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TIME TO RECRUITMENT IN AN ORGANIZATION THROUGH THREE PARAMETER GENERALIZED EXPONENTIAL MODEL

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Abstract

Recruiting employees is an important task for management team in all sectors. The expected time needs to be predicted to make decisions on recruitment. In this paper, statistical model developed to obtain the expected time using three parameters generalized exponential models for reaching threshold level. In the time of recruitment, the assumptions were assumed that independent and identically distributed (i.i.d) random variables for time between decision epochs.Existing numbers at each epoch's time and threshold level is been calculated. The simulation results were studied to illustrate the proposed model.

Key Words: Three parameter generalized exponential distribution, Expected time, Recruitment, Threshold.

1. Introduction

The three-parameter generalized exponential (GE) distribution wasintroduced by Gupta and Kundu (1999). Recently the two-parameter generalized exponential (GE) distribution has been proposed bythe authors. It has been studied extensively by Gupta and Kundu (1999, 2001, 2001a, 2005), The GE has aunimodal and right skewed density function. Model is obtained for the expected time of breakdown point to reach the threshold level through,three parameter generalized exponential distributions.One can see for more detail in Esary et al., (1973),Pandiyan et al., (2010), Pandiyan et al., (2010) about the expected time to cross the threshold level of the organization.

The threshold level is represented by a random variable following a three parameter generalized exponential distributions. At every epoch a random number of persons quit the organization. The organization is exposed to a break down situation when the number of exits of personnel exceeds a "threshold level". The organization takes decisions on revising policies at random times, where the inter-decision times, which are called epochs, are i.i.d random variable.

2. Notations

X_1 : a continuous random variable denoting the amount of damage/depletion caused to the system due to the exit of persons on the ith occasion of policy announcement, $i = 1,2,3,\ldots,k$ and $X_i's$ are i.i.d and $X_i = X$ for all *i*.

Y : a continuous random variable denoting the threshold level having three parameter generalized Exponential distribution.

$g(.)$: The probability density functions (p.d.f) of X_i.

$g_k(.)$: The k- fold convolution of $g(.)$i.e., p.d.f. of $\sum_{i=1}^{k} X_i$

$g*(.)$: Laplace transform of $g(.)$; $g_k^*(.)$: Laplace transform of$g_k(.)$

$h(.)$: The probability density functions of random threshold level which has three parameter generalized Exponential distribution and $H(.)$is the corresponding Probability generating function.

U : a continuous random variable denoting the inter-arrival times between decision epochs.

$f(.)$: p.d.f. of random variable U with corresponding Probability generating function.

$V_k(t)$: $F_k(t) - F_{k+1}(t)$

$F_k(t)$: Probability that there are exactly 'k' policies decisions in (0, t]

$S(.)$: The survivor function i.e. $P[T > t]$;$1 - S(t) = L(t)$

3. Model Description

The three-parameter generalized Exponential distribution has the following cumulative distribution function (CDF) is

$$F(x; \alpha, \lambda, \theta) = \left[1 - e^{-\lambda(x-\theta)}\right]^{\alpha}; \qquad x > \theta, \quad \alpha, \lambda > 0 \tag{1}$$

and the corresponding probability density function (PDF) is

$$f(x; \alpha, \lambda, \theta) = \alpha\lambda \left(1 - e^{-\lambda(x-\theta)}\right)^{\alpha-1} e^{-\lambda(x-\theta)}; \qquad x > \theta, \quad \alpha, \lambda > \tag{2}$$

The corresponding survival function is

$$\bar{H}(x) = 1 - \left[1 - e^{-\lambda(x-\theta)}\right]$$
$$= e^{-\lambda(x-\theta)} \tag{3}$$

We consider a system that is subject to shocks, where each shock reduces the effectiveness of the system and makes it more expensive to run. Assume that shocks occur randomly in time in accordance with a three parameter generalized Exponential distribution. Taking the shape parameter as $\alpha = 1$.

$$P(X_i < Y) = \int_0^{\infty} g_{(x)}^{*} \bar{H}(x)\, dx$$
$$= \left[g^* \lambda(1-\theta)\right]^k \tag{4}$$

The survival function which gives the probability that the cumulative threshold will fail only after time t.

