RELIABILITY OF A MULTI COMPONENT STRESS STRENGTH MODEL WITH STANDBY SYSTEM USING MIXTURE OF TWO EXPONENTIAL DISTRIBUTIONS

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Abstract

In the present work, a multi component standby system of stress-strength model is considered to find the reliability. Reliability has been derived when stress-strength follow exponential distribution and mixture of two exponential distributions. The general expression for the reliability of a multi component standby system is obtained and the system reliability is computed numerically for different values of the stress and strength parameters. The reliability of a multi component standby system has been developed when the initial stress distribution is arbitrary.

Key Words: Exponential Distribution, Laplace Transform, Mixture Of Two Exponential Distributions, Standby System, Reliability, Stress-Strength Model.

1. Introduction

Reliability of a system is the probability that a system will adequately perform its intended purpose for a given period of time under stated environmental conditions [3]. In some cases system failures occur due to certain type of stresses acting on them. Thus system composed of random strengths will have its strength as random variable and the stress applied on it will also be a random variable. A system fails whenever an applied stress exceeds strength of the system. The reliability of an n-cascade system with stress attenuation was proposed by Pandit and Sriwastav [5]. Raghava char et al. [6] studied the reliability of a cascade system with normal stress and strength distribution.

In reliability theory, there are lots of real life situations where the concept of mixture distributions can be applied. For example, in life testing experiments, the systems will be failed due to different causes and the times to failure due to different reasons are likely to follow different distributions. Knowledge of these distributions is essential to eliminate cause of failures and thereby to improve the reliability. Maya, T. Nair [4] described the estimation of reliability based on finite mixture of pareto and beta distributions. Detailed explanation of mixture distributions is given in [7]. Doloi and Borah [1] have considered a cascade system when stress-strength follows mixture of two distributions. Gogoi and Borah [2] have studied the estimation of reliability for multi component systems using exponential, gamma and lindley stress-strength distributions.

In the present paper, explicit expressions for n-standby system reliability are obtained; a general recursive rule is indicated for obtaining these expressions. We derive the reliability, when the stress-strength follows exponential and mixture of two exponential distributions. Numerical calculations for R(i) and R_i, i=1,2,3 have been done for the three particular cases when stresses and strengths are identically distributed. i.e., $\lambda_1 = \lambda_2 = \cdots = \lambda_n = \lambda$ and $\mu_1 = \mu_2 = \cdots = \mu_n = \mu$.

2. Notations

X_i	Strength of i th component
Y_i	Stress of i th component
R(n)	Marginal reliability of n th component
R_n	System Reliability of n components
λ_i	Strength parameter of i th component
μ_i	Stress parameter of ith component

3. Assumptions and Model Description

The assumptions taken in this model are

- (i) The random variables X and Y are independent.
- (ii) The values of stress and strength are non-negative.

If X denotes the strength of the component and Y is the stress imposed on it, then the reliability of the component is given by

$$R=P(X>Y)$$

$$= \int_{0}^{\infty} \int_{0}^{x} g(y)f(x)dydx$$
(1)

where f(x) and g(y) are probability density functions of strength and stress respectively.

Consider a system of n- components, out of which only one is working under impact of stresses and the remaining (n-1) are standbys. Whenever the working component fails, one from standby components takes the place of a failed component and is subjected to impact of stress then the system works. The system fails when all the components fail. Let X_1, X_2, \ldots, X_n be the strengths of the n components arranged in order of activation in the system. And let Y_1, Y_2, \ldots, Y_n be the stresses on the n components respectively then the system reliability R_n of the system is given by

$$R_n = R(1) + R(2) + R(3) + \dots + R(n)$$
(2)

Where the marginal reliability R(n) is the reliability of the system by the n^{th} component and defined as

$$R(n) = P[X_1 < Y_1, X_2 < Y_2, \dots, X_{n-1} < Y_{n-1}, X_n > Y_n]$$

$$R(n) = P\left[\left(\bigcap_{i=1}^{n-1} X_i < Y_i\right) \cap (X_n > Y_n)\right]$$
(3)

Let $f_i(x)$ and $g_i(y)$ be the probability density functions of $X_i \& Y_i$ i=1,2,....n respectively then

$$R(n) = \int_{-\infty}^{\infty} F_1(y) g_1(y) dy \int_{-\infty}^{\infty} F_2(y) g_2(y) dy \dots \dots \dots \int_{-\infty}^{\infty} \overline{F_n}(y) g_n(y) dy$$
(4)

where
$$F_i(y) = \int_0^\infty f_i(x)dx$$
 and $\overline{F}_i(y) = 1 - F_i(y)$ (5)

