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A CLASS OF RATIO-CUM-DUAL TO RATIO ESTIMATOR OF POPULATION VARIANCE

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Abstract

In this paper, we have proposed a class of ratio-cum-dual to ratio estimators using known values of some population parameters of the auxiliary variable to estimate the population variance of the study variable. The expressions for the bias and mean square error of the proposed estimators have been derived upto the first order of approximation. A comparison has been made with some well known estimators, such as sample variance (s_y^2), Isaki ratio type (t_R) and the dual to ratio type ($t^{(d)}$) estimators of the population variance and it is shown that the proposed estimator is better than the mentioned estimators under the optimum condition. For illustration, an empirical study has been carried out.

Key words: Class of estimators, Dual to ratio estimator, Bias, Mean square error.

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1. Introduction

The use of auxiliary information has been widely discussed in sampling theory. Auxiliary variables are used in survey sampling to obtain improved sampling designs and to achieve higher precision in the estimates of some population parameters such as the mean or the variance of the study variable. This information may be used at both the stage of designing (leading for instance, to stratification, systematic or probability proportional to size sampling designs) and estimation stage. It is well established that when the auxiliary information is to be used at the estimation stage, the ratio, product and regression methods of estimation are widely used in many situations.

The estimation of the population variance is a burning issue in sampling theory and many efforts have been made to improve the precision of the estimates. In survey sampling literature, a great variety of techniques using the auxiliary information by means of ratio, product and regression methods have been used. Particularly, in the presence of multi-auxiliary variables, a wide variety of estimators have been proposed, following different ideas, and linking together ratio, product or regression estimators, each one exploiting the variables one at a time. In the present paper, we have proposed a dual to ratio and a ratio cum dual to ratio estimators for improved estimation of the population variance. The main aim of this paper is to develop a new ratio estimator and to improve the efficiency of ratio type estimators for the population variance.

2. Notations And Traditional Methods

Let a finite population U consists of N units $U_1, U_2, \ldots\ldots\ldots, U_N$ and a sample of n units is drawn from this population using the simple random sampling without replacement (SRSWOR) technique. Let Y and X be the study and the auxiliary variables, respectively, and assume that these variables are highly positively correlated as we are interested in ratio estimators in this paper.

Isaki [2] proposed the ratio type estimator for the population variance of the study variable as

$$t_R = s_y^2 \left(\frac{S_x^2}{s_x^2} \right), \tag{2.1}$$

where

$$s_y^2 = \frac{1}{n-1}\sum_{i=1}^{n}(y_i - \bar{y})^2, \; s_x^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2, S_x^2 = \frac{1}{N-1}\sum_{i=1}^{N}(X_i - \bar{X})^2$$

$$\bar{X} = \frac{1}{N}\sum_{i=1}^{N} X_i, \quad \bar{Y} = \frac{1}{N}\sum_{i=1}^{N} Y_i, \quad \bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i, \quad \bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i .$$

The mean square error (MSE) of the estimator in (2.1) upto the first order of approximation is given as

$$MSE(t_R) = f\, S_y^4 \left[(\lambda_{40} - 1) + (\lambda_{04} - 1) - 2(\lambda_{22} - 1) \right], \tag{2.2}$$

where $\lambda_{rs} = \dfrac{\mu_{rs}}{\mu_{r0}^{r/2}\mu_{0s}^{s/2}}$, $\mu_{rs} = \dfrac{1}{N-1}\sum_{i=1}^{N}(Y_i - \bar{Y})^r (X_i - \bar{X})^s$, and $f = \dfrac{1}{N} - \dfrac{1}{n}$.

3. Proposed Estimator

From the dual estimators for the population mean in literature, a dual to ratio type estimator for the population variance of the study variable can be given by

$$t^{(d)} = s_y^2 \left(\frac{s_x^{*2}}{S_x^2} \right), \tag{3.1}$$

where $s_x^{*2} = \dfrac{NS_x^2 - ns_x^2}{N-n} = (1+g)S_x^2 - gs_x^2$, $g = \dfrac{n}{N-n}$.

