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A MARKOV CHAIN ANALYSIS OF DAILY RAINFALL OCCURRENCE AT WESTERN ORISSA OF INDIA

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Abstract

This paper makes an attempt to investigate the pattern of occurrence of rainfall such as sequences of wet and dry spells, expected length of such spells, expected length of weather cycle etc. in the western part of Orissa state of India by fitting a 2-state Markov chain probability model to the collected daily rainfall data for a period of 29 years. The key assumption behind reliability of this model is that the occurrence of a wet or a dry day is dependent on the weather condition of the previous day.

Key words: Chi-square test, Dry spell, Goodness of fit, Markov chain probability model, Steady state probability, Transition probability, Weather cycle, Wet spell.

1. Introduction

There has been a growing operational demand for modeling daily rainfall data using various stochastic models. Because, once the rainfall process is adequately and appropriately modeled, the model can be used to provide prior knowledge of the structural characteristics of varying rainfall systems which are very much essential for agricultural and hydrological planning, industrial and water resource management, and climate change studies. As the distribution of rainfall varies over space and time, it is required to analyze the data covering long periods and recorded at various locations to obtain reliable information. It is also natural to imagine that the total agricultural production in any region depends not only on the total rainfall in a season, but also on its pattern of occurrence such as spells of rainy and dry days, expected number of dry days between two rainy days etc. Thus, a model-based scientific study of the pattern of occurrence of daily rainfall at regional level is therefore crucial for solving various water management problems and to assess the crop failure due to deficiency or excess of rainfall.

The Markov chain model for defining probable occurrences of dry and wet spells has already been achieved widespread use starting from the pioneering work of Gabriel and Neumann (1962). Because other models for constant probabilities are not able to describe the daily persistence of wet and dry conditions. Caskey (1963), Weiss (1964), Hopkins and Robillard (1964), Katz (1974), Todorovic and Woolhiser (1975), Basu (1971), Bhargava et al. (1973), Sundararaj and Ramachandra (1975), Aneja and Srivastava (1986), Rahman (1999a,1999b), Ravindranan and Dani (1993), Akhter and

Hossian (1998), Rahman et al. (2002), Banik et al. (2002), among others analyzed several situations on applying the Markov chain process.

Western Orissa is a less developed and agro-based region of India. More than 80% of annual rainfall over this region is received from the south-west monsoon. The cultivation of all agricultural crops of the Western Orissa region mainly depends on the occurrences of rainfall (natural irrigation), as sufficient supplementary irrigation facilities are not available in the most part of the region. Agricultural production has very often been observed to be seriously affected either by excessive or insufficient rainfall. Clear understanding of the effects of various rainfall characteristics in this region is therefore necessary for planning response measures. Some of the previous studies on the rainfall over the state of Orissa are either for the state as a whole or for isolated stations in the state. To our knowledge, there is no such study for the Western Orissa region.

This paper presents a statistical analysis in order to identify the pattern of occurrence of rainfall by applying a 2-state Markov chain probability model to the data on daily rainfall occurrence for 29 years in the western part of Orissa during the monsoon season.

2. Materials and Methods

Source and Nature of Data

There is not any clear cut boundary line to define the Western Orissa region. However, we consider three districts of the Orrisa state namely, Jharsuguda, Sambalpur and Bolangir because all the four meteorological stations of the Indian Meteorological Department viz., Jharsuguda, Sambalpur, Titilagarh and Bolangir of the Western Orissa are confined to these districts only. In the present study, the daily rainfall data of the said four meteorological stations for 29 years (from 1977 to 2005) were collected from the Meteorological Centre, Bhubaneswar, Orissa. As the monsoon rainfall in the Western Orissa region mainly ranges from June to November, the period considered for the study was taken from 1st June to 30th November which also coincides with the growth season of the major crops in the tract.