A mixture of two exponential distributions with probability density function of the form

$$f(x) = p\lambda_1 \exp(-\lambda_1 x) + (1 - p)\lambda_2 \exp(-\lambda_2 x), 0 0 \ (i = 1, 2)$$
(6)

The r^{th} moment of the mixture of two exponential distributions

$$E(x^{r}) = \int_{0}^{0} x^{r} [p_{1}\lambda_{1}exp(-\lambda_{1}x) + (1-p_{1})\lambda_{2}exp(-\lambda_{2}x)] dx$$
$$= p_{1}\lambda_{1} \frac{\Gamma(r+1)}{\lambda_{1}^{r+1}} + (1-p_{1})\lambda_{2} \frac{\Gamma(r+1)}{\lambda_{2}^{r+1}}$$
(7)

When r = 1

$$E(x) = \frac{p_1}{\lambda_1} + \frac{1 - p_1}{\lambda_2}, 0 < p_1 < 1, \lambda_i > 0, i = 1, 2$$
(8)

When r = 2

$$E(x^{2}) = \frac{2p_{1}}{\lambda_{1}^{2}} + \frac{2(1-p_{1})}{\lambda_{2}^{2}}$$
(9)

Thus the variance is given by

$$V(x) = \frac{p_1(2-p_1)}{\lambda_1^2} + \frac{(1-p_1)(2-p_1)}{\lambda_2^2} - \frac{2p_1(1-p_1)}{\lambda_1\lambda_2}$$
(10)

In this paper we are considering three cases. They are

(1) Stress and strength follows exponential distribution.

- (2) Stress follows exponential distribution and strength follows mixture of two exponential distributions.
- (3) Stress and strength follows mixture of two exponential distributions.

4. Reliability computations

Case (i) Stress and strength follow exponential distribution:

Let $f_i(x)$ be the probability density function of exponential strength with mean $\frac{1}{\lambda_i}$ is given by $f_i(x) = \lambda_i exp(-\lambda_i x)$ $\lambda_i > 0, x > 0, i = 1, 2, \dots, n$ and $g_i(y)$ be the probability density function of exponential stress with mean $\frac{1}{\mu_i}$ is given by $g_i(y) = \mu_i exp(-\mu_i y)$ $\mu_i > 0, y > 0, i = 1, 2, \dots, n$ Then $R(1) = \int_0^{\infty} \overline{F_1}(y)g_1(y)dy = \int_0^{\infty} exp(-\lambda_1 y)\mu_1 exp(-\mu_1 y) dy$ $R(1) = \frac{\mu_1}{\mu_1 + \lambda_1}$ (11) $R(2) = \left[\int_0^{\infty} F_1(y)g_1(y)dy\right] \left[\int_0^{\infty} \overline{F_2}(y)g_2(y)dy\right]$ $= \left[\int_0^{\infty} [1 - exp(-\lambda_1 y)]\mu_1 exp(-\mu_1 y)\right] \left[\int_0^{\infty} [exp(-\lambda_2 y)]\mu_2 exp(-\mu_2 y)\right]$ $R(2) = \left[1 - \frac{\mu_1}{\lambda_1 + \mu_1}\right] \left[\frac{\mu_2}{\lambda_2 + \mu_2}\right]$ (12)

$$R(3) = \left[\int_{0}^{\infty} [1 - \exp(-\lambda_{1}y)] \mu_{1} \exp(-\mu_{1}y) \right] \\ \times \left[\int_{0}^{\infty} [1 - \exp(-\lambda_{2}y)] \mu_{2} \exp(-\mu_{2}y) \right] \times \left[\int_{0}^{\infty} [\exp(-\lambda_{3}y)] \mu_{3} \exp(-\mu_{3}y) \right] \\ R(3) = \left[1 - \frac{\mu_{1}}{\lambda_{1} + \mu_{1}} \right] \left[1 - \frac{\mu_{2}}{\lambda_{2} + \mu_{2}} \right] \left[\frac{\mu_{3}}{\lambda_{3} + \mu_{3}} \right]$$
(13)
Therefore in general,
$$R(n) = \left[\prod_{i=1}^{n-1} \left[1 - \frac{\mu_{i}}{\lambda_{i} + \mu_{i}} \right] \right] \left[\frac{\mu_{n}}{\lambda_{n} + \mu_{n}} \right]$$
(14)
And
$$R_{n} = R(1) + R(2) + R(3) + \dots + R(n) = \sum_{i=1}^{n} R(i)$$

