigh distribution				
↓	λ =0.13	λ =0.15	λ_1=0.20	θ_1=2.5	p=0.3, q=0.7	λ =0.13	λ =0.15	λ_1=0.20	θ_1=2.5	p=0.3, q=0.7
5	4529.4	4516.6	4517.5	4545.7	4590.5	3683.6	3660.7	3679.7	3698.5	3883.7
10	4527.5	4514.7	4515.0	4543.7	4587.4	3643.6	3624.0	3633.6	3658.7	3827.7
15	4526.8	4514.0	4514.2	4543.0	4586.2	3641.1	3621.7	3630.7	3656.2	3823.6
20	4526.5	4513.6	4513.7	4542.7	4585.7	3639.8	3620.5	3629.1	3654.9	3821.6
25	4526.3	4513.4	4513.5	4542.5	4585.3	3639.0	3619.8	3628.2	3654.1	3820.3
30	4526.1	4513.3	4513.3	4542.3	4585.1	3638.4	3619.3	3627.6	3653.6	3819.5
35	4526.0	4513.2	4513.2	4542.2	4584.9	3638.1	3618.9	3627.1	3653.2	3818.9
40	4525.9	4513.1	4513.1	4542.1	4584.8	3637.8	3618.7	3626.8	3652.9	3818.4
45	4525.9	4513.1	4513.0	4542.1	4584.7	3637.6	3618.5	3626.6	3652.7	3818.0

Table 3: Comparison between the effects of the exponential and Rayleigh distributions with respect to maximum rate of operation (α) and other parameters (λ =0.13, λ_1=0.17, λ_2=0.21, λ_3=0.27, θ=3.7, θ_1=2.1, θ_2=2.7, p=0.7, q=0.3, α=10, K_1=5000, K_2=500, K_3=100, K_4=75, K_5=50, K_6=25) on the profit of the system