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ON ESTIMATION OF STRESS STRENGTH MODEL FOR GENERALIZED INVERTED EXPONENTIAL DISTRIBUTION

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Abstract

In this paper, the estimation of R=Pr(Y < Y), when X and Y are two generalized inverted exponential distributions with different parameters is considered. The maximum likelihood estimator (MLE) of R and its asymptotic distribution are obtained. Exact and asymptotic confidence intervals of R are constructed using both exact and asymptotic distributions. Assuming that the common scale parameter is known, MLE, Bayes estimators and confidence intervals of R are investigated. Bayes estimators are based on informative and non-informative priors of the unknown parameters. Monte Carlo simulations are performed to compare and to validate the different proposed estimators.

Key Words: Generalized Exponential Distribution, System Reliability, Stress-Strength, Bayes, Maximum Likelihood.

1. Introduction

The estimation of system reliability in statistical applications is very common and has much attention in literature. The most widely approach used for reliability estimation is the well-known stress-strength model. This model is used in many applications of physics and engineering such as strength failure and system collapse. In stress-strength modeling, R=Pr(Y < X) is a measure of component reliability when it is subjected to random stress Y and has strength X. In this context, R can be considered as a measure of system performance and naturally arise in electrical and electronic systems. Other interpretation can be that, the reliability, R, of the system is the probability that the system is strong enough to overcome the stress imposed on it. It may be mentioned that R is of greater interest than just reliability since it provides a general measure of the difference between two populations and has applications in many area. For example, if X is the response for a control group, and Y refers to a treatment group, R is a measure of the effect of the treatment. In addition, it may be mentioned that R equals the area under the receiver operating characteristic (ROC) curve for diagnostic test or biomarkers with continuous outcome (Bamber, 1975). The ROC curve is widely used, in biological, medical and health service research, to evaluate the ability of diagnostic tests or biomarkers to distinguish between two groups of subjects, usually non-diseased and diseased subjects. For more details, one can be advised to Kotz et. al., (2003).

Many authors have studied the stress-strength parameter R. Gogoi and Borah (2012) deals with the stress vs. strength problem incorporating multi-component for systems viz. standby redundancy in the case of Exponential, Gamma and Lindley distributions. Singh et. al., (2011) have developed a re-modeling of stress-strength system reliability where they have defined the probability that the system is capable to withstand the maximum operated stress at its minimum strength when both stress and strength variables are Weibull distributed. Barbiero (2013) studied statistical inference for the reliability of stress-strength models when stress and strength are independent Poisson random variables, whereas, Ali et. al. (2010) have investigated the estimation of Pr(X < Y), when X and Y belong to different distribution families. Wong (2012) has constructed an asymptotic confidence interval for Pr(Y < X) where X and Y are two independent generalized Pareto random variables with a common scale parameters. Furthermore, Rubio and Steel, (2012) studied Bayesian estimation of the stress-strength model in the case when the marginal distributions of X and Y are independent /dependent random variables that belong to classes of distributions obtained by skewing scale mixtures of normal distributions and when the variable.

In this paper, estimation of the system reliability, *R*, when X and Y are independent but not identically distributed generalized inverted exponential distribution (GIED) variables is considered. The GIED distribution has the following cumulative distribution function (*cdf*) and probability density function (*pdf*) for X > 0:

$$F(x) = 1 - [1 - e^{-\lambda/x}]^{\alpha}, \qquad x > 0 \ , \tag{1}$$

with

$$f(x) = \alpha \frac{\lambda}{x^2}\, e^{-\lambda/x}[1 - e^{-\lambda/x}]^{\alpha-1}, \qquad x > 0\,, \tag{2}$$

where $\lambda > 0$ is the scale parameter and $\alpha > 0$ is the shape parameter (Abouammoh and Alshingiti, 2009).