Markov Chain Model and Estimation

A rainy or wet day (a dry day) has been defined as one with ≥ 2.5 mm (< 2.5 mm) of rainfall according to the definition proposed by the Indian Meteorological Department [cf., Basu (1971), Reddy et al. (1986)]. This gives a sequence of occurrence of wet and dry days. Further, under the assumption that the occurrence of a wet or a dry day is influenced only by the weather condition of the previous day, the process of occurrence of wet and dry days can be described by a 2 – state Markov chain with wet and dry days as the two states. The transition probability matrix P, which describes the 2 – state Markov chain model is

$$P = \begin{bmatrix} P_{00} & P_{01} \\ P_{10} & P_{11} \end{bmatrix}, \tag{2.1}$$

with $P_{00} + P_{01} = 1$ and $P_{10} + P_{11} = 1$, where P_{00} , P_{01} , P_{10} and P_{11} are the transition probabilities i.e., they are respectively the probabilities of occurrence of the following conditional events :

E_{00}: A day is a dry day given that the preceding day was a dry day
E_{01}: A day is a wet day given that the preceding day was a dry day
E_{10}: A day is a dry day given that the preceding day was a wet day
E_{11}: A day is a wet day given that the preceding day was a wet day

Suppose that each day from 1st June to 30th November is classified according to the occurrence of the four events E_{00} , E_{01} , E_{10} and E_{11} such that 1st June is classified on the consideration of weather condition (wet or dry) of 31st May. Then, repeating this process for each year, frequencies of the occurrences of E_{00} , E_{01} , E_{10} and E_{11} are counted. Let these observed frequencies be denoted by a, b, c and d respectively with $a + b = n_0$ and $c + d = n_1$. Then the maximum likelihood estimates of the unknown transition probabilities P_{01} and P_{11} *i.e.*, the parameters of the model are obtained as

$$\hat{P}_{01} = p_{01} = \frac{b}{a+b} = \frac{b}{n_0} \text{ and } \hat{P}_{11} = p_{11} = \frac{d}{c+d} = \frac{d}{n_1},$$

with estimated variances given by

$$v(p_{01}) = \frac{p_{01}(1-p_{01})}{n_0} = \frac{p_{01}p_{00}}{n_0} \tag{2.2}$$

and

$$v(p_{11}) = \frac{p_{11}(1-p_{11})}{n_1} = \frac{p_{11}p_{10}}{n_1}, \tag{2.3}$$

respectively.

The transition probabilities are conditional probabilities. But, the probability of a dry day (P_0) and the probability of a wet day (P_1) are also estimated from the observed frequencies of the conditional events as follows:

$$\hat{P}_0 = p_0 = \frac{a+c}{n_0+n_1} \text{ and } \hat{P}_1 = p_1 = \frac{b+d}{n_0+n_1}.$$

These unconditional probabilities are also called binomial probabilities treating a rainy day as a success and a dry day as a failure in the system.

In order to test that the occurrence of a wet or dry day is influenced by the immediately preceding day's weather, so that the Markov chain model works reasonably well, a normal test can be employed by computing the usual normal deviate test statistic

$$Z = \frac{p_{01}-p_{11}}{\text{est. S.E. of } (p_{01}-p_{11})} = \frac{p_{01}-p_{11}}{\sqrt{p(1-p)\left(\frac{1}{n_0}-\frac{1}{n_1}\right)}}, \tag{2.4}$$

where $p = \frac{n_0 p_{01}+n_1 p_{11}}{n_0+n_1}$ [cf., Bhargava et al. (1973)].

Separate estimations of the transition probabilities P_{01} and P_{11} for the four meteorological stations encourage us to test the homogeneity of these stations with respect to these parameters. This will obviously lead to yield common estimates of these parameters for the Western Orissa region under study. For this purpose, let the frequencies of occurrences of E_{00} , E_{01} , E_{10} and E_{11} for the i^{th} station be denoted by a_i , b_i , c_i and d_i respectively with $a_i + b_i = n_{0i}$ and $c_i + d_i = n_{1i}$. Then, the estimates of P_{01} and P_{11}, pooled over the four stations, are defined by $\bar{p}_{01} = \frac{\sum b_i}{\sum n_{0i}}$ and $\bar{p}_{11} = \frac{\sum d_i}{\sum n_{1i}}$, respectively. Taking these estimates as the expected probabilities, we can apply two chi-square tests for each station, to test the discrepancies between the observed and the expected values of p_{01} and p_{11}. For the i^{th} station, the concerned chi-square statistics, each at 1 degree of freedom (df), are defined by

$$\chi^2(p_{01}) = \frac{a_i^2}{n_{0i}(1-\bar{p}_{01})} + \frac{b_i^2}{n_{0i}\bar{p}_{01}} - n_{0i}\ , \quad (2.5)$$

and

$$\chi^2(p_{11}) = \frac{c_i^2}{n_{1i}(1-\bar{p}_{11})} + \frac{d_i^2}{n_{1i}\bar{p}_{11}} - n_{1i}\ , i = 1,2,3,4, \quad (2.6)$$