Case (ii) Strength X follows exponential and stress Y follows mixture of two exponential

Strength follows exponential distribution with probability density function $f_i(x) = \lambda_i exp(-\lambda_i x)$ $\lambda_i > 0, x > 0$ Stress follows a mixture of two exponential distributions with probability density function

$$g_{i}(y) = p_{2i-1}\mu_{2i-1}\exp(-\mu_{2i-1}y) + (1-p_{2i-1})\mu_{2i}\exp(-\mu_{2i}y)$$

$$0 < p_{2i-1} < 1, \ \mu_{2i-1} > 0, \ \mu_{2i} > 0 \qquad (i = 1, 2, \dots, n)$$

Then

$$R(1) = \int_{0}^{\infty} \exp(-\lambda_{1}y) \left[p_{1}\mu_{1} \exp(-\mu_{1}y) + (1-p_{1})\mu_{2} \exp(-\mu_{2}y)\right] dy$$

$$= p_{1}\mu_{1} \int_{0}^{\infty} \exp(-(\lambda_{1}+\mu_{1})y) dy + (1-p_{1})\mu_{2} \int_{0}^{\infty} \exp(-(\lambda_{1}+\mu_{2})y) dy$$

$$R(1) = \frac{p_{1}\mu_{1}}{\lambda_{1}+\mu_{1}} + (1-p_{1})\frac{\mu_{2}}{\lambda_{1}+\mu_{2}}$$
(15)
$$R(2) = \left[\int_{0}^{\infty} F_{1}(y)g_{1}(y)dy\right] \left[\int_{0}^{\infty} \overline{F}_{2}(y)g_{2}(y)dy\right]$$

$$= \int_{0}^{\infty} \left[1-\exp(-\lambda_{1}y)\right] \left[p_{1}\mu_{1} \exp(-\mu_{1}y) + (1-p_{1})\mu_{2} \exp(-\mu_{2}y)\right] dy$$

$$\times \int_{0}^{\infty} \exp(-\lambda_{2}y) \left[p_{3}\mu_{3} \exp(-\mu_{3}y) + (1-p_{3})\mu_{4} \exp(-\mu_{4}y)\right] dy$$

$$R(2) = \left\{1-\frac{p_{1}\mu_{1}}{\lambda_{1}+\mu_{1}} - (1-p_{1})\frac{\mu_{2}}{\lambda_{1}+\mu_{2}}\right\} \left\{\frac{p_{3}\mu_{3}}{\lambda_{2}+\mu_{3}} + (1-p_{3})\frac{\mu_{4}}{\lambda_{2}+\mu_{4}}\right\}$$
(16)
$$R(3) = \left[\int_{0}^{\infty} F_{1}(y)g_{1}(y)dy\right] \left[\int_{0}^{\infty} F_{2}(y)g_{2}(y)dy\right] \left[\int_{0}^{\infty} \overline{F}_{3}(y)g_{3}(y)dy\right]$$

$$R(3) = \left\{1-\frac{p_{1}\mu_{1}}{\lambda_{1}+\mu_{1}} - (1-p_{1})\frac{\mu_{2}}{\lambda_{1}+\mu_{2}}\right\} \left\{1-\frac{p_{3}\mu_{3}}{\lambda_{2}+\mu_{3}} - (1-p_{3})\frac{\mu_{4}}{\lambda_{2}+\mu_{4}}\right\}$$

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$$\times \left\{ \frac{p_5 \mu_5}{\lambda_3 + \mu_5} + (1 - p_5) \frac{\mu_6}{\lambda_3 + \mu_6} \right\}$$
(17)

In general,

$$R(n) = \left[\int_{0}^{\infty} \prod_{i=1}^{n-1} F_{i}(y)g_{i}(y)dy\right] \left[\int_{0}^{\infty} \bar{F}_{n}(y)g_{n}(y)dy\right]$$

$$R(n) = \left[\frac{p_{2n-1}\mu_{2n-1}}{\lambda_{n} + \mu_{2n-1}} + (1 - p_{2n-1})\frac{\mu_{2n}}{\lambda_{n} + \mu_{2n}}\right]$$

$$\times \left[\prod_{k=1}^{n-1} \left[1 - \frac{p_{2k-1}\mu_{2k-1}}{\lambda_{k} + \mu_{2k-1}} - (1 - p_{2k-1})\frac{\mu_{2k}}{\lambda_{k} + \mu_{2k}}\right]\right]$$

$$R_{n} = R(1) + R(2) + R(3) + \cdots + R(n) = \sum_{i=1}^{n} R(i)$$
(18)

