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MIXED SAMPLING PLANS FOR MARKOFF MODEL UNDER INSPECTION ERROR

Mujahida Sayyed[1] and J. R. Singh[2]

[1]College of Agriculture, GanjBasoda, (M.P.), India
[2]S. S. in Statistics, Vikram University, Ujjain (M.P.), India
Email: mujahida.sayyed@rediffmail.com

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Abstract

The Markoff model is examined to cost light on its physical interpretation and to facilitate its use. The purpose of this paper is, therefore, to determine and illustrate the effects of inspection error on the OC and ASN functions for independent and dependent mixed acceptance-sampling plans. Where in variable sampling plans, random error terms are considered to be according to Markoff model for coefficient of variation (CV) and attribute sampling plans analysis with regard to the choice of a sampling plan taking inspection error into consideration. A comparison between the independent and dependent mixed plan have been made in respect of OC and ASN functions under inspection error.

Key words: OC, ASN, CV, AR(1), Inspection error, Sampling Plan.

1. Introduction

The choice between acceptance sampling by attributes and by variables has commonly been considered a first step in the application of sampling plans to specific problems in industry. The dichotomy is more apparent than real, however, since other alternative exist in the combination of both attributes and variables results to determine the disposition of the lot. One useful alternative is embodied in the so-called "combined" variables-attributes acceptance sampling plan (mixed sampling plan). The mixed sampling plan were proposed by Dodge (1932) and later dealt with by Bowker and Goode (1952). Savage (1955) developed mixed plans for the case where the process characteristic is exponentially distributed. Kao (1966) used both attribute and variable characteristics for single sampling plans to control item variability.

Schilling (1967) proposed a method for determining the operating characteristics of mixed variable attribute sampling plans, single sided specification and standard deviation known using the normal approximation. Later Adams and Mirkhani (1976) developed mixed plans for the case of unknown standard deviation. Suresh and Devaarul (2000) developed and designed special purpose mixed sampling plans with chain sampling as attribute plan. Suresh and Devaarul (2002,2003) provided an approximate method for reducing the cost of inspection and also find the procedure of multidimensional mixed sampling plan. Recently Radhakrishan and Sampath (2007, 2009) construct and compare the mixed sampling plan with attribute sampling plan. Radhakrishnan and Mallika (2010) **c**onstructed Double Sampling Plan Using Convex

Combination of AOQL and MAAOQ. Sampath et al. (2012) also studied mixed sampling plan.

Mixed plans are of two types, so-called “independent” and “dependent” plans. Independent mixed plans maintain stochastic independence between the probabilities of the variables and attributes constituents of the procedure. Dependent mixed plans are those in which the probabilities of the variables and attributes constituents of the procedure are made dependent.

Statistical process control for auto correlated processes has received a great deal of attention, due in part to the increasing prevalence of autocorrelation in process inspection data. With improvements in measurement and data collection technology, processes can be sampled at higher rates, which often lead to data autocorrelation. The modal used by Markoff was called a first order auto regressive model (AR(1)).However, it is quite common that the data obtained from processes, such as chemical processes are Markoff modal and thus, the standard design tables for sampling plans are invalid.

Apley and Tsung (2002) proposed the autoregressive chart for auto correlated processes. Montgomery and Mastragelo (1991) discuss some methods for auto correlated data and Zou et al. (2008) used the autocorrelated data for variable sampling plan.

The effect of coefficient of variation on statistical processes have been found by many like Alberche et al. (2006), Tian (2005), Verrill (2003) and recently Mahmoudvand et al. (2007) obtained the bounds for the population of variation on some distributions.

In this paper we present a method for evaluating joint probabilities necessary to determine the OC and ASN for independent and dependent mixed plans under inspection error for the case of single specification limits and known CV, assuming a normal distribution. An example is given to demonstrate the use of the plans.

