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NESTED BALANCED TERNARY DESIGNS AND THEIR PB ARRAYS

H.L. Sharma
Department of Mathematics and Statistics,
College of Agriculture, JN Agricultural University, Jabalpur (M.P.) INDIA
Email: drhlsharma_jnkvv@rediffmail.com

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Abstract

This paper is concerned with the generalization of the parameters of nested balanced ternary designs (NBTD) through tactical configurations. A four symbol PB arrays of varying strength 2m or (2m+1) has been constructed. In view of this, an example of PB arrays in four symbols of strength three has been included. Two orthogonal arrays (OA) (9, 3, 3, 2) of index unity and (18, 4, 3, 2) of index 2 has been developed through NBTD. The new designs that can be obtained through the PB arrays have also been included which are useful for intercropping experiments in relation to practical situations. One actual example of intercropping experiments with six intercrops has been added.

Key Words: Nested Balanced Ternary Design, Partially Balanced (PB) Arrays, Tactical Configurations, Balanced Incomplete Block (BIB) Design, Doubly Balanced Incomplete Block (DBIB) Design, Strength.

1. Introduction

Initially, the balanced ternary designs were introduced by Tocher (1952). He obtained some balanced ternary designs by trial and error. A number of authors have studied these designs during the past decades (Billington, 1984; Donovan, 1988; Patwardhan and Sharma, 1988; Sarvate, 1990; Tyagi and Rizwi, 1979).

Nested balanced incomplete block (NBIB) designs were defined by Preece (1967) for statistical situations where there are two sources of variability and one source is nested within the other. That is, an NBIB design has two systems of blocks, the second nested within the first (each block from the first system, called super blocks, consisting of some blocks, called sub-blocks from the second), such that ignoring either system leaves a BIB design where blocks are those of the other system.

Partially balanced arrays are important as fractional design in Statistics. The construction of such arrays was generalized by Chakravarti (1961) and later Dey *et al.* (1972) have constructed PB arrays of strength two and three with three symbols using BIB and DBIB designs.. Hedayat and Wallis (1978) have used Hadamard matrices to construct these designs. In general, PB arrays of strength 2, 3 and 4 are identical with balanced fractional factorial designs of resolution 3, 4 and 5, respectively.

It might be important to stress here that PB arrays not only provide a mathematically challenging field of research which unites various branches of the combinatorial theory of design of experiments, they are also urgently needed for practical problems arising in factorial experimentation. Further, Nigam (1985) constructed a series of (n+1) symbol PB arrays of strength two from regular group divisible designs.

A tactical configuration, introduced by Sprott (1955) is a generalized structure of a BIB design. Sharma and Chandak (1999) obtained a tactical configuration of order (2m+1) from a tactical configuration of order 2m. Gupta *et al.* (1995) have given the construction of NBTD using two BIB designs under certain restrictions.

Sharma (2005) constructed three symbol PB arrays of strength (2m+1). Recently, Sharma *et al.* (2009) have constructed a series of two associate partially balanced ternary designs and partially balanced arrays through group divisible (GD) and L_2 designs. They have also constructed a five symbol PB arrays of strength two for the first and second associates.

The purpose of this paper is to derive the expression for the generalization of the parameters of NBTD and their constructed PB arrays of strength (2m+1) or 2m in four symbols using tactical configuration (α,β,k,v) converted into design parameters by standard relationship.

2. Definitions and Notations

2.1 Balanced Ternary Design (BTD)

A balanced *n-ary* design with parameters V,B,R,K,Λ and incidence matrix $N=((n_{ij}))$ is an arrangement of V treatments in B blocks, each of cardinality $K(K \le V)$ such that (i) the i^{th} treatment appears n_{ij} times in the j^{th} block where n_{ij} can take any of the values 0,1,2,…n-1.(ii)each treatment occurs R times, and (iii) $\sum_{j=1}^{j=B} n_{ij}\, n_{i'j} = \Lambda$ for all $i \neq i'=1,2,\ldots V$. Note that $\sum_{j=1}^{j=B} n_{ij} = R$ for all i,and $\sum_{j=1}^{j=V} n_{ij} = K$ for all j. For n=2 (binary) the design is called a BIB design with the usual coincidence number $\lambda = \Lambda$. When n=3 we use the term " balanced ternary design". Thus, a balanced ternary design is a collection of B blocks, each of cardinality $K(K \le V)$, chosen from a set of size V in such a way that each of the V treatments occurs R times altogether, each of the treatments occurring once in precisely in Q_1 blocks and twice in precisely Q_2 blocks, and with incidence matrix having inner product of any two rows Λ is denoted by BTD (V, B, Q_1, Q_2, R, K, Λ). It is to be noted that $Q_1+2Q_2=R$ [Gupta *et el.*,1995;Sarvate and Seberry, 1993].

