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RELIABILITY ANALYSIS OF A TWO UNIT COLD STANDBY SYSTEM USING MARKOV PROCESS

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Abstract

This paper deals with the reliability analysis of a system having four components arranged in series. Subsystems A, B, C have single unit whereas subsystem D has three units where one unit is active and the other two are cold standby arranged in parallel. System can completely fail either due to the failure of subsystems A, B and C or due the failure of all units of subsystem D. All failure rates are constant and all repair rates follow the general time distribution. The analysis is carried out using the supplementary variable technique and Laplace transformation for evaluating the reliability measures.

Key Words: System Availability, System Reliability, Sensitivity Analysis, Cost Analysis, Cold Standby.

1. Introduction

System reliability occupies increasingly more important issues in power plants, manufacturing systems, industrial systems, engineering systems, standby systems, etc. Maintaining a high or required level of reliability is often an essential requirement of the systems. The study of repairable systems is an important component in reliability analysis. Also repairman is one of the essential parts of repairable systems, and can affect the economy of the systems, directly or indirectly. The primary goal of reliability engineering is to improve the performance of a system. In the initial design activity, the redundancy allocation is a direct way of enhancing the reliability of any system. There are two types of redundancy strategies, active and standby. If all the redundant components operate simultaneously from time zero, even though the system needs only one at any given time, such an arrangement is called active redundancy. On the other side there are three variants of standby redundancy such as cold, warm and hot. In the cold standby redundancy, the component does not fail before it operates. In warm standby redundancy, the component is more prone to failure before operation than the cold standby components. In the hot standby redundancy, the failure pattern of component does not depend whether the component is inactive or in operation. The mathematical models for hot standby and active standby arrangements are the same. Lots of work has been done by many researchers in this area. Nakagawa, Osaki (1975) and Okumato (1997) have studied the behavior of a two unit redundant system under the assumption that whenever the operating unit fails, it goes to repair. Gopalan and Naidu (1981) investigated the stochastic behavior of a two unit repairable system under different inspection strategies. Singh and Srinivasu (1987) have analyzed a two unit cold standby system with the concept of preparation time for repair. Gupta and Bansal (1990) have analyzed the three unit standby system. Gupta and Sharma (1993) have analyzed a two unit standby system with two types of repairs. Recently, Ram et al. (2013) have analyzed a standby system with waiting for repair strategy.

This traditional system reliability measures including reliability, availability, mean time to failure and cost analysis. These are effective and efficient tool for probabilistic risk assessment in system design, operation and maintenance. Some earlier researchers including Gupta and Sharma (1993), Philip and Deans (1997), Bhardwaj and Malik (2011), El-Damcese and Temraz (2012) developed different mathematical models with identical unit systems, common cause failure and they have computed the reliability measures such as availability, reliability, mean time to failure (MTTF), mean time between failure (MTBF) and profit function of complex engineering system with different type of failures [Guo and Yang (2008)] and one type of repair. Furthermore, Yeh (2011) studied reliability measures with different assumptions for repairable and non-repairable systems. Ram and Singh (2009, 2012) have analyzed the reliability characteristics by using the concept of copula.

The present paper deliberates the concept of a repairable system, which can fail completely due to failure of its subsystems and failure of its last component (*D*) and both the cold standby units connected in parallel with that component.

2. Mathematical Model Details

2.1 Nomenclature

s	Laplace transform variable
t	Time scale
λ_i	Failure rates of the system, where *i*=1,2,3,4
μ_j	Repair rates of the system from failed state to good state, where *j*=1,2,3,4
$P^0{}_{ABCD}(t)$	Transition state probability of the state S_0 when all the subsystems *A, B, C* and *D* are in good condition
$P^1{}_{\bar{A}BCD}(t), P^2{}_{A\bar{B}CD}(t), P^3{}_{AB\bar{C}D}(t)$	Transition state probabilities of the state S_1, S_2, S_3 when subsystems *A*, *B* and *C* are in failed condition respectively.
$P^4{}_{ABCd_1}(t), P^5{}_{\bar{A}BCd_1}(t), P^6{}_{A\bar{B}Cd_1}(t), P^7{}_{AB\bar{C}d_1}(t), P^8{}_{ABCd_2}(t), P^9{}_{\bar{A}BCd_2}(t), P^{10}{}_{A\bar{B}Cd_2}(t), P^{11}{}_{AB\bar{C}d_2}(t), P^{12}{}_{ABC\bar{D}}(t)$	Transition state probabilities of states S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{12} respectively.
$E_p(t)$	Expected profit during the interval (0, *t*).
K_1, K_2	Revenue and service cost per unit time respectively.

2.2 Model Description and Assumptions

In the present paper, we have analyzed a repairable system which consists of four sub-systems namely *A, B, C* and *D* in series. Subsystems *A, B, C* are single unit arranged in series. Failure of any one of these causes the complete failure of the system.

Subsystem D consists of three units. One unit of subsystem D is active and other two are in cold standby mode. Complete failure of the system will occur due to subsystem D when one active unit and two standby units of subsystem D failed at a time. We have assumed that the system can be repaired in both the cases. For the failures, the repairs are done absolutely, so after the repair every subsystem is as good as new. The state transition diagram of the proposed model has been shown in Fig. 1. Failure rates are assumed to be constant in general, while the repairs pursue general distribution. With the help of Supplementary variable technique and Laplace transformation, following reliability measures of the system have been evaluated:

(i) Transition state probabilities of the system.
(ii) A series of measures such as availability, reliability, MTTF, sensitivity analysis and cost effectiveness of the system.