It helps to bring out the fact that some of the age old yardsticks used for the choice of an appropriate sampling plan are not insensitive to the factor of inspection error and these yardsticks do get affected in varying degrees depending on the actual extent of error committed during the performance of the inspection task.

2. Model Description

Consider a Markoff process given by the following model

$$X_t = \mu + \xi_t, \qquad t = 1,2,\ldots n \tag{1}$$

where X_t is the response at time t, μ is a population mean and ξ_t can be expressed as

$$\xi_t = \alpha_1 \xi_{t-1} + \varepsilon_t \tag{2}$$

where

$$\left.\begin{array}{ll}\text{(i)} & \varepsilon_t \sim N\left(0,\sigma_\varepsilon^2\right) \\ \text{(ii)} & Cov\left(\varepsilon_t,\varepsilon_\tau\right)=\begin{cases}\sigma_\varepsilon^2 & t=\tau \\ 0 & t\neq\tau.\end{cases}\end{array}\right\}$$

When the correlation is present in the data, we have for the distribution of the sample mean $\overline{x}$,

$$\left.\begin{array}{l} E(\overline{x})=\mu \\ Var(\overline{x})=\dfrac{\sigma^2}{n}\lambda(\alpha_1,n), \end{array}\right\}$$

where

$$\lambda(\alpha_1,n) = \left[\frac{1+\alpha_1}{1-\alpha_1}-\frac{2\alpha_1}{n}\frac{\left(1-\alpha_1^n\right)}{\left(1-\alpha_1\right)^2}\right],$$

and variance of Markoff's model is

$$\sigma^2=\frac{\sigma_\varepsilon^2}{1-\alpha_1^2}$$

Suppose a random sample of n observations $x_1, x_2,..., x_n$ is taken from a normal distribution with unknown mean and known coefficient of variation (v=$\dfrac{\sigma}{\mu}$). An estimator $\overline{x}'=\dfrac{\sum_{j=1}^{n} x_j}{n}$ has been constructed and chosen so that MSE $\left(\overline{x}'-\mu\right)^2$ is minimum and

$$\text{min.MSE}(\overline{x}')=\frac{\sigma^2}{[(n/T^2)+v^2]},$$

where $T^2=\lambda\left(\alpha_1,n\right)$ and $v=\dfrac{\sigma}{\mu}$.

Let r be the probability that nonconformity is correctly noted by the inspector and let c_f be the average number of false alarms per part. If c is the true average number of nonconformities per part and c_0 is the average number per part observed by the inspector, then

$$c_0 = rc + c_f \tag{3}$$

with both r and c_f estimated. Every effort should be made to eliminate both types of errors, i.e. to get r close to one and c_f close to zero.

The expression for OC function of the independent mixed plan under measurement error can be obtained as:

$$P_a = \Pr ob.(\bar{x}' \le A) + \Pr ob.(\bar{x}' > A) \sum_{j=0}^{C_2} P_{c_0}(j, n_2),$$

$$P_a = P_{a_1} + \Pr ob.(\bar{x}' > A) \sum_{j=0}^{C_2} P_{c_0}(j, n_2), \tag{4}$$

where

$$P_{c_0}(j, n_2) = rc + c_f$$

and

$$P_{a_1} = \Phi\left[\left(\frac{n_1}{T^2} + v^2\right)^{1/2} \left(k_p - k\right)\right], \tag{5}$$

where

$$\Phi(t) = \left(\sqrt{2\pi}\right)^{-1} \int_{-\infty}^{t} \exp\left(-\frac{1}{2}u^2\right) du.$$

The term P_{a_1} gives the probability of acceptance on variable criterion basis on the first sample. Similarly the ASN function under measurement error of independent mixed plan is obtained by the following expression

$$\text{ASN}_{c_0} = n_1 + n_2\left(1 - P_{a_1}\right). \tag{6}$$

The equations (4) and (6) will give OC and ASN functions respectively under measurement error for independent mixed plans for known CV.

