Journal of Reliability and Statistical Studies; ISSN (Print): 0974-8024, (Online):2229-5666
Vol. 6, Issue 1 (2013): 87-100

A TWO-UNIT STANDBY SYSTEM WITH TWO OPERATIVE MODES OF THE UNITS AND PREPARATION TIME FOR REPAIR

Rakesh Gupta and Parul Bhardwaj

Department of Statistics, Ch. Charan Singh University, Meerut-250004

(Received March 16, 2013)

Abstract

The paper deals with a stochastic behavior of a two-identical unit cold standby system model assuming three modes- normal, partially failure and total failure of the units. A totally failed unit needs some preparation work before going into repair and after completion of preparation, the unit is sent for repair. A partially failed unit may operate with reduced efficiency while it is undergoing repair. A single repairman plays the triple role – the repair of partially failed unit, the preparation for repair of totally failed unit and repair of totally failed unit. The repair discipline is FCFS in respect of above three jobs. The various measures of system effectiveness are obtained by using regenerative point technique.

Keywords: Regenerative point, Reliability, MTSF, Availability, Busy period of repairman, Net expected profit.

1. Introduction

Two-unit standby redundant system models have been analyzed widely in the literature of reliability by many authors [1,5,7]. They have assumed two modes of a unit- normal (N) and total failure (F). Sometimes we observe that an operating unit doesn't operate with its full efficiency i.e. it works with reduced efficiency so that it is said to work in partial failure mode. Keeping this fact in view some authors [2,3,4,6,9,10] analyzed the system models with three modes of a unit- normal (N), partial failure (P) and total failure (F). The above authors have assumed that a totally failed unit immediately enters into repair facility for its repair maintenance. In real situations, it has been observed so many times that a failed unit needs some preparation time before starting its repair. For example: in case of failure of an automobile the repairman arranges some material including faulty parts before starting the repair of a failed automobile. Singh and Srinivasu [8] analyzed a two-unit standby system model with two modes of a unit assuming that a failed unit first goes for preparation before entering into repair. The preparation and repair work is performed by a single repairman.

The purpose of the present paper is to analyze a two identical unit cold standby system model with three modes of a unit (N, P and F) out of which N and P are the operative modes where the unit operates with full (90%-100%) and reduced (70%-90%) efficiencies respectively. It has been also assumed that a failed unit requires a significant preparation time before entering into repair. The preparation time is taken as a random variable having some probability distribution. The following economic related measures of system effectiveness have been obtained by using regenerative point technique-

i. Transition probabilities and mean sojourn times in various states.
ii. Reliability and Mean time to system failure.
iii. Point-wise and steady-state availabilities of the system as well as expected up time of the system during time interval (0, t).
iv. Expected busy period of repairman in repair of partially failed unit, repair of a totally failed unit and preparation for repair of a totally failed unit during time interval (0, t).
v. Net expected profit in time interval (0, t) and in steady-state.

2. Model Description and Assumptions

i. The system comprises of two identical units. Initially, one unit is operative and other is kept into cold standby.
ii. Each unit of the system has three modes- normal (N), partial failure (P) and total failure (F). An operating unit in N-mode first enters into P-mode and then into F-mode i.e. a unit can't enter into F-mode directly from N-mode. A unit is known as N, P or F-unit in its respective mode.
iii. The standby unit is switched on for its operation only when the operative unit fails completely. The switching being instantaneous, perfect and without any damage to the system.
iv. In P-mode, a unit is working as well as undergoing repair. On getting repaired, the unit enters the N-mode. It is also possible that the partially operating unit during its repair deteriorates further and enters into the F-mode.
v. As soon as a unit enters into F-mode, it needs some preparation work before starting its repair. The preparation time is a random variable.
vi. A single repairman is always available with the system to repair a partially failed unit, to preparation for repair of a totally failed unit and to repair of a totally failed unit. The service discipline of repairman is FCFS in respect of above three jobs.
vii. In case the N-unit fails partially or P-unit fails totally while the other unit is already in F-mode and is under preparation or repair, the later waits for repair till the preparation or repair of the earlier totally failed unit is completed.
viii. The failure time distributions of a N- unit or P- unit are taken exponential whereas preparation and both repair time distributions are general.

3. Notations and States of The System

a) Notations

E :Set of regenerative states i.e. S_0 to S_3, S_5, S_7

α_1 : Constant failure rate of an operating unit.

α_2 : Constant failure rate of a partially operating unit.

$G_1(\cdot), g_1(\cdot)$: Cdf and pdf of repair time of a partially failed unit.

$G_2(\cdot), g_2(\cdot)$: Cdf and pdf of repair time of a totally failed unit.

$H(\cdot), h(\cdot)$: Cdf and pdf of preparation time of a totally failed unit for its repair.

$q_{ij}(\cdot)$: Pdf of transition time from state S_i to S_j .

p_{ij} : Steady state probability that the system transits from state S_i to S_j .

ψ_i : Mean sojourn time in state S_i

†n : Mean repair time of totally failed unit $= \int t dG_2(t)$

m : Mean preparation time for repair of a totally failed unit $= \int t dH(t)$

*, ~: Symbols for Laplace and Laplace-Stieltjes transforms.

b) Symbols for the states of the systems:

N_o, N_s : Unit in normal mode and operative/standby.

P_{or}, P_{ow} : Unit in partially operative mode and under repair/waiting for repair.

F_r, F_{pr} : Unit in total failure mode and under repair/ preparation for repair.

F_{wp} : Unit in total failure mode and waiting for preparation for repair.

† Limits of integration are taken 0 to ∞ whenever they are not mentioned

TRANSITION DIAGRAM