Some numerical examples are also presented to illustrate the model mathematically. The state specification of the system is given in Table below:

State	State description
S_0	The system is in good working condition.
S_1	The system is in failed state due to failure of subsystem *A*.
S_2	The system is in failed state due to failure of subsystem *B*.
S_3	The system is in failed state due to failure of subsystem *C*.
S_4	The system is in good state due to failure of one of the unit of *D*.
S_5	The system is in failed state due to failure of subsystem *A* and active unit of subsystem *D*.
S_6	The system is in failed state due to failure of subsystem *B* and active unit of subsystem *D*.
S_7	The system is in failed state due to failure of subsystem C and active unit of subsystem *D*.
S_8	The system is in good state due to failure of one of the standby unit of subsystem *D*.
S_9	The system is in good state due to failure of subsystem *A* and one of the standby units of subsystem *D*.
S_{10}	The system is in failed state due to failure of subsystem *B* and one of the standby units of subsystem *D*.
S_{11}	The system is in failed state due to failure of subsystem *C* and one of the standby units of subsystem *D*.
S_{12}	The system is in failed state due to complete failure of subsystem *D*.

The following assumptions are associated with the model:

(i) Initially the system is in good state.
(ii) The system has two states namely good and failed.
(iii) The system has completely failed after the failure of subsystems *A, B* and *C* and failure of all the three units of subsystem *D*.
(iv) All failure and repair rates are constant.
(v) The system can be repaired, when it is in completely failed mode.
(vi) The repaired system works like a new one.

2.3 State Transition Diagram of Model

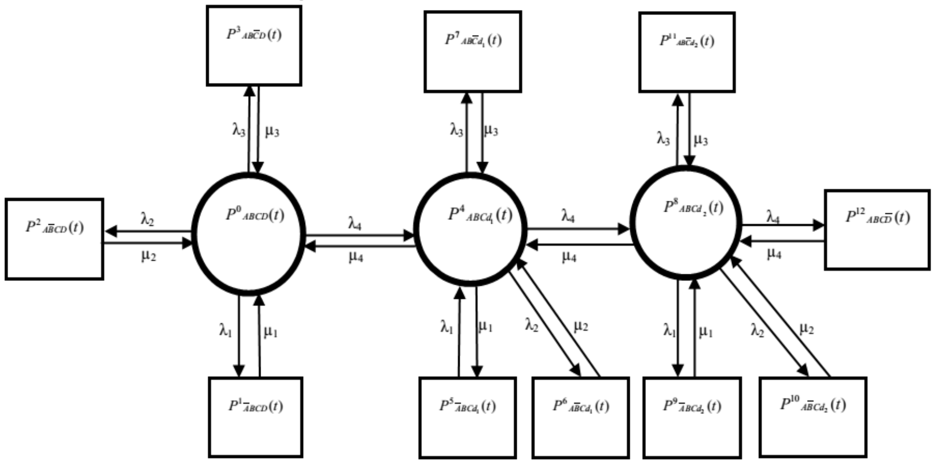

Fig. 1: State Transition Diagram

2.4 Formulation and Solution of the Mathematical Model

By the probability of the considerations and continuity arguments, we can obtain the following set of difference differential equations governing the present mathematical model

$$\left[\frac{\partial}{\partial t}+\lambda_1+\lambda_2+\lambda_3+\lambda_4\right]P^0{}_{ABCD}(t)=\mu_1 P^1{}_{\bar{A}BCD}(t)+\mu_2 P^2{}_{A\bar{B}CD}(t) \quad \ldots$$
$$+\mu_3 P^3{}_{AB\bar{C}D}(t)+\mu_4 P^4{}_{ABCd_1}(t)$$

(1)

$$\left[\frac{\partial}{\partial t}+\mu_1\right]P^1{}_{\bar{A}BCD}(t)=\lambda_1 P^0{}_{ABCD}(t) \quad \ldots (2)$$

$$\left[\frac{\partial}{\partial t}+\mu_2\right]P^2{}_{A\bar{B}CD}(t)=\lambda_2 P^0{}_{ABCD}(t) \quad \ldots (3)$$

$$\left[\frac{\partial}{\partial t}+\mu_3\right]P^3{}_{AB\bar{C}D}(t)=\lambda_3 P^0{}_{ABCD}(t) \quad \ldots (4)$$

$$\left[\frac{\partial}{\partial t}+\lambda_1+\lambda_2+\lambda_3+\lambda_4+\mu_4\right]P^4{}_{ABCd_1}(t)=\mu_1 P^5{}_{\bar{A}BCd_1}(t)+\mu_2 P^6{}_{A\bar{B}Cd_1}(t)$$
$$+\mu_3 P^7{}_{AB\bar{C}d_1}(t)+\mu_4 P^8{}_{ABCd_2}(t)+\lambda_4 P^0{}_{ABCD}(t) \quad \ldots (5)$$

$$\left[\frac{\partial}{\partial t}+\mu_1\right]P^5{}_{\bar{A}BCd_1}(t)=\lambda_1 P^4{}_{ABCd_1}(t) \quad \ldots (6)$$

$$\left[\frac{\partial}{\partial t}+\mu_2\right]P^6{}_{A\bar{B}Cd_1}(t)=\lambda_2 P^4{}_{ABCd_1}(t) \quad \ldots (7)$$

$$\left[\frac{\partial}{\partial t}+\mu_3\right]P^7{}_{AB\overline{C}d_1}(t)=\lambda_3P^4{}_{ABCd_1}(t) \quad \dots (8)$$