2.2 Partially Balanced Arrays

Let A be an m×N matrix, with elements 0, 1, 2… or s-1. Let us consider s^t (1×t) vectors, $X' = (x_1, x_2, \ldots, x_t)$, which can be formed from t-rowed submatrix of A, and associate with each (tx1) vector X a positive integer $\boldsymbol{\mu}(x_1, x_2, \ldots, x_t)$, which is invariant under permutations of $(x_1, x_2, \ldots, x_t)$, where $x_i = 0,1,2,\ldots, s-1$; $i=1,2,\ldots,t$. If for every t-rowed submatrix of A, the s^t distinct (t×1) vectors X occur as column $\boldsymbol{\mu}(x_1, x_2, \ldots, x_t)$ times, then the matrix A is called a partially balanced (PB) array of strength t in N assemblies with m constraints, s symbols and the specified $\mu(x_1, x_2, \ldots, x_t)$ parameters and is represented as the PB array (m, N, s, t) with index set $\Lambda_{s,t}$.

2.3 Nested Balanced Ternary Design

A nested balanced ternary design with parameters V, B_1, B_2, Q_1, Q_2, R, K_1, K_2, Λ_1, Λ_2,m) is an arrangement of V treatments each replicated R times with two systems of blocking such that:

(i) the second system is nested within the first with each block from the first system (subsequently referred to as whole block) contained exactly m blocks from the second system (sub-block);

(ii) ignoring the second system leaves a BTD with B_1 blocks each of K_1 units and with Λ_1 concurrences;

(iii) ignoring the first system leaves a NBTD with B_2 blocks each of K_2 units and with Λ_2 concurrences, and

Thus, $VR = B_1K_1$, $VR = B_2K_2$, $\Lambda_1 (V-1)= R_1(K_1-1)-2Q_1$, $\Lambda_2 (V-1)= R_2(K_2-1)-2Q_2$ where Q_1 and Q_2 are multiplicities of '2' in BTD and NBTD respectively.

2.4 Tactical configuration

Given a set Ω of v elements, and given positive integers k, β (β ≤ k ≤ v) and α, we designate by a tactical configuration c(α-β-k-v), a system of blocks (subset of Ω), having k elements each such that every subset of Ω having β elements is included in exactly α blocks. If α =1, then the configuration is called the Steiner system i.e., it is a complete (1-β-k-v) configuration of v elements arranged in blocks of k so that each set of β elements occurs exactly once. The symbol λ_t denotes the frequency of the number of blocks in which any t treatments a, b,c,…, occur together. It is very obvious that t=1,2,…,β (β may be odd or even) and λ_1= r (number of replication), λ_0 = b = number of blocks. Sharma and Chandak (1999) have demonstrated that a configuration of order (2m+1) can always be constructed for all positive integral values of m.

Let $\mu^{e\ f\ g\ h}_{i\ j\ k\ l}$ denote the frequency of the t - plet in the t×b (t≤v) sub array of the b×v array in four symbols i, j, k, l with frequencies e, f, g and h respectively, such that e+f+g+h=t.

3. Construction of NBTD and Partially Balanced Arrays

3.1 Construction of NBTD through tactical configuration

Gupta *et al.* (1995) have constructed NBTD using two BIB designs D_1 and D_2 with parameters (v, b_1, r_1, k_1, λ_1) and (k_1, b_2, r_2, k_2, λ_2) respectively. They obtained D as NBTD with parameters v, b_1 b_2, $r_1(b_2 + r_2)$, $k_1+ k_2$, $\Lambda= (2r_2 + b_2 + \lambda_2)\lambda_1$. It is to be noted that the number of treatments in D_2 equals k_1, the block size of D_1.On the similar lines of Gupta *et al.* (1995), we have generalized the parameters of NBTD through tactical configurations and thus we have the following theorem:

Theorem 3.1

The existence of tactical configurations (λ_β,- β- k- v) and (λ_β'- β- k_1- k) respectively where k_1<k implies the existence of a nested balanced ternary design D with parameters V=v,

$$B=\lambda_\beta \lambda_\beta{}' \binom{v}{\beta} \Big/ \binom{k_1}{\beta}, \quad Q_1=\lambda_\beta \binom{v-1}{\beta-1} \Big/ \binom{k-1}{\beta-1},$$

$$Q_2=\lambda_\beta \lambda_\beta{}' \binom{v-1}{\beta-1} \Big/ \binom{k_1-1}{\beta-1}, \quad R=Q_1+2Q_2=\lambda_\beta \lambda_\beta{}' \binom{v-1}{\beta-1}\left(\frac{k+k_1}{k_1}\right) \Big/ \binom{k_1-1}{\beta-1},$$

