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RELIABILITY ANALYSIS OF A COMPLEX REPAIRABLE SYSTEM COMPOSED OF TWO 2-OUT-OF-3: G SUBSYSTEMS CONNECTED IN PARALLEL

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Abstract
This paper deals with the reliability analysis of a complex repairable system. The considered system consists of two repairable subsystems L and M connected in parallel configuration. Subsystem L and M both are 2-out-of-3: G consists of 3 type-A and 3 type-B components respectively which are in parallel configuration. A hot spare of type-A and type-B is connected to the L and M subsystem respectively. System has two states: Good and Failed. Supplementary variable technique has been used for mathematical formation of model. With the help of Gumbel-Hougaard family of copula reliability and cost analysis of the system is evaluated. The system is studied by using the supplementary variable technique to obtain various related measures such as mean time to failure, steady state probability, availability and cost analysis. At last some particular cases of the system are taken to highlight the different possibilities.

Key Words: System, Reliability, Availability, MTTF, Sensitivity, k-out-of-m: G, Gumbel-Hougaard Copula.

1. Introduction
In the present competitive market cutting down production costs and improving productivity and delivery performance of manufacturing systems are the key objectives of Industries. Most of the industries and manufacturers are trying to introduce more complex mechanization and automation in their industrial process to compete in the rate race of manufacturing more sophisticated equipments which leads to more complexities in the system, subsequently chances of failures of such systems increase. Reliability and its related issues have therefore become very relevant for manufacturing system to meet the mission success. Various researchers have made their contribution in the development of different techniques to find the reliability characteristics of complex system. They focused themselves on the analysis of the various repairable and non-repairable systems. Some of them used the concept of copula to find the joint distribution of failure and repair rates. They also applied reliability techniques to analyze the different industries. Reliability and availability analysis make it possible to evolve different alternative for the improvement of the system design and configuration. With the advent of modern age and development of the complex engineering systems, an improvement in the design to improve the reliability of the systems became a challenge for the engineers. Engineers are interested in the development of such systems

whose reliability and profit are maximum with minimum cost. For this, it is essential to develop such systems whose performances are failure free.

A lot of studies dealing with the reliability and availability of k-out-of-n systems (standby systems) have been carried out. The systems of n units in which k units are sufficient to perform the entire function of the system are called k-out-of-n redundant systems. These systems have wide application in the real world. The communication system with three transmitters can be sited as a good example of 2-out-of-3 redundant system. Many useful results [8, 3 and 5] have been published by researchers (Moustafa; Hassett *et al.*; and Szidarovszky *et al.*) regarding various types of failure in a system. In recent past some good works have been done in reliability modelling with the application of copulas. Lindskog [6] applied the copula into modelling dependence. Nailwal *et al.* [9] have studied performance evaluation and reliability analysis of a complex system with three possibilities in repair with the application of copula. Nailwal *et al.* [10] have applied copula in reliability measures and sensitivity analysis of a complex matrix system including power failure. Kumar *et al.* [4] delt with profit analysis of two unit non-identical system with degradation and replacement. Angus [1] has studied k-out-of-n: G system in different configurations. Malik *et al*. [7] have studied the profit analysis of a stochastic model of 2-out-of-3 redundant system with inspection by a server who appears and disappears randomly. Goel *et al*. [2] have analyzed stochastic behavior of a two unit parallel system with partial and catastrophic failures and preventive maintenance. Ram and Singh [14, 13 and 12] have analyzed the reliability characteristics of various complex repairable systems by using Gumbel-Hougaard family of copula. Pandey *et al.* [11] has analyzed reliability and cost of a system with multiple components using copula.

Many research results have been reported on reliability of 2-out-of-3 redundant systems involving 2-out-of-3 system as main system, but a little attention has been paid for the case where 2-out-of-3 system is used as a subsystem. Keeping this fact in view, here we have focused on this issue while developing reliability model. In the present model we have considered a system consisting of two 2-out-of-3: G subsystems L and M. Subsystems L and M are connected in parallel configuration which consist of 3 type-A and 3 type-B components respectively. A hot spare of type-A and type-B is connected to the subsystems L and M respectively. SA and SB are two different types of spares that can replace only own type components (SA can replace only A, SB can replace only B) used in model. In the transition state diagram (Figure 2) of the system, we denote $A^{x}B^{y}SA^{z}SB^{w}$ by the joint state that there x type-A components, y type-B components, z type-A spare component and w type-B spare component are functional (x; $y = k, k+1, \ldots, m$; z; $w = 0, 1$). Each component of the system has two modes- good and failed. Failure rates of components of type-A and type-B are constant. All components of type-A/type-B are repairable and repair rates follow general distribution in all the cases. We have used Gumbel-Hougaard family of copula to find joint distribution of repairs whenever both the subsystems are being repaired simultaneously with two different repair rates. The repair of the failed component is perfect. By the help of Laplace transforms and supplementary variable technique the following reliability characteristics of the system have been analyzed in this model:

(i) Transition state probabilities.
(ii) Asymptotic behaviour of system.

(iii) Various reliability measures such as availability, reliability, mean time to failure and sensitivity with respect to different parameter.

At last, some special cases of the system are taken to highlight the reliability characteristics of the system. These are as follows:

A. Repairable and non-identical.
B. Repairable and identical.
C. Non-repairable and non-identical.
D. Non-repairable and identical.

The state specification chart of the considered system is given in Table 1. Block diagram and transition state diagram of investigated system are shown in Figure 1 and Figure 2 respectively.

2. Assumptions

(i) Initially the system is in perfectly good state i.e. all the components are functioning perfectly.
(ii) At t=0 all the components are perfectly well and at $t > 0$ they start operating.
(iii) The system consists of two subsystems L and M connected in parallel.
(iv) Subsystems L is 2-out-of-3: G system of 3 components of type-A and M is 2-out-of-3: G system of 3 components of type-B.
(v) A hot spare of type-A and type-B is connected to the subsystems L and M respectively. When a component fails in subsystem, the hot spare is switched into operation.
(vi) Each component is either functional or failed.
(vii) Failure rates of type-A component and type-B component are assumed as constant.
(viii) Each subsystem on complete failure goes for repair.
(ix) The repaired subsystem is as good as new and is immediately reconnected to the system.
(x) Transition from the completely failed state S_{11} to the initial state S_{44} follows two different distributions.
(xi) Joint probability distribution of repair rate from S_{11} to the initial state S_{44} is computed by Gumbel-Hougaard family of copula.
(xii) If both units fail, the system fails completely.

3. Nomenclature

λ_A / λ_B : Failure rate of component of type-A/type-B.
$\eta(x)$: Repair rate of type-A component.
$\psi(y)$: Repair rate of type-B component.
$p_{uv}(t)$: Probability that the system is in S_{uv} state at instant t for u; v = 4 to 1.
$\overline{p}_{uv}(s)$: Laplace transform of $P_{uv}(t)$.
$p_{uv}(j,t)$: The pdf (system is in state S_{uv} and is under repair; elapsed repair time is j, t), where $j = x, y, z$.
$\xi(z)$: Coupled repair rate.

Considering $u_1 = \eta(x)$ and $u_2 = \psi(y)$, the expression for joint probability (failed state S_{11} to good state S_{44}) according to Gumbel-Hougaard family of copula is given by

$$\xi(z) = \exp[(\log(u_1))^{\theta} + (\log(u_2))^{\theta}]^{\frac{1}{\theta}}$$

4. State Specification

G = Good state, F = Failed state

States	State of subsystem A	State of subsystem B	State of system
S_{44}	G	G	G
S_{34}	G	G	G
S_{43}	G	G	G
S_{24}	G	G	G
S_{33}	G	G	G
S_{42}	G	G	G
S_{14}	F	G	G
S_{23}	G	G	G
S_{32}	G	G	G
S_{41}	G	F	G
S_{13}	F	G	G
S_{22}	G	G	G
S_{31}	G	F	G
S_{12}	F	G	G
S_{21}	G	F	G
S_{11}	F	F	F

Table 1: State Specification

5. Block and State Transition Diagram

Figure 1 and 2 represent the block diagram and the state transition diagram of investigated system respectively.