The dependent mixed plans have been further generalized by subjecting the first sample to both the variable and the attribute inspection criteria and the mixed sample only to attribute inspection. The formulae for the OC and ASN functions when inspection procedure is assumed to be under measurement error are given as
The OC function (P_a')

$$P_a' = \Pr ob.(\bar{x}' \le A) + \sum_{i=0}^{C_1} \sum_{j=0}^{C_2 - i} P_{n_1}(i, \bar{x}' > A) P(j; n_2) \tag{7}$$

$$= P_{a_1} + \sum_{i=0}^{C_1} \sum_{j=0}^{C_2 - i} P_{n_1}\left[i, \bar{Z} > \left(K_p - k\right)\right] P_{c_0}(j; n_2), \tag{8}$$

where $P_{n_1}\left\{i, \bar{Z} > \left(K_p - k\right)\right\}$ is the joint probability of defectives in the sample of size n_1 and $\bar{Z}$ exceeding (K_p-k) and

$P_{c_0}(j; n_2)$ = Probability of j defectives in a sample of size n_2
= rc + c_f

The ASN Function

$$ASN'_{c_0} = n_1 + n_2 \left\{ \sum_{i=0}^{C_1} P_{n_1}\left(i, \overline{Z} > \left(K_p - k\right)\right) \right\} . \tag{9}$$

Therefore, for the same probability of acceptance, the independent plan requires a larger second sample size. But even if the second sample of size n_2, of the dependent plan is kept the same as that of the independent plan, the ASN of the dependent plan, will be lower. Thus, the dependent plan is superior to the independent plan in terms of the same protection with a smaller sample size.

3. Numerical Illustration and Discussion of Results

We shall describe the procedure for evaluating the characteristics of the independent and dependent mixed plans under inspection error for known CV as

N	=	1000,	k	=	2.0
n_1	=	5,	n_2	=	20
c_1	=	0,	c_2	=	0.

Following equation (4), (8) and for different incoming lot quality, the P_a and P'_a for independent and dependent mixed plans have been calculated. The ASN_{c_0} and ASN'_{c_0} for independent and dependent mixed plans have been calculated from the equations (6) and (9).

The error rates r and c_f must of course be estimated. Initially, inspection error rates r and c_f will be considered defective, while this is reasonable in many automated inspection tasks, it may also apply to human inspection tasks. It is evident from figures 1(a),1(b) and 1(c),the OC function shows that the inspection error causes decrease in the probability of acceptance for independent mixed plans for known CV and from figures 2(a), 2(b) and 2(c) similar pattern have been for dependent mixed plan under inspection error.

ASN curves study shows that the inspection error causes an increase in the ASN for lots of good quality and decreases for lots of bad quality for independent mixed plan. Inspection error causes serious effect on ASN curves.

Lastly it will be worthwhile to mention that a true quality control practitioner has to ascertain the validity of various assumptions especially about the product distribution, the requirement of specifications keeping utility in view etc. Before he really proceeds to any of the acceptance sampling inspection plans and takes precautionary measure to correct the inspection error.