And

Case (iii) Strength and stress follows mixture of two exponential distributions:

Let Strength follows a mixture of two exponential distributions with probability density function

$$f_i(x) = p_{2i-1}\lambda_{2i-1}\exp(-\lambda_{2i-1}x) + (1-p_{2i-1})\lambda_{2i}\exp(-\lambda_{2i}x)$$

$$0 < p_{2i-1} < 1, \ \lambda_{2i-1} > 0, \ \lambda_{2i} > 0 \qquad (i = 1, 2, \dots, n)$$

 $0 < p_{2i-1} < 1, \lambda_{2i-1} > 0, \lambda_{2i} > 0$ (l = 1, 2, ..., n)Stress follows a mixture of two exponential distributions with probability density function

$$\begin{split} g_{i}(y) &= q_{2i-1}\mu_{2i-1}\exp(-\mu_{2i-1}y) + (1-q_{2i-1})\mu_{2i}\exp(-\mu_{2i}y) \\ &= 0 \quad (i = 1, 2, \dots, n) \\ \text{Then } f_{1}(x) &= p_{1}\lambda_{1}\exp(-\lambda_{1}x) + (1-p_{1})\lambda_{2}\exp(-\lambda_{2}x) \\ \text{and the cumulative distribution function is given by} \\ F_{1}(y) &= \int_{0}^{y} f_{1}(x)dx = \int_{0}^{y} [p_{1}\lambda_{1}\exp(-\lambda_{1}x) + (1-p_{1})\lambda_{2}\exp(-\lambda_{2}x)]dx \\ F_{1}(y) &= 1 - p_{1}\exp(-\lambda_{1}y) - (1-p_{1})\exp(-\lambda_{2}y) \\ F_{1}(y) &= p_{1}\exp(-\lambda_{1}y) + (1-p_{1})\exp(-\lambda_{2}y) \\ \text{And} \quad g_{1}(y) &= q_{1}\mu_{1}\exp(-\mu_{1}y) + (1-q_{1})\mu_{2}\exp(-\mu_{2}y) \\ \text{R}(1) &= \int_{0}^{\infty} \overline{F_{1}}(y)g_{1}(y)dy \\ \text{R}(1) &= \int_{0}^{\infty} [p_{1}\exp(-\lambda_{1}y) + (1-p_{1})\exp(-\lambda_{2}y)][q_{1}\mu_{1}\exp(-\mu_{1}y) \\ &+ (1-q_{1})\mu_{2}\exp(-\mu_{2}y)]dy \\ &= \int_{0}^{\infty} [p_{1}q_{1}\mu_{1}\exp(-(\lambda_{1}+\mu_{1})y) + \mu_{1}q_{1}(1-p_{1})\exp(-(\lambda_{2}+\mu_{1})y) + p_{1}\mu_{2}(1-q_{1}) \\ &\times \exp(-(\lambda_{1}+\mu_{2})y) \\ &+ \mu_{2}(1-p_{1})(1-q_{1})\exp(-(\lambda_{2}+\mu_{2})y)]dy \end{split}$$