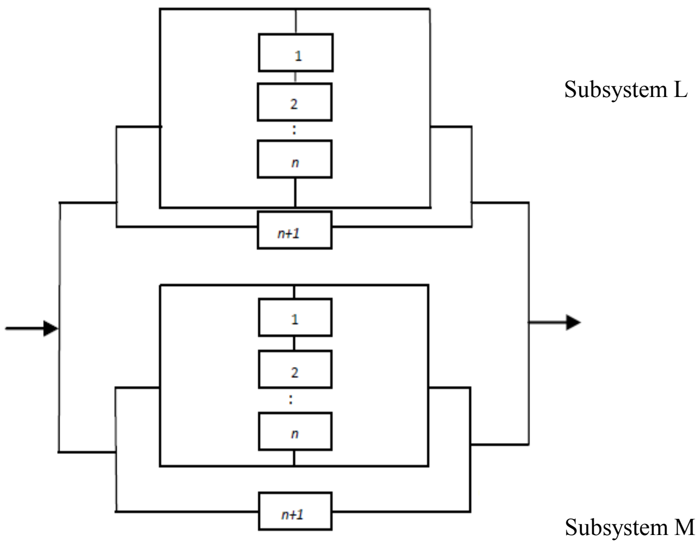

Figure 1: Block diagram of system

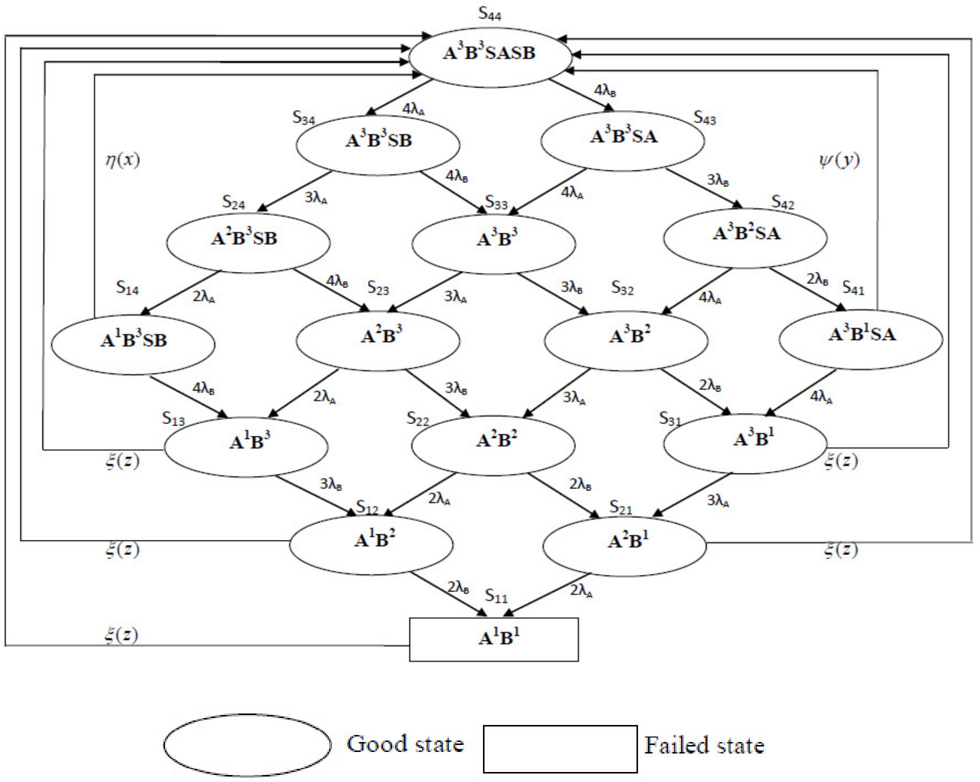

Figure 2: Transition State Diagram

6. Formulation of Mathematical Model

By probability consideration and continuity arguments, we obtain the following set of integro-differential equations governing the behavior of the system.

$$\left[\frac{d}{dt}+4\lambda_A+4\lambda_B\right]p_{44}(t)=\int_0^\infty p_{41}(y,t)\psi(y)dy+\int_0^\infty p_{14}(x,t)\eta(x)dx+\int_0^\infty p_{13}(z,t)\xi(z)dz+\int_0^\infty p_{31}(z,t)\xi(z)dz$$
$$+\int_0^\infty p_{12}(z,t)\xi(z)dz+\int_0^\infty p_{21}(z,t)\xi(z)dz+\int_0^\infty p_{11}(z,t)\xi(z)dz \tag{1}$$

$$\left[\frac{d}{dt}+3\lambda_A+4\lambda_B\right]p_{34}(t)=4\lambda_A p_{44}(t) \tag{2}$$

$$\left[\frac{d}{dt}+4\lambda_A+3\lambda_B\right]p_{43}(t)=4\lambda_B p_{44}(t) \tag{3}$$

$$\left[\frac{d}{dt}+2\lambda_A+4\lambda_B\right]p_{24}(t)=3\lambda_A p_{34}(t) \tag{4}$$

$$\left[\frac{d}{dt}+3\lambda_A+3\lambda_B\right]p_{33}(t)=4\lambda_A p_{43}(t)+4\lambda_B p_{34}(t) \tag{5}$$

$$\left[\frac{d}{dt}+4\lambda_A+2\lambda_B\right]p_{42}(t)=3\lambda_B p_{43}(t) \tag{6}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+4\lambda_B+\eta(x)\right]p_{14}(x,t)=0 \tag{7}$$

$$\left[\frac{d}{dt}+2\lambda_A+3\lambda_B\right]p_{23}(t)=3\lambda_A p_{33}(t)+4\lambda_B p_{24}(t) \tag{8}$$

$$\left[\frac{d}{dt}+3\lambda_A+2\lambda_B\right]p_{32}(t)=3\lambda_B p_{33}(t)+4\lambda_A p_{42}(t) \tag{9}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial y}+4\lambda_A+\psi(y)\right]p_{41}(y,t)=0 \tag{10}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial z}+3\lambda_B+\xi(z)\right]p_{13}(z,t)=0 \tag{11}$$

$$\left[\frac{d}{dt}+2\lambda_A+2\lambda_B\right]p_{22}(t)=3\lambda_B p_{23}(t)+3\lambda_A p_{32}(t) \tag{12}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial z}+3\lambda_A+\xi(z)\right]p_{31}(z,t)=0 \tag{13}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial z}+2\lambda_B+\xi(z)\right]p_{12}(z,t)=0 \tag{14}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial z}+2\lambda_A+\xi(z)\right]p_{21}(z,t)=0 \tag{15}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial z}+\xi(z)\right]p_{11}(z,t)=0 \tag{16}$$

Boundary conditions

$$p_{14}(0,t)=2\lambda_A p_{24}(t) \tag{17}$$

$$p_{41}(0,t)=2\lambda_B p_{42}(t) \tag{18}$$

$$p_{13}(0,t)=4\lambda_B p_{14}(t)+2\lambda_A p_{23}(t) \tag{19}$$

$$p_{31}(0,t) = 2\lambda_B p_{32}(t) + 4\lambda_A p_{41}(t) \tag{20}$$
$$p_{12}(0,t) = 3\lambda_B p_{13}(t) + 2\lambda_A p_{22}(t) \tag{21}$$
$$p_{21}(0,t) = 2\lambda_B p_{22}(t) + 3\lambda_A p_{31}(t) \tag{22}$$
$$p_{11}(0,t) = 2\lambda_B p_{12}(t) + 2\lambda_A p_{21}(t) \tag{23}$$

Initial condition

$p_{44}(0) = 1$ and other probabilities are zero at t=0. (24)

Solution of the model

Solving equations (1-23) with the help of Laplace transforms and using equation (24), we get

$$[s + 4\lambda_A + 4\lambda_B]\overline{p}_{44}(s) = 1 + \int_0^\infty \overline{p}_{41}(y,s)\psi(y)dy + \int_0^\infty \overline{p}_{14}(x,s)\eta(x)dx + \int_0^\infty \overline{p}_{13}(z,s)\xi(z)dz + \int_0^\infty \overline{p}_{31}(z,s)\xi(z)dz$$
$$+ \int_0^\infty \overline{p}_{12}(z,s)\xi(z)dz + \int_0^\infty \overline{p}_{21}(z,s)\xi(z)dz + \int_0^\infty \overline{p}_{11}(z,s)\xi(z)dz \tag{25}$$