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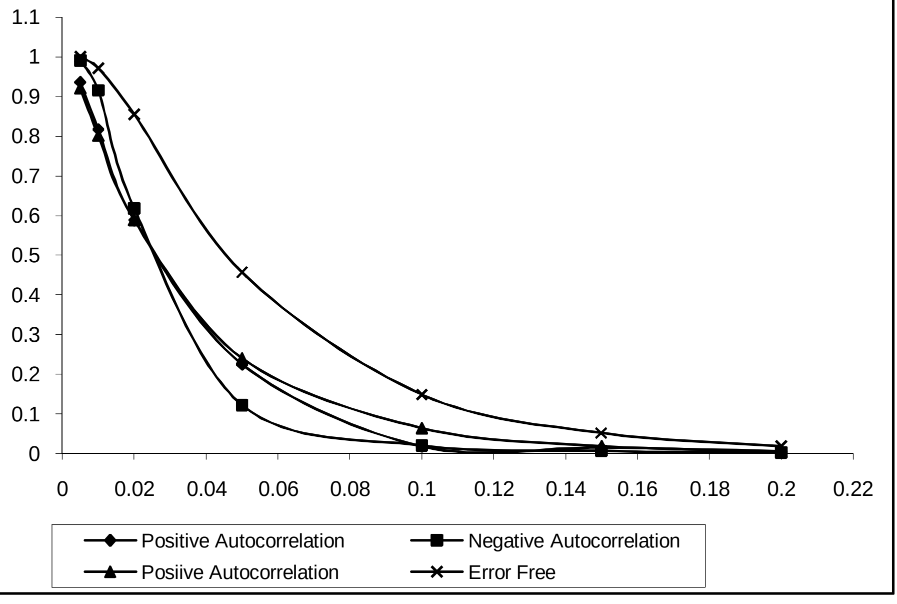
Fig. 1(a) OC Curve for Independent Combined Plans for v=2