The rest of the paper is organized as follows. In section 2, the system reliability is derived and in section 3, the maximum likelihood estimation of *R* is discussed. In section 4, asymptotic confidence interval of *R* is obtained while section 5 is devoted to the Bayesian estimation of *R*. Numerical solutions and performance studies of the estimators are investigated ion section 6. Finally the paper is conclude.

2. System Reliability (*R*)

Let X and Y be two independent GIED random variables with parameters (λ,α) and (λ,β) respectively. The reliability of the system is defined as follows

$$R = P(Y < X)$$

$$= \int_0^{\infty}\int_0^{x} \alpha \frac{\lambda}{x^2}\, e^{-\lambda/x}(1 - e^{-\lambda/x})^{\alpha-1}\, \beta \frac{\lambda}{y^2}\, e^{-\lambda/y}(1 - e^{-\lambda/y})^{\beta-1}\, dy\, dx \ ,$$

$$= 1 - \frac{\alpha}{\alpha+\beta} = \frac{\beta}{\alpha+\beta} \ . \tag{3}$$

3. Maximum likelihood estimation

Assume that two independent random samples $(X_1, X_2, ..., X_n)$ and $(Y_1, Y_2, ..., Y_m)$ are observed from GIED(λ,α), and GIED(λ,β) respectively. The likelihood function of λ,α and β for the observed samples is

$$L(data;\lambda,\alpha,\beta) = \lambda^n\, \alpha^n\, [\prod_{i=1}^{n} x_i^2]^{-1}\, e^{-\sum_{i=1}^{n}\lambda/x_i} \prod_{i=1}^{n}(1 - e^{-\lambda/x_i})^{\alpha-1}$$

$$\times\ \lambda^m \beta^m [\prod_{j=1}^{m} y_j^2]^{-1}\, e^{-\sum_{j=1}^{m}\lambda/y_j} \prod_{j=1}^{m}(1-e^{-\lambda/y_j})^{\beta-1} \tag{4}$$

Therefore, the log-likelihood function of λ, α and β will be

$$\log L = (m+n)\log\lambda + n\log\alpha + m\log\beta - 2\sum_{i=1}^{n}\log x_i - \sum_{i=1}^{n}\frac{\lambda}{x_i} - 2\sum_{j=1}^{m}\log y_j - \sum_{j=1}^{m}\frac{\lambda}{y_j}$$
$$+(\alpha-1)\sum_{i=1}^{n}\log(1-e^{\lambda/x_i}) + (\beta-1)\sum_{j=1}^{m}\log(1-e^{\lambda/y_j}). \tag{5}$$

The estimators $\hat{\lambda}$, $\hat{\alpha}$ and $\hat{\beta}$ of the parameters λ, α and β respectively can be obtained as the solution of the likelihood equations

$$\frac{(n+m)}{\lambda} - \sum_{i=1}^{n}\frac{1}{x_i} - \sum_{j=1}^{m}\frac{1}{y_j} + (\alpha-1)\sum_{j=1}^{m}\frac{e^{-\lambda/x_i}}{x_i\,(1-e^{-\lambda/x_i})} + (\beta-1)\sum_{j=1}^{m}\frac{e^{-\lambda/y_j}}{y_j\,(1-e^{-\lambda/y_j})} = 0 \tag{6}$$

$$\frac{n}{\alpha} + \sum_{i=1}^{n}\log(1-e^{-\lambda/x_i}) = 0 \tag{7}$$

$$\frac{m}{\beta} + \sum_{j=1}^{m}\log(1-e^{-\lambda/y_j}) = 0 \tag{8}$$

From Equations (7) and (8), the estimators of α and β are give by

$$\hat{\alpha} = -\frac{n}{\sum_{i=1}^{n}\log[1-e^{-\hat{\lambda}/x_i}]}, \tag{9}$$

and

$$\hat{\beta} = -\frac{m}{\sum_{j=1}^{m}\log[1-e^{-\hat{\lambda}/y_j}]} \tag{10}$$