$$= p_{1}q_{1}\frac{\mu_{1}}{\lambda_{1} + \mu_{1}} + q_{1}(1 - p_{1})\frac{\mu_{1}}{\lambda_{2} + \mu_{1}} + p_{1}(1 - q_{1})\frac{\mu_{2}}{\mu_{2} + \lambda_{1}} \\ + (1 - p_{1})(1 - q_{1})\frac{\mu_{2}}{\mu_{2} + \lambda_{2}} \\ R(1) = p_{1}\left[q_{1}\frac{\mu_{1}}{\lambda_{1} + \mu_{1}} + (1 - q_{1})\frac{\mu_{2}}{\lambda_{1} + \mu_{2}}\right] \\ + (1 - p_{1})\left[q_{1}\frac{\mu_{1}}{\lambda_{2} + \mu_{1}} + (1 - q_{1})\frac{\mu_{2}}{\lambda_{2} + \mu_{2}}\right]$$
(19)
$$R(2) = \left[\int_{0}^{\infty} F_{1}(y)g_{1}(y)dy\right]\left[\int_{0}^{\infty} \bar{F}_{2}(y)g_{2}(y)dy\right] \\ = \left[\int_{0}^{\infty} [1 - p_{1}\exp(-\lambda_{1}y) - (1 - p_{1})\exp(-\lambda_{2}y)][q_{1}\mu_{1}\exp(-\mu_{1}y) \\ + (1 - q_{1})\mu_{2}\exp(-\mu_{2}y)]dy\right] \\ \times \left[\int_{0}^{\infty} [p_{3}\exp(-\lambda_{3}y) + (1 - p_{3})\exp(-\lambda_{4}y)][q_{3}\mu_{3}\exp(-\mu_{3}y) \\ + (1 - q_{3})\mu_{4}\exp(-\mu_{4}y)]dy\right] \\ R(2) = \left\{1 - p_{1}\left[q_{1}\frac{\mu_{1}}{\lambda_{1} + \mu_{1}} + (1 - q_{1})\frac{\mu_{2}}{\lambda_{1} + \mu_{2}}\right] \\ - (1 - p_{1})\left[q_{1}\frac{\mu_{1}}{\lambda_{2} + \mu_{1}} + (1 - q_{3})\frac{\mu_{4}}{\lambda_{3} + \mu_{4}}\right] \\ + (1 - p_{3})\left[q_{3}\frac{\mu_{3}}{\lambda_{4} + \mu_{3}} + (1 - q_{3})\frac{\mu_{4}}{\lambda_{4} + \mu_{4}}\right]\right\}$$
(20)
In general

In general,

$$R(n) = \prod_{i=1}^{n-1} \left\{ 1 - p_{2i-1} \left[q_{2i-1} \frac{\mu_{2i-1}}{\lambda_{2i-1} + \mu_{2i-1}} + (1 - q_{2i-1}) \frac{\mu_{2i}}{\lambda_{2i-1} + \mu_{2i}} \right] \right\}$$
$$-(1 - p_{2i-1}) \left[q_{2i-1} \frac{\mu_{2i-1}}{\lambda_{2i-1} + \mu_{2i-1}} + (1 - q_{2i-1}) \frac{\mu_{2i}}{\lambda_{2i-1} + \mu_{2i}} \right]$$
$$\times \left\{ p_{2n-1} \left[q_{2n-1} \frac{\mu_{2n-1}}{\lambda_{2n-1} + \mu_{2n-1}} + (1 - q_{2n-1}) \frac{\mu_{2n}}{\lambda_{2n-1} + \mu_{2n}} \right] + (1 - p_{2n-1}) \left[q_{2n-1} \frac{\mu_{2n-1}}{\lambda_{2n} + \mu_{2n-1}} + (1 - q_{2n-1}) \frac{\mu_{2n}}{\lambda_{2n-1} + \mu_{2n}} \right] \right\}$$
$$+ (1 - q_{2n-1}) \left[q_{2n-1} \frac{\mu_{2n}}{\lambda_{2n} + \mu_{2n-1}} \right] \right\}$$
(21)

And $R_n = R(1) + R(2) + R(3) + \dots + R(n) = \sum_{i=1}^n R(i)$

5. When the initial stress distribution is arbitrary:

We now consider the components strength to have exponential distribution as in case(i), but allow the stress Y_1 on the first component to have a general probability distribution function $g_1(y)$ over $0 < Y_1 < \infty$. The cdf of the stress Y_1 is

$$G_1(y) = \int_0^y g_1(t)dt \tag{22}$$

The marginal reliability functions R(i) are

$$R(1) = \int_{0}^{0} \exp(-\lambda_1 y) g_1(y) dy$$
(23)

$$R(2) = \left[\int_{0}^{\infty} [1 - \exp(-\lambda_1 y)] g_1(y) dy\right] \left[\int_{0}^{\infty} [\exp(-\lambda_2 y)] g_2(y) dy\right]$$
(24)
$$R(3) = \left[\int_{0}^{\infty} [1 - \exp(-\lambda_1 y)] g_1(y) dy\right] \times \left[\int_{0}^{\infty} [1 - \exp(-\lambda_2 y)] g_2(y) dy\right]$$
$$\times \left[\int_{0}^{\infty} [\exp(-\lambda_3 y)] g_3(y) dy\right]$$
(25)