K=k+k_1

$$\Lambda=\lambda_\beta \lambda_\beta{}' \binom{v-2}{\beta-2}\left(\frac{(k+k_1)^2-3k_1-k}{k_1(k-1)}\right) \Big/ \binom{k_1-2}{\beta-2}$$

Proof: Let us construct a tactical configuration (($\lambda_\beta{}'$- β- k_1- k) using the treatment labels in the ith block of tactical configuration (λ_β,- β- k- v) converting into design parameters and add the ith block of tactical configuration (λ_β,- β- k- v) to each of the blocks of this tactical configuration. Then, the resulting design is a NBTD. Each parameter can be verified on the basis of similar lines of Gupta *et.al.* (1995). The strength of PB arrays through NBTD will depend on the strength of tactical configurations i.e. if β is even, then strength of PB arrays will be even, otherwise odd.

Theorem 3.2 The column of **A'** when treated as assemblies give rise to a balanced array with v treatments, 2b assemblies, four symbols and strength β, β may be odd (2m+1) or even (2m) where **A'** is given by **A' = [N' / M']** and **A'** denotes the transpose of **A**.

Proof:

Let us consider a NBTD with usual parameters V, B, Q_1, Q_2, R, K and Λ constructed through two tactical configurations with certain condition mentioned in Theorem 3.1. Again,

let **N** = (n_{ij}) be the incidence matrix of this NBTD design,
where n_{ij} = 2, if the j^{th} treatment occurs in the i^{th} block twice
= 1, if the j^{th} treatment occurs in the i^{th} block once
= 0, otherwise

Evidently, **N** is a **b**×**v** array of symbols (0, 1, 2). Let any assembly of this array be designated by a row vector **z**= (**z_1, z_2,…,z_v**), z_i=0,1 or 2. Then we define the "image" of **z** as **z*** given by **z* = (z_1*, z_2*,…,z_v*)**, zi+zi*≡3(mod4) for all i=1,2,…, v . Now let **M** be a **b**×**v** array of "images" of each of the assemblies of **N.** The frequency of the ordered t-plet. (2, 2, 2,…, β) i.e. $\mu^{0\ 0\ \beta\ *}_{0\ 1\ 2\ 3}$ in any t-columned sub-array of N is obviously the number of blocks in which any β treatments a, b, c,…, occur together and is therefore equal to $\lambda_\beta' \times \lambda_\beta$. The frequency of the other t-plet (1, 2, 2,…β), i.e. $\mu^{0\;1\;\beta-1\;*}_{0\;1\;2\;3}$ in any t columned sub-array of N is the number of blocks in which all treatments occur twice with merely one treatment once. Clearly the number of such blocks is $(\lambda'_{\beta-1}-\lambda'_\beta)\lambda_\beta$ because $(\lambda'_{\beta-1}-\lambda'_\beta)$ blocks are genuine with λ_β to create the blocks in which all treatments occur twice with merely one treatment once. Similarly the frequency of the blocks having the occurrence of one treatment two times and rest, all twice i.e., $\mu^{0\;2\;\beta-2\;*}_{0\;1\;2\;3}$ then clearly the number of such blocks is $(\lambda'_{\beta-2} - 2\lambda_{\beta-1} + \lambda'_\beta)\lambda_\beta$

Proceeding like this

$$\mu^{0\;3\;\beta-3\;*}_{0\;1\;2\;3} = \left(\lambda'_{\beta-3} - {}^{3}C_1\lambda'_{\beta-2} + {}^{3}C_2\lambda'_{\beta-1} - \lambda'_\beta\right)\lambda_\beta$$

… … …

$$\mu^{0\;\beta\;0\;*}_{0\;1\;2\;3} = \left(\lambda'_0 - {}^{\beta}C_1\lambda'_1 + {}^{\beta}C_2\lambda'_2 + \ldots(-1)^\beta \binom{\beta}{\beta}\lambda'_\beta\right)\lambda_\beta$$

In this way, we have in N

$$\mu^{0\;0\;\beta\;*}_{0\;1\;2\;3},\ \mu^{0\;1\;\beta-1\;*}_{0\;1\;2\;3},\ \mu^{0\;2\;\beta-2\;*}_{0\;1\;2\;3},\ \ldots,\ \mu^{0\;\beta\;0\;*}_{0\;1\;2\;3}$$

$$\mu^{1\;0\;\beta-1\;*}_{0\;1\;2\;3},\ \mu^{2\;1\;\beta-2\;*}_{0\;1\;2\;3},\ \mu^{1\;2\;\beta-3\;*}_{0\;1\;2\;3},\ \ldots,\ \mu^{1\;\beta-1\;0\;*}_{0\;1\;2\;3}$$

$$\mu^{2\;0\;\beta-2\;*}_{0\;1\;2\;3},\ \mu^{2\;1\;\beta-3\;*}_{0\;1\;2\;3},\ \mu^{2\;2\;\beta-4\;*}_{0\;1\;2\;3},\ \ldots,\ \mu^{2\;\beta-2\;0\;*}_{0\;1\;2\;3}$$

$$\mu^{3\;0\;\beta-3\;*}_{0\;1\;2\;3},\ \mu^{3\;1\;\beta-4\;*}_{0\;1\;2\;3},\ \mu^{3\;2\;\beta-5\;*}_{0\;1\;2\;3},\ \ldots,\ \mu^{3\;\beta-3\;0\;*}_{0\;1\;2\;3}$$

