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# ESTIMATION OF PARAMETERS OF POISSON TYPE EXPONENTIAL CLASS MODEL USING NON-INFORMATIVE AND GAMMA PRIOR

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## Abstract

This paper concerns with estimation of parameter of Poisson type exponential class model of software evaluation. The Bayes estimators of parameters i. e. number of failures β0 and failure rate β1 have been obtained using non – informative and gamma prior respectively under squared error loss function. The obtained estimators have been compared with corresponding maximum likelihood estimators on the basis of their simulated risks (average loss).

**Key words:** Software Reliability, Poisson Type Exponential Class, Non – Homogeneous Poisson Process, Bayes Estimator, Maximum Likelihood Estimator, Non Informative Prior and Gamma Prior.

## 1. Introduction:

A software reliability model has the form of a random process, which describes the behavior of failure with time and it is generally specified as a function of time. The time involved in the characterization of model is a cumulative time and the origin is the start of the system.

The first study of software reliability has been done by Hudson (1967) and he has viewed software development as a birth and death process. Jelinski and Moranda (1975) have proposed a model which assumes that hazard rate for failure is piecewise constant and proportional to the number of faults remaining. Further, Moranda has obtained maximum likelihood estimators for total number of faults existing in the software [See also Moranda (1975), Shooman (1972) and Shooman and Natrajan (1976)].

The Poisson process provides a good approximation to the occurrence of many real life events. Under Poisson type model, software failure process is studied using non homogeneous Poisson Process [NHPP] with failure intensity λ (t). Let M (t) denote failure experienced by time‘t’ & M (t) increases by 1 whenever a failure occurs.

$$P_m(t) = P[\, M(t) = m]$$

$$P_m(t) = \frac{[\mu(t)]^m \exp[-\mu(t)]}{m!} \qquad \text{...(1.1)}$$

Where, $$\mu(t) = \int_0^t \lambda(x)\, dx \qquad ...(1.2)$$

and λ (x) is a failure intensity function .The NHPP { M(t);t≥0} has mean and variance equal to mean value function μ(t) of the process.

Musa and Okumoto (1983) described a classification scheme for software reliability models. The model is nominated according to the distribution of failures experienced by time‘t’ as [ type ] and functional form of failure intensity as [ class ]. In this paper we have considered Poisson type exponential class model. [ For details see Musa (1975), Schneidewind (1975), Moranda(1975), Goel & Okumoto(1979), Musa et al (1987), Malviya et al (1992), Musa(2004) and Pham(2006) ]. Ikemoto and Dohi has proposed exponential type software reliability model using regression technique.

## Poisson type –exponential class model (PTEC)

The exponential distribution is the first model in the field of reliability and life testing for which statistical methods have been developed. The initial work related to this model has been done by Sukhatme (1937) & later Epstein (1958).Let us assume that time to failure of an individual faults has an exponential distribution and per fault hazard rate is constant. The mean value function for this model is,

$$\mu(t) = \beta_0[1 - \exp(-\beta_1 t)] \qquad ... (1.3)$$

and failure intensity function is,

$$\lambda(t) = \beta_0\, \beta_1 \exp(-\beta_1 t) \qquad ; \ \beta_0, \beta_1 > 0 \qquad ... (1.4)$$

Where, $\beta_0$ and $\beta_1$ define total number of failures and failure rate respectively [see Musa et al (1987)].

Consider $t_e$ is the total testing time and $m_e$ failures are experienced at time $t_1, t_2, t_3 ... t_{m_e}$ . Since the conditional density function of $T_i$ [time interval between failure $T_i = t_i - t_{i-1}$] depends only on previous failure time $T_{i-1}$, and hence the likelihood function takes the form,

$$L(\beta_0, \beta_1) = \left[\prod_{i=1}^{m_e} \lambda(t_i)\right] \exp[-\mu(t_e)] \qquad ... (1.5)$$

[For more details refer Musa et al (1987)].

Now, substituting $\lambda(t_i)$ and $\mu(t_e)$ from equation (1.3) and (1.4) respectively the likelihood function becomes

$$L(\beta_0, \beta_1) = \beta_0^{m_e}\ \beta_1^{m_e}\ e^{-\beta_1 T}\ \exp[-\beta_0\{1 - \exp(-\beta_1 t_e)\}] \qquad ... (1.6)$$

Where, $$T = \sum_{i=1}^{m_e} t_i$$

The maximum likelihood estimators of $\beta_0$ and $\beta_1$ can be obtained and are the solution of given equations (1.7) and (1.8)

$$\hat{\beta}_{L_0} = \frac{m_e}{1 - \exp(-\hat{\beta}_{L_1} m_e)} \qquad ...(1.7)$$

$$\frac{m_e}{\hat{\beta}_{L_1}} - \frac{m_e\, t_e}{\exp\,(\hat{\beta}_{L_1} t_e) - 1} - \sum_{i=1}^{m_e} t_i = 0 \qquad ...(1.8)$$

## Need of Bayes approach

Bayesian analysis provides better quality of inference, it reduces the sample size and thus time. Moreover, under natural identification and measurability conditions, Bayes estimators are consistent for all most all parameter values and very often enjoy small sample properties too. Perhaps, due to these reasons, this method is currently getting popularity in virtually all areas of statistical applications. [See for more details, Berger (1985)].