[cf., Rohatgi and Saleh (2000, p. 502)].

Equilibrium or Steady State Probabilities of the Markov Chain

According to Cox and Miller (1967), since the sequence of wet and dry days can be considered as an infinite sequence on time axis, we can take any starting point with an initial day as wet or dry. Then, the system, after a sufficiently long period of time, is expected to settle down to a condition of statistical equilibrium with steady state or equilibrium probabilities which are independent of the initial conditions. These probabilities corresponding to dry and wet days are given by

$$\pi_0 = \frac{1-P_{11}}{1+P_{01}-P_{11}} \text{ and } \pi_1 = \frac{P_{01}}{1+P_{01}-P_{11}},$$

respectively. The number of days after which the state of equilibrium *i.e.*, the original state is attained is equal to the number of steps or times the P–matrix is powered so that its diagonal elements become equal to π_0 and π_1 [cf., Cox and Miller (1967)].

Expected Lengths of Wet and Dry Spells

The wet and dry spell (or run) lengths are very important statistical descriptors of wet and dry periods in a geographical area. Assuming that the lengths of wet and dry spells (denoted by W and D respectively) follow geometric distribution [cf., Bhargava et al. (1973), Sundararaj and Ramachandra (1986), Ravindran and Dani (1993)], the probability of a wet spell of length x is given by

$$P(W = x) = P_{10}P_{11}^{x-1},\ x = 1, 2, \ldots\ldots, \quad (2.7)$$

and therefore, the expected length of the wet spell is obtained as

$$E(W) = \sum_{x=1}^{\infty} x\, P_{10}P_{11}^{x-1} = \frac{1}{P_{10}}. \quad (2.8)$$

The probability of a dry spell of length y is

$$P(D = y) = P_{01}P_{00}^{y-1},\ y = 1,2, \ldots\ldots \quad (2.9)$$

and the expected length of the dry spell is given as

$$E(D) = \frac{1}{P_{01}}. \quad (2.10)$$

If we denote the length of weather cycle as, then the expected length of weather cycle i.e., $E(C)$ is given by

$$E(C) = E(W) + E(D) = \frac{1}{P_{10}} + \frac{1}{P_{01}}. \quad (2.11)$$

To test the strength of fitting of the geometric distribution for describing the distributions of dry and wet spell lengths under the Markovian preconditions of dependence, a chi-square goodness of fit test can be performed with the help of the test statistic

$$\chi_g^2 = \sum_k \frac{(\text{observed frequency - expected frequency})^2}{\text{expected frequency}}, \quad (2.12)$$

which is asymptotically distributed as chi-square with $k-1$ df, where $k=$ number of spells.

As discussed in Cox and Miller (1967), the occurrence of the wet and dry days can be easily treated as dependent Bernoullian trials. Then, the expected values of the number of wet and dry days in a n–day period, denoted by W_n and D_n respectively, are obtained as

$$E(W_n) = n\pi_1 \text{ and } E(D_n) = n\pi_0\,, \tag{2.13}$$

[cf., Reddy et al. (1986)]. Assuming n to be large, the asymptotic variance of the number of wet (or dry) days in a n - day period is given by

$$V_n \sim \frac{nP_{01}(1-P_{11})(1+P_{11}-P_{01})}{(1-P_{11}+P_{01})^3} = \frac{nP_{01}P_{10}(P_{00}+P_{11})}{(P_{10}+P_{01})^3}\,. \tag{2.14}$$

The maximum likelihood estimates of $\pi_0, \pi_1, E(W), E(D), E(C), E(W_n), (D_n)$ and V_n are obtained in the usual way replacing P_{01} and P_{11} by p_{01} and p_{11} respectively. These estimates are denoted by $\hat{\pi}_0, \hat{\pi}_1, \hat{E}(W), \hat{E}(D), \hat{E}(C), \hat{E}(W_n), \hat{E}(D_n)$ and $\hat{V}_n$.