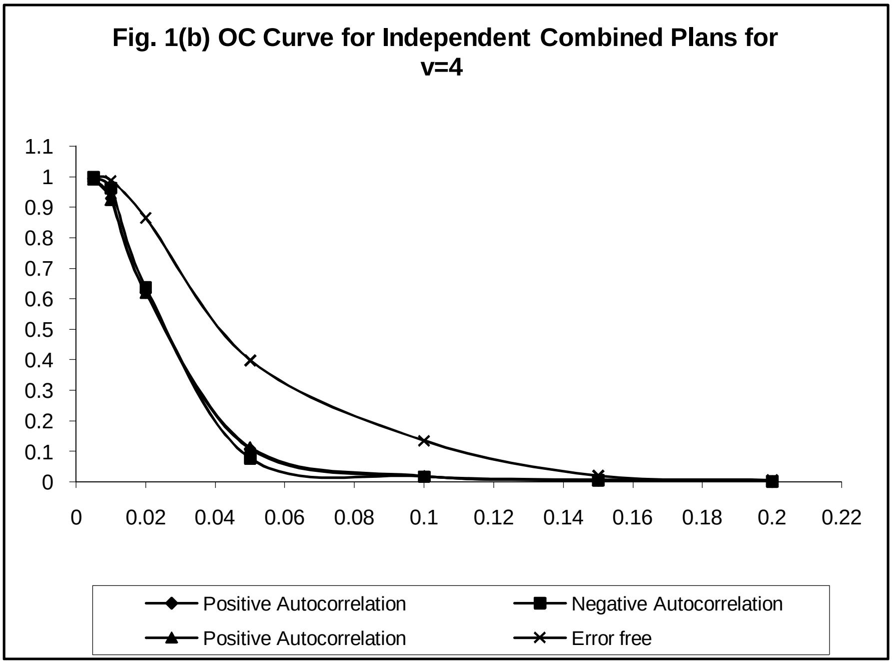


Fig. 1(b) OC Curve for Independent Combined Plans for v=4

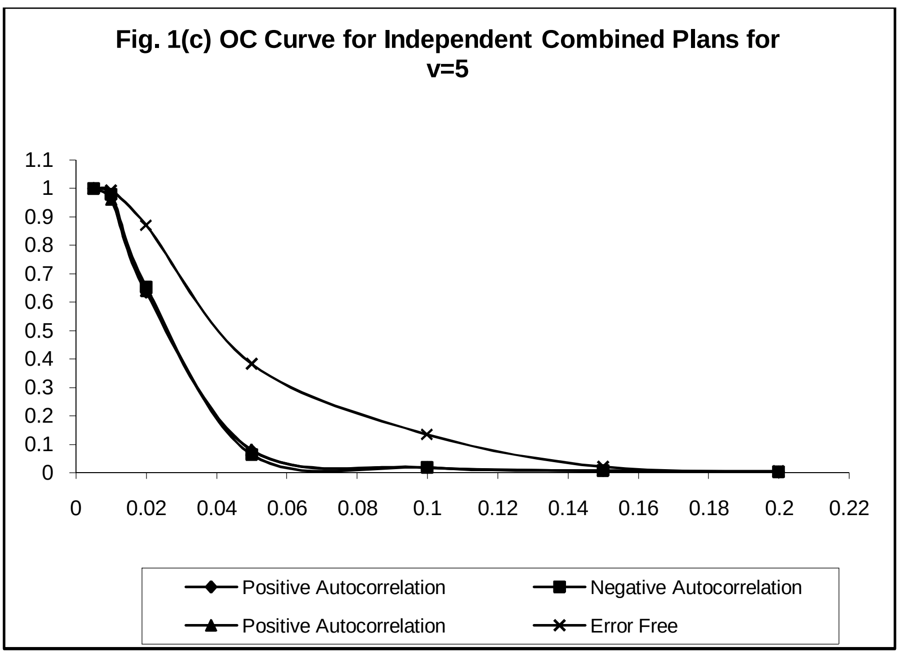


Fig. 1(c) OC Curve for Independent Combined Plans for v=5

Fig. 2(a) OC Curve for Dependent Combined Plans for v=2