The quantity $\int_0^\infty \exp(-\lambda_1 y) g_1(y) dy = \tilde{g}_1(\lambda_1)$ denotes laplace transform of the function $g_1(y)$ and it exists as long as function $g_1(y)$ is sectionally continuous and is of exponential order i.e., a constant A can be found such that

 $|\exp(-\lambda_1 y) g_1(y)| < A$, $\forall y \in (0, \infty)$ (26) Hence the marginal reliability functions R(1), R(2), R(3) can be easily expressed in terms of Laplace transform $\tilde{g}_i(\lambda)$ of the stress probability distribution function $g_i(y)$ In fact

$$R(1) = \tilde{g}_1(\lambda_1)$$
(27)

$$R(2) = [\tilde{g}_1(0) - \tilde{g}_1(\lambda_1)][\tilde{g}_2(\lambda_2)]$$
(28)

$$R(3) = [\tilde{g}_1(0) - \tilde{g}_1(\lambda_1)][\tilde{g}_2(0) - \tilde{g}_2(\lambda_2)][\tilde{g}_3(\lambda_3)]$$
Hence, we have
(29)

$$R_2 = \tilde{g}_1(\lambda_1) + [\tilde{g}_1(0) - \tilde{g}_1(\lambda_1)][\tilde{g}_2(\lambda_2)]$$
(30)

$$R_3 = \tilde{g}_1(\lambda_1) + [\tilde{g}_1(0) - \tilde{g}_1(\lambda_1)] [\tilde{g}_2(\lambda_2) + [\tilde{g}_2(0) - \tilde{g}_2(\lambda_2)] [\tilde{g}_3(\lambda_3)]] \quad (31)$$

λ_1	R(1)	R(2)	R(3)	R ₂	R ₃
0.01	0.909091	0.082645	0.007513	0.991736	0.999249
0.02	0.833333	0.138889	0.023148	0.972222	0.99537
0.03	0.769231	0.177515	0.040965	0.946746	0.987711
0.04	0.714286	0.204082	0.058309	0.918367	0.976676
0.05	0.666667	0.222222	0.074074	0.888889	0.962963
0.06	0.625	0.234375	0.087891	0.859375	0.947266
0.07	0.588235	0.242215	0.099735	0.83045	0.930185
0.08	0.555556	0.246914	0.109739	0.802469	0.912209
0.09	0.526316	0.249307	0.118093	0.775623	0.893716

Table1: Marginal reliabilities R(1), R(2), R(3) and system reliabilities R₂, R₃ when stress and strength follow exponential distribution for $\mu_i = 0.1$, i =

1, 2, *n* :

μ_1	R(1)	R(2)	R(3)	R ₂	R ₃
0.1	0.5	0.25	0.125	0.75	0.875
0.2	0.666667	0.222222	0.074074	0.888889	0.962963
0.3	0.75	0.1875	0.046875	0.9375	0.984375
0.4	0.8	0.16	0.032	0.96	0.992
0.5	0.833333	0.138889	0.023148	0.972222	0.99537
0.6	0.857143	0.122449	0.017493	0.979592	0.997085
0.7	0.875	0.109375	0.013672	0.984375	0.998047
0.8	0.888889	0.098765	0.010974	0.987654	0.998628
0.9	0.9	0.09	0.009	0.99	0.999

Table2. Marginal reliabilities R(1), R(2), R(3) and system reliabilities R_2 , R_3 when stress and strength follow exponential distribution for $\lambda_i = 0.1$, $i = 1, 2 \dots n$:



Fig.1: System reliabilities R_1, R_2, R_3 for different strength parameters



Fig.2: System reliabilities R1, R2, R3 for different stress parameters

6. Conclusion

In the present work, numerical calculations for R(i) and R_i, i=1,2,3 have been done for the three particular cases when stresses and strengths are identically distributed. It has been observed by the computations and figures that if the strength parameter increases then the value of reliability decreases and if the stress parameter increases then the value of reliability increases. Tables 1 and 2 illustrate the variations of R(n) and R_n, for n<4 for some values of stress and strength parameters. These variations are very instructive with a careful scrutiny. Thus for instance it is noticed that for a given $\mu_1 = 0.1$, $\lambda_1 = 0.05$ in tables R₁=0.66667, R₂=0.88889, R₃=0.96296. This indicates that by addition of one standby component to a 2-standby system, the system reliability is improved by about 33%, whereas, for 3-standby system the reliability improvement is 8%.

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