$$\left[\frac{\partial}{\partial t}+\lambda_1+\lambda_2+\lambda_3+\lambda_4+\mu_4\right]P^8{}_{ABCd_2}(t)=\mu_1P^9{}_{\overline{A}BCd_2}(t)+\mu_2P^{10}{}_{A\overline{B}Cd_2}(t)$$
$$+\mu_3P^{11}{}_{AB\overline{C}d_2}(t)+\mu_4P^{12}{}_{ABC\overline{D}}(t)+\lambda_4P^4{}_{ABCd_1}(t) \quad \dots (9)$$

$$\left[\frac{\partial}{\partial t}+\mu_1\right]P^9{}_{\overline{A}BCd_2}(t)=\lambda_1P^8{}_{ABCd_2}(t) \quad \dots (10)$$

$$\left[\frac{\partial}{\partial t}+\mu_2\right]P^{10}{}_{A\overline{B}Cd_2}(t)=\lambda_2P^8{}_{ABCd_2}(t) \quad \dots (11)$$

$$\left[\frac{\partial}{\partial t}+\mu_3\right]P^{11}{}_{AB\overline{C}d_2}(t)=\lambda_3P^8{}_{ABCd_2}(t) \quad \dots (12)$$

$$\left[\frac{\partial}{\partial t}+\mu_4\right]P^{12}{}_{ABC\overline{D}}(t)=\lambda_4P^8{}_{ABCd_2}(t) \quad \dots (13)$$

Initial condition

$P^0_{ABCD}(t)=1$ at t=0 and all other state probabilities are zero initially … (14)

Taking Laplace transformation of equations (1-13), we get

$$[s+\lambda_1+\lambda_2+\lambda_3+\lambda_4]\overline{P}^0{}_{ABCD}(s)=1+\mu_1\overline{P}^1{}_{\overline{A}BCD}(s)+\mu_2\overline{P}^2{}_{A\overline{B}CD}(s)$$
$$+\mu_3\overline{P}^3{}_{AB\overline{C}D}(s)+\mu_4\overline{P}^4{}_{ABCd_1}(s) \quad \dots (15)$$

$$[s+\mu_1]\overline{P}^1{}_{\overline{A}BCD}(s)=\lambda_1\overline{P}^0{}_{ABCD}(s) \quad \dots$$

(16)

$$\left[\frac{\partial}{\partial t}+\mu_2\right]P^2{}_{A\overline{B}CD}(s)=\lambda_2P^0{}_{ABCD}(s) \quad \dots (17)$$

$$[s+\mu_3]\overline{P}^3{}_{AB\overline{C}D}(s)=\lambda_3\overline{P}^0{}_{ABCD}(s) \quad \dots (18)$$

$$[s+\lambda_1+\lambda_2+\lambda_3+\lambda_4+\mu_4]\overline{P}^4{}_{ABCd_1}(s)=\mu_1\overline{P}^5{}_{\overline{A}BCd_1}(s)+\mu_2\overline{P}^6{}_{A\overline{B}Cd_1}(s)$$
$$+\mu_3\overline{P}^7{}_{AB\overline{C}d_1}(s)+\mu_4\overline{P}^8{}_{ABCd_2}(s)+\lambda_4\overline{P}^0{}_{ABCD}(s) \quad \dots (19)$$

$$[s+\mu_1]P^5{}_{\overline{A}BCd_1}(s)=\lambda_1\overline{P}^4{}_{ABCd_1}(s) \quad \dots (20)$$

$$[s+\mu_2]\overline{P}^6{}_{A\overline{B}Cd_1}(s)=\lambda_2\overline{P}^4{}_{ABCd_1}(s) \quad \dots (21)$$

$$[s+\mu_3]\overline{P}^7{}_{AB\overline{C}d_1}(s)=\lambda_3\overline{P}^4{}_{ABCd_1}(s) \quad \dots (22)$$

$$[s+\lambda_1+\lambda_2+\lambda_3+\lambda_4+\mu_4]\overline{P}^8{}_{ABCd_2}(s)=\mu_1\overline{P}^9{}_{\overline{A}BCd_2}(s)+\mu_2\overline{P}^{10}{}_{A\overline{B}Cd_2}(s)$$
$$+\mu_3\overline{P}^{11}{}_{AB\overline{C}d_2}(s)+\mu_4\overline{P}^{12}{}_{ABC\overline{D}}(s)+\lambda_4\overline{P}^4{}_{ABCd_1}(s) \quad \dots (23)$$

$$[s+\mu_1]\overline{P}^9{}_{\overline{A}BCd_2}(s)=\lambda_1\overline{P}^8{}_{ABCd_2}(s) \quad \dots (24)$$

$$[s+\mu_2]\overline{P}^{10}{}_{A\overline{B}Cd_2}(s)=\lambda_2\overline{P}^8{}_{ABCd_2}(s) \quad \dots (25)$$

$$[s+\mu_3]\overline{P}^{11}{}_{AB\overline{C}d_2}(s)=\lambda_3\overline{P}^8{}_{ABCd_2}(s) \quad \dots (26)$$

$$[s+\mu_1]\overline{P}^{12}{}_{ABC\overline{D}}(s)=\lambda_4\overline{P}^8{}_{ABCd_2}(s) \quad \dots (27)$$