where $\hat{\lambda}$ is the solution of the nonlinear equation

$$\frac{(n+m)}{\hat{\lambda}} - \sum_{i=1}^{n}\frac{1}{x_i} - \sum_{j=1}^{m}\frac{1}{y_j} - \left(\frac{n}{\sum_{i=1}^{n}\log[1-e^{-\hat{\lambda}/x_i}]} + 1\right)\sum_{j=1}^{m}\frac{e^{-\hat{\lambda}/x_i}}{x_i\,(1-e^{-\hat{\lambda}/x_i})}$$
$$-\left(\frac{m}{\sum_{j=1}^{m}\log(1-e^{-\hat{\lambda}/y_j})} + 1\right)\sum_{j=1}^{m}\frac{e^{-\hat{\lambda}/y_j}}{y_j\,(1-e^{-\hat{\lambda}/y_j})} = 0 \tag{11}$$

Once the estimators of α and β, are derived and using the invariance property of the MLEs, the MLE of R becomes

$$\hat{R} = \frac{\hat{\beta}}{\hat{\alpha}+\hat{\beta}} = \frac{[m\sum_{i=1}^{n}\log(1-e^{-\hat{\lambda}/x_i})]}{[m\sum_{i=1}^{n}\log(1-e^{-\hat{\lambda}/x_i}) + n\sum_{j=1}^{m}\log(1-e^{-\hat{\lambda}/y_j})]}. \tag{12}$$

Assuming that the scale parameter λ is known, we have

$$\hat{\alpha} = -\frac{n}{\sum_{i=1}^{n}\log[1-e^{-\lambda/x_i}]}, \tag{13}$$

$$\hat{\beta} = -\frac{m}{\sum_{j=1}^{m}\log[1-e^{-\lambda/y_j}]}, \tag{14}$$

and

$$\hat{R} = \frac{[m\sum_{i=1}^{n}\log(1-e^{-\lambda/x_i})]}{[m\sum_{i=1}^{n}\log(1-e^{-\lambda/x_i}) + n\sum_{j=1}^{m}\log(1-e^{-\lambda/y_j})]} . \tag{15}$$

When the scale parameter λ is known and equal to one, it can be easily shown that the random variable $U_i = -\log[1-e^{-1/x_i}]$ is distributed as exponential random variable with mean (1/α). Similarly, $V_i = -\log[1-e^{-1/y_j}]$ is distributed as exponential random variable with mean (1/β). Therefore, $2\alpha\sum_{i=1}^{n}\log[1-e^{-1/x_i}] \sim \chi^2_{2n}$ and $2\beta\sum_{j=1}^{m}\log[1-e^{-1/y_j}] \sim \chi^2_{2m}$ (Gupta and Kundu, 2002). Accordingly, $\hat{R}$ can be written as $\hat{R} \approx 1/(1+(\alpha/\beta)F)$, where F has a Fisher distribution with $2n$ and $2m$ degrees of freedom respectively and therefore, the *pdf* of $\hat{R}$ is given by

$$f_{\hat{R}}(u) = \frac{\Gamma(n+m)}{\Gamma(n)\Gamma(m)}\left(\frac{n\alpha}{m\beta}\right)^n\left(\frac{1}{u^2}\right)\frac{\left(\frac{1-u}{u}\right)^{n-1}}{\left(1+\frac{n\alpha}{m\beta}\left(\frac{1-u}{u}\right)\right)^{n+m}} , \tag{16}$$

where $0 < u < 1$. Based on this information a $(1-\tau)$ 100% confidence interval of R can be obtained as follows

$$\left[\frac{1}{1+\left(\frac{1}{\hat{R}}-1\right)F_{2n,2m,1-\tau/2}}, \frac{1}{1+\left(\frac{1}{\hat{R}}-1\right)F_{2n,2m,\tau/2}}\right], \tag{17}$$

where $F_{2n,2m,\tau/2}$ and $F_{2n,2m,1-\tau/2}$ are the lower and upper τ /2th percentile of a Fisher distribution with $2n$ and $2m$ degrees of freedom respectively