… … … …

$$\mu^{\beta-1\;0\;1\;*}_{0\;1\;2\;3},\ \mu^{\beta-1\;1\;0\;*}_{0\;1\;2\;3},\ \mu^{\beta\;0\;0\;*}_{0\;1\;2\;3}$$

Since the assemblies of M are "images" those of N, it follows that in any t-column sub-array of M, the frequency of the ordered t-plets will be corresponding to N (i.e.) the symbols will be in M are:

$$\mu^{*\;\beta\;0\;0}_{0\;1\;2\;3},\ \mu^{*\;\beta-1\;1\;0}_{0\;1\;2\;3},\ \mu^{*\;\beta-2\;2\;0}_{0\;1\;2\;3},\ \ldots,\ \mu^{*\;0\;\beta\;0}_{0\;1\;2\;3}$$

$$\mu^{*\;\beta-1\;0\;1}_{0\;1\;2\;3},\ \mu^{*\;\beta-2\;1\;1}_{0\;1\;2\;3},\ \mu^{*\;\beta-3\;2\;1}_{0\;1\;2\;3},\ \ldots,\ \mu^{*\;0\;\beta-1\;1}_{0\;1\;2\;3}$$

$$\mu^{*\;\beta-2\;0\;2}_{0\;1\;2\;3},\ \mu^{*\;\beta-3\;1\;2}_{0\;1\;2\;3},\ \mu^{*\;\beta-4\;2\;2}_{0\;1\;2\;3},\ \ldots,\ \mu^{*\;0\;\beta-2\;2}_{0\;1\;2\;3}$$

$$\mu^{*\;\beta-3\;0\;3}_{0\;1\;2\;3},\ \mu^{*\;\beta-4\;1\;3}_{0\;1\;2\;3},\ \mu^{*\;\beta-5\;2\;3}_{0\;1\;2\;3},\ \ldots,\ \mu^{*\;0\;\beta-3\;3}_{0\;1\;2\;3}$$

$$\ldots,\ \mu^{*\;1\;0\;\beta-1}_{0\;1\;2\;3}\quad \mu^{*\;0\;1\;\beta-1}_{0\;1\;2\;3}\quad \mu^{*\;0\;0\;\beta}_{0\;1\;2\;3}.$$

Therefore, in the whole array A, the frequencies of all ordered t-plets are given by

$$\mu^{0\;0\;0\;\beta}_{0\;1\;2\;3} = \mu^{\beta\;0\;0\;0}_{0\;1\;2\;3} = \lambda'_0\left[\lambda_0 - {}^{\beta}C_1\lambda_1 + {}^{\beta}C_2\lambda_2 + \ldots(-1)^\beta\binom{\beta}{\beta}\lambda_\beta\right]$$

$$\mu^{0\;0\;1\;\beta-1}_{0\;1\;2\;\;3} = \mu^{\beta-1\;1\;0\;0}_{0\;\;1\;2\;3}$$
$$= \left(\lambda_0' - \lambda_1'\right) \left[\lambda_1 - {}^{\beta-1}C_1\lambda_1 + {}^{\beta-1}C_2\lambda_2 + \ldots(-1)^{\beta-1}\binom{\beta-1}{\beta-1}\lambda_\beta\right]$$
$$\mu^{0\;0\;2\;\beta-2}_{0\;1\;2\;\;3} = \mu^{\beta-2\;2\;0\;0}_{0\;\;1\;2\;3}$$
$$= \left(\lambda_0' - 2\lambda_1' + \lambda_2'\right) \left[\lambda_2 - {}^{\beta-2}C_1\lambda_3 + \ldots(-1)^{\beta-2}\binom{\beta-2}{\beta-2}\lambda_\beta\right]$$