$S(t) = P(T > t) =$ Probability that the total damage survives beyond *t*

$$= \sum_{k=0}^{\infty} P\,\{\text{there are exactly k contacts in } (0, t]$$
$$* P\,(\text{the total cumulative threshold } (0, t]\}$$

It is also known from renewal process that

$$P(\text{exactly k policy decesions in } (0, t]) = F_k(t) - F_{K+1}(t) \text{ with } F_0(t) = 1$$

$$P(T > t) = \sum_{k=0}^{\infty} V_k(t) P(X_i < Y) \tag{5}$$

Now, the life time is given by

$P(T < t) = L(t) =$ the distribution function of life time (T)

$L(t) = 1 - S(t)$

Taking Laplace transformation L(T), We get

$$= 1 - \{\sum_{k=0}^{\infty}[F_k(t) - F_{k+1}(t)][g^*(\lambda - \lambda\theta)]^k\} \quad (6)$$

Let the random variable U denoting inter arrival time which follows exponential with parameter. Now $f^*(s) = \left(\frac{c}{c+s}\right)$, substituting in the below equation we get

$$l^*(s) = \frac{[1 - g^*(\lambda - \lambda\theta)\,]\ f^*(s)}{[1 - g^*(\lambda - \lambda\theta)\ \ f^*(s)]}$$

$$= \frac{[1 - g^*(\lambda - \lambda\theta)]\frac{c}{c+s}}{\left[1 - g^*(\lambda - \lambda\theta)\frac{c}{c+s}\right]} = \frac{c[1 - g^*(\lambda - \lambda\theta)]}{[c + s - g^*(\lambda - \lambda\theta)c]}$$

(7)

$$E(T) = -\frac{d}{ds} l^*(s) \text{given } S = 0$$

$$E(T) = \frac{1}{c[1 - g^*(\lambda - \lambda\theta)]}$$

$g^*(.) \sim \exp(\mu)$, $g^*(\lambda) \sim \exp\left(\frac{\mu}{\mu + \lambda}\right)$, $g^*(\lambda\theta) \sim \exp\left(\frac{\mu}{\mu + \lambda\theta}\right)$

$$E(T) = \frac{1}{c\left[1 - \left(\frac{\mu}{\mu+\lambda} - \frac{\mu}{\mu+\lambda\theta}\right)\right]}$$

On simplification we get

$$= \frac{\mu^2 + \mu\lambda\theta + \mu\lambda + \lambda^2\theta}{c[\mu^2 + 2\mu\lambda + \lambda^2\theta]}$$

(8)

$$E(T^2) = -\frac{d^2}{ds^2} l^*(s) \text{given } S = 0$$

$$= \frac{2}{c^2[1 - g^*(\lambda - \lambda\theta)]^2} \quad (9)$$

$g^*(.) \sim \exp(\mu)$, $g^*(\lambda) \sim \exp\left(\frac{\mu}{\mu + \lambda}\right)$, $g^*(\theta) \sim \exp\left(\frac{\mu}{\mu + \lambda\theta}\right)$

$$= \frac{2}{c^2\left[1-\left(\frac{\mu}{\mu+\lambda}-\frac{\mu}{\mu+\lambda\theta}\right)\right]^2} \tag{10}$$

On simplification we get

$$E(T^2) = \frac{2[\mu^2+\mu\lambda\theta+\mu\lambda+\lambda^2\theta]^2}{c^2[\mu^2+2\mu\lambda+\lambda^2\theta]^2} \tag{11}$$

$$V(T) = E(T^2) - [E(T)]^2$$

$$V(T) = \frac{2[\mu^2+\mu\lambda\theta+\mu\lambda+\lambda^2\theta]^2}{c^2[\mu^2+2\mu\lambda+\lambda^2\theta]^2} - \frac{[\mu^2+\mu\lambda\theta+\mu\lambda+\lambda^2\theta]^2}{c^2[\mu^2+2\mu\lambda+\lambda^2\theta]^2}$$

$$= \frac{[\mu^2+\mu\lambda\theta+\mu\lambda+\lambda^2\theta]^2}{c^2[\mu^2+2\mu\lambda+\lambda^2\theta]^2} \tag{12}$$