This approach is much more direct, that is deductive than classical approach which uses inductive reasoning. In fact, the use of past experience makes Bayes inference more informative particularly in those situations where prior distribution accurately reflects the verification of the parameter. In the case of Poisson type exponential class model, if prior information about $\beta_0$ & $\beta_1$ are available certainly then obtained Bayes estimator will out perform over MLE's. Thus, in view of this, suitable priors are selected and the Bayes estimators are proposed.

Here in this paper for the considered PTEC model, the prior and posterior distribution has been established in the next section. The Bayes estimator of the parameters namely, total number of failures and failure rate has been obtained in section 3. To study the performance of the proposed Bayes estimator, these have been compared with corresponding MLE's in section 4, and the last section provides the finding in the form of conclusion.

## 2. The Prior and Posterior distributions:

### 2.1 Prior distribution

Apriori information in the form of a prior distribution plays an important role in Bayesian analysis. The Bayesian approach, in addition to sample data, utilizes information about the parameter depending upon past experience in the form of prior. Generally, it is difficult to choose a prior which may be appropriate in all circumstances. But , a general class of priors called non-informative priors used in the cases when a little or no information about the parameter is known apriori. [See Martz and Waller(1982) Box and Tiao (1973) Berger(1985)].Suppose , in the present study, the information about the number of failures in the software is not available then, the general class of non – informative prior for $\beta_0$ becomes more suitable and it is,

$$g_1(\beta_0) = \frac{1}{\beta_0^c}\ ; \qquad \beta_0 > 0 \qquad ...(2.1)$$

Where, c is constant and greater than zero.

In most of the testing problems one may have sufficient information about the scale parameter. These may come from previous data personal experience or from other relevant sources. In such situation, one would like to use an informative prior. However, one may restrict to the class of possible prior distribution those allow easy computations. In statistical literature a number of informative priors have been suggested by authors such as Zellner(1982), Leamer (1978) a conjugate prior by Raiffa Schlaffer (1961) etc. Perhaps the most widely used informative prior is conjugate prior. It may be noted that a prior distribution is said to be a conjugate prior if posterior distribution and prior distribution both belong to same family of distributions.

Authors such as Apostolakis and Mosleh (1979), Grohowski et. al. (1976) and other have found Gamma distribution to be sufficiently versatile for particle reliability applications in life testing. Here, an attempt is made to apply the same for the estimation of $\beta_1$ of software reliability. Such Gamma prior distribution for $\beta_1$ is,

$$g_2(\beta_1) = \frac{a^b}{\Gamma b} e^{-a\beta_1} \beta_1^{b-1} \qquad ; a, b > 0, \quad \beta_1 > 0 \qquad \ldots (2.2)$$

Where, ‘a’ and ‘b’ are the parameters of prior distribution. Therefore joint apriori for $\beta_0$ and $\beta_1$ will be

$$g(\beta_0, \beta_1) = \frac{1}{\beta_0^c} \frac{a^b}{\Gamma b} e^{-a\beta_1} \beta_1^{b-1} \qquad ; a, b, c > 0 \qquad \ldots (2.3)$$

$$0 < \beta_0 < \infty, \; 0 < \beta_1 < \infty$$

## 2.2 The Posterior Distribution

Combining the likelihood function (1.6) with joint prior distribution (2.3), using Bayes theorem the joint posterior distribution of $\beta_0$ and $\beta_1$ on given $\underline{t}$ is,

$$\pi(\beta_0, \beta_1 / \underline{t}) = [m_1(\underline{t})]^{-1} \beta_0^{f} \beta_1^{d-1} e^{-\beta_1(a+T)} \exp[-\beta_0(1 - \exp(-\beta_1 t_e))]$$

$$\ldots (2.4)$$

$$m_e \leq \beta_0 < \infty$$
$$0 < \beta_1 < \infty$$

where $d = m_e + b$, $f = m_e - c$ and

$$m_1(t) = \sum_{i=0}^{\infty} \frac{(d-1)!}{i!} \frac{\Gamma((f+i+1), m_e)}{[T+a+it_e]^d}$$

$$\text{Since } \left| \frac{(d-1)!}{i![T+a+it_e]^d} \right| < 1 \text{ and } \Gamma[(f+i+1), m_e] < \Gamma(f+i+1)$$

:e.