$$[s + 3\lambda_A + 4\lambda_B]\overline{p}_{34}(s) = 4\lambda_A \overline{p}_{44}(s) \tag{26}$$
$$[s + 4\lambda_A + 3\lambda_B]\overline{p}_{43}(s) = 4\lambda_B \overline{p}_{44}(s) \tag{27}$$
$$[s + 2\lambda_A + 4\lambda_B]\overline{p}_{24}(s) = 3\lambda_A \overline{p}_{34}(s) \tag{28}$$
$$[s + 3\lambda_A + 3\lambda_B]\overline{p}_{33}(s) = 4\lambda_A \overline{p}_{43}(s) + 4\lambda_B \overline{p}_{34}(s) \tag{29}$$
$$[s + 4\lambda_A + 2\lambda_B]\overline{p}_{42}(s) = 3\lambda_B \overline{p}_{43}(s) \tag{30}$$
$$[s + 2\lambda_A + 3\lambda_B]\overline{p}_{23}(s) = 3\lambda_A \overline{p}_{33}(s) + 4\lambda_B \overline{p}_{24}(s) \tag{31}$$
$$[s + 3\lambda_A + 2\lambda_B]\overline{p}_{32}(s) = 4\lambda_A \overline{p}_{42}(s) + 3\lambda_B \overline{p}_{33}(s) \tag{32}$$
$$[s + 2\lambda_A + 2\lambda_B]\overline{p}_{22}(s) = 3\lambda_B \overline{p}_{23}(s) + 3\lambda_A \overline{p}_{32}(s) \tag{33}$$
$$\left[s + \frac{\partial}{\partial x} + 4\lambda_B + \eta(x)\right]\overline{p}_{14}(x,s) = 0 \tag{34}$$
$$\left[s + \frac{\partial}{\partial y} + 4\lambda_A + \psi(y)\right]\overline{p}_{41}(y,s) = 0 \tag{35}$$
$$\left[s + \frac{\partial}{\partial z} + 3\lambda_B + \xi(z)\right]\overline{p}_{13}(z,s) = 0 \tag{36}$$
$$\left[s + \frac{\partial}{\partial z} + 3\lambda_A + \xi(z)\right]\overline{p}_{31}(z,s) = 0 \tag{37}$$
$$\left[s + \frac{\partial}{\partial z} + 2\lambda_B + \xi(z)\right]\overline{p}_{12}(z,s) = 0 \tag{38}$$
$$\left[s + \frac{\partial}{\partial z} + 2\lambda_A + \xi(z)\right]\overline{p}_{21}(z,s) = 0 \tag{39}$$
$$\left[s + \frac{\partial}{\partial z} + \xi(z)\right]p_{11}(z,s) = 0 \tag{40}$$

Boundary conditions

$$\overline{p}_{14}(0,s) = 2\lambda_A \overline{p}_{24}(s) \tag{41}$$
$$\overline{p}_{41}(0,s) = 2\lambda_B \overline{p}_{42}(s) \tag{42}$$
$$\overline{p}_{13}(0,s) = 4\lambda_B \overline{p}_{14}(s) + 2\lambda_A \overline{p}_{23}(s) \tag{43}$$

$$\overline{p}_{31}(0,s) = 2\lambda_B \overline{p}_{32}(s) + 4\lambda_A \overline{p}_{41}(s) \tag{44}$$

$$\overline{p}_{12}(0,s) = 3\lambda_B \overline{p}_{13}(s) + 2\lambda_A \overline{p}_{22}(s) \tag{45}$$

$$\overline{p}_{21}(0,s) = 2\lambda_B \overline{p}_{22}(s) + 3\lambda_A \overline{p}_{31}(s) \tag{46}$$

$$p_{11}(0,t) = 2\lambda_B p_{12}(t) + 2\lambda_A p_{21}(t) \tag{47}$$

The transition state probabilities for the system can be viewed as a result of solving the set of equations (25- 40) with the help of (41- 47)

$$\overline{p}_{44}(s) = \frac{1}{D(s)} \tag{48}$$

$$\overline{p}_{34}(s) = \frac{4\lambda_A}{D(s)(s+3\lambda_A+4\lambda_B)} \tag{49}$$

$$\overline{p}_{43}(s) = \frac{4\lambda_B}{D(s)(s+4\lambda_A+3\lambda_B)} \tag{50}$$

$$\overline{p}_{24}(s) = \frac{3.4\lambda_A^{\;2}}{D(s)(s+3\lambda_A+4\lambda_B)(s+2\lambda_A+4\lambda_B)} \tag{51}$$

$$\overline{p}_{33}(s) = \frac{4.4\lambda_A\lambda_B}{D(s)(s+3\lambda_A+3\lambda_B)}\left[\frac{1}{s+3\lambda_A+4\lambda_B}+\frac{1}{s+4\lambda_A+3\lambda_B}\right] \tag{52}$$

$$\overline{p}_{42}(s) = \frac{3.4\lambda_B^{\;2}}{D(s)(s+4\lambda_A+3\lambda_B)(s+4\lambda_A+2\lambda_B)} \tag{53}$$

$$\overline{p}_{23}(s) = \frac{3.4.4\lambda_A^{\;2}\lambda_B}{D(s)A(s)} \tag{54}$$

$$\overline{p}_{32}(s) = \frac{3.4.4\lambda_A\lambda_B^{\;2}}{D(s)B(s)} \tag{55}$$

$$[s+2\lambda_A+2\lambda_B]\overline{p}_{22}(s) = 3\lambda_B\overline{p}_{23}(s) + 3\lambda_A\overline{p}_{32}(s) \tag{56}$$

$$\overline{p}_{14}(x,s) = \frac{2.3.4\lambda_A^{\;3}[1-S_\eta(s+4\lambda_B)]}{D(s)(s+3\lambda_A+4\lambda_B)(s+2\lambda_A+4\lambda_B)(s+4\lambda_B)} \tag{57}$$

$$\overline{p}_{41}(x,s) = \frac{2.3.4\lambda_B^{\;3}[1-S_\psi(s+4\lambda_A)]}{D(s)(s+4\lambda_A+3\lambda_B)(s+4\lambda_A+2\lambda_B)(s+4\lambda_A)} \tag{58}$$

$$\overline{p}_{13}(x,s) = \frac{2.3.4.4\lambda_A^{\;3}\lambda_B}{D(s)E(s)} \tag{59}$$

$$\overline{p}_{31}(x,s) = \frac{2.3.4.4\lambda_A\lambda_B^{\;3}}{D(s)F(s)} \tag{60}$$

$$\overline{p}_{12}(z,s) = \frac{2.3.3.4.4\lambda_A^{\;3}\lambda_B^{\;2}\left(1-\overline{S}_\xi(s+2\lambda_B)\right)}{D(s)(s+2\lambda_B)}\left[\frac{1}{C(s)}+\frac{1}{E(s)}\right] \tag{61}$$

$$\overline{p}_{21}(z,s) = \frac{2.3.3.4.4\lambda_A^{\;2}\lambda_B^{\;3}\left(1-\overline{S}_\xi(s+2\lambda_A)\right)}{D(s)(s+2\lambda_A)}\left[\frac{1}{C(s)}+\frac{1}{F(s)}\right] \tag{62}$$

$$\overline{p}_{11}(z,s) = \frac{2.2.3.3.4.4\lambda_A^{\;3}\lambda_B^{\;3}}{sD(s)}\left[\frac{\left(1-\overline{S}_\xi(s+2\lambda_B)\right)}{(s+2\lambda_B)}\left(\frac{1}{C(s)}+\frac{1}{E(s)}\right)+\frac{\left(1-\overline{S}_\xi(s+2\lambda_A)\right)}{(s+2\lambda_A)}\left(\frac{1}{C(s)}+\frac{1}{F(s)}\right)\right] \tag{63}$$

where

$$D(s)=(s+4\lambda_A+4\lambda_B)-\left[\frac{2.3.4\lambda_A{}^3\eta(x)}{(s+4\lambda_B+\eta(x))}+\frac{2.3.4\lambda_B{}^3\psi(y)}{(s+4\lambda_A+\psi(y))}+\frac{2.3.4.4\lambda_A{}^3\lambda_B\xi(z)}{(s+3\lambda_B+\xi(z))}+\frac{2.3.4.4\lambda_A\lambda_B{}^3\xi(z)}{(s+3\lambda_A+\xi(z))}\right]$$
$$+\frac{2.3.3.4.4\lambda_A{}^3\lambda_B{}^2\xi(z)}{(s+2\lambda_B+\xi(z))}+\frac{2.3.3.4.4\lambda_A{}^2\lambda_B{}^3\xi(z)}{(s+2\lambda_A+\xi(z))}+\frac{2.2.3.3.4.4\lambda_A{}^3\lambda_B{}^3\xi(z)}{(s+\xi(z))} \quad (64)$$

$$\frac{1}{A(s)}=\frac{1}{(s+2\lambda_A+3\lambda_B)}\left[\frac{1}{(s+3\lambda_A+4\lambda_B)(s+2\lambda_A+4\lambda_B)}+\frac{1}{(s+3\lambda_A+3\lambda_B)(s+3\lambda_A+4\lambda_B)}\right.$$
$$\left.+\frac{1}{(s+3\lambda_A+3\lambda_B)(s+4\lambda_A+3\lambda_B)}\right] \quad (65)$$

$$\frac{1}{B(s)}=\frac{1}{(s+3\lambda_A+2\lambda_B)}\left[\frac{1}{(s+3\lambda_A+3\lambda_B)(s+3\lambda_A+4\lambda_B)}+\frac{1}{(s+3\lambda_A+3\lambda_B)(s+4\lambda_A+3\lambda_B)}+\frac{1}{(s+4\lambda_A+3\lambda_B)(s+4\lambda_A+2\lambda_B)}\right] \quad (66)$$