4. The Special Case of r^{th} Moment

The r^{th} moment three parameter generalized Exponential distribution about origin is given by

$$\mu'_r = \int_0^\infty x^r\, \alpha\lambda\, (1-e^{-\lambda(x-\theta)})^{\alpha-1}\, e^{-\lambda(x-\theta)}\, dx$$

On simplification we get

$$\mu'_r = \lambda\, e^{\lambda\theta} \int_0^\infty x^{r+1-1} e^{-\lambda x}$$

$$= \frac{e^{\lambda\theta}\Gamma r+1}{\lambda^r} \tag{13}$$

The special cases of these rth moment three parameter generalized Exponential distribution

$$\mu'_1 = \frac{e^{\lambda\theta}}{\lambda}, \quad \mu'_2 = \frac{2\,e^{\lambda\theta}}{\lambda^2}, \quad \mu'_3 = \frac{6\,e^{\lambda\theta}}{\lambda^3}, \quad \mu'_4 = \frac{24\,e^{\lambda\theta}}{\lambda^4}$$

5. Numerical Illustration

On the basis of the expressions derived for the expected time and variance, the behaviour of the same due to the change in different parameters is shown in Figures 1a to 3b.

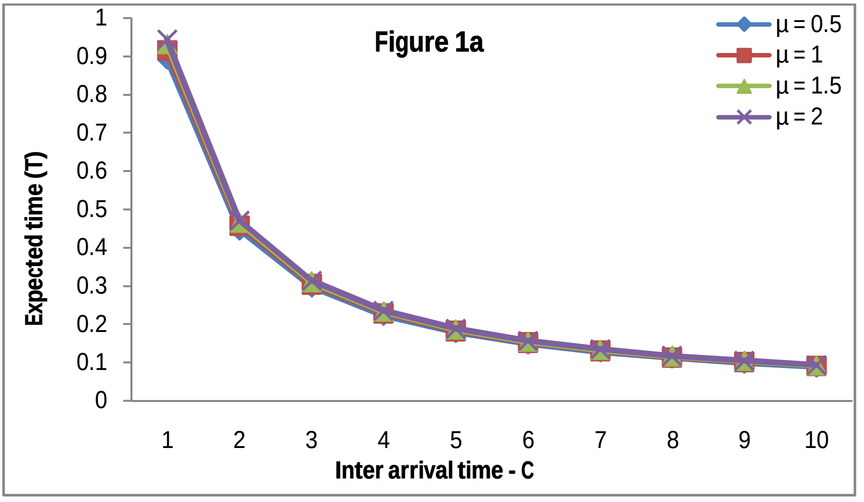


Figure 1a
Expected time (T)
Inter arrival time - C
μ = 0.5
μ = 1
μ = 1.5
μ = 2
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
1 2 3 4 5 6 7 8 9 10

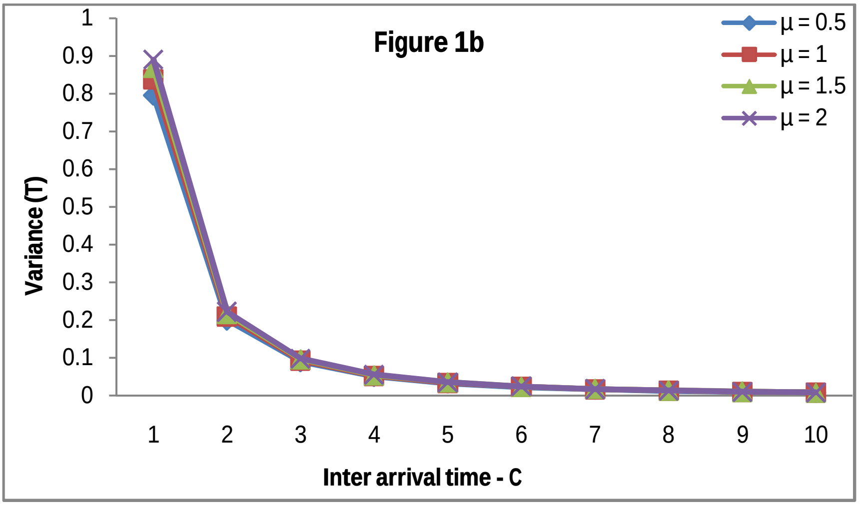


Figure 1b
Variance (T)
Inter arrival time - C
μ = 0.5
μ = 1
μ = 1.5
μ = 2
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
1 2 3 4 5 6 7 8 9 10