Now from joint posterior (2.4) of $\beta_0$ and $\beta_1$, the marginal posterior distribution of $\beta_0$ can be obtained by integrating (2.4) over the whole range of $\beta_1$ i. e. 0 to $\infty$ and we get,

$$\pi(\beta_0 / \underline{t}) = [m_1(\underline{t})]^{-1} (d-1)! \sum_{i=0}^{\infty} \frac{\beta_0^{f+i} e^{-\beta_0}}{i![T+a+it_e]^d} \qquad ; \quad m_e < \beta_0 < \infty$$

$$\ldots (2.5)$$

Similarly, the marginal posterior distribution of $\beta_1$, obtained from the joint posterior distribution (2.4) by integrating out $\beta_0$ is,

$$\pi\ (\beta_1 / \underline{t}) = \ [\, m_1(\underline{t})\,]^{-1}\ \beta_1^{d-1}\ e^{-\beta_1(a+T)}\ \frac{\Gamma((f+1), Wm_e)}{W^{f+1}}\ ;\quad 0<\beta_1<\infty \qquad ...(2.6)$$

Where,

$$W = [1-\exp\ (-\beta_1 t_e)\,]$$

## 3. Bayes Estimators

The Bayes estimators of $\beta_0$ and $\beta_1$ can be obtained with the help of posterior distributions given by equation (2.5) and (2.6) after selecting the appropriate loss function. Now consider in both the cases, the loss function is squared error loss the Bayes estimator becomes posterior mean. The consideration of squared error loss function can be justified by its simplicity thus

$$\widetilde{\beta}_0 = [m_1(\ \underline{t}\ )]^{-1}\,(d-1)!\sum_{i=0}^{\infty}\frac{\Gamma(f+i+2, m_e)}{i!\,[T+a+it_e]^d}\quad ;\quad d>1 \quad ...\ (3.1)$$

Similarly, the Byes estimation of $\beta_1$ under the similar loss function is,

$$\widetilde{\beta}_1 = [m_1(\underline{t})]^{-1}\sum_{i=0}^{\infty}\ \frac{\mathrm{d}!}{\mathrm{i}!}\ \frac{\Gamma((f+i+1), m_e)}{[T+a+it_e]^{d+1}} \qquad ...\ (3.2)$$

## 4. Comparison

The proposed Bayes estimators $\widetilde{\beta}_0$ and $\widetilde{\beta}_1$ have been compared with the corresponding maximum likelihood estimators $\hat{\beta}_1$ and $\hat{\beta}_0$ in this section. This comparison is based on the risk efficiencies. The risk efficiencies of $\widetilde{\beta}_0$ i.e. $RE'_1$ and of $\widetilde{\beta}_1$ i.e. $RE'_2$ have been calculated on the basis of Monte-Carlo simulation technique generating 1000 random samples for $t_e = 125$, $\beta_0 = 20\ (5)\ 40$, $\beta_1 = 0.007\ (0.001)\ 0.014$, a = 0.5 , b = 0.5,2,5 c = 0.25, 1.5, 2.25, which are summarized in table 1 to 9.

The tables 1, 2 and 3 give the risk efficiencies of $\widetilde{\beta}_0$ and $\widetilde{\beta}_1$ for the different values of $\beta_0$ ( = 20 (5) 40) , $\beta_1$ (= 0.007 (0.001) 0.014) , a = 0.5 , b = 0.5, 2, 5 , $t_e$ = 125 and c = 0.25. We can see from these tables that the risk efficiency of proposed estimator of $\beta_0$ is more than1 for all the choices of $\beta_0$ and $\beta_1$. The risk efficiency first increases then decreases after attaining maxima for the variation of $\beta_1$. This is true for all the choices of $\beta_0$. The maximum risk efficiency is, when $\beta_1$ is approximately

0.01 for all the values of $\beta_0$. As $\beta_0$ increases, the risk efficiency $RE'_1$ decreases for $\beta_1 < 0.01$ and increases for $\beta_1 \geq 0.01$. This means the risk efficiency $RE'_1$ will be maximum if $\beta_0$ and $\beta_1$ both are small. The risk efficiency $RE'_2$ of $\tilde{\beta}_1$ increases as $\beta_0$ and $\beta_1$ increase. Here, it can be noted that the performance of the proposed estimator $\tilde{\beta}_1$ is nearly same as MLE. The performance of $\tilde{\beta}_1$ can further be improved if the true values of $\beta_0$ and $\beta_1$ are very large i.e. number of failures in a software is sufficiently large and failure rate is high.

From the table 1, 2 and 3, we see that as the value of 'b' increases the performance of both the estimators decline. Therefore, for a small value of 'b' the proposed estimator $\tilde{\beta}_0$ and $\tilde{\beta}_1$ will perform better than MLE. The variation of 'a' has also been considered and seen by calculating the risk efficiencies for different choices but it has been observed that the variation of 'a' does not affect much the performance of both the proposed estimators, therefore, such tables are not presented here.