$$\frac{1}{C(s)}=\frac{1}{(s+2\lambda_A+2\lambda_B)}\left[\frac{1}{(s+2\lambda_A+3\lambda_B)(s+3\lambda_A+4\lambda_B)}\left(\frac{1}{(s+2\lambda_A+4\lambda_B)}+\frac{1}{(s+3\lambda_A+3\lambda_B)}\right)\right.$$
$$+\frac{1}{(s+3\lambda_A+2\lambda_B)}\left(\frac{1}{(s+3\lambda_A+3\lambda_B)(s+3\lambda_A+4\lambda_B)}+\frac{1}{(s+4\lambda_A+2\lambda_B)(s+4\lambda_A+3\lambda_B)}\right)$$
$$\left.+\frac{1}{(s+3\lambda_A+3\lambda_B)(s+4\lambda_A+3\lambda_B)}\left(\frac{1}{(s+2\lambda_A+3\lambda_B)}+\frac{1}{(s+3\lambda_A+2\lambda_B)}\right)\right] \quad (67)$$

$$\frac{1}{E(s)}=\frac{\left(1-\overline{S}_\xi(s+3\lambda_B)\right)}{(s+3\lambda_B)}\left[\frac{\left(1-\overline{S}_\eta(s+4\lambda_B)\right)}{(s+3\lambda_A+4\lambda_B)(s+2\lambda_A+4\lambda_B)(s+4\lambda_B)}+\frac{1}{A(s)}\right] \quad (68)$$

$$\frac{1}{F(s)}=\frac{\left(1-\overline{S}_\xi(s+3\lambda_A)\right)}{(s+3\lambda_A)}\left[\frac{\left(1-\overline{S}_\eta(s+4\lambda_A)\right)}{(s+4\lambda_A+3\lambda_B)(s+4\lambda_A+2\lambda_B)(s+4\lambda_A)}+\frac{1}{B(s)}\right] \quad (69)$$

Also

$$\overline{p}_{up}(s)=\overline{p}_{44}(s)+\overline{p}_{34}(s)+\overline{p}_{43}(s)+\overline{p}_{24}(s)+\overline{p}_{33}(s)+\overline{p}_{42}(s)+\overline{p}_{23}(s)+\overline{p}_{32}(s)+\overline{p}_{22}(s)+\overline{p}_{14}(x,s)$$
$$+\overline{p}_{41}(y,s)+\overline{p}_{13}(z,s)+\overline{p}_{31}(z,s)+\overline{p}_{12}(s)+\overline{p}_{21}(z,s) \quad (70)$$

$$\overline{p}_{down}(s)=\overline{p}_{11}(z,s) \quad (71)$$

7. Asymptotic Behaviour of the System

Using Abel's lemma in Laplace transforms,

$$\lim_{s\to 0}\{s\overline{F}(s)\}=\lim_{t\to\infty}F(t)$$

provided the limit on the right hand side exits, the time independent operational probabilities are obtained as follows:

$$p_{44}=\frac{1}{D(0)} \quad (72)$$

$$p_{34}=\frac{4\lambda_A}{D(0)(3\lambda_A+4\lambda_B)} \quad (73)$$

$$p_{43}=\frac{4\lambda_B}{D(0)(4\lambda_A+3\lambda_B)} \quad (74)$$

$$p_{24} = \frac{3.4\lambda_A^{\,2}}{D(0)(3\lambda_A + 4\lambda_B)(2\lambda_A + 4\lambda_B)} \tag{75}$$

$$p_{33} = \frac{4.4\lambda_A\lambda_B}{D(0)(3\lambda_A + 3\lambda_B)}\left[\frac{1}{3\lambda_A + 4\lambda_B} + \frac{1}{4\lambda_A + 3\lambda_B}\right] \tag{76}$$

$$p_{42} = \frac{3.4\lambda_B^{\,2}}{D(0)(4\lambda_A + 3\lambda_B)(4\lambda_A + 2\lambda_B)} \tag{77}$$

$$p_{23} = \frac{3.4.4\lambda_A^{\,2}\lambda_B}{D(0)A(0)} \tag{78}$$

$$p_{32} = \frac{3.4.4\lambda_A\lambda_B^{\,2}}{D(0)B(0)} \tag{79}$$

$$p_{22} = \frac{3.3.4.4\lambda_A^{\,2}\lambda_B^{\,2}}{D(0)C(0)} \tag{80}$$

$$p_{14} = \frac{2.3.4\lambda_A^{\,3}}{D(0)(3\lambda_A + 4\lambda_B)(2\lambda_A + 4\lambda_B)(4\lambda_B + \eta(x))} \tag{81}$$

$$p_{41} = \frac{2.3.4\lambda_B^{\,3}}{D(0)(4\lambda_A + 3\lambda_B)(4\lambda_A + 2\lambda_B)(4\lambda_A + \psi(y))} \tag{82}$$

$$p_{13} = \frac{2.3.4.4\lambda_A^{\,3}\lambda_B}{D(0)E(0)} \tag{83}$$

$$p_{31} = \frac{2.3.4.4\lambda_A\lambda_B^{\,3}}{D(0)F(0)} \tag{84}$$

$$p_{12} = \frac{2.3.3.4.4\lambda_A^{\,3}\lambda_B^{\,2}}{D(0)(2\lambda_B + \xi(z))}\left[\frac{1}{C(0)} + \frac{1}{E(0)}\right] \tag{85}$$

$$p_{21} = \frac{2.3.3.4.4\lambda_A^{\,2}\lambda_B^{\,3}}{D(0)(2\lambda_A + \xi(z))}\left[\frac{1}{C(0)} + \frac{1}{F(0)}\right] \tag{86}$$

$$p_{11} = \frac{2.2.3.3.4.4\lambda_A^{\,3}\lambda_B^{\,3}}{D(0)\xi(z)}\left[\frac{1}{(2\lambda_B + \xi(z))}\left(\frac{1}{C(0)} + \frac{1}{E(0)}\right) + \frac{1}{(2\lambda_A + \xi(z))}\left(\frac{1}{C(0)} + \frac{1}{F(0)}\right)\right] \tag{87}$$

8. Special Cases

When repair follows exponential distribution. In this case the result can be derived by putting

$$\bar{S}_\mu(s) = \frac{\eta(x)}{s + \eta(x)}, \bar{S}_\psi(s) = \frac{\psi(y)}{s + \psi(y)}, \bar{S}_\xi(s) = \frac{\exp\left\{(\log\eta(x))^\theta + (\log\psi(x))^\theta\right\}^{\frac{1}{\theta}}}{s + \exp\left\{(\log\eta(x))^\theta + (\log\psi(x))^\theta\right\}^{\frac{1}{\theta}}} \tag{88}$$

A. Repairable and Non Identical

When system is repairable and all the units of the system are non-identical then the transition state probabilities corresponding to system can be obtained as

$$\bar{p}_{44}(s) \equiv \frac{1}{D(s)} \tag{89}$$

$$\bar{p}_{34}(s)=\frac{4\lambda_A}{(s+3\lambda_A+4\lambda_B)}\bar{p}_{44}(s) \tag{90}$$

$$\bar{p}_{43}(s)=\frac{4\lambda_B}{(s+4\lambda_A+3\lambda_B)}\bar{p}_{44}(s) \tag{91}$$

$$\bar{p}_{24}(s)=\frac{3.4\lambda_A^{\,2}}{(s+3\lambda_A+4\lambda_B)(s+2\lambda_A+4\lambda_B)}\bar{p}_{44}(s) \tag{92}$$

$$\bar{p}_{33}(s)=\frac{4.4\lambda_A\lambda_B\bar{p}_{44}(s)}{(s+3\lambda_A+3\lambda_B)}\left[\frac{1}{s+3\lambda_A+4\lambda_B}+\frac{1}{s+4\lambda_A+3\lambda_B}\right] \tag{93}$$

$$\bar{p}_{42}(s)=\frac{3.4\lambda_B^{\,2}}{(s+4\lambda_A+3\lambda_B)(s+4\lambda_A+2\lambda_B)}\bar{p}_{44}(s) \tag{94}$$

$$\bar{p}_{23}(s)=\frac{3.4.4\lambda_A^{\,2}\lambda_B}{A(s)}\bar{p}_{44}(s) \tag{95}$$

$$\bar{p}_{32}(s)=\frac{3.4.4\lambda_A\lambda_B^{\,2}}{B(s)}\bar{p}_{44}(s) \tag{96}$$

$$\bar{p}_{22}(s)=\frac{3.3.4.4\lambda_A^{\,2}\lambda_B^{\,2}}{C(s)}\bar{p}_{44}(s) \tag{97}$$

$$\bar{p}_{14}(s)=\frac{2.3.4\lambda_A^{\,3}}{(s+3\lambda_A+4\lambda_B)(s+2\lambda_A+4\lambda_B)(s+4\lambda_B+\eta(x))}\bar{p}_{44}(s) \tag{98}$$

$$\bar{p}_{41}(s)=\frac{2.3.4\lambda_B^{\,3}}{(s+4\lambda_A+3\lambda_B)(s+4\lambda_A+2\lambda_B)(s+4\lambda_A+\psi(y))}\bar{p}_{44}(s) \tag{99}$$