Solving (15-27), one may get

$$\overline{P}^{0}{}_{ABCD}(s)=\frac{1}{\left(A-\dfrac{\mu_4\lambda_4}{\left\{B-\dfrac{\mu_4\lambda_4}{C}\right\}}\right)} \qquad \ldots(28)$$

$$\overline{P}^{1}{}_{\overline{A}BCD}(s)=\frac{\lambda_1}{(s+\mu_1)}\overline{P}^{0}{}_{ABCD}(s) \qquad \ldots(29)$$

$$\overline{P}^{2}{}_{A\overline{B}CD}(s)=\frac{\lambda_2}{(s+\mu_2)}\overline{P}^{0}{}_{ABCD}(s) \qquad \ldots(30)$$

$$\overline{P}^{3}{}_{AB\overline{C}D}(s)=\frac{\lambda_3}{(s+\mu_3)}\overline{P}^{0}{}_{ABCD}(s) \qquad \ldots(31)$$

$$\overline{P}^{4}{}_{ABCd_1}(s)=\frac{\lambda_4}{\left\{B-\dfrac{\mu_4\lambda_4}{C}\right\}}\overline{P}^{0}{}_{ABCD}(s) \qquad \ldots(32)$$

$$\overline{P}^{5}{}_{\overline{A}BCd_1}(s)=\frac{\lambda_1}{(s+\mu_1)}\frac{\lambda_4}{\left\{B-\dfrac{\mu_4\lambda_4}{C}\right\}}\overline{P}^{0}{}_{ABCD}(s) \qquad \ldots(33)$$

$$\overline{P}^{6}{}_{A\overline{B}Cd_1}(s)=\frac{\lambda_2}{(s+\mu_2)}\frac{\lambda_4}{\left\{B-\dfrac{\mu_4\lambda_4}{C}\right\}}\overline{P}^{0}{}_{ABCD}(s) \qquad \ldots(34)$$

$$\overline{P}^{7}{}_{AB\overline{C}d_1}(s)=\frac{\lambda_3}{(s+\mu_3)}\frac{\lambda_4}{\left\{B-\dfrac{\mu_4\lambda_4}{C}\right\}}\overline{P}^{0}{}_{ABCD}(s) \qquad \ldots(35)$$

$$\overline{P}^{8}{}_{ABCd_2}(s)=\frac{\lambda_4}{\left\{\dfrac{BC}{\lambda_4}-\mu_4\right\}}\overline{P}^{0}{}_{ABCD}(s) \qquad \ldots(36)$$

$$\overline{P}^{9}{}_{\overline{A}BCd_2}(s)=\frac{\lambda_1}{(s+\mu_1)}\frac{\lambda_4}{\left\{\dfrac{BC}{\lambda_4}-\mu_4\right\}}\overline{P}^{0}{}_{ABCD}(s) \qquad \ldots(37)$$

$$\overline{P}^{10}{}_{A\overline{B}Cd_2}(s)=\frac{\lambda_2}{(s+\mu_2)}\frac{\lambda_4}{\left\{\dfrac{BC}{\lambda_4}-\mu_4\right\}}\overline{P}^{0}{}_{ABCD}(s) \qquad \ldots(38)$$

$$\overline{P}^{11}{}_{AB\overline{C}d_2}(s)=\frac{\lambda_3}{(s+\mu_3)}\frac{\lambda_4}{\left\{\dfrac{BC}{\lambda_4}-\mu_4\right\}}\overline{P}^{0}{}_{ABCD}(s) \qquad \ldots(39)$$

$$\overline{P}^{12}{}_{ABC\overline{D}}(s)=\frac{\lambda_4}{(s+\mu_4)}\frac{\lambda_4}{\left\{\dfrac{BC}{\lambda_4}-\mu_4\right\}}\overline{P}^{0}{}_{ABCD}(s) \qquad \ldots(40)$$

Where

$$A=\left((s+\lambda_1+\lambda_2+\lambda_3+\lambda_4)-\frac{\mu_1\lambda_1}{(s+\mu_1)}-\frac{\mu_2\lambda_2}{(s+\mu_2)}-\frac{\mu_3\lambda_3}{(s+\mu_3)}\right)$$

$$B=\left((s+\lambda_1+\lambda_2+\lambda_3+\lambda_4+\mu_4)-\frac{\mu_1\lambda_1}{(s+\mu_1)}-\frac{\mu_2\lambda_2}{(s+\mu_2)}-\frac{\mu_3\lambda_3}{(s+\mu_3)}\right)$$

$$C=\left((s+\lambda_1+\lambda_2+\lambda_3+\lambda_4+\mu_4)-\frac{\mu_1\lambda_1}{(s+\mu_1)}-\frac{\mu_2\lambda_2}{(s+\mu_2)}-\frac{\mu_3\lambda_3}{(s+\mu_3)}-\frac{\mu_4\lambda_4}{(s+\mu_4)}\right)$$

The Laplace transformations of the probabilities that the system is in the up (i. e. good state) and failed state at any time are as follows

$$\overline{P}_{up}(s)=\overline{P}^{0}{}_{ABCD}(s)+\overline{P}^{4}{}_{ABCd_1}(s)+\overline{P}^{8}{}_{ABCd_2}(s)$$

$$\overline{P}_{up}(s)=\left(1+\frac{\lambda_4}{\left\{B-\dfrac{\mu_4\lambda_4}{C}\right\}}+\frac{\lambda_4}{\left\{\dfrac{BC}{\lambda_4}-\mu_4\right\}}\right)\overline{P}^{0}{}_{ABCD}(s) \qquad \ldots (41)$$

$$\overline{P}_{down}(s)=\overline{P}^{1}{}_{\overline{A}BCD}(s)+\overline{P}^{2}{}_{A\overline{B}CD}(s)+\overline{P}^{3}{}_{AB\overline{C}D}(s)+\overline{P}^{5}{}_{\overline{A}BCd_1}(s)+\overline{P}^{6}{}_{A\overline{B}Cd_1}(s)+\overline{P}^{7}{}_{AB\overline{C}d_1}(s)$$
$$+\overline{P}^{9}{}_{\overline{A}BCd_2}(s)+\overline{P}^{10}{}_{A\overline{B}Cd_2}(s)+\overline{P}^{11}{}_{AB\overline{C}d_2}(s)+\overline{P}^{12}{}_{ABC\overline{D}}(s)$$