The tables 4, 5 and 6 gives the risk efficiencies of $\tilde{\beta}_0$ and $\tilde{\beta}_1$ for the different values of $\beta_0$ (= 20(5) 40), $\beta_1$ = (0.007 (0.001) 0.014) , a = 0.5, b = 0.5, 2, 5 , $t_e$ = 125 and c = 1.5. Similar trend is observed as in table 1,2 and 3. For the choices of $\beta_0$ and $\beta_1$ the risk efficiencies $RE'_1$ of proposed estimator $\tilde{\beta}_0$ is more than 1. The risk efficiencies first increases, attains maxima for the variation of $\beta_1$, and then decreases. Similar trend is observed for other choices of $\beta_0$. The maximum risk efficiency is, when $\beta_1$ is near to 0.01 for all the values of $\beta_0$. As $\beta_0$ increases, the risk efficiency $RE'_1$ decreases for $\beta_1 < 0.01$ and increases for $\beta_1 \geq 0.01$.

The risk efficiency $RE'_2$ of $\tilde{\beta}_1$ increases as $\beta_0$ and $\beta_1$ increase, here it can be noted that the performance of proposed estimator $\tilde{\beta}_1$ (for b = 0.5) is nearly same as MLE. The estimator $\tilde{\beta}_1$ is as good as MLE if the values of $\beta_0$ and $\beta_1$ are very large. We can see from table 5 and 6 that as 'b' increases (b = 2, 5), MLE is better than proposed estimator $\tilde{\beta}_1$. Hence, the use of proposed estimator $\tilde{\beta}_1$ is suggested for large values of $\beta_0$, $\beta_1$ and small 'b'.

The table 7 , 8 and 9 present the risk efficiencies of $\tilde{\beta}_0$ and $\tilde{\beta}_1$ for the different values of $\beta_0$ (= 20 (5) 40), $\beta_1$ (= 0.007 (0.001) 0.014), a = 0.5 , and b = 0.5, 2.0, 5.0 of the proposed estimator of $\beta_0$ is more than 1 for all choices of $\beta_0$ and $\beta_1$.

The risk efficiency first increases then decreases after attaining a maxima for the variation of $\beta_1$. For all the choices of $\beta_0$ , the trend is same. As $\beta_0$ increases, the risk efficiency $RE'_1$ decreases for $\beta_1 < 0.01$ and increases for $\beta_1 \geq 0.01$.

We can see that as $\beta_0$ and $\beta_1$ increase, the risk efficiency $RE'_2$ of $\tilde{\beta}_1$ increases. For b = 0.5, it can be noted that the performance of proposed estimator $\tilde{\beta}_1$ is nearly same as MLE, also it can be improved further if $\beta_0$ and $\beta_1$ are very large. From the table 8 and 9, we see that as the values of 'b' increases the performance of both the estimators decline. Therefore, for small values of 'b', $\tilde{\beta}_0$ and $\tilde{\beta}_1$ performs better than MLE.

The tables 1 , 4 and 7 give the risk efficiencies for $\tilde{\beta}_0$ and $\tilde{\beta}_1$ for different values of c = 0.25, 1.5, 2.25 , a = 0.5 , b = 0.5 , $\beta_0$ (= 20 (5) 40) and $\tilde{\beta}_1$ = (0.007 (0.001) 0.014). As 'c' increases the risk efficiency $RE'_1$ decreases, that is, for small 'c', $\tilde{\beta}_0$ performs much better than M.L.E. Similar trend has been observed for a = 0.5, b = 2.0 and 5.0. The variation in 'c' does not affect risk efficiency $RE'_2$, that is, the performance of $\tilde{\beta}_1$ is unchanged due to change in 'c'.

## 5. Conclusion

The performances of the proposed estimators $\tilde{\beta}_0$ and $\tilde{\beta}_1$ in comparison with $\hat{\beta}_{L_0}$ and $\hat{\beta}_{L_1}$ respectively, have been discussed in the previous section. On the basis of these discussions, we may conclude that the use of the proposed estimators $\tilde{\beta}_0$ for total number of failures $\beta_0$ may be recommended, for small 'c'.

For small 'b', and large $\beta_0$ , $\beta_1$, the proposed estimator $\tilde{\beta}_1$ of failure rate $\beta_1$ performs as good as MLE. Hence, when number of failures is sufficiently large, failure rate is high and 'b' is small , the use of proposed estimators can be recommended.