$$\bar{p}_{13}(s)=\frac{2.3.4.4\lambda_A^{\,3}\lambda_B}{E(s)}\bar{p}_{44}(s) \tag{100}$$

$$\bar{p}_{31}(s)=\frac{2.3.4.4\lambda_A\lambda_B^{\,3}}{F(s)}\bar{p}_{44}(s) \tag{101}$$

$$\bar{p}_{21}(s)=\frac{2.3.3.4.4\lambda_A^{\,2}\lambda_B^{\,3}\bar{p}_{44}(s)}{(s+2\lambda_A+\xi(z))}\left[\frac{1}{C(s)}+\frac{1}{F(s)}\right] \tag{102}$$

$$\bar{p}_{12}(s)=\frac{2.3.3.4.4\lambda_A^{\,3}\lambda_B^{\,2}\bar{p}_{44}(s)}{(s+2\lambda_B+\xi(z))}\left[\frac{1}{C(s)}+\frac{1}{E(s)}\right] \tag{103}$$

$$\bar{p}_{11}(s)=\frac{2.2.3.3.4.4\lambda_A^{\,3}\lambda_B^{\,3}\bar{p}_{44}(s)}{(s+\xi(z))}\left[\frac{1}{(s+2\lambda_B+\xi(z))}\left(\frac{1}{C(s)}+\frac{1}{E(s)}\right)+\frac{1}{(s+2\lambda_A+\xi(z))}\left(\frac{1}{C(s)}+\frac{1}{F(s)}\right)\right] \tag{104}$$

B. Repairable and Identical

When the considered system is taken to be repairable and units are identical then transition state probabilities in this case transition state probabilities can be obtained by putting $\lambda_A=\lambda_B=\lambda$ in equations (48-63), which are given by

$$\bar{p}_{44}(s)\equiv\frac{1}{D(s)} \tag{105}$$

$$\overline{p}_{34}(s) = \frac{4\lambda}{(s+7\lambda)}\overline{p}_{44}(s) \tag{106}$$

$$\overline{p}_{43}(s) = \frac{4\lambda}{(s+7\lambda)}\overline{p}_{44}(s) \tag{107}$$

$$\overline{p}_{24}(s) = \frac{3.4\lambda^2}{(s+7\lambda)(s+6\lambda)}\overline{p}_{44}(s) \tag{108}$$

$$\overline{p}_{33}(s) = \frac{2.4.4\lambda^2\overline{p}_{44}(s)}{(s+6\lambda)(s+7\lambda)} \tag{109}$$

$$\overline{p}_{42}(s) = \frac{3.4\lambda^2}{(s+7\lambda)(s+6\lambda)}\overline{p}_{44}(s) \tag{110}$$

$$\overline{p}_{23}(s) = \frac{3.4.4\lambda^3}{A(s)}\overline{p}_{44}(s) \tag{111}$$

$$\overline{p}_{32}(s) = \frac{3.4.4\lambda^3}{B(s)}\overline{p}_{44}(s) \tag{112}$$

$$\overline{p}_{14}(s) = \frac{2.3.4\lambda^3}{(s+7\lambda)(s+6\lambda)(s+4\lambda+\eta(x))}\overline{p}_{44}(s) \tag{113}$$

$$\overline{p}_{41}(s) = \frac{2.3.4\lambda^3}{(s+7\lambda)(s+6\lambda)(s+4\lambda+\psi(y))}\overline{p}_{44}(s) \tag{114}$$

$$\overline{p}_{13}(s) = \frac{2.3.4.4\lambda^4}{E(s)}\overline{p}_{44}(s) \tag{115}$$

$$\overline{p}_{31}(s) = \frac{2.3.4.4\lambda^4}{F(s)}\overline{p}_{44}(s) \tag{116}$$

$$\overline{p}_{22}(s) = \frac{3.3.4.4\lambda^4}{C(s)}\overline{p}_{44}(s) \tag{117}$$

$$\overline{p}_{12}(s) = \frac{2.3.3.4.4\lambda^5\overline{p}_{44}(s)}{(s+2\lambda+\xi(z))}\left[\frac{1}{C(s)}+\frac{1}{E(s)}\right] \tag{118}$$

$$\overline{p}_{21}(s) = \frac{2.3.3.4.4\lambda^5\overline{p}_{44}(s)}{(s+2\lambda+\xi(z))}\left[\frac{1}{C(s)}+\frac{1}{F(s)}\right] \tag{119}$$

$$\overline{p}_{11}(s) = \frac{2.2.3.3.4.4\lambda^6\overline{p}_{44}(s)}{(s+\xi(z))}\left[\frac{1}{(s+2\lambda+\xi(z))}\left(\frac{1}{E(s)}+\frac{2}{C(s)}+\frac{1}{F(s)}\right)\right] \tag{120}$$

C. Non Repairable and Non Identical

When the considered system is taken to be non-repairable (i.e. $\eta(x)=\psi(y)=\xi(z)=0$) and units are non-identical then the transition state probabilities corresponding to the present system are given by

$$\overline{p}_{44}(s) \equiv \frac{1}{D(s)} \tag{121}$$

$$\overline{p}_{34}(s) = \frac{4\lambda_A}{(s+3\lambda_A+4\lambda_B)}\overline{p}_{44}(s) \tag{122}$$

$$\overline{p}_{43}(s)=\frac{4\lambda_B}{(s+4\lambda_A+3\lambda_B)}\overline{p}_{44}(s) \tag{123}$$

$$\overline{p}_{24}(s)=\frac{3.4\lambda_A{}^2}{(s+3\lambda_A+4\lambda_B)(s+2\lambda_A+4\lambda_B)}\overline{p}_{44}(s) \tag{124}$$

$$\overline{p}_{33}(s)=\frac{4.4\lambda_A\lambda_B\overline{p}_{44}(s)}{(s+3\lambda_A+3\lambda_B)}\left[\frac{1}{s+3\lambda_A+4\lambda_B}+\frac{1}{s+4\lambda_A+3\lambda_B}\right] \tag{125}$$

$$\overline{p}_{42}(s)=\frac{3.4\lambda_B{}^2}{(s+4\lambda_A+3\lambda_B)(s+4\lambda_A+2\lambda_B)}\overline{p}_{44}(s) \tag{126}$$

$$\overline{p}_{23}(s)=\frac{3.4.4\lambda_A{}^2\lambda_B}{A(s)}\overline{p}_{44}(s) \tag{127}$$

$$\overline{p}_{32}(s)=\frac{3.4.4\lambda_A\lambda_B{}^2}{B(s)}\overline{p}_{44}(s) \tag{128}$$

$$\overline{p}_{14}(s)=\frac{2.3.4\lambda_A{}^3}{(s+3\lambda_A+4\lambda_B)(s+2\lambda_A+4\lambda_B)(s+4\lambda_B)}\overline{p}_{44}(s) \tag{129}$$

$$\overline{p}_{41}(s)=\frac{2.3.4\lambda_B{}^3}{(s+4\lambda_A+3\lambda_B)(s+4\lambda_A+2\lambda_B)(s+4\lambda_A)}\overline{p}_{44}(s) \tag{130}$$

$$\overline{p}_{13}(s)=\frac{2.3.4.4\lambda_A{}^3\lambda_B}{E(s)}\overline{p}_{44}(s) \tag{131}$$

$$\overline{p}_{22}(s)=\frac{3.3.4.4\lambda_A{}^2\lambda_B{}^2}{C(s)}\overline{p}_{44}(s) \tag{132}$$

$$\overline{p}_{31}(s)=\frac{2.3.4.4\lambda_A\lambda_B{}^3}{F(s)}\overline{p}_{44}(s) \tag{133}$$

$$\overline{p}_{12}(s)=\frac{2.3.3.4.4\lambda_A{}^3\lambda_B{}^2\overline{p}_{44}(s)}{(s+2\lambda_B)}\left[\frac{1}{C(s)}+\frac{1}{E(s)}\right] \tag{134}$$

$$\overline{p}_{21}(s)=\frac{2.3.3.4.4\lambda_A{}^2\lambda_B{}^3\overline{p}_{44}(s)}{(s+2\lambda_A)}\left[\frac{1}{C(s)}+\frac{1}{F(s)}\right] \tag{135}$$

$$\overline{p}_{11}(s)=\frac{2.2.3.3.4.4\lambda_A{}^3\lambda_B{}^3\overline{p}_{44}(s)}{s}\left[\frac{1}{(s+2\lambda_B)}\left(\frac{1}{C(s)}+\frac{1}{E(s)}\right)+\frac{1}{(s+2\lambda_A)}\left(\frac{1}{C(s)}+\frac{1}{F(s)}\right)\right] \tag{136}$$