4. Asymptotic distribution and confidence interval of *R*

Based on the asymptotic properties and the general conditions of the MLEs $\hat{\lambda}$, $\hat{\alpha}$ and $\hat{\beta}$ (Lehmann, 1999), the asymptotic distribution of the MLEs immediately follows from the Fisher information matrix of λ, α and β. That is, when $n \to \infty, m \to \infty$ and $n/m \to p, 0 < p < 1$, it follows that

$$\left(\sqrt{n}(\hat{\alpha}-\alpha), \sqrt{m}(\hat{\beta}-\beta), \sqrt{n}(\hat{\lambda}-\lambda)\right) \xrightarrow{D} N_3(0,\Sigma_3) \tag{18}$$

where

$$\Sigma_3 = I^{-1}(\Omega) = \begin{pmatrix} I_{11} & I_{12} & I_{13} \\ I_{21} & I_{22} & I_{23} \\ I_{31} & I_{32} & I_{33} \end{pmatrix}^{-1} , \tag{19}$$

and the matrix $I(\Omega)$ is the Fisher information matrix of the parameter vector $\Omega = (\alpha, \beta, \lambda)$, and the ij^{th} element is given by the second partial derivatives

$I_{ij} = \partial^2 \ln L(\Omega) / \partial\omega_i \partial\omega_j$, $i, j = 1,2,3$. From the asymptotic properties of the MLEs of λ, α and β, one can easily get,

$$\sqrt{n}\left(\hat{R} - R\right) = \sqrt{n}\left(\frac{\hat{\beta}}{\hat{\alpha} + \hat{\beta}} - \frac{\beta}{\alpha + \beta}\right) \xrightarrow{D} N(0, \psi^2) \tag{20}$$

where

$$\psi^2 = E\left(\sqrt{n}\left(\hat{R} - R\right)\right)^2 = E\left(\left(\frac{\hat{\beta}\sqrt{n}\,(\alpha + \beta)}{(\hat{\alpha} + \hat{\beta})} - \frac{\beta\sqrt{n}\,(\hat{\alpha} + \hat{\beta})}{(\alpha + \beta)}\right)\right)^2, \tag{21}$$

A (1-τ)100% approximate confidence interval of R can be constructed based on the asymptotic results obtained. This asymptotic confidence interval is given by

$$\hat{R} \pm Z_{1-\tau/2}\, \hat{\psi}\,, \tag{22}$$

where $\hat{\psi}$ is the asymptotic standard deviation of $\hat{R}$.

5. Bayesian Estimation of R

In this section, the Bayes estimator of R denoted as $\hat{R}_{BS}$ is obtained under the assumption that the shape parameters α and β are independent random variables with prior distributions $\Gamma(a_1, b_1)$ and $\Gamma(a_2, b_2)$ with *pdf*'s respectively

$$\pi(\alpha) = \frac{b_1^{a_1}}{\Gamma(a_1)}\, \alpha^{a_1 - 1}\, e^{-b_1 \alpha}; \qquad \alpha > 0, \tag{23}$$

and

$$\pi(\beta) = \frac{b_2^{a_2}}{\Gamma(a_2)}\, \beta^{a_2 - 1}\, e^{-b_2 \beta}; \qquad \beta > 0. \tag{24}$$

Based on the above assumptions and from Equation (4), the joint density of the data, α and β can be obtained as

$$L(data, \alpha, \beta) = L(data; \alpha, \beta)\; \pi(\beta)\; \pi(\beta). \tag{25}$$

Therefore, the joint posterior density of these data, α and β given the data can be obtained as follows

$$L(\alpha, \beta / data) = \frac{L(data, \alpha, \beta)\; \pi(\beta)\; \pi(\beta)}{\int_0^\infty \int_0^\infty L(data, \alpha, \beta)\; \pi(\beta)\; \pi(\beta)\; d\alpha\, d\beta}, \tag{26}$$