3. Results and Discussions

Estimation of Model Parameters

At the first step towards the fitting of a 2-state Markov chain model to our data, the raw data on the daily rainfall are classified into four classes according to the conditional events E_{00} , E_{01} , E_{10} and E_{11}. From the actual frequencies of these classes, the corresponding relative frequencies are computed in order to obtain the maximum likelihood estimates of the transition (conditional) probabilities P_{00}, P_{01}, P_{10} and P_{11} along with the unconditional binomial probabilities P_0 and P_1 for the four meteorological stations.

From the calculated value of the Z - statistic defined in (2.4), it is found that $|Z| > 3$ for all stations. This high significant value shows that the weather of a day is influenced by the weather of the previous day. As such, the occurrences of wet and dry days in the tract can be rightly described by a 2-state Markov chain model.

In order to test for the differences in p_{01} and p_{11} from station to station, χ^2-tests for the homogeneity of rainfall between the stations were run by using formulae (2.5) and (2.6). It is found that the calculated χ^2 values for all four stations are insignificant for both the parameters P_{01} and P_{11} at 5 % as well as 1 % levels of significance. Therefore, the patterns of the occurrence of rainfall at these four meteorological stations are regarded as similar. Hence, their daily rainfall amounts are grouped together (pooled) in the usual manner in order to obtain a single estimate of the daily rainfall amount, to compose common estimates the model parameters and to study various rainfall characteristics for the Western Orissa climatic situation.

Estimated values of the various conditional and unconditional probabilities associated with our 2-state Markov chain model are displayed in Table 3.1. Entries of this table clearly indicate that the probabilities of the rainfall are maximum and minimum in August and November respectively at the Western Orissa. The conditional probabilities for October and November show how rapidly and markedly the dry conditions establish themselves. The probabilities of wet conditions for these months are negligibly low, indicating totally dry conditions. As no useful information on these probabilities can be expected from the said two months, they are excluded from the further discussion and we consider the period from June to September which is known as the rainy (or kharif) season of Orissa.

Months	Conditional Probabilities			Unconditional Probabilities		
	p_{00}	p_{01}	p_{10}	p_{11}	p_0	p_1
June	0.7820	0.2180	0.4703	0.5297	0.6810	0.3190
July	0.5983	0.4017	0.3922	0.6078	0.4922	0.5078
August	0.6121	0.3879	0.3598	0.6402	0.4803	0.5197
September	0.7411	0.2589	0.4749	0.5251	0.6466	0.3534
October	0.9214	0.0786	0.6632	0.3368	0.8935	0.1065
November	0.9793	0.0207	0.7113	0.2887	0.9718	0.0282
June to September	0.6946	0.3054	0.4134	0.5866	0.5736	0.4264
June to November	0.8100	0.1900	0.4326	0.5674	0.6930	0.3070

Table 3.1: Estimates of Conditional and Unconditional Probabilities (1977-2005)

Statistical Descriptors	Months				
	June	July	August	September	June to September
$\hat{\pi}_0$	0.6833	0.4940	0.4812	0.6472	0.5751
$\hat{\pi}_1$	0.3167	0.5060	0.5188	0.3528	0.4249
$\hat{E}(D_n)$	20.4983≅(20)	15.3145≅(15)	14.9175≅(15)	19.4154≅(19)	70.1653≅(70)
$\hat{E}(W_n)$	9.5017≅(10)	15.6855≅(16)	16.0825≅(16)	10.5846≅(11)	51.8347≅(52)
$\hat{E}(D)$	4.5872	2.4894	2.5780	3.8625	3.2744
$\hat{E}(W)$	2.1263	2.5497	2.7793	2.1057	2.4190
$\hat{E}(C)$	6.7135	5.0391	5.3573	5.9682	5.6934
S. D. of dry or wet days	3.5174	3.4311	3.6002	3.4381	7.2895
No. of days to equilibrium	13	09	10	10	11

Table 3.2: Statistical Descriptors of the Markov Chain Probability Model (1977-2005)

Estimation of Expected Dry and Wet Days (With Spell Lengths)

Various statistical descriptors of the Markov chain model *viz.*, estimated values of the expected number of dry and wet days and their spell lengths, length of weather cycle, S.D. of the estimated number of wet or dry days, steady state probabilities and number of days required for equilibrium, as explained in the preceding section, are computed and are compiled in Table 3.2.