$$=\left[\frac{\lambda_1}{(s+\mu_1)}+\frac{\lambda_2}{(s+\mu_2)}+\frac{\lambda_3}{(s+\mu_3)}+\frac{\lambda_1}{(s+\mu_1)}\frac{\lambda_4}{\left\{B-\dfrac{\mu_4\lambda_4}{C}\right\}}\right.$$
$$+\frac{\lambda_2}{(s+\mu_2)}\frac{\lambda_4}{\left\{B-\dfrac{\mu_4\lambda_4}{C}\right\}}+\frac{\lambda_3}{(s+\mu_3)}\frac{\lambda_4}{\left\{B-\dfrac{\mu_4\lambda_4}{C}\right\}}+\frac{\lambda_1}{(s+\mu_1)}\frac{\lambda_4}{\left\{\dfrac{BC}{\lambda_4}-\mu_4\right\}}$$
$$\left.+\frac{\lambda_2}{(s+\mu_2)}\frac{\lambda_4}{\left\{\dfrac{BC}{\lambda_4}-\mu_4\right\}}+\frac{\lambda_3}{(s+\mu_3)}\frac{\lambda_4}{\left\{\dfrac{BC}{\lambda_4}-\mu_4\right\}}+\frac{\lambda_4}{(s+\mu_4)}\frac{\lambda_4}{\left\{\dfrac{BC}{\lambda_4}-\mu_4\right\}}\right]\overline{P}^{0}{}_{ABCD}(s)$$

... (42)

3. Particular Cases

3.1 Availability Analysis

Availability is the probability that the system is operating at a specified time *t*. It is always associated with the concept of maintainability. Availability depends upon both failure and repair rates. Taking the values of different parameters as $\lambda_1=0.10, \lambda_2=0.20, \lambda_3=0.30, \lambda_4=0.40, \mu_1=\mu_2=\mu_3=\mu_4=1$ and putting all these values in (41) and then taking the inverse Laplace transform, we get

$$\begin{aligned} P_{up}(t) = {} & 0.60937500\ 00 + 0.14556887\ 39e^{(-3.119081\ 163t)} - 0.0513187315\ 0e^{(-2.254930100\ t)} \\ & + 0.4110932321\ e^{(1.61971453\ 3t)} - 0.00293118\ 7789e^{(-0.6717400\ 169t)} \\ & + 0.01922479\ 986e^{(-0.3345341\ 867t)} \end{aligned} \quad \ldots (43)$$

Now, varying time unit t from 0 to 10 in (43), we obtain Table 1 and correspondingly Fig. 2 representing the behavior of availability of the system with respect to time.

Time (t)	$P_{up}(t)$
0	1.00000
1	0.69827
2	0.63402
3	0.61916
4	0.61484
5	0.61300
6	0.61193
7	0.61120
8	0.61068
9	0.61031
10	0.61004

Table 1: Availability as function of time

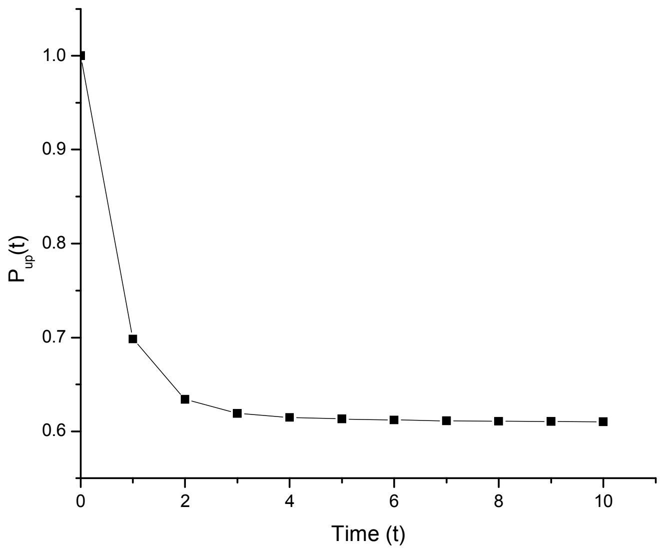

Fig. 2: Availability as function of time

3.2 Reliability Analysis

Reliability is defined as the probability that a device will perform its intended function during a specified period of time under stated conditions. It is always a function of time. It is also depends on environmental conditions which may or may not vary with time. Taking all repairs equal to zero in (41) and after taking inverse Laplace transform, one may get

$$R(t) = (1/2) \times e^{-(\lambda_1+\lambda_2+\lambda_3+\lambda_4)t}(\lambda_4^{\,2}t^2 + 2\lambda_4 t + 2) \quad \text{... (44)}$$

Let us fix the failure rates as $\lambda_1 = 0.10, \lambda_2 = 0.20, \lambda_3 = 0.30, \lambda_4 = 0.40$. By putting all these values in (44) and varying time unit t from 0 to 10, one can obtain Table 2 and Fig. 3, which represents the reliability variation of the system.