The MSE of the estimator in (3.1) upto the first order of approximation can be obtained by

$$MSE\left(t^{(d)}\right) = f\, S_y^4 \left[(\lambda_{40} - 1) + g^2(\lambda_{04} - 1) - 2g(\lambda_{22} - 1) \right] \tag{3.2}$$

Using the Sharma and Tailor [4] estimator for the population mean defined by

$$\bar{y}_{ST} = \bar{y}\left[\alpha\left(\frac{\bar{X}}{\bar{x}} \right) + (1-\alpha)\left(\frac{\bar{x}^*}{\bar{X}} \right) \right],$$

we propose the ratio-cum-dual to ratio type estimator for the population variance of the study variable as

$$t_{pr} = s_y^2\left[\alpha\left(\frac{S_x^2}{s_x^2}\right)+(1-\alpha)\left(\frac{s_x^{*2}}{S_x^2}\right)\right], \tag{3.3}$$

where α is a suitably chosen constant to be determined such that MSE of the proposed estimator in (3.3) is minimum. For $\alpha = 1$, the proposed estimator reduces to the ratio estimator in (2.1) and for $\alpha = 0$, it turns to the dual to ratio estimator in (3.1). Thus, the mentioned estimators are the particular cases of the proposed estimator, t_{pr}.

In order to study the large sample properties of the proposed family of estimators, we define $s_y^2 = S_y^2(1+\varepsilon_0)$ and $s_x^2 = S_x^2(1+\varepsilon_1)$ such that $E(\varepsilon_i) = 0$ for i = 0,1 and $E(\varepsilon_0^2) = f(\lambda_{40}-1)$, $E(\varepsilon_1^2) = f(\lambda_{04}-1)$, $E(\varepsilon_0\varepsilon_1) = f(\lambda_{22}-1)$.
Expressing (3.3) in terms of ε_i, $(i = 0, 1)$, we have

$$t_{pr} = S_y^2(1+\varepsilon_0)\left[\alpha(1+\varepsilon_1)^{-1} + (1-\alpha)(1-g\varepsilon_1)\right]. \tag{3.4}$$

Expanding terms on right hand side of (3.4), we get

$$t_{pr} = S_y^2(1+\varepsilon_0)\left[\alpha\left(1-\varepsilon_1+\varepsilon_1^2+\ldots\right)+(1-\alpha)(1-g\varepsilon_1)\right]$$

$$= S_y^2\left[\alpha\left(1+\varepsilon_0-\varepsilon_1+\varepsilon_1^2-\varepsilon_0\varepsilon_1+\ldots\right)+(1-\alpha)\left(1+\varepsilon_0-g\varepsilon_1-g\varepsilon_0\varepsilon_1\right)\right] \tag{3.5}$$

Taking expectation, after subtracting S_y^2 on both sides of (3.5), upto the first order of approximation, we get

$$B(t_{pr}) = f\,S_y^2\left\{\alpha(\lambda_{04}-1)-[g+(1-g)\alpha](\lambda_{22}-1)\right\}$$

$$= f\,S_y^2\left[\alpha(\lambda_{04}-1)-\alpha_1(\lambda_{22}-1)\right], \tag{3.6}$$

where $\alpha_1 = [g+(1-g)\alpha]$.
From (3.5), we have

$$t_{pr} - S_y^2 \cong S_y^2(\varepsilon_0 - \alpha_1\varepsilon_1)$$

$$\left(t_{pr} - S_y^2\right)^2 \cong S_y^4(\varepsilon_0 - \alpha_1\varepsilon_1)^2$$

$$= S_y^4\left(\varepsilon_0^2 + \alpha_1^2\varepsilon_1^2 - 2\alpha_1\varepsilon_0\varepsilon_1\right) \tag{3.7}$$

Taking expectation on both sides of (3.7), we get the MSE of the proposed estimator upto the first order of approximation as follows:

$$MSE(t_{pr}) = f\,S_y^4\left[(\lambda_{40}-1)-\alpha_1^2(\lambda_{04}-1)-2\alpha_1(\lambda_{22}-1)\right]. \tag{3.8}$$

The MSE of the proposed estimator is minimized for the optimum value of α as

$$\alpha = \frac{K-g}{1-g} \text{ or let } \alpha_1 = K = \alpha_1^{opt}\ (say), \tag{3.9}$$

where $K = \dfrac{\lambda_{22} - 1}{\lambda_{04} - 1}$. Substituting this value of α_1^{opt} in (3.8), we get the minimum MSE of the proposed estimator as follows:

$$MSE_{\min}(t_{pr}) = f\, S_y^4 \left[\lambda_{40} - 1 - \frac{(\lambda_{22} - 1)^2}{\lambda_{04} - 1} \right]. \tag{3.10}$$

If K is not known in practice, it is estimated by $\hat{K} = \dfrac{\hat{\lambda}_{22} - 1}{\hat{\lambda}_{04} - 1}$ and putting this we get the minimum MSE of t_{pr}, equal to (3.10).