| $\beta_0$ \ $\beta_1$ | | 0.007 | 0.008 | 0.009 | 0.010 | 0.011 | 0.012 | 0.013 | 0.014 |
|---|---|---|---|---|---|---|---|---|---|
| 20 | $RE'_1$ | 2.1139 | 2.2047 | 2.1778 | 2.1141 | 1.9336 | 1.6503 | 1.4335 | 1.3391 |
| | RE'2 | 0.9332 | 0.9357 | 0.9398 | 0.9422 | 0.9454 | 0.9480 | 0.9499 | 0.9500 |
| 25 | $RE'_1$ | 1.9331 | 2.0443 | 2.0947 | 2.1984 | 2.1549 | 2.0214 | 1.7087 | 1.6852 |
| | $RE'_2$ | 0.9451 | 0.9481 | 0.9524 | 0.9542 | 0.9545 | 0.9580 | 0.9599 | 0.9645 |
| 30 | $RE'_1$ | 1.8253 | 1.9465 | 2.0175 | 2.1978 | 2.1658 | 2.0762 | 1.8948 | 1.7343 |
| | $RE'_2$ | 0.9542 | 0.9577 | 0.9605 | 0.9626 | 0.9846 | 0.9665 | 0.9678 | 0.9697 |
| 35 | $RE'_1$ | 1.7542 | 1.9130 | 2.0194 | 2.0861 | 2.1554 | 2.1807 | 2.0798 | 1.9884 |
| | $RE'_2$ | 0.9611 | 0.9648 | 0.9671 | 0.9693 | 0.9706 | 0.9722 | 0.9732 | 0.9769 |
| 40 | $RE'_1$ | 1.6891 | 1.8189 | 1.9344 | 2.0642 | 2.1500 | 2.1404 | 2.1100 | 2.1039 |
| | $RE'_2$ | 0.9668 | 0.9697 | 0.9723 | 0.9738 | 0.9753 | 0.9766 | 0.9775 | 0.9803 |

**Table 1: Risk efficiencies of $\tilde{\beta}_0$ and $\tilde{\beta}_1$ for the different values of $\beta_0$ and $\beta_1$ with $t_e$ =125, a = 0.5, b = 0.5 and c = 0.25**

| $\beta_0$ \ $\beta_1$ | | 0.007 | 0.008 | 0.009 | 0.010 | 0.011 | 0.012 | 0.013 | 0.014 |
|---|---|---|---|---|---|---|---|---|---|
| 20 | $RE'_1$ | 2.0216 | 2.1409 | 2.1527 | 2.0811 | 1.9267 | 1.5652 | 1.3355 | 1.3011 |
| | $RE'_2$ | 0.7324 | 0.7509 | 0.7599 | 0.7725 | 0.7805 | 0.7922 | 0.7980 | 0.7998 |
| 25 | $RE'_1$ | 1.9275 | 2.0413 | 2.1212 | 2.1178 | 2.0749 | 1.9332 | 1.6693 | 1.5683 |
| | $RE'_2$ | 0.7795 | 0.7945 | 0.8022 | 0.8135 | 0.8205 | 0.8258 | 0.8321 | 0.8366 |
| 30 | $RE'_1$ | 1.8082 | 1.9471 | 2.1021 | 2.2140 | 2.1036 | 2.0618 | 1.8640 | 1.7509 |
| | $RE'_2$ | 0.8126 | 0.8240 | 0.8354 | 0.8412 | 0.8486 | 0.8532 | 0.8578 | 0.8611 |
| 35 | $RE'_1$ | 1.7445 | 1.8946 | 2.0033 | 2.1147 | 2.1658 | 2.1514 | 2.0484 | 1.8767 |
| | $RE'_2$ | 0.8363 | 0.8485 | 0.8570 | 0.8637 | 0.8684 | 0.8732 | 0.8766 | 0.8834 |
| 40 | $RE'_1$ | 1.6965 | 1.8109 | 1.9280 | 2.0952 | 2.1791 | 2.1943 | 2.0620 | 1.9189 |
| | $RE'_2$ | 0.8562 | 0.8653 | 0.8737 | 0.8804 | 0.8844 | 0.8848 | 0.8918 | 0.8999 |

**Table 2: Risk efficiencies of $\tilde{\beta}_0$ and $\tilde{\beta}_1$ for the different values of $\beta_0$ and $\beta_1$ with $t_e$ =125, a = 0.5, b = 2 and c = 0.25**