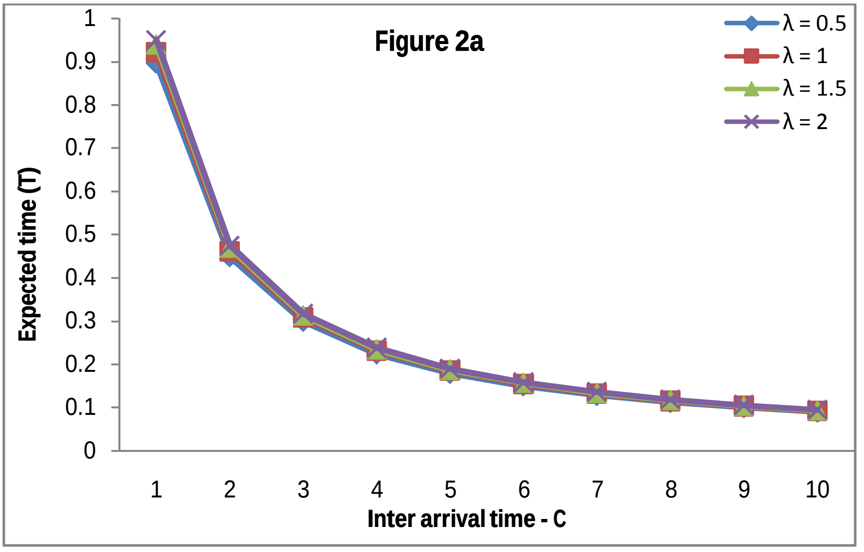


Figure 2a
Expected time (T)
Inter arrival time - C
λ = 0.5
λ = 1
λ = 1.5
λ = 2
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
1
2
3
4
5
6
7
8
9
10