4. Efficiency Comparisons

It is well known that the variance of the sample variance estimator, $t_0 = s_y^2$, is given by

$$V(t_0) = f\, S_y^4 (\lambda_{40} - 1). \tag{4.1}$$

Then, the proposed estimator, t_{pr}, is better than the estimator, t_0, if

$$MSE(t_0) > MSE_{\min}(t_{pr}),$$

$$\frac{(\lambda_{22} - 1)^2}{\lambda_{04} - 1} > 0,$$

$$\lambda_{04} > 1. \tag{4.2}$$

The proposed estimator is more efficient than the estimator, t_R, under the condition

$$MSE(t_R) > MSE_{\min}(t_{pr}),$$

$$[(\lambda_{04} - 1) - (\lambda_{22} - 1)]^2 > 0,$$

$$[1 - K]^2 > 0. \tag{4.3}$$

Note that (4.3) is always satisfied.

The proposed estimator is better than the estimator, $t^{(d)}$, if

$$MSE(t^{(d)}) > MSE_{\min}(t_{pr}),$$

$$[g(\lambda_{04} - 1) - (\lambda_{22} - 1)]^2 > 0,$$

$$[g - K]^2 > 0. \tag{4.4}$$

Note that (4.4) is always satisfied. Therefore, the proposed estimator, t_{pr}, is more efficient than the estimators, t_R and $t^{(d)}$, under all conditions.

5. Numerical Examples

To analyze the performance of various estimators for the population variance of the study variable, we consider five data sets as follows:

Data 1. Source: Cochran [1], page 152
Y: Number of inhabitants in 1930.
X: Number of inhabitants in 1920.
$N = 196$, $n = 49$, $\lambda_{40} = 8.5362$, $\lambda_{04} = 7.3617$, $\lambda_{22} = 7.8780$, $\rho = 0.9820$

Data 2. Source: Sukhatme and Sukhatme [9], page 185
Y: Wheat acreage in 1937.
X: Wheat acreage in 1936.
$N = 170$, $n = 10$, $\lambda_{40} = 3.1842$, $\lambda_{04} = 2.2030$, $\lambda_{22} = 2.5597$, $\rho = 0.9770$

Data 3. Source: Singh *et al.* [5]
Y: The number of agriculture labourers for1971.
X: The number of agriculture labourers for1961.
$N = 278$, $n = 30$, $\lambda_{40} = 24.8969$, $\lambda_{04} = 37.8898$, $\lambda_{22} = 25.8142$, $\rho = 0.7273$

Data 4. Source: Singh [6]
Y: Amount (in $ 1000) of real estate farm loans in different states during 1997.
X: Amount (in $ 1000) of non real estate farm loans in different states during 1997.
$N = 50$, $n = 08$, $\lambda_{40} = 3.5822$, $\lambda_{04} = 4.5247$, $\lambda_{22} = 2.8411$, $\rho = 0.8038$

Data 5. Source: Kadilar and Cingi [3]
Y: The level of apple production (1 unit = 100 tonnes).
X: The number of apple trees (1 unit = 100 trees).
$N = 106$, $n = 20$, $\lambda_{40} = 80.13$, $\lambda_{04} = 25.71$, $\lambda_{22} = 33.30$, $\rho = 0.8200$

6. Results And Conclusion

The percentage relative efficiency (PRE) of estimators $t_0 = s_y^2$, t_R, $t^{(d)}$, and t_{pr} with respect to s_y^2 have been computed and given in the Table 1.

Table 1: Efficiencies of different estimators with respect to s_y^2

Estimator	PRE Values				
	Data1	Data2	Data3	Data4	Data5
$t_0 = s_y^2$	100.0000	100.0000	100.0000	100.0000	100.0000
t_R	5310.9232	815.6100	214.1600	106.5000	201.6600
$t^{(d)}$	206.0366	109.5400	129.6400	128.5500	120.9100
t_{pr}	7536.3206	1347.9800	331.6500	159.3400	214.3900

From the results of the empirical study and theoretical discussions, it is inferred that the proposed estimator, t_{pr}, for estimating the population variance of the study variable under the optimum condition performs better than the sample variance estimator, s_y^2, traditional ratio type estimator, t_R, and the dual to ratio type estimator, $t^{(d)}$, therefore it should be preferred for the estimation of population variance.

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