The posterior pdf's of α and β are

$$\pi(\alpha / x) \sim \text{Gamma}\left(n + a_1, b_1 - T_1\right), \tag{27}$$

and

$$\pi(\beta / x) \sim \text{Gamma}\left(m + a_2, b_2 - T_2\right). \tag{28}$$

respectively, where $T_1 = \sum_{i=1}^{n} \log[1 - e^{-\lambda / x_i}]$ and $T_2 = \sum_{j=1}^{m} \log[1 - e^{-\lambda / y_j}]$. Since α and β are assumed to be independent and using Equation (26-28), the posterior *pdf* of *R* becomes

$$\pi(r / x, y) = K\, \frac{r^{m + a_2 - 1}\, (1 - r)^{n + a_1 - 1}}{[(1 - r)(b_1 - T_1) + r\, (b_2 - T_2)]^{n + m + a_1 + a_2}}, \quad 0 < r < 1, \tag{29}$$

where

$$K = \frac{\Gamma(n + m + a_1 + a_2 - 1)}{\Gamma(n + a_1)\Gamma(m + a_2)} (b_1 - T_1)^{n + a_1} (b_2 - T_2)^{m + a_2}.$$

Therefore, the Bayesian estimator of R under squared error loss function is given by

$$\hat{R}_{BS} = E(R / x, y) = \int_0^1 r\,\pi(r / x, y)\,dr\ . \tag{30}$$

The Bayes estimate of R under squared error loss cannot be computed analytically. Alternatively, numerical solution based on MATHEMATICA program is employed to evaluate $\hat{R}_{BS}$ for different values of the parameters.

6. Simulation study

In this section, Monte Carlo simulation is performed to test the behavior of the proposed estimators for different sample sizes and for different parameter values. The performances of the MLEs and the Bayes estimates are compared in terms of biases and mean squares errors (MSEs). Bayes estimates, are computed based on two type of priors, (i) non-informative priors, where $a_1 = a_2 = b_1 = b_2 = 0.0001$, (Congdon, 2001, Kundu and Gupta, 2005). (ii) Informative priors, where it is assumed that there are some prior information about the parameters and $a_1, a_2, b_1, b_2 > 0$ (for example, $a_1 = a_2 = 3$, $b_1 = b_2 = 2$). The simulations are based on 1000 replications and the results are presented in Tables 1. In Table 2, we obtain both CI_{EX} and CI_{AS} using Equations (17) and (22) respectively. Both confidence intervals are based on MLEs of *R*. the results are shown in table 2. All simulations are based on the following sample sizes; *n, m* = 15, 25, and 50 and we assume $\beta = 0.5, 1.5, 2.5, 4.5$, and $\alpha = 1.5$, respectively. Table 3 shows the results of Bayes estimation of *R*.

It can be noted that even for small sample sizes, the performance of the Bayes estimator is better than the MLE of *R* in terms of biases and MSEs. It is also observed that when (*n,m*) increases, the MSE and biases decrease for both MLE and Bayesian estimators. In addition, it is noted that for fixed sample sizes and as the parameter β increases MSE and biases decrease for both MLE and Bayesian estimation methods. The confidence intervals CI_{AS}, performs quite well as the sample sizes increases, while CI_{EX} have larger interval length comparing to CI_{AS}. It is also observed that for the MLE of *R* there is overestimation for parameter values of $0 < R < 0.5$, while for values $0.5 \le R \le 1$, there is underestimation of the true value of *R*.

Table 1: MLE estimation of R when α is fixed and equal to one and scale parameter is known (λ=1).