Time (t)	Reliability *R(t)*
0	1.00000
1	0.54446
2	0.28691
3	0.14537
4	0.07106
5	0.03368
6	0.01556
7	0.00703
8	0.00312
9	0.00136
10	0.00059

Table 2: Reliability as function of time

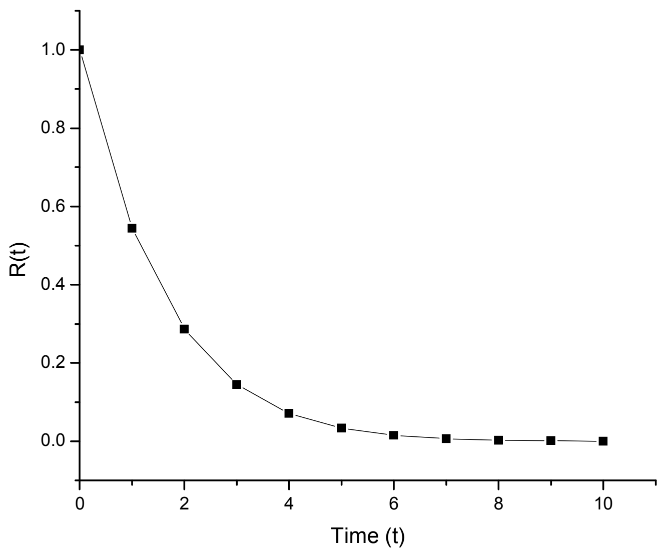

Fig. 3: Reliability as function of time

3.3 Mean Time to Failure (MTTF) Analysis

Mean time to failures (MTTF) is the predicted elapsed time between inherent failures of a system during operation. MTTF can be calculated as the average time between failures of a system. Taking all repairs to be zero in (41) as s tends to zero, one can obtain the MTTF as:

$$MTTF = \lim_{s \to 0} \overline{P}_{up}(s)$$

$$= \left(\frac{1 + \dfrac{\lambda_4}{(\lambda_1+\lambda_2+\lambda_3+\lambda_4)} + \dfrac{{\lambda_4}^2}{(\lambda_1+\lambda_2+\lambda_3+\lambda_4)^2}}{(\lambda_1+\lambda_2+\lambda_3+\lambda_4)} \right) \quad \ldots (45)$$

Setting $\lambda_1 = 0.10, \lambda_2 = 0.20, \lambda_3 = 0.30, \lambda_4 = 0.40$ and varying $\lambda_1, \lambda_2, \lambda_3, \lambda_4$ one by one respectively at 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, one may obtain the variation of MTTF with respect to failure rates.

Variation in $\lambda_1, \lambda_2, \lambda_3, \lambda_4$	MTTF with respect to failure rates			
	λ_1	λ_2	λ_3	λ_4
0.1	1.56000	1.82441	2.18750	1.66180
0.2	1.35987	1.56000	1.82441	1.64062
0.3	1.20370	1.35987	1.56000	1.60493
0.4	1.07874	1.20370	1.35987	1.56000
0.5	0.97667	1.07874	1.20370	1.51014
0.6	0.89185	0.97667	1.07874	1.45833
0.7	0.82031	0.89185	0.97667	1.40646
0.8	0.75921	0.82031	0.89185	1.35568
0.9	0.70644	0.75921	0.82031	1.30666

Table 3: MTTF as function of failure rates

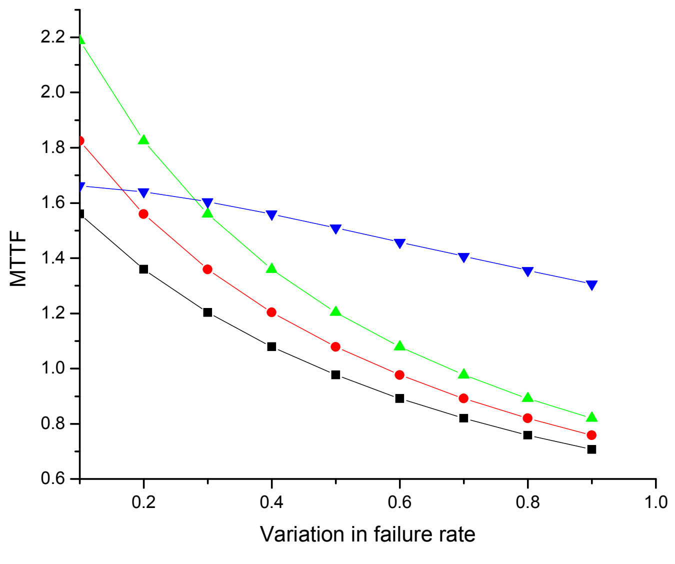

Fig.4: MTTF as function of failure rates

3.4 Expected Profit

Cost control is critical to maintain product reliability. Clearly reliability alone will not guarantee product viability. Similarly arbitrary cost cutting can be detrimental to profit when the relating system reliabilities too low. Let the service facility be always available, then expected profit during the interval (0, *t*] is given as

$$E_p(t) = K_1 \int_0^t P_{up}(t)dt - tK_2 \qquad \ldots (46)$$

Using (43) expected profit for the same set of parameters is given by

$$\begin{aligned} E_p(t) = K_1\big[&0.60937500\,00t - 0.46670434\,75e^{(-3.119081\,163t)} + 0.22758457\,79e^{(-2.254930\,100t)} \\ &- 0.253805978\,6e^{(-1.619714\,533t)} + 0.43635747\,69e^{(-.6717400\,169t)} \\ &- 0.57467369\,93e^{(-.3345341\,867t)} + 0.28881835\,94 - tK_2 \qquad \ldots (47) \end{aligned}$$

Setting K_1= 1 and K_2= 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, respectively in (47), one can get the Table 4.