D. Non Repairable and Identical

When the considered system is taken to be non-repairable and units are identical then the transition state probabilities corresponding to present system are given by

$$\overline{p}_{44}(s)\equiv\frac{1}{D(s)} \tag{137}$$

$$\overline{p}_{34}(s)=\frac{4\lambda}{(s+7\lambda)}\overline{p}_{44}(s) \tag{138}$$

$$\overline{p}_{43}(s)=\frac{4\lambda}{(s+7\lambda)}\overline{p}_{44}(s) \tag{139}$$

$$\bar{p}_{24}(s) = \frac{3.4\lambda^2}{(s+7\lambda)(s+6\lambda)}\bar{p}_{44}(s) \tag{140}$$

$$\bar{p}_{33}(s) = \frac{2.4.4\lambda^2\bar{p}_{44}(s)}{(s+6\lambda)(s+7\lambda)} \tag{141}$$

$$\bar{p}_{42}(s) = \frac{3.4\lambda^2}{(s+7\lambda)(s+6\lambda)}\bar{p}_{44}(s) \tag{142}$$

$$\bar{p}_{23}(s) = \frac{3.4.4\lambda^3}{A(s)}\bar{p}_{44}(s) \tag{143}$$

$$\bar{p}_{32}(s) = \frac{3.4.4\lambda^3}{B(s)}\bar{p}_{44}(s) \tag{144}$$

$$\bar{p}_{14}(s) = \frac{2.3.4\lambda^3}{(s+7\lambda)(s+6\lambda)(s+4\lambda)}\bar{p}_{44}(s) \tag{145}$$

$$\bar{p}_{41}(s) = \frac{2.3.4\lambda^3}{(s+7\lambda)(s+6\lambda)(s+4\lambda)}\bar{p}_{44}(s) \tag{146}$$

$$\bar{p}_{13}(s) = \frac{2.3.4.4\lambda^4}{E(s)}\bar{p}_{44}(s) \tag{147}$$

$$\bar{p}_{22}(s) = \frac{3.3.4.4\lambda^4}{C(s)}\bar{p}_{44}(s) \tag{148}$$

$$\bar{p}_{31}(s) = \frac{2.3.4.4\lambda^4}{F(s)}\bar{p}_{44}(s) \tag{149}$$

$$\bar{p}_{12}(s) = \frac{2.3.3.4.4\lambda^5\bar{p}_{44}(s)}{(s+2\lambda)}\left[\frac{1}{C(s)}+\frac{1}{E(s)}\right] \tag{150}$$

$$\bar{p}_{21}(s) = \frac{2.3.3.4.4\lambda^5\bar{p}_{44}(s)}{(s+2\lambda)}\left[\frac{1}{C(s)}+\frac{1}{F(s)}\right] \tag{151}$$

$$\bar{p}_{11}(s) = \frac{2.2.3.3.4.4\lambda^6\bar{p}_{44}(s)}{s}\left[\frac{1}{(s+2\lambda)}\left(\frac{1}{E(s)}+\frac{2}{C(s)}+\frac{1}{F(s)}\right)\right] \tag{152}$$

9. Numerical Computations

The Maple software has been used to analyze reliability, availability, MTTF, cost effectiveness and sensitivity of the system.

(I) Reliability Analysis

Let the failure rates of the components are λ_A=0.2 and λ_B=0.1, repair rates $\eta(x)=\psi(y)=\xi(z)=0$, $\theta = 1$ and $x = y = z = 1$. Also let the repair follows exponential distribution, i.e. equation (88) holds.

Putting all these values in equation (70), taking inverse Laplace transformation and setting t = 0, 10, 20, 30, 40, 50, 60, 70, 80, 90, 100, one can obtain Table 2 and Figure 3 which represent how reliability varies with respect to the time.

(II) Availability Analysis

Let the repair rates $\eta(x) = \psi(y) = \xi(z) = 1$, failure rates λ_A=0.2, λ_B=0.1, and x = y = z = 1. Putting all values in equation and taking inverse Laplace transformation, we get

$$P_{up}(t)=38.38159801/\exp(0.06t)+10.62986542/\exp(0.1197555005t) -27.49443874/\exp(0.11t)-6.67929548/\exp(0.1t) +70.62209955/\exp(0.09t) -32.7352613/\exp(0.08t)-51.72454434/\exp(0.07t) - 0.00005582017207/\exp(1.080025132t)+0.0002142911854/\exp(1.060002062t)- 0.0006288670057/\exp(1.040208788t)+0.001818175244/\exp(1.03000826t) - 0.001370908301/\exp(1.020000253t)+0.000000005310688936/\exp(1.000000005t) \quad (153)$$

Now setting t=0, 10, 20, 30, 40, 50, 60, 70, 80, 90, 100, one can get Table 1 and Figure 4 which show the variation of availability with respect to time.

(III) MTTF Analysis

Let us suppose that repair follows exponential distribution then using equation (88) and MTTF of the system is given by

$$\text{MTTF} = \lim_{s \to 0} P_{up}(s) \quad (154)$$

We have the following three cases when repair rates $\eta(x) = \psi(y) = \xi(z) = 0$, θ = 1 and x = y = z = 1:

(a) Assuming failure rate λ_A=0.06 and varying the value of λ_B as 0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.07, 0.08, 0.09, 0.10, we obtain the change of MTTF with respect to λ_B.

(b) Varying λ_A as 0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.07, 0.08, 0.09, 0.10, one can find changes of MTTF with respect to λ_A.

(c) Let us increase the value of λ_A and λ_B from 0.01 to 0.10, we obtain manner in which MTTF varies with respect to λ_A and λ_B simultaneously. Table 4 and Figure 5 show how MTTF varies with respect to time.

(IV) Cost Analysis

Let the failure rates λ_A=0.2, λ_B=0.1, repair rates $\eta = \psi = \xi = 1$ and x = y = z = 1. Putting all these values and taking inverse Laplace transforms, one can obtain equation (156). If the repair facility is always available, then expected profit during the interval (0, 100] is given by

$$E_P(t) = c_1 \int_0^t P_{up}(t)dt - c_2 t \quad (155)$$

where c_1 and c_2 are revenue rate per unit time and service cost per unit time respectively, then

$$E_p(t) = c_1(-639.6933002/\exp(.6000000000e-1t) -88.76306621/\exp(.1197555005t)+249.9494431/ \exp(.1100000000t) +66.79295480/\exp(.1000000000t)-784.6899950/\exp(.9000000000e-1t) +409.1907664/\exp(.8000000000e-1t) +738.9220620/\exp(.7000000000e-1t) +.5168414180e-4/\exp(1.080025132t)-.2021611024e-3/\exp(1.060002062t) +.6045584434e-3/\exp(1.040208788t)-0.1765204528e-2/\exp(1.030008260t)$$

$$+0.1344027413e\text{-}2/\exp(1.020000253t) -0.5310688909e\text{-}8/\exp(1.000000005t)+48.29110270-c_2t \tag{156}$$

Taking $c_1 = 1$ and $c_2 = 0.1, 0.2, 0.3, 0.4, 0.5$ and using equation (88) one can compute the variation of $E_P(t)$ with respect to time. The computational values obtained are given in Table 5 and shown in Figure 6.

(V) Sensitivity Analysis

Assuming that the equation (88) holds. We first perform a sensitivity analysis for changes in R(t) resulting from changes in system parameters λ_A and λ_B which yields

$$\frac{\partial R(t)}{\partial \lambda_A} = \begin{array}{l} -96t\exp(-(3y+4x)t)+192t\exp(-(3x+3y)t)-96t\exp(-(2x+3y)t) \\ -144t\exp(-(2y+3x)t)+36t\exp(-(2x+4y)t)+72t\exp(-(2y+2x)t)-12t\exp(-2xt) \\ -12t\exp(-4xt)+72t\exp(-(4x+2y)t)+24t\exp(-3xt)+4((-18xt-1)\exp((3x+4y)t) \\ +(9xt+1)\exp(-(4x+4y)t)+2\sinh(1/2xt)\exp((-7/2x-4y)t))/x \end{array} \tag{157}$$

$$\frac{\partial R(t)}{\partial \lambda_B} = \begin{array}{l} 4((-18ty-1)\exp(-(3y+4x)t)+(9ty+1)\exp(-(4x+4y)t) \\ +2\sinh(1/2ty)\exp((-4x-7/2y)t))/y+192t\ \exp((3x+3y)t)-144t\exp(-(2x+3y)t) \\ -96t\exp(-(2y+3x)t)+72t\exp(-(2x+4y)t)-96t\exp(-(3x+4y)t)+72t\exp(-(2y+2x)t) \\ +24t\exp(-3ty)+36t\exp(-(4x+2y)t)-12t\exp(-2ty)-12t\exp(-4ty) \end{array} \tag{158}$$

Tables 6 and 7 are corresponding to the sensitivity analysis of the system reliability with respect to change in λ_A and λ_B respectively. The nature of sensitivity has been shown in Figures 7 and 8. One can see that sensitivity of the system reliability decreases with the increase in the values of λ_A and λ_B.