… … …

$$\mu^{0\;0\;\beta\;0}_{0\;1\;2\;3} = \left[\lambda_0' - {}^{\beta}C_1\lambda_1' + {}^{\beta}C_2\lambda_2' + \ldots(-1)^{\beta}\binom{\beta}{\beta}\lambda_\beta'\right]\lambda_\beta + \lambda_\beta' \times \lambda_\beta$$
$$\mu^{0\;1\;0\;\beta-1}_{0\;1\;2\;\;3} = \mu^{\beta-1\;0\;1\;0}_{0\;\;1\;2\;3} = \lambda_1' \left[\lambda_1 - {}^{\beta-1}C_1\lambda_2 + \ldots(-1)^{\beta-1}\binom{\beta-1}{\beta-1}\lambda_\beta\right]$$
$$\mu^{\beta-2\;1\;1\;0}_{0\;\;1\;2\;3} = \mu^{0\;1\;1\;\beta-2}_{0\;1\;2\;\;3} = \left(\lambda_1' - \lambda_2'\right) \left[\lambda_2 - {}^{\beta-2}C_1\lambda_3 + \ldots(-1)^{\beta-2}\binom{\beta-2}{\beta-2}\lambda_\beta\right]$$
$$\mu^{0\;\beta-1\;1\;0}_{0\;\;1\;\;2\;3} = \mu^{0\;\beta-1\;1\;0}_{0\;\;1\;\;2\;3} = \left(\lambda_{\beta-1}' - \lambda_\beta'\right)\times\lambda_\beta + \left[\lambda_{\beta-2}' - 2\lambda_{\beta-1}' + \lambda_\beta'\right]\times\lambda_\beta$$
$$\mu^{0\;\beta\;0\;0}_{0\;1\;2\;3} = \mu^{0\;0\;\beta\;0}_{0\;1\;2\;3}$$
$$= \lambda'_\beta \times \lambda_\beta + \left(\lambda_{\beta-3}' - \binom{\beta}{1}\lambda'_1 + \binom{\beta}{2}\lambda_2' + \ldots(-1)^{\beta}\binom{\beta}{\beta}\lambda'_\beta\right)\lambda_\beta$$

Thus A is a four symbol partially balanced arrays of strength (2m+1) for all positive integral values of m. The frequency of all other t - plets combinations is zero.
Hence the theorem.
The result of Gupta *et al.* (1995) becomes a particular case of this theorem.

4. Illustrative Examples

Example 4.1

Consider the two tactical configurations (1-3-4-10) and (1-3-3-4), then we have a NBTD with parameters as V=10, B=120, Q_1=12, Q_2=36, R=84, K=7, Λ=48 , and applying the construction method given in Section 3 of this paper, we get a balanced array (v=10, b=240, s=4, t=3) with index set $\Lambda_{4,3}$.

$$\mu^{0030}_{0123} = \mu^{0300}_{0123} = \lambda_3 ' \times \lambda_3 = 1(Combination01)$$
$$\mu^{0120}_{0123} = \mu^{0210}_{0123} = (\lambda_2 ' - \lambda_3 ') \times \lambda_3 = 1(Combinations03)$$
$$\mu^{0210}_{0123} = \mu^{0120}_{0123} = (\lambda_1 ' - 2\lambda_2 ' + \lambda_3 ') \times \lambda_3 = 0(Combinations03)$$
$$\mu^{0300}_{0123} = \mu^{0030}_{0123} = (\lambda_0 ' - 3\lambda_1 ' + 3\lambda_2 ' - \lambda_3 ') \times \lambda_3 = 0(Combination01)$$
$$\mu^{1020}_{0123} = \mu^{0201}_{0123} = \lambda_2 '(\lambda_2 - \lambda_3) = 6(Combinations03)$$
$$\mu^{1110}_{0123} = \mu^{0111}_{0123} = (\lambda_1 ' - \lambda_2 ')(\lambda_2 - \lambda_3) = 3(Combinations06)$$
$$\mu^{1200}_{0123} = \mu^{0021}_{0123} = (\lambda_0 ' - 2\lambda_1 ' + \lambda_2 ')(\lambda_2 - \lambda_3) = 0(Combination01)$$
$$\mu^{2010}_{0123} = \mu^{0102}_{0123} = \lambda_1 '(\lambda_1 - 2\lambda_2 + \lambda_3) = 15(Combinations03)$$
$$\mu^{2100}_{0123} = \mu^{0012}_{0123} = (\lambda_0 ' - \lambda_1 ')(\lambda_1 - 2\lambda_2 + \lambda_3) = 5(Combinations03)$$
$$\mu^{3000}_{0123} = \mu^{0003}_{0123} = \lambda_0 '(\lambda_0 - 3\lambda_1 + 3\lambda_2 - \lambda_3) = 20(Combination01)$$