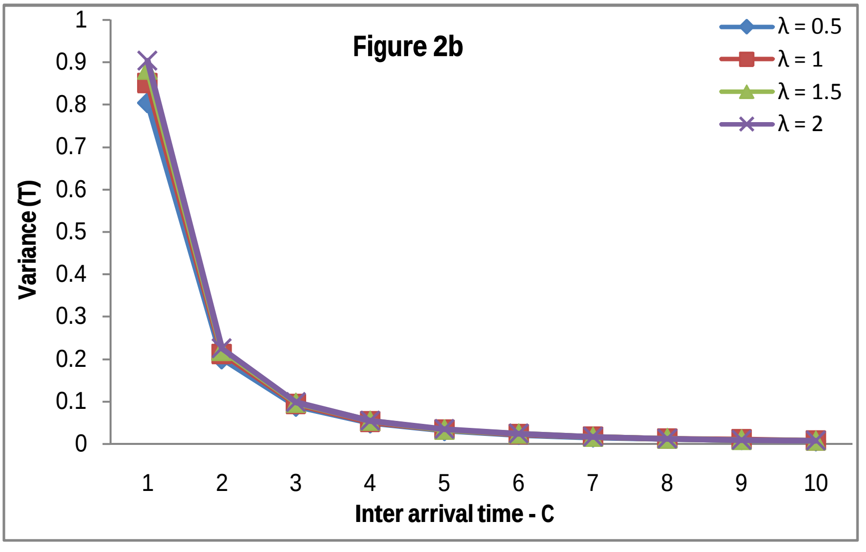
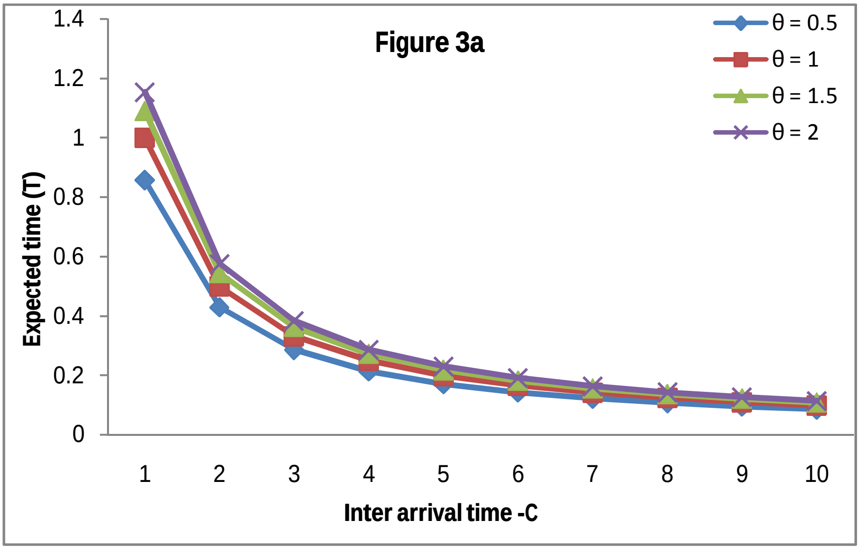
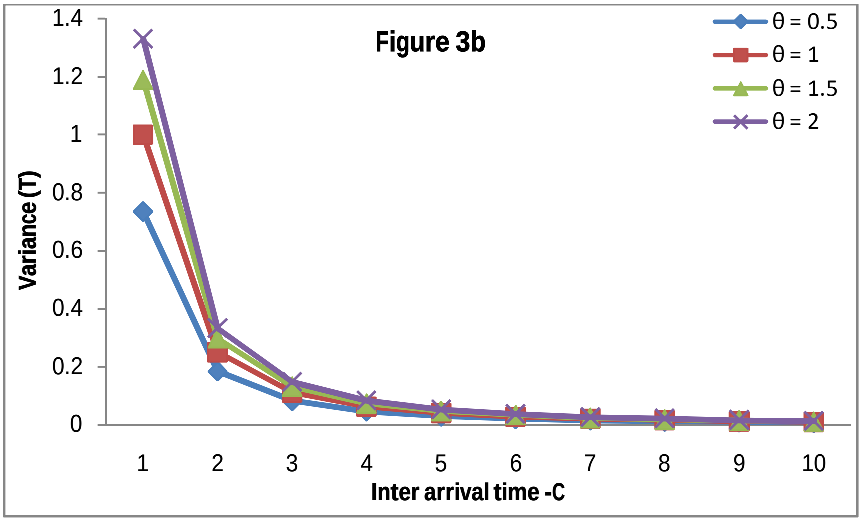


Figure 2b
Variance (T)
Inter arrival time - C
λ = 0.5
λ = 1
λ = 1.5
λ = 2
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
1
2
3
4
5
6
7
8
9
10





6. Conclusion

The mathematical models have been discussed by various authors taking into consideration, many hypothetical assumptions. Such models provide the possible clues relating to the consequences of infections, the time taken for recruitment etc.

When μ is kept fixed, the inter-arrival time $'c'$ which follows exponential distribution, is an increasing parameter. Therefore, the value of the expected time $E(T)$ to cross the threshold of recruitment is decreasing, for all cases of the parameter value $\mu = 0.5, 1, 1.5, 2$. When the value of the parameter μ increases, the expected time is also found decreasing, this is observed in Figure 1a,and 1b. The same case is found in Variance $V(T)$ which is observed in Figure 1a, and1b.

When λ is kept fixed and the inter-arrival time $'c'$ increases, the value of the expected time $E(T)$ to cross the threshold of recruitment is found to be decreasing, in all the cases of the parameter value $\lambda = 0.5, 1, 1.5, 2$. When the value of the parameter λ increases, the expected time is found increasing. This is indicated in Figure 2a and 2b. The same case is observed in the threshold of recruitment of Variance $V(T)$ which is observed in Figure 2a and 2b.

When θ is kept fixed and the inter-arrival time $'c'$ increases, the value of the expected time $E(T)$ to cross the threshold of recruitment is found to be decreasing, in all the cases of the parameter value $\theta = 0.5, 1, 1.5, 2$. When the value of the parameter θ increases, the expected time is found increasing. This is indicated in Figure 3a and 3b. The same case is observed in the threshold of recruitment of Variance $V(T)$ which is observed in Figure 3a and 3b.

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