Time (t)	Expected Profit $E_p(t)$					
	K_2= 0.1	K_2=0.2	K_2=0.3	K_2=0.4	K_2=0.5	K_2=0.6
0	0	0.	0	0	0	0
1	0.71123	0.61123	0.51123	0.41123	0.31123	0.21123
2	1.26956	1.06956	0.86956	0.66956	0.46956	0.26956
3	1.79451	1.49451	1.19451	0.89451	0.59451	0.29451
4	2.31115	1.91115	1.51115	1.11115	0.71115	0.31115
5	2.82497	2.32497	1.82497	1.32497	0.82497	0.32497
6	3.33740	2.73740	2.13740	1.53740	0.93740	0.33740
7	3.84895	3.14895	2.44895	1.74895	1.04895	0.34895
8	4.35988	3.55988	2.75988	1.95988	1.15988	0.35988
9	4.87037	3.97037	3.07037	2.17037	1.27037	0.37037

Table 4: Expected profit as function of time

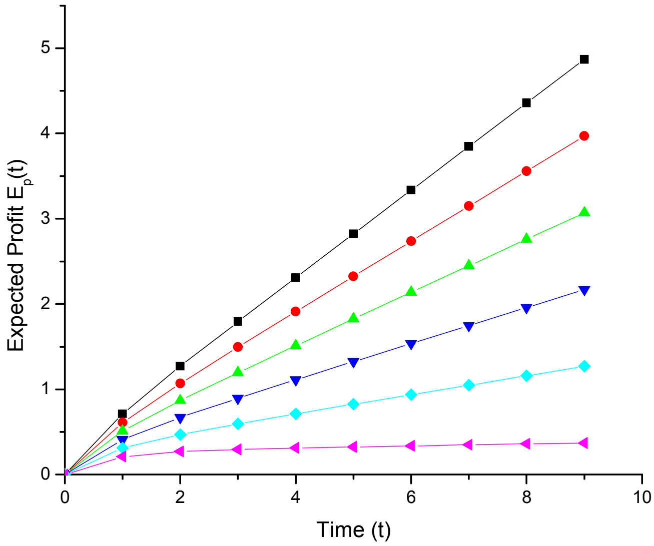

Fig. 5: Expected profit as function of time

3.5 Sensitivity Analysis

3.5.1 Sensitivity of Reliability

Sensitivity analysis is a technique to predict the conclusion of a decision if a state of affairs turns out to be different compared to the key prediction. Sensitivity analysis is very useful when attempting to determine the impact the actual outcome of a fastidious variable will have if it differs from what was previously assumed. Sensitivity analysis is used to determine how sensitive a model is to changes in the value of the parameters of the model and to change in the structure of the model. We first perform the sensitivity analysis for changes in reliability resulting from changes in the system parameters λ_1, λ_2, λ_3 and λ_4 by differentiating (44) with respect to failure rates $\lambda_1, \lambda_2, \lambda_3, \lambda_4$ respectively and by putting $\lambda_1 = 0.10, \lambda_2 = 0.20, \lambda_3 = 0.30, \lambda_4 = 0.40$ we get the numerical values of $\frac{\partial R(t)}{\partial \lambda_1}, \frac{\partial R(t)}{\partial \lambda_2}, \frac{\partial R(t)}{\partial \lambda_3}, \frac{\partial R(t)}{\partial \lambda_4}$.

Now taking t=0 to 10 units of time in the partial derivatives of reliability with respect to different failure rates, we have obtained the Table 5 and Fig. 6 respectively.

Time (t)	$\frac{\partial R(t)}{\partial \lambda_1}$	$\frac{\partial R(t)}{\partial \lambda_2}$	$\frac{\partial R(t)}{\partial \lambda_3}$	$\frac{\partial R(t)}{\partial \lambda_4}$
0	0	0	0	0
1	-0.54446	-0.54446	-0.54446	-0.02943
2	-0.57382	-0.57382	-0.57382	-0.08661
3	-0.43613	-0.43613	-0.43613	-0.10754
4	-0.28425	-0.28425	-0.28425	-0.09377
5	-0.16844	-0.16844	-0.16844	-0.06737
6	-0.09339	-0.09339	-0.09339	-0.04283
7	-0.04927	-0.04927	-0.04927	-0.02502
8	-0.02501	-0.02501	-0.02501	-0.01374
9	-0.01230	-0.01230	-0.01230	-0.00719
10	-0.00590	-0.00590	-0.00590	-0.00363

Table 5: Sensitivity of Reliability as function of time

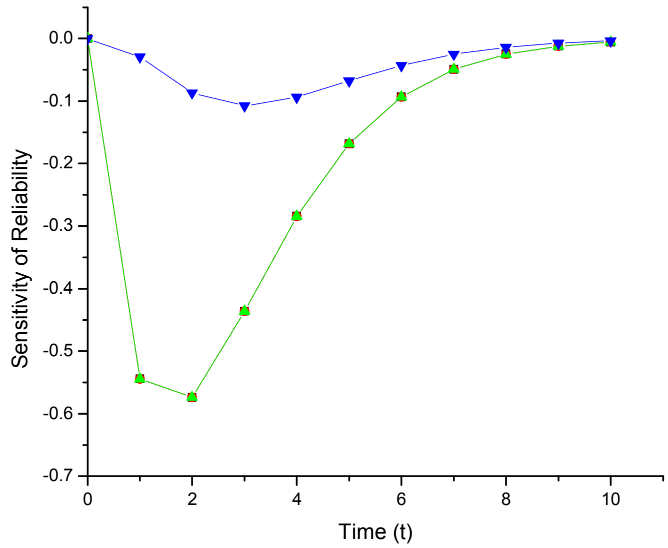

Fig. 6: Sensitivity of Reliability as function of time

3.5.2 Sensitivity of MTTF

Sensitivity analysis for changes in MTTF resulting from changes in system parameters i.e. system failure rates λ_1, λ_2, λ_3, λ_4. By Differentiating (45) with respect to failure rates λ_1, λ_2, λ_3, λ_4 respectively and putting the values as $\lambda_1 = 0.10$, $\lambda_2 = 0.20,\ \lambda_3 = 0.30,\ \lambda_4 = 0.40$, we get the values of $\frac{\partial MTTF}{\partial \lambda_1}$, $\frac{\partial MTTF}{\partial \lambda_2}$, $\frac{\partial MTTF}{\partial \lambda_3}$,