2	2	2	0	1	0	0	0	0	0			1	1	1	3	2	3	3	3	3	3
1	2	2	0	2	0	0	0	0	0			2	1	1	3	1	3	3	3	3	3
2	1	2	0	2	0	0	0	0	0			1	2	1	3	1	3	3	3	3	3
2	2	1	0	2	0	0	0	0	0			1	1	2	3	1	3	3	3	3	3
0	2	2	2	0	1	0	0	0	0			3	1	1	1	3	2	3	3	3	3
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2	0	2	0	0	0	0	2	1	0			1	3	1	3	3	3	3	1	2	3
1	0	2	0	0	0	0	2	2	0			2	3	1	3	3	3	3	1	1	3

2	0	1	0	0	0	0	2	2	0			1	3	2	3	3	3	3	1	1	3
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2	2	0	1	0	0	0	0	2	0			1	1	3	2	3	3	3	3	1	3
2	0	0	2	2	0	0	1	0	0			1	3	3	1	1	3	3	2	3	3
1	0	0	2	2	0	0	2	0	0			2	3	3	1	1	3	3	1	3	3
2	0	0	1	2	0	0	2	0	0			1	3	3	2	1	3	3	1	3	3
2	0	0	2	1	0	0	2	0	0			1	3	3	1	2	3	3	1	3	3
0	2	0	0	2	2	0	0	1	0			3	1	3	3	1	1	3	3	2	3
0	1	0	0	2	2	0	0	2	0			3	2	3	3	1	1	3	3	1	3
0	2	0	0	1	2	0	0	2	0			3	1	3	3	2	1	3	3	1	3
0	2	0	0	2	1	0	0	2	0			3	1	3	3	1	2	3	3	1	3
2	0	2	0	0	2	1	0	0	0			1	3	1	3	3	1	2	3	3	3
1	0	2	0	0	2	2	0	0	0			2	3	1	3	3	1	1	3	3	3
2	0	1	0	0	2	2	0	0	0			1	3	2	3	3	1	1	3	3	3
2	0	2	0	0	1	2	0	0	0			1	3	1	3	3	2	1	3	3	3
0	2	0	2	0	0	2	1	0	0			3	1	3	1	3	3	1	2	3	3
0	1	0	2	0	0	2	2	0	0			3	2	3	1	3	3	1	1	3	3
0	2	0	1	0	0	2	2	0	0			3	1	3	2	3	3	1	1	3	3
0	2	0	2	0	0	1	2	0	0			3	1	3	1	3	3	2	1	3	3
0	0	2	0	2	0	0	2	1	0			3	3	1	3	1	3	3	1	2	3
0	0	1	0	2	0	0	2	2	0			3	3	2	3	1	3	3	1	1	3
0	0	2	0	1	0	0	2	2	0			3	3	1	3	2	3	3	1	1	3
0	0	2	0	2	0	0	1	2	0			3	3	1	3	1	3	3	2	1	3
2	0	0	2	0	2	0	0	1	0			1	3	3	1	3	1	3	3	2	3
1	0	0	2	0	2	0	0	2	0			2	3	3	1	3	1	3	3	1	3
2	0	0	1	0	2	0	0	2	0			1	3	3	2	3	1	3	3	1	3
2	0	0	2	0	1	0	0	2	0			1	3	3	1	3	2	3	3	1	3
2	2	0	0	2	0	1	0	0	0			1	1	3	3	1	3	2	3	3	3
1	2	0	0	2	0	2	0	0	0			2	1	3	3	1	3	1	3	3	3
2	1	0	0	2	0	2	0	0	0			1	2	3	3	1	3	1	3	3	3
2	2	0	0	1	0	2	0	0	0			1	1	3	3	2	3	1	3	3	3
0	2	2	0	0	2	0	1	0	0			3	1	1	3	3	1	3	2	3	3
0	1	2	0	0	2	0	1	0	0			3	2	1	3	3	1	3	2	3	3
0	2	1	0	0	2	0	1	0	0			3	1	2	3	3	1	3	2	3	3
0	2	2	0	0	1	0	2	0	0			3	1	1	3	3	2	3	1	3	3
0	0	2	2	0	0	2	0	1	0			3	3	1	1	3	3	1	3	2	3
0	0	1	2	0	0	2	0	2	0			3	3	2	1	3	3	1	3	1	3
0	0	2	1	0	0	2	0	2	0			3	3	1	2	3	3	1	3	1	3
0	0	2	2	0	0	1	0	2	0			3	3	1	1	3	3	2	3	1	3
2	2	0	2	0	0	0	0	0	1			1	1	3	1	3	3	3	3	3	2
1	2	0	2	0	0	0	0	0	2			2	1	3	1	3	3	3	3	3	1
2	1	0	2	0	0	0	0	0	2			1	2	3	1	3	3	3	3	3	1
2	2	0	1	0	0	0	0	0	2			1	1	3	2	3	3	3	3	3	1
0	2	2	0	2	0	0	0	0	1			3	1	1	3	1	3	3	3	3	2