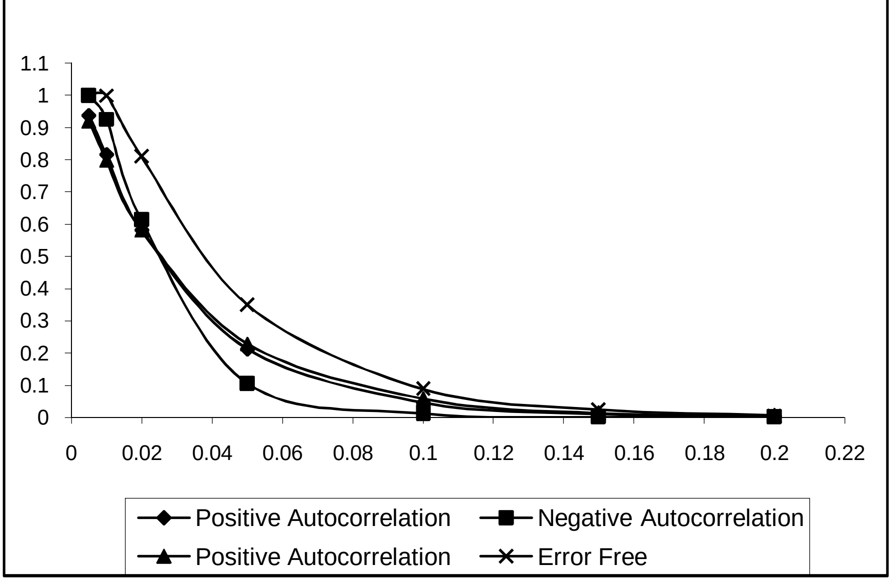


Fig. 2(b) OC Curve for Dependent Combined Plans for v=4

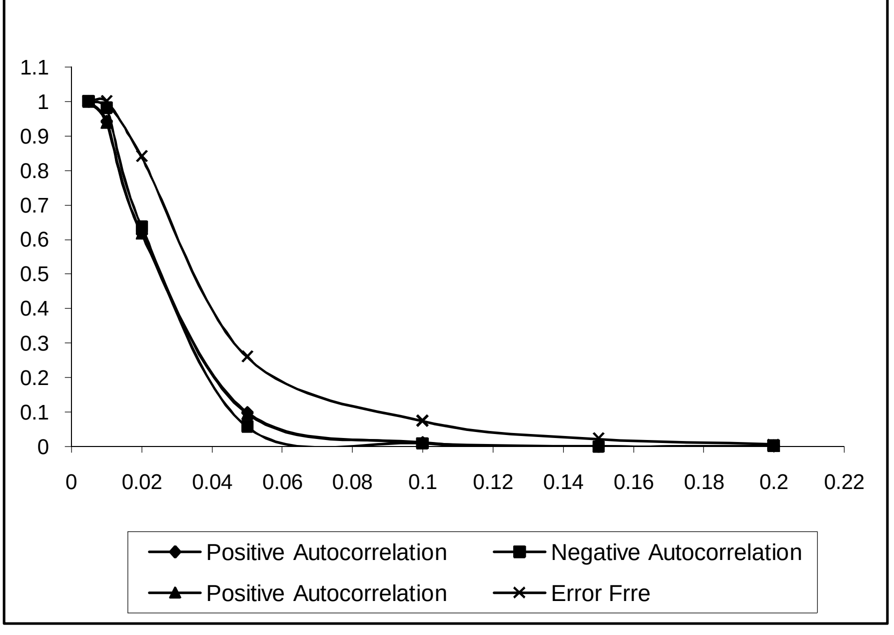


Fig. 2(c) OC Curve for Dependent Combined Plans for v=5

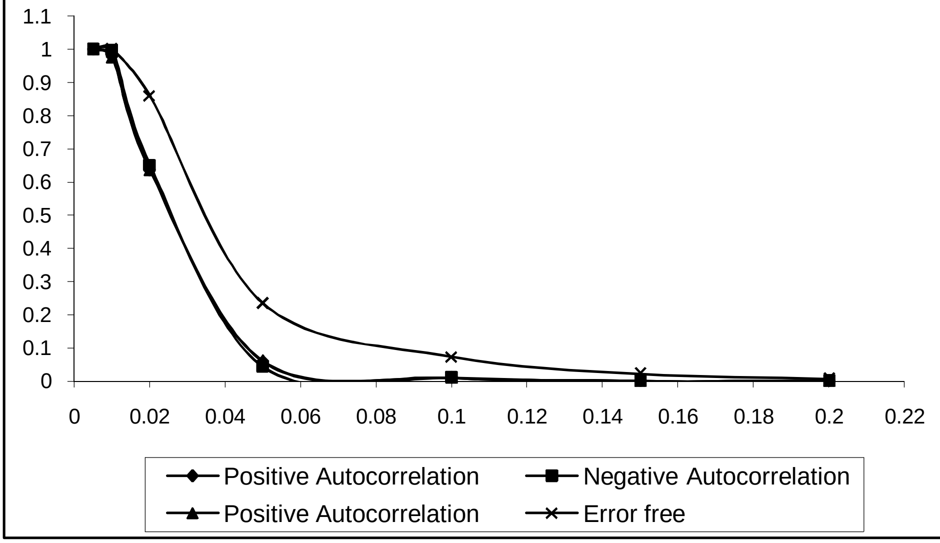
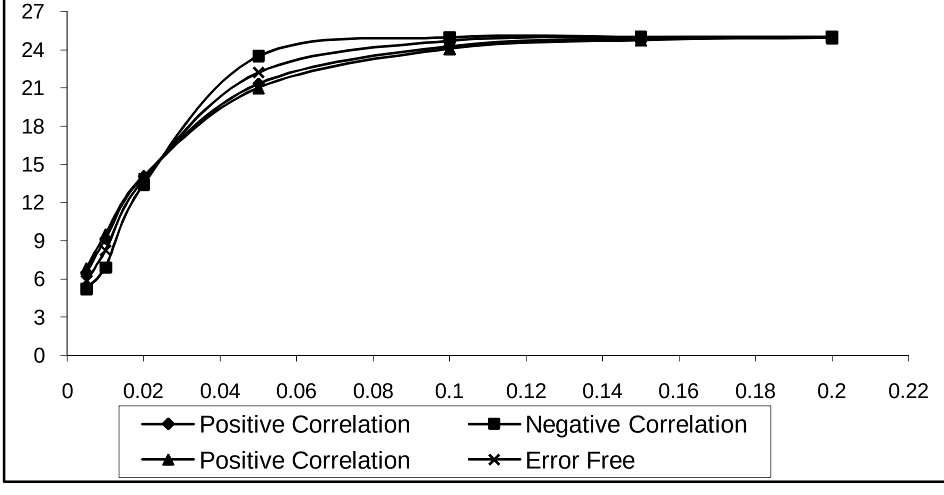