| $\beta_0$ \ $\beta_1$ | | 0.007 | 0.008 | 0.009 | 0.010 | 0.011 | 0.012 | 0.013 | 0.014 |
|---|---|---|---|---|---|---|---|---|---|
| 20 | $RE'_1$ | 2.0095 | 2.0847 | 2.1372 | 2.0314 | 1.9151 | 1.6383 | 1.5380 | 1.4899 |
| | $RE'_2$ | 0.4857 | 0.5117 | 0.5287 | 0.5447 | 0.5580 | 0.5714 | 0.5940 | 0.6120 |
| 25 | $RE'_1$ | 1.9253 | 2.0502 | 2.1343 | 2.1034 | 2.0681 | 1.9203 | 1.6483 | 1.5136 |
| | $RE'_2$ | 0.5577 | 0.5768 | 0.5977 | 0.6061 | 0.6210 | 0.6301 | 0.6548 | 0.6698 |
| 30 | $RE'_1$ | 1.8210 | 1.9741 | 2.0541 | 2.1614 | 2.1498 | 2.1460 | 2.1220 | 2.0081 |
| | $RE'_2$ | 0.6097 | 0.6322 | 0.6433 | 0.6463 | 0.6562 | 0.6760 | 0.6890 | 0.6989 |
| 35 | $RE'_1$ | 1.7408 | 1.8889 | 2.0282 | 2.1621 | 2.1533 | 2.1471 | 2.1366 | 2.0117 |
| | $RE'_2$ | 0.6493 | 0.6683 | 0.6845 | 0.6976 | 0.7054 | 0.7153 | 0.7211 | 0.7338 |
| 40 | $RE'_1$ | 1.6893 | 1.8117 | 1.9287 | 2.0469 | 2.1428 | 2.1485 | 2.1405 | 2.1229 |
| | $RE'_2$ | 0.6829 | 0.7011 | 0.7167 | 0.7270 | 0.7375 | 0.7448 | 0.7585 | 0.7689 |

**Table 3: Risk efficiencies of $\widetilde{\beta}_0$ and $\widetilde{\beta}_1$ for the different values of $\beta_0$ and $\beta_1$ with $t_e$ =125, a = 0.5, b = 5.0 and c = 0.25**

| $\beta_0$ \ $\beta_1$ | | 0.007 | 0.008 | 0.009 | 0.010 | 0.011 | 0.012 | 0.013 | 0.014 |
|---|---|---|---|---|---|---|---|---|---|
| 20 | $RE_1$ | 1.8178 | 1.8675 | 1.9093 | 1.8679 | 1.7808 | 1.6476 | 1.3910 | 1.3521 |
| | $RE_2$ | 0.9313 | 0.9361 | 0.9392 | 0.9424 | 0.9448 | 0.9476 | 0.9500 | 0.9511 |
| 25 | $RE_1$ | 1.7239 | 1.8464 | 1.9614 | 1.9724 | 1.9943 | 1.7698 | 1.6850 | 1.5585 |
| | $RE_2$ | 0.9446 | 0.9482 | 0.9516 | 0.9544 | 0.9562 | 0.9585 | 09599 | 0.9611 |
| 30 | $RE_1$ | 1.6847 | 1.7951 | 1.8941 | 1.9951 | 2.0084 | 1.8972 | 1.7610 | 1.6621 |
| | $RE_2$ | 0.9542 | 0.9578 | 0.9604 | 0.9633 | 0.9648 | 0.9664 | 0.9677 | 0.9743 |
| 35 | $RE_1$ | 1.6406 | 1.7487 | 1.8445 | 1.9330 | 2.0081 | 1.9905 | 1.9062 | 1.8871 |
| | $RE_2$ | 0.9661 | 0.9645 | 0.9669 | 0.9687 | 0.9703 | 0.9721 | 0.9733 | 0.9789 |
| 40 | $RE_1$ | 1.5883 | 1.6926 | 1.8329 | 1.9166 | 1.9892 | 2.0669 | 1.9723 | 1.9131 |
| | $RE_2$ | 0.9622 | 0.9697 | 0.9722 | 0.9738 | 0.9751 | 0.9764 | 09777 | 0.9787 |

**Table 4: Risk efficiencies of $\widetilde{\beta}_0$ and $\widetilde{\beta}_1$ for the different values of $\beta_0$ and $\beta_1$ with $t_e$ =125, a = 0.5, b = 0.5 and c = 1.5**

| $\beta_0$ \ $\beta_1$ | | 0.007 | 0.008 | 0.009 | 0.010 | 0.011 | 0.012 | 0.013 | 0.014 |
|---|---|---|---|---|---|---|---|---|---|
| 20 | $RE'_1$ | 1.7633 | 1.8631 | 1.9326 | 1.8935 | 1.7697 | 1.6636 | 1.3886 | 1.2331 |
| | $RE'_2$ | 0.7315 | 0.7490 | 0.7624 | 0.7718 | 0.7822 | 0.7868 | 0.7966 | 0.8012 |
| 25 | $RE'_1$ | 1.7203 | 1.8266 | 1.9305 | 1.9632 | 1.9356 | 1.8199 | 1.6495 | 1.5841 |
| | $RE'_2$ | 0.7804 | 0.7934 | 0.8028 | 0.8129 | 0.8202 | 0.8251 | 0.8319 | 0.8491 |
| 30 | $RE'_1$ | 1.6728 | 1.7785 | 1.8973 | 1.9727 | 1.9945 | 1.9507 | 1.8121 | 1.7301 |
| | $RE'_2$ | 0.8101 | 0.8237 | 0.8333 | 0.8427 | 0.8487 | 0.8502 | 0.8575 | 0.8593 |
| 35 | $RE'_1$ | 1.6341 | 1.7268 | 1.8374 | 1.9535 | 2.0060 | 1.9258 | 1.8705 | 1.8157 |
| | $RE'_2$ | 0.8365 | 0.8472 | 0.8559 | 0.8616 | 0.8678 | 0.8736 | 0.8772 | 0.8899 |
| 40 | $RE'_1$ | 1.5889 | 1.7179 | 1.8155 | 1.9462 | 1.9804 | 2.0178 | 1.9894 | 1.9157 |
| | $RE'_2$ | 0.8551 | 0.8661 | 0.8738 | 0.8791 | 0.8845 | 0.8885 | 0.8920 | 0.8981 |