From the Table 3.2, it can be seen that the expected length of dry spells varies from 2.4894 to 4.5872 days whereas that of wet spells varies from 2.1057 to 2.7793 days. This means that after every 2 to 3 consecutive wet days, a dry day is likely to occur and after every 2 to 5 consecutive dry days, a wet day is likely to occur. However, computed overall expected values of the spell lengths indicate that after every 2 to 3 consecutive wet days, a dry day is expected and after every 3 consecutive dry days, a wet day is expected during the rainy season i.e., during June to September. Hence, for this period the expected length of weather cycle varies from 5 to 6 days.

It is also evident from the Table 3.2 that the months July and August possess the highest number of expected rainy days i.e., 16 days and the lowest number of expected dry days i.e., 15 days. Assuming that the variables W_n and D_n follow normal distribution, we have computed 95% confidence intervals for $E(W_n)$ and $E(D_n)$. From these confidence intervals we have concluded that the rainy days (dry days) are expected to lie between 38 to 66 days (56 to 84 days) during the period of 122 days of the rainy season. Apparently, there is not much overlapping between the two distributions.

For the month of August, $\hat{\pi}_0$ and $\hat{\pi}_1$values are respectively smaller and larger than other months and for the consolidated period from June to September these values are 0.5751 and 0.4249 respectively. As the number of days to equilibrium for the months varies from 9 to 13 days, this proves that after 9 to 13 days, during the rainy season, the probability of the day being wet or being dry is independent of the initial weather conditions.

Spell Length (Days)	Wet Spell		Dry Spell	
	Observed Frequency	Expected Frequency	Observed Frequency	Expected Frequency
1	1155	1211	932	886
2	680	710	652	616
3	415	405	400	428
4	251	234	288	298
5	159	139	183	207
6	98	82	122	143
7	65	58	97	99
8	53	47	64	69
9	21	17	45	47
10	20	13	46	36
11	13*	14*	20	22
12	-	-	15	16
13	-	-	19	12
14	-	-	10	11
15	-	-	7**	10**
Calculated Value of χ^2-Statistic (χ_g^2)	17.731		21.110	
Degrees of Freedom	10		14	
1% Critical Value of χ^2 ($\chi_{0.01}^2$)	23.209		29.141	
5% Critical Value of χ^2 ($\chi_{0.05}^2$)	18.307		23.685	

* Frequencies corresponding to spell length ≥ 11

** Frequencies corresponding to spell length ≥ 15

Table 3.3: Observed and Expected Frequencies of Wet and Dry Spells (June-September)

To study the closeness of fitting of the Markov based geometric distribution to the lengths of dry and wet spells, the χ^2-test for goodness of fit at 1% and 5% levels of significance has been applied on using the test statistic defined in (2.12). The test results for the different months are more or less similar and they provide evidence for quite good fit in each case. So, to save space, we do not present the results relating to the goodness of fit test for the individual months but these results for the consolidated months are presented in Table 3.3. The insignificant values of the calculated χ^2 statistic prove that the lengths of wet and dry spells can be described by the Markov based geometric distribution. The graphical representations of the fitted probability distribution for the wet and dry spells are shown in Figures 3.1 and 3.2 respectively.