Fig. 3(a) ASN Curve for Independent Combined Plans for v=2



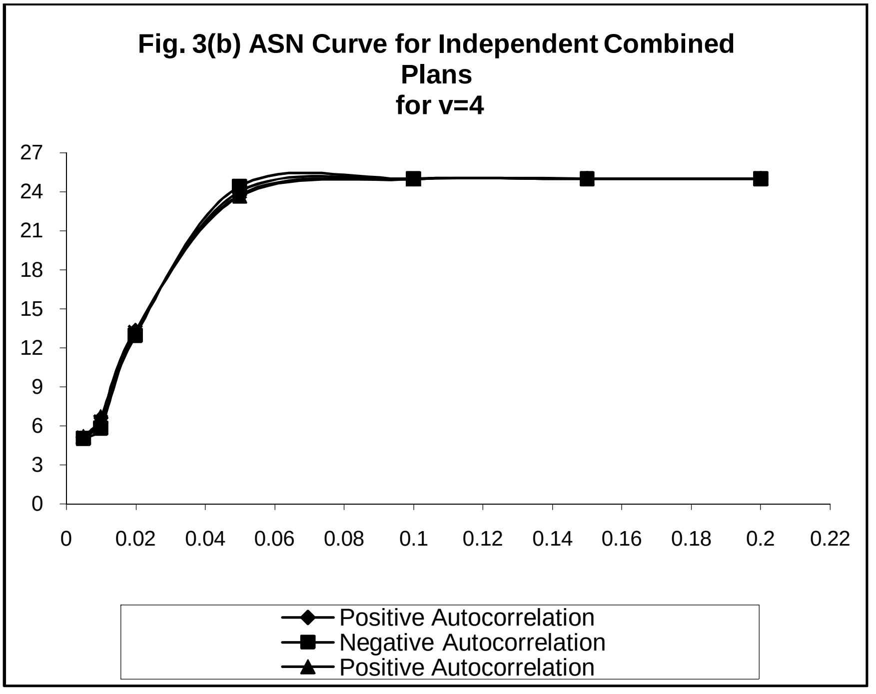


Fig. 3(b) ASN Curve for Independent Combined Plans for v=4
27
24
21
18
15
12
9
6
3
0
0
0.02
0.04
0.06
0.08
0.1
0.12
0.14
0.16
0.18
0.2
0.22
Positive Autocorrelation
Negative Autocorrelation
Positive Autocorrelation