**Table 5: Risk efficiencies of $\tilde{\beta}_0$ and $\tilde{\beta}_1$ for the different values of $\beta_0$ and $\beta_1$ with $t_e$ =125, a = 0.5, b = 2 and c = 1.5**

| $\beta_0$ \ $\beta_1$ | | 0.007 | 0.008 | 0.009 | 0.010 | 0.011 | 0.012 | 0.013 | 0.014 |
|---|---|---|---|---|---|---|---|---|---|
| 20 | $RE'_1$ | 1.8128 | 1.8405 | 1.8580 | 1.9318 | 1.7618 | 1.7318 | 1.3315 | 1.2065 |
| | $RE'_2$ | 0.4931 | 0.5107 | 0.5284 | 05437 | 0.5584 | 0.5652 | 0.5844 | 0.5931 |
| 25 | $RE'_1$ | 1.7321 | 1.8229 | 1.9267 | 1.9652 | 1.9001 | 1.8670 | 1.6508 | 1.4387 |
| | $RE'_2$ | 0.5558 | 0.5749 | 0.5963 | 0.6087 | 0.6245 | 0.6300 | 0.6389 | 0.6456 |
| 30 | $RE'_1$ | 1.6758 | 1.7835 | 1.9176 | 2.0101 | 1.9675 | 1.9442 | 1.8090 | 1.7765 |
| | $RE'_2$ | 0.6087 | 0.6302 | 0.6446 | 0.6577 | 0.6683 | 0.6767 | 0.6860 | 0.6944 |
| 35 | $RE'_1$ | 1.6292 | 1.7518 | 1.8556 | 1.9373 | 1.9552 | 1.9508 | 1.8933 | 1.8661 |
| | $RE'_2$ | 0.6505 | 1.6685 | 0.6846 | 0.6971 | 0.7084 | 0.7153 | 0.7224 | 0.7385 |
| 40 | $RE'_1$ | 1.5889 | 1.6988 | 1.8239 | 1.9295 | 2.004 | 1.9994 | 1.99709 | 1.9615 |
| | $RE'_2$ | 0.6840 | 0.7023 | 0.7163 | 0.7278 | 0.7366 | 0.7453 | 0.7505 | 0.7654 |

**Table 6: Risk efficiencies of $\tilde{\beta}_0$ and $\tilde{\beta}_1$ for the different values of $\beta_0$ and $\beta_1$ with $t_e$ =125, a = 0.5, b = 5.0 and c = 1.5**

| $\beta_0$ \ $\beta_1$ | | 0.007 | 0.008 | 0.009 | 0.010 | 0.011 | 0.012 | 0.013 | 0.014 |
|---|---|---|---|---|---|---|---|---|---|
| 20 | $RE'_1$ | 1.5385 | 1.6310 | 1.6962 | 1.6578 | 1.6300 | 1.4810 | 1.3374 | 1.2330 |
| | $RE'_2$ | 0.9314 | 0.9358 | 0.9394 | 0.9426 | 0.9448 | 0.9479 | 0.9504 | 0.9519 |
| 25 | $RE'_1$ | 1.5190 | 1.5936 | 1.6539 | 1.6806 | 1.7099 | 1.6178 | 1.5313 | 1.3747 |
| | $RE'_2$ | 0.9441 | 0.9486 | 0.9521 | 0.9544 | 0.9563 | 0.9587 | 0.9601 | 0.9620 |
| 30 | $RE'_1$ | 1.4695 | 1.5458 | 1.6127 | 1.6730 | 1.7391 | 1.7431 | 1.6515 | 1.5266 |
| | $RE'_2$ | 0.9530 | 0.9573 | 0.9604 | 0.9633 | 0.9648 | 0.9661 | 0.9676 | 0.9689 |
| 35 | $RE'_1$ | 1.4505 | 1.5268 | 1.5976 | 1.6441 | 1.7017 | 1.7107 | 1.6840 | 1.6429 |
| | $RE'_2$ | 0.9614 | 0.9442 | 0.9674 | 0.9690 | 0.9722 | 0.9722 | 0.9733 | 0.9743 |
| 40 | $RE'_1$ | 1.4229 | 1.5134 | 1.5862 | 1.6323 | 1.7036 | 1.7236 | 1.7446 | 1.7322 |
| | $RE'_2$ | 0.9669 | 0.9700 | 0.9723 | 0.9739 | 0.9768 | 0.9768 | 0.9777 | 0.9784 |