0	1	2	0	2	0	0	0	0	2			3	2	1	3	1	3	3	3	3	1
0	2	1	0	2	0	0	0	0	2			3	1	2	3	1	3	3	3	3	1
0	2	2	0	1	0	0	0	0	2			3	1	1	3	2	3	3	3	3	1
0	0	2	2	0	2	0	0	0	1			3	3	1	1	3	1	3	3	3	2
0	0	1	2	0	2	0	0	0	2			3	3	2	1	3	1	3	3	3	1
0	0	2	1	0	2	0	0	0	2			3	3	1	2	3	1	3	3	3	1
0	0	2	2	0	1	0	0	0	2			3	3	1	1	3	2	3	3	3	1
0	0	0	2	2	0	2	0	0	1			3	3	3	1	1	3	1	3	3	2
0	0	0	1	2	0	2	0	0	2			3	3	3	2	1	3	1	3	3	1
0	0	0	2	1	0	2	0	0	2			3	3	3	1	2	3	1	3	3	1
0	0	0	2	2	0	1	0	0	2			3	3	3	1	1	3	2	3	3	1
0	0	0	0	2	2	0	2	0	1			3	3	3	3	1	1	3	1	3	2
0	0	0	0	1	2	0	2	0	2			3	3	3	3	2	1	3	1	3	1
0	0	0	0	2	1	0	2	0	2			3	3	3	3	1	2	3	1	3	1
0	0	0	0	2	2	0	1	0	2			3	3	3	3	1	1	3	2	3	1
2	0	0	0	0	2	2	0	0	1			1	3	3	3	3	1	1	3	3	2
1	0	0	0	0	2	2	0	0	2			2	3	3	3	3	1	1	3	3	1
2	0	0	0	0	1	2	0	0	2			1	3	3	3	3	2	1	3	3	1
2	0	0	0	0	2	1	0	0	2			1	3	3	3	3	1	2	3	3	1
0	2	0	0	0	0	2	2	0	1			3	1	3	3	3	3	1	1	3	2
0	1	0	0	0	0	2	2	0	2			3	2	3	3	3	3	1	1	3	1
0	2	0	0	0	0	1	2	0	2			3	1	3	3	3	3	2	1	3	1
0	2	0	0	0	0	2	1	0	2			3	1	3	3	3	3	1	2	3	1
2	0	2	0	0	0	0	2	0	1			1	3	1	3	3	3	3	1	3	2
1	0	2	0	0	0	0	2	0	2			2	3	1	3	3	3	3	1	3	1
2	0	1	0	0	0	0	2	0	2			1	3	2	3	3	3	3	1	3	1
2	0	2	0	0	0	0	1	0	2			1	3	1	3	3	3	3	2	3	1
2	0	0	0	2	0	0	0	2	1			1	3	3	3	1	3	3	3	1	2
1	0	0	0	2	0	0	0	2	2			2	3	3	3	1	3	3	3	1	1
2	0	0	0	1	0	0	0	2	2			1	3	3	3	2	3	3	3	1	1
2	0	0	0	2	0	0	0	1	2			1	3	3	3	1	3	3	3	2	1
0	2	0	0	0	2	0	0	2	1			3	1	3	3	3	1	3	3	1	2
0	1	0	0	0	2	0	0	2	2			3	2	3	3	3	1	3	3	1	1
0	2	0	0	0	1	0	0	2	2			3	1	3	3	3	2	3	3	1	1
0	2	0	0	0	2	0	0	1	2			3	1	3	3	3	1	3	3	2	1
0	0	2	0	0	0	2	0	2	1			3	3	1	3	3	3	1	3	1	2
0	0	1	0	0	0	2	0	2	2			3	3	2	3	3	3	1	3	1	1
0	0	2	0	0	0	1	0	2	2			3	3	1	3	3	3	2	3	1	1
0	0	2	0	0	0	2	0	1	2			3	3	1	3	3	3	1	3	2	1
0	0	0	2	0	0	0	2	2	1			3	3	3	1	3	3	3	1	1	2
0	0	0	1	0	0	0	2	2	2			3	3	3	2	3	3	3	1	1	1
0	0	0	2	0	0	0	1	2	2			3	3	3	1	3	3	3	2	1	1
0	0	0	2	0	0	0	2	1	2			3	3	3	1	3	3	3	1	2	1