, $\frac{\partial MTTF}{\partial \lambda_4}$. Varying the failure rates one by one respectively as 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9 in the partial derivatives of MTTF with respect to different failure rates, one can obtain the Table 6 and Fig. 7.

Variation in $\lambda_1, \lambda_2, \lambda_3, \lambda_4$	$\frac{\partial MTTF}{\partial \lambda_1}$	$\frac{\partial MTTF}{\partial \lambda_2}$	$\frac{\partial MTTF}{\partial \lambda_3}$	$\frac{\partial MTTF}{\partial \lambda_4}$
0.1	-2.28000	-3.06355	-4.29687	-0.12494
0.2	-1.75534	-2.28000	-3.06355	-0.29296
0.3	-1.38888	-1.75534	-2.28000	-0.41152
0.4	-1.12391	-1.38888	-1.75534	-0.48000
0.5	-0.92669	-1.12391	-1.38888	-0.51226
0.6	-0.77629	-0.92669	-1.12391	-0.52083
0.7	-0.65917	-0.77629	-0.92669	-0.51468
0.8	-0.56632	-0.65917	-0.77629	-0.49979
0.9	-0.49154	-0.56632	-0.65917	-0.47999

Table 6: Sensitivity of MTTF as function of failure rates

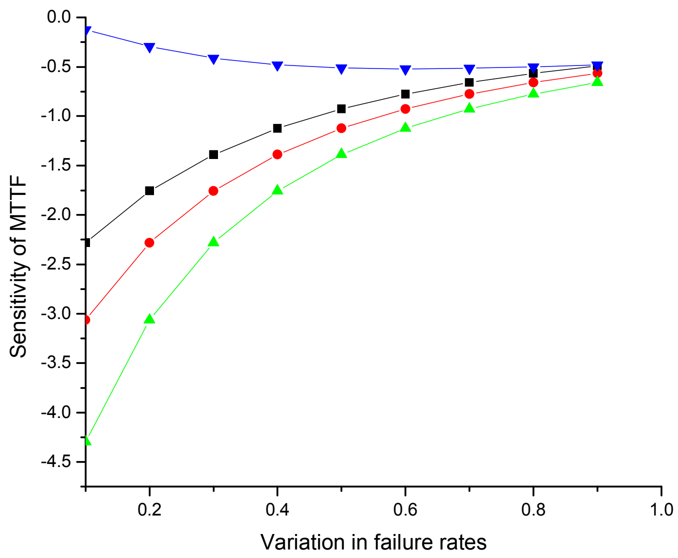

Fig. 7: Sensitivity of MTTF as function of failure rates

4. Conclusion

In this paper, we have evaluated various reliability indices such as availability, reliability, MTTF, cost function, sensitivity analysis, for the considered system by employing Markov Process. From the results and analysis of the designed system, one can conclude the following:

(i) Analysis of Table 1 gives us the idea of the availability of the stated system with respect to time *t*. Critical examination of corresponding Fig. 2 yields that the values of the availability decreases approximately in an even manner with the increment in time.
(ii) Table 2 shows the trends of reliability of the designed system with respect to the time when all the failure and repair rates have some fixed values. From the graph (Fig. 3), we concluded that the reliability of the system decreases more sharply with the passage of time. Reliability may be improved by clarity of expression, lengthening the measure, and other informal means.
(iii) Table 3 shows that MTTF of the above stated system with respect to various failure rates. A critical examination of Fig. 4 shows that the MTTF decreases with increment in failure rates $\lambda_1, \lambda_2, \lambda_3, \lambda_4$.
(iv) Table 4 and corresponding Fig. 5 represent the cost function vs. time. Here, one can easily observe that increasing service cost leads decrement into expected profit. The study shows that minimum service cost leads to maximum expected profit on the other hand maximum service cost leads to minimum profit. From this one may conclude that by controlling service cost, high profit could be attained.
(v) Furthermore, we evaluate sensitivity of reliability and MTTF of the system. The sensitivity analysis of the described system reliability with respect to $\lambda_1, \lambda_2, \lambda_3, \lambda_4$ are shown in Table 5 and Fig.6. It reveals that sensitivity increases as time passes. It is clear from the graph that system reliability is more sensitive with respect to λ_4. Finally, Table 6 and Fig. 7 show the sensitivity of MTTF with respect to $\lambda_1, \lambda_2, \lambda_3, \lambda_4$ which show that it increases with increment in failure rates $\lambda_1, \lambda_2, \lambda_3, \lambda_4$. Critical observation of the graph point out that MTTF of the system is more sensitive with respect to λ_4. From this we can conclude that the system can be made less sensitive by controlling its failure rates.

From the hypothetical point of view, the research of this paper is based mainly on system reliability theory, and stochastic processes. The results achieved in this paper are valuable in a study of improving the reliability of the systems. Additionally, they can be extensively used in many engineering disciplines.

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