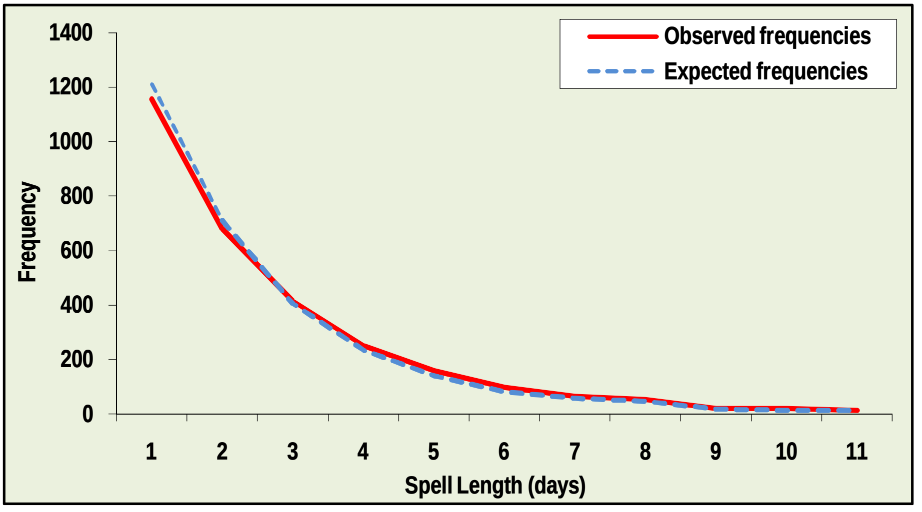

Fig. 3.1: Observed and Expected Frequencies of Wet Spells

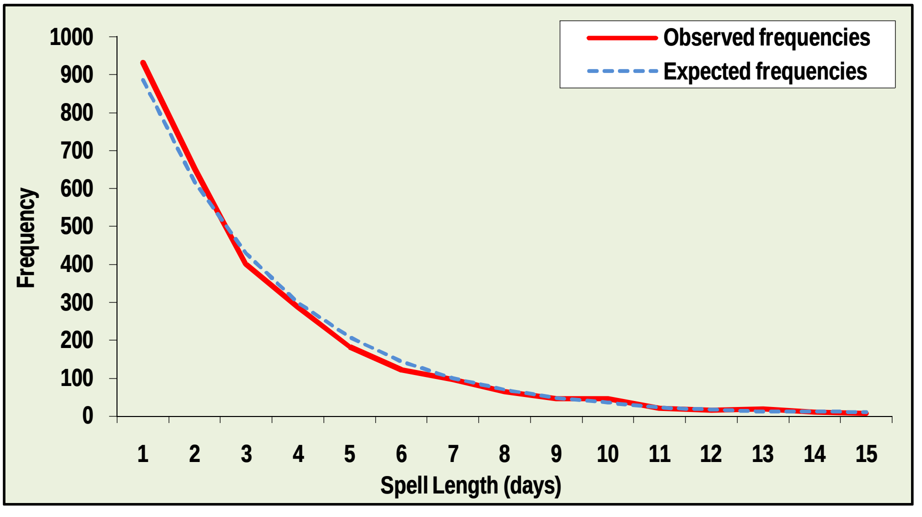

Fig. 3.2: Observed and Expected Frequencies of Dry Spells

4. Conclusions

From this study on the different aspects of the pattern of daily rainfall occurrence at Western Orissa leads to the following tentative conclusions:

(i) The Markov chain probability model appears to provide a good approximation for describing the occurrence of the sequence of wet and dry days.

(ii) The rainfall distributions of the four meteorological stations of the Western Orissa exhibits more or less similar pattern.

(iii) On the whole, as judged by the χ^2 - test of goodness of fit, the geometric distribution model under the assumption of Markovian dependence of weather occurrence seems to be satisfactory for representing the distributions of wet and dry spells.

(iv) During the rainy season, the expected length of dry spells is about 3 days and that of wet spells varies from 2 to 3 days, and the system settles down after about 9 to 13 days to a condition of statistical equilibrium in which the occupation probabilities are independent of the initial conditions. The estimated ranges for the expected numbers of dry days and rainy days during the period of 122 days are 56-84 and 38-66 respectively.

Although the Markov chain model provides a satisfactory fit to our daily rainfall data for computing probability of occurrence of the sequence of wet or dry days, we stress on further investigations with the help of other models and with new definitions of wet and dry days as well as other goodness of fit tests. Studies involving Time Series models such as ARCH or GARCH models can also be carried out for volatility of the data.

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