Example 4.2
Case I

Rao and Rao (2001) have introduced new design using orthogonal array (8,4,2,3) of strength 3 for conducting intercropping experiments when the intercrops are sub-divided into classes (groups) based on agronomic, cultural, plant protection, economic considerations besides the main crop. In the similar fashion, let us consider NBTD with parameters V=3 , B=6, Q_1=2, Q_2=2, R=6,K=3, Λ=4, so that **N′** of Example 4.1 can be performed. Taking the image of **N′** as **M′** using **zi+zi*** ≡ 3 (mod 4) for all i=1,2,…,v treatments. The blocks are given below:

$$\mathbf{A'}=\left(\begin{array}{c|c} 2\,1\,0\,0\,2\,1 & 1\,2\,3\,3\,1\,2 \\ 1\,2\,2\,1\,0\,0 & 2\,1\,1\,2\,3\,3 \\ 0\,0\,1\,2\,1\,2 & 3\,3\,2\,1\,2\,1 \end{array}\right)$$

Two OA and new designs that can be obtained through the PB arrays have been given below:

1. The **N'** and its images **M'** are PB arrays of strength (2m+1) with four symbols with index set $\Lambda_{4,3}$ constructed by author in the present paper. In the similar fashion, $\Lambda_{4,5}$ can be constructed having a large number of blocks. Hence, we have constructed PB arrays in four symbols of strength 3.

2. Using image method zi+zi* ≡ 3(mod 4) on the constructed PB arrays can be used for conducting intercropping experiments when intercrops are subdivided into various groups based on agronomic practices including main crop assuming that some of the interaction of intercrops are negligible. We construct design for experiments where each plot consists of main crop p and q intercrops such that each of these intercrops is selected from a group of r intercrops following Rao and Rao (2001).

Let us consider an intercropping experiment using a main crop p and 12 intercrops where the intercrops are divided into three groups S_1, S_2 and S_3 with 4 in each group viz., S_1=[1,2,3,4], S_2=[5,6,7,8] and S_3=[9,10,11,12]. Let us designate the symbols 0,1,2,3 of first row of PB array with intercrops 1,2,3,4 of S_1, second row with intercrops 5,6,7,8 of S_2 and third row with intercrops 9,10,11,12 of S_3. Taking the column of the array as the plots of the intercrop experiment in addition to the main crop ‘p’ in each plot, the resulting intercropping experiment will have the following 12 plots:

(p,3,6,9), (p,2,7,9), (p,1,7,10), (p,1,6,11), (p,3,5,10), (p,2,5,11)
(p,2,7,12), (p,3,6,12), (p,4,6,11), (p,4,7,10), (p,2,8,11), (p,3,8,10)

It is to be mentioned that this method provides intercropping design with one main crop and 12 intercrops divided into three groups of four intercrops each assuming that sum of the interaction of the intercrops are negligible.

Case II

Based on the parameters of the NBTD i.e. V=3, B=6, Q_1=2, Q_2=2, R=6, K=3, Λ=4 . Let us consider an intercropping experiment using a main crop p and 9 intercrops where the intercrops are divided into three groups S_1, S_2 and S_3 with 3 in each group viz., S_1=[1,2,3,], S_2=[4,5,6] and S_3=[7,8,9]. Let us designate the symbols 0, 1, 2 of first row of PB array with intercrops 1, 2, 3 of S_1, second row with intercrops 4, 5, 6 of S_2 and third row with intercrops 7,8,9 of S_3. Taking the column of the array as the plots of the intercrop experiment in addition to the main crop 'p' in each plot, the resulting intercropping experiment will have the following plots: (p,3,5,7), (p,2,6,7), (p,1,6,8), (p,1,5,9), (p,2,4,9), (p,3,4,8). This intercropping design consists of fewer constraints and intercrops rather than that of Rao and Rao (2001).

Case III

Let us consider NBTD with parameters i.e. V=3, B=6, Q_1= 2, Q_2= 2,R=6, K=3, Λ=4, and addition of one column of **0**'s,**1**'s and **2**'s, of order 3x1, it becomes an orthogonal array (9,3,3,2) of index unity. This can be used in fractional factorial and for intercropping experiment in addition to 3 plots into 6 plots considered in Case II. This also consists of fewer constraints than Rao and Rao (2001).

Case IV

Based on the parameters of the NBTD i.e. V=4, B=12, Q_1=3, Q_2=6, R=15, K=5, Λ=16.This parameter of NBTD can be considered for intercropping design taking 12 intercrops into four groups S_1, S_2, S_3 and S_4 with 3 in each group. On the pattern of Case I and II, the resulting plots can be developed.

In addition to the above parameters of NBTD (Case IV) and two columns of **1**'s of order 4x1 and one identity matrix of the order 4x4, we have an orthogonal array (18, 4, 3, 2) of index 2. This array can be used in fractional factorial design. Sharma (2005) has recently considered an intercropping experiment using a main crop 'p' and 9 intercrops on the basis of PB arrays. In the context of an actual example of intercropping experiment, Pandey *et al.,* (2003) have studied the effect of maize (*Zea mays* L.) based intercropping systems on maize yield as main crop and 6 intercrops.

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