Time	Reliability
0	1
10	0.999934
20	9.98E-01
30	9.98E-01
40	9.58E-01
50	9.09E-01
60	8.42E-01
70	7.64E-01
80	6.80E-01
90	5.96E-01
100	5.17E-01

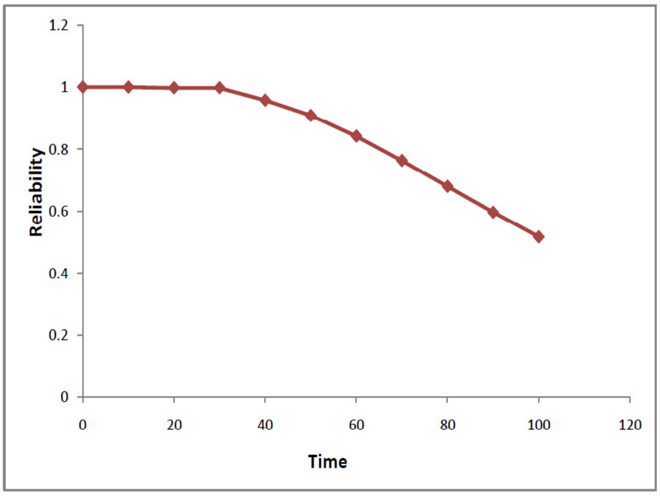

Table 2: Time vs. Reliability

Figure 3: Time vs. Reliability

Time	Availability
0	1
10	0.982733
20	8.88E-01
30	7.33E-01
40	5.60E-01
50	4.03E-01
60	2.77E-01
70	1.83E-01
80	1.17E-01
90	7.34E-02
100	4.50E-02

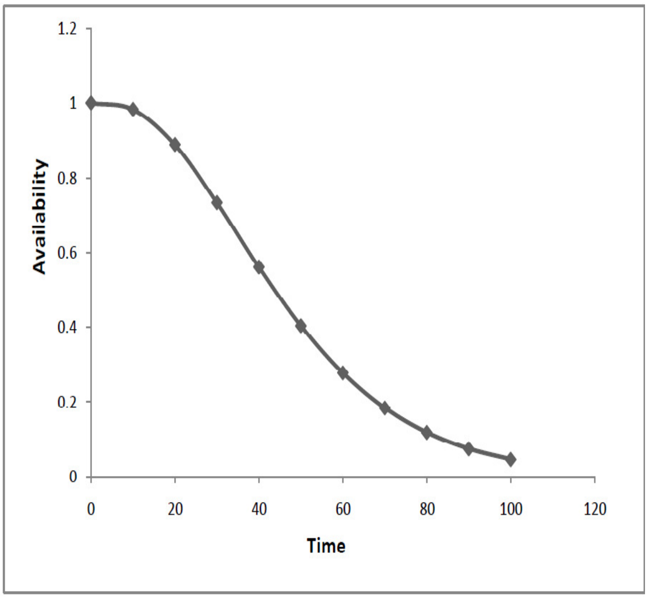

Table 1: Time vs. Availability **Figure 4: Time vs. Availability**

λ_A	MTTF	λ_B	MTTF	λ_A and λ_B	MTTF
0.01	108.8022	0.01	108.5903	0.01	143.2143
0.02	56.07281	0.02	55.30102	0.02	71.60714
0.03	39.78768	0.03	38.42833	0.03	47.7381
0.04	32.4933	0.04	30.64013	0.04	35.80357
0.05	28.64286	0.05	26.40345	0.05	28.64286
0.06	26.40345	0.06	23.86905	0.06	23.86905
0.07	25.01197	0.07	22.25332	0.07	20.45918
0.08	24.10386	0.08	21.17413	0.08	17.90179
0.09	23.48797	0.09	20.42675	0.09	15.9127
0.1	23.057	0.1	19.89384	0.1	14.32143

Table 4: Failure rates vs. MTTF

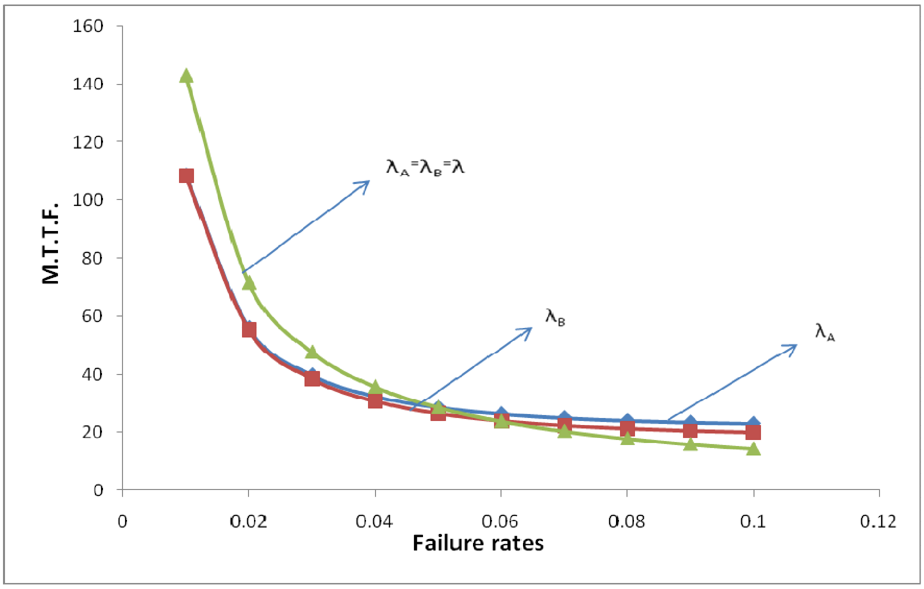

Figure 5: Failure rates vs. MTTF

Time	$Ep(t)$				
	$C_2 = 0.1$	$C_2 = 0.2$	$C_2 = 0.3$	$C_2 = 0.4$	$C_2 = 0.5$
0	0	0	0	0	0
10	0.900029	0.800029	0.700029	0.600029	0.500029
20	1.80E+00	1.600141	1.400141	1.200141	1.000141
30	2.70E+00	2.40023	2.10023	1.80023	1.50023
40	3.60E+00	3.200008	2.800008	2.400008	2.000008
50	4.50E+00	3.999035	3.499035	2.999035	2.499035
60	5.40E+00	4.796752	4.196752	3.596752	2.996752
70	6.29E+00	5.592498	4.892498	4.192498	3.492498
80	7.19E+00	6.385535	5.585535	4.785535	3.985535
90	8.08E+00	7.175069	6.275069	5.375069	4.475069
100	8.96E+00	7.96026	6.96026	5.96026	4.96026

Table 5: Time vs. expected profit

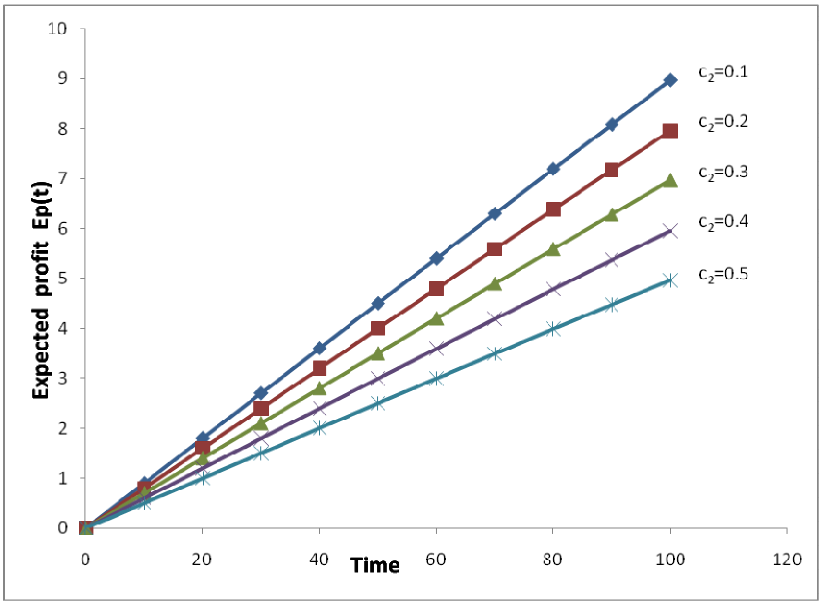

Figure 6: Time vs. expected profit

Time	Value of $\partial R(t)/\partial \lambda_A$		
0	0	0	0
10	-0.14829	-0.24821	-0.3288
20	-2.84902	-3.57691	-3.57157
30	-10.2393	-9.72114	-7.39799
40	-19.0896	-13.813	-8.09578
50	-25.396	-14.1113	-6.4309
60	-27.7481	-11.9242	-4.26197
70	-26.7918	-8.96338	-2.5318
80	-23.8387	-6.24727	-1.4037
90	-20.0516	-4.13932	-0.74398
100	-16.2038	-2.64835	-0.38255