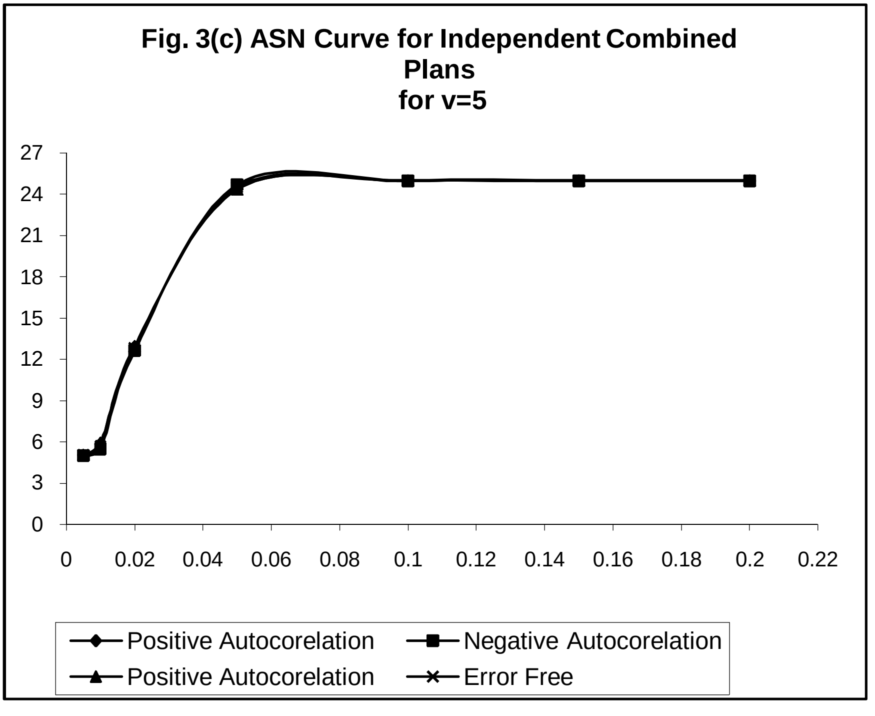


Fig. 3(c) ASN Curve for Independent Combined Plans for v=5
27
24
21
18
15
12
9
6
3
0
0
0.02
0.04
0.06
0.08
0.1
0.12
0.14
0.16
0.18
0.2
0.22
Positive Autocorelation
Negative Autocorelation
Positive Autocorelation
Error Free

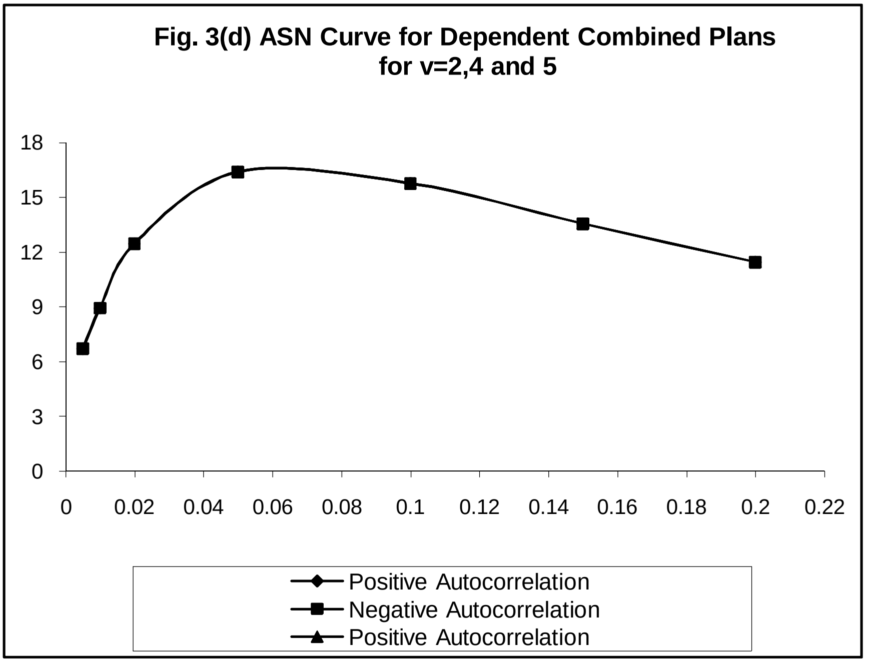


Fig. 3(d) ASN Curve for Dependent Combined Plans
for v=2,4 and 5
18
15
12
9
6
3
0
0
0.02
0.04
0.06
0.08
0.1
0.12
0.14
0.16
0.18
0.2
0.22
Positive Autocorrelation
Negative Autocorrelation
Positive Autocorrelation