**Table 7: Risk efficiencies of $\tilde{\beta}_0$ and $\tilde{\beta}_1$ for the different values of $\beta_0$ and $\beta_1$ with $t_e$ =125, a = 0.5, b = 0.5 and c = 2.25**

| $\beta_0$ \ $\beta_1$ | | 0.007 | 0.008 | 0.009 | 0.010 | 0.011 | 0.012 | 0.013 | 0.014 |
|---|---|---|---|---|---|---|---|---|---|
| 20 | $RE'_1$ | 1.5481 | 1.5973 | 1.6609 | 1.6688 | 1.6361 | 1.5300 | 1.3278 | 1.221 |
| | $RE'_2$ | 0.7323 | 0.7483 | 0.7666 | 0.7725 | 0.7814 | 0.7899 | 0.7981 | 0.8024 |
| 25 | $RE'_1$ | 1.5079 | 1.5826 | 1.6426 | 1.6827 | 1.6684 | 1.6297 | 1.5703 | 1.3988 |
| | $RE'_2$ | 0.7791 | 0.7935 | 0.8044 | 0.8134 | 0.8203 | 0.8248 | 0.8301 | 0.8343 |
| 30 | $RE'_1$ | 1.4794 | 1.5452 | 1.6078 | 1.6858 | 1.7024 | 1.7294 | 1.6392 | 1.5139 |
| | $RE'_2$ | 0.8120 | 0.8235 | 0.8334 | 0.8426 | 0.8479 | 0.8525 | 0.8568 | 0.8614 |
| 35 | $RE'_1$ | 1.4469 | 1.5307 | 1.5982 | 1.6743 | 1.7308 | 1.7297 | 1.6892 | 1.6007 |
| | $RE'_2$ | 0.8371 | 0.8479 | 0.8565 | 0.8623 | 0.8678 | 0.8722 | 0.8777 | 0.8801 |
| 40 | $RE'_1$ | 1.4167 | 1.5039 | 1.5897 | 1.6494 | 1.7308 | 1.7477 | 1.7424 | 1.6980 |
| | $RE'_2$ | 0.8546 | 0.8664 | 0.8740 | 0.8803 | 0.8845 | 0.8881 | 0.8921 | 0.8950 |

**Table 8: Risk efficiencies of $\tilde{\beta}_0$ and $\tilde{\beta}_1$ for the different values of $\beta_0$ and $\beta_1$ with $t_e$ =125, a = 0.5, b = 2.0 and c = 2.25**

| $\beta_0$ \ $\beta_1$ | | 0.007 | 0.008 | 0.009 | 0.010 | 0.011 | 0.012 | 0.013 | 0.014 |
|---|---|---|---|---|---|---|---|---|---|
| 20 | $RE'_1$ | 1.5489 | 1.6045 | 1.6604 | 1.6557 | 1.5811 | 1.5151 | 1.3279 | 1.1365 |
| | $RE'_2$ | 0.4881 | 0.5115 | 05304 | 0.5434 | 0.5633 | 0.5730 | 0.5831 | 0.5980 |
| 25 | $RE'_1$ | 1.4961 | 1.5798 | 1.6472 | 1.6753 | 1.6837 | 1.6542 | 1.5525 | 1.4024 |
| | $RE'_2$ | 0.5572 | 0.5791 | 0.5936 | 0.6116 | 0.6221 | 0.6316 | 0.6401 | 0.6480 |
| 30 | $RE'_1$ | 1.4776 | 1.5437 | 1.6271 | 1.7103 | 1.7120 | 1.6725 | 1.6199 | 1.5094 |
| | $RE'_2$ | 0.6108 | 0.6306 | 0.6454 | 0.6564 | 0.6706 | 0.6788 | 0.6868 | 0.6932 |
| 35 | $RE'_1$ | 1.4514 | 1.5243 | 1.5944 | 1.6627 | 1.7255 | 1.7307 | 1.7112 | 1.6168 |
| | $RE'_2$ | 0.6508 | 0.6707 | 0.6839 | 0.6952 | 0.7061 | 0.7147 | 0.7209 | 0.7283 |
| 40 | $RE'_1$ | 1.4212 | 1.5003 | 1.5870 | 1.6480 | 1.7123 | 1.7408 | 1.7530 | 1.6897 |
| | $RE'_2$ | 0.6823 | 0.7018 | 0.7154 | 0.7286 | 0.7361 | 0.7440 | 0.7504 | 0.7560 |

**Table 9; Risk efficiencies of $\tilde{\beta}_0$ and $\tilde{\beta}_1$ for the different values of $\beta_0$ and $\beta_1$ with $t_e$=125, a = 0.5, b = 5.0 and c= 2.25**

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