Table 6: Sensitivity analysis of the system MTTF w. r. t. λ_A

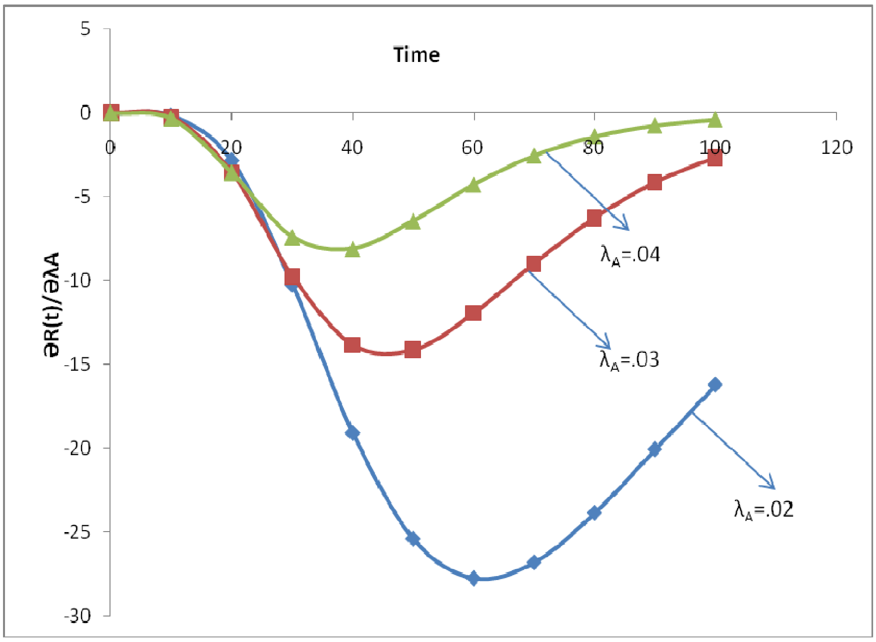

Figure 7: Sensitivity of system MTTF with respect to different values of λ_A

Time	Value of $\partial R(t)/\partial \lambda_B$		
0	0	0	0
10	-0.14829	-0.24821	-0.3288
20	-2.84902	-3.57691	-3.57157
30	-10.2393	-9.72114	-7.39799
40	-19.0896	-13.813	-8.09578
50	-25.396	-14.1113	-6.4309
60	-27.7481	-11.9242	-4.26197
70	-26.7918	-8.96338	-2.5318
80	-23.8387	-6.24727	-1.4037
90	-20.0516	-4.13932	-0.74398
100	-16.2038	-2.64835	-0.38255

Table 7: Sensitivity analysis of the system MTTF w. r. t. λ_B

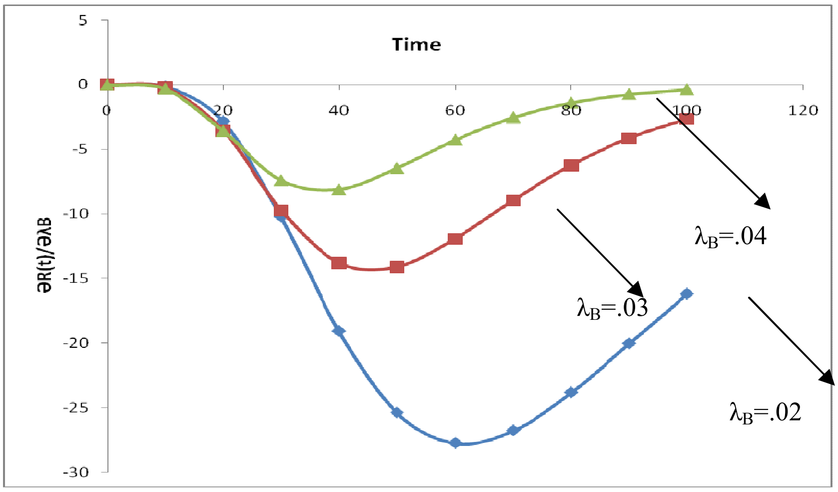

Figure 8: Sensitivity of system MTTF with respect to different values of λ_B

10. Interpretation of the Result and Conclusion

For a more concrete study of the system behaviour, curves for reliability, availability, MTTF, expected profit with respect to time and sensitivity with respect to λ_A and λ_B have been plotted.

The Table 2 gives the variation of reliability with respect to the time. By critical examination of the Figure 3 we conclude that the reliability of the system decreases as time increases and attains a value 0.5170511226 at t=100.

Figure 4 depicts the behaviour of availability with respect to time and its value has been given in Table 3. It is clear from the Figure 4 that availability decreases as the time increases from 0 to 100 and attains a value 0.04501027844 at t=100.

Figures 5 shows the variation of system MTTF with respect to λ_A, λ_B and λ ($\lambda_A = \lambda_B$). The corresponding values have been given in Tables 4. Observations of this figure reveal that in each case MTTF of considered system decreases as failure rates increases from 0.01 to 0.1. It varies from 108.8022-23.057, 108.5903-19.89384 and 143.2143-14.32143 with respect to λ_A, λ_B and λ respectively. Also, MTTF of the system is greater with respect to λ than λ_A and λ_B. One of the interesting facts is that at failure rate 0.05, MTTF with respect to λ_A and λ are same. Prior to failure rate 0.05, MTTF is higher with respect to λ than λ_A and afterwards situation got reversed. We also observe that prior to failure rate 0.06, value of the MTTF is higher with respect to λ than λ_B and after this the value of MTTF got reversed. It is worth mentioning that the value of MTTF with respect to λ_B and λ are the same at failure rate 0.06.

For the cost analysis of the system we keep revenue cost per unit time at 1 and vary service cost from 0.1 to 0.5. The behaviour of expected profit can be observed from Figure 6. We can see from the figure that the profit goes on decreasing with the increase in service cost. One can draw an important conclusion from Figure 6 that system becomes more profitable when service cost is 0.1. The highest value of expected profit is 8.96026.

Figures 7 and 8 represent how sensitivity of the system reliability varies with respect to parameters λ_A and λ_B. It is clear from these figures that sensitivity of system reliability initially decreases and then increases as time passes with respect to λ_A and λ_B and increases with the increase in failure rates λ_A and λ_B from 0.2 to 0.4. It is interesting to note that sensitivity of the system reliability is same with respect to λ_A and λ_B. By studying the graphs we can conclude that the system can be made less sensitive by decreasing its failure rates.

References

1. Angus, J. E. (1988). On computing MTBF for a k-out-of-n: G repairable system, IEEE Transactions on Reliability, 37(3), p. 312-313.
2. Goel L. R. and Gupta, P. (1984). Stochastic analysis of a two unit parallel system with partial and catastrophic failures and preventive maintenance, Microelectron. Reliab., 24(3), p. 881-883.
3. Hassett, T. F., Dietrich, D. L. and Szidarovszky, F. (1995). Time varying failure rates in the availability and reliability analysis of repairable systems, IEEE Transactions on Reliability, 44 (1), p. 155-160.
4. Kumar, J. and Kadyan, M. S. (2012). Profit analysis of a system of non identical units with degradation and replacement, International Journal of Computer Application, 40(3), p. 19-25.
5. Lim, Tae-Jin and Lie, C. H. (2000) Analysis of system reliability with dependent repair modes, IEEE Transactions on Reliability, 49 (2), p. 153-162.
6. Lindskog, F. (2000). Modelling dependence with copulas, ETH, Zurich, Risklab report.
7. Malik, S. C. and Bhardwaj, R. K. (2008). Profit analysis of a stochastic model of 2-out-of-3 redundant system with inspection by a server who appears and disappears randomly, International Journal of Business and Engineering Research, 1(1), p. 49-58.
8. Moustafa, M. S. (2001). Reliability analysis of systems with time varying failure rates, Economic Quality Control, 16(2), p. 183-191.
9. Nailwal, B. and Singh, S. B. (2011). Performance evaluation and reliability analysis of a complex system with three possibilities in repair with the application of copula, International Journal of Reliability and Application. 12(1), p. 15-39.
10. Nailwal, B. and Singh, S. B. (2012). Reliability measures and sensitivity analysis of a Complex matrix system including power failure, IJE Transactions A: Basics, 25(2), p. 115-130.

11. Pandey, S. B., Singh, S. B. and Sharma, S. (2008). Reliability and cost analysis of a system with multiple components using copula, Journal of Reliability and Statistical Studies, 1(1), 25-32.
12. Ram, M and Singh, S. B. (2009). Analysis of reliability characteristics of a complex engineering system under copula, Journal of Reliability and Statistical Studies, 2(1), p. 91-102.
13. Ram, M and Singh, S. B. (2012). Cost benefit analysis of a system under head of-line repair approach using Gumbel-Hougaard Family copula, Journal of Reliability and Statistical Studies, 5(2), p. 105-118.
14. Ram, M. and Singh, S. B. (2008). Availability and cost analysis of a parallel redundant complex system with two types of failure under preemptive-resume repair discipline using Gumbel-Hougaard family of copula in repair, International Journal of Reliability, Quality & Safety Engineering, 15 (4), p. 341-365.