(n,m)	β	*R*	$\hat{R}_{ML}$	*Bias*	*MSE*
(10,10)	0.25	0.2000	0.2101	0.0101	0.1061
	0.50	0.3333	0.3411	0.0078	0.0872
	1.00	0.5000	0.5005	0.0005	0.0843
	2.00	0.6667	0.6599	-0.0068	0.0626
	3.00	0.7500	0.7410	-0.0090	0.0527
(10,15)	0.25	0.2000	0.2087	0.0087	0.0485
	0.50	0.3333	0.3432	0.0099	0.0365
	1.00	0.5000	0.5019	0.0019	0.0640
	2.00	0.6667	0.6641	-0.0026	0.0401
	3.00	0.7500	0.7457	-0.0043	0.0337
(15,15)	0.25	0.2000	0.2044	0.0044	0.0232
	0.50	0.3333	0.3357	0.0024	0.0204
	1.00	0.5000	0.4977	-0.0023	0.0373
	2.00	0.6667	0.6603	-0.0064	0.0218
	3.00	0.7500	0.7427	-0.0073	0.0183
(25,25)	0.25	0.2000	0.2063	0.0063	0.0176
	0.50	0.3333	0.3403	0.0073	0.0148
	1.00	0.5000	0.5001	0.0001	0.0246
	2.00	0.6667	0.6600	-0.0070	0.0148
	3.00	0.7500	0.7427	-0.0073	0.0056
(25,50)	0.25	0.2000	0.2033	0.0033	0.0119
	0.50	0.3333	0.3345	0.0015	0.0098
	1.00	0.5000	0.5035	0.0035	0.0091
	2.00	0.6667	0.6664	-0.0003	0.0056
	3.00	0.7500	0.7456	-0.0044	0.0042
(50,50)	0.25	0.2000	0.2049	0.0049	0.0084
	0.50	0.3333	0.3350	0.0020	0.0077
	1.00	0.5000	0.4996	-0.0004	0.0063
	2.00	0.6667	0.6647	-0.0023	0.0035
	3.00	0.7500	0.7476	-0.0024	0.0028

Table 2: Exact and asymptotic confidence intervals of R based on MLEs and at significance level 0.05 and scale parameter is known (λ=1)

(n,m)	*R*	CI_{EX}	CI_{AS}
(10,10)	0.2000	(0.1113, 0.3610)	(0.1317, 0.2885)
	0.3333	(0.1960, 0.5237)	(0.2666, 0.4156)
	0.5000	(0.3525, 0.6803)	(0.4279, 0.5730)
	0.6667	(0.4774, 0.8047)	(0.5952, 0.7246)
	0.7500	(0.5739, 0.8587)	(0.6822, 0.7998)
(10,15)	0.2000	(0.1201, 0.3497)	(0.1401, 0.2773)
	0.3333	(0.2129, 0.5159)	(0.2824, 0.4039)
	0.5000	(0.3428, 0.6726)	(0.4451, 0.5587)
	0.6667	(0.5058, 0.8013)	(0.6131, 0.7151)
	0.7500	(0.6029, 0.8567)	(0.7026, 0.7888)
(15,15)	0.2000	(0.1225, 0.3211)	(0.1456, 0.2632)
	0.3333	(0.2154, 0.4819)	(0.2808, 0.3906)
	0.5000	(0.3499, 0.6459)	(0.4487, 0.5467)
	0.6667	(0.5136, 0.7816)	(0.6172, 0.7034)
	0.7500	(0.6106, 0.8416)	(0.7035, 0.7819)

Table 2 cont.: Exact and asymptotic confidence intervals of R based on MLEs and at significance level 0.05 and scale parameter is known (λ=1)

(n,m)	R	CI_{EX}	CI_{AS}
(25,25)	0.2000	(0.1398, 0.2937)	(0.1534, 0.2592)
	0.3333	(0.2439, 0.4521)	(0.2972, 0.3834)
	0.5000	(0.3848, 0.6154)	(0.4589, 0.5413)
	0.6667	(0.5483, 0.7564)	(0.6228, 0.6972)
	0.7500	(0.6434, 0.8220)	(0.7133, 0.7721)
(25,50)	0.2000	(0.1439, 0.2835)	(0.1641, 0.2425)
	0.3333	(0.2487, 0.4380)	(0.3012, 0.3678)
	0.5000	(0.4004, 0.6113)	(0.4741, 0.5329)
	0.6667	(0.5682, 0.7560)	(0.6409, 0.6919)
	0.7500	(0.6587, 0.8197)	(0.7241, 0.7672)
(50,50)	0.2000	(0.1562, 0.2640)	(0.1755, 0.2343)
	0.3333	(0.2658, 0.4122)	(0.3134, 0.3566)
	0.5000	(0.4177, 0.5815)	(0.4819, 0.5172)
	0.6667	(0.5875, 0.7340)	(0.6490, 0.6804)
	0.7500	(0.6803, 0.8048)	(0.7378, 0.7574)

Table 3: Bayesian estimation of R when the population value of α is one and the scale parameter is known (λ=1)

(n,m)	β	R	Non-informative priors $a_1 = a_2 = b_1 = b_2 = 0.0001$		Informative priors $a_1 = a_2 = 3$, $b_1 = b_2 = 2$	
			Bias	*(MSE)*	*Bias*	*(MSE)*
(10,10)	0.25	0.2000	0.0143	0.0148	0.0111	0.0130
	0.50	0.3333	0.0113	0.0144	0.0098	0.0104
	1.00	0.5000	0.0075	0.0113	0.0090	0.0091
	2.00	0.6667	0.0034	0.0081	0.0082	0.0085
	3.00	0.7500	0.0041	0.0075	0.0062	0.0077
(10,15)	0.25	0.2000	0.0073	0.0096	0.0094	0.0109
	0.50	0.3333	0.0084	0.0122	0.0089	0.0084
	1.00	0.5000	0.0061	0.0081	0.0075	0.0078
	2.00	0.6667	0.0025	0.0060	0.0051	0.0072
	3.00	0.7500	0.0021	0.0047	0.0046	0.0062
(15,15)	0.25	0.2000	0.0056	0.0072	0.0084	0.0096
	0.50	0.3333	0.0061	0.0067	0.0071	0.0077
	1.00	0.5000	0.0038	0.0053	0.0062	0.0074
	2.00	0.6667	0.0022	0.0041	0.0045	0.0064
	3.00	0.7500	0.0017	0.0030	0.0038	0.0051
(25,25)	0.25	0.2000	0.0044	0.0050	0.0069	0.0082
	0.50	0.3333	0.0058	0.0036	0.0053	0.0074
	1.00	0.5000	0.0030	0.0030	0.0047	0.0068
	2.00	0.6667	0.0021	0.0019	0.0038	0.0049
	3.00	0.7500	0.0019	0.0016	0.0031	0.0052
(25,50)	0.25	0.2000	0.0020	0.0035	0.0049	0.0073
	0.50	0.3333	0.0040	0.0028	0.0044	0.0057
	1.00	0.5000	0.0021	0.0019	0.0026	0.0041
	2.00	0.6667	0.0010	0.0016	0.0016	0.0036
	3.00	0.7500	0.0007	0.0011	0.0009	0.0030
(50,50)	0.25	0.2000	0.0021	0.0024	0.0041	0.0056
	0.50	0.3333	0.0016	0.0016	0.0033	0.0042
	1.00	0.5000	0.0012	0.0012	0.0018	0.0035
	2.00	0.6667	0.0007	0.0008	0.0011	0.0017
	3.00	0.7500	0.0007	0.0007	0.0007	0.0020

7. Conclusion

In this paper, the problem of estimating Pr(Y <X) for the generalized inverted exponential distribution has been addressed. The asymptotic distribution of the maximum likelihood estimator has been used to construct confidence intervals which function well even for small sample sizes. It has been observed that the Bayes estimators behave quite similarly to the MLEs. Moreover, the MSE of the estimates of R decreases as the parameter β increases for fixed (m,n). Further, when (m,n), increases the MSEs of all the estimators decreases rapidly. The performance of the Bayes estimators is also quite well and the MSEs of the Bayes estimators are smaller than the MSEs of MLEs. Finally, the average lengths of all intervals decrease as (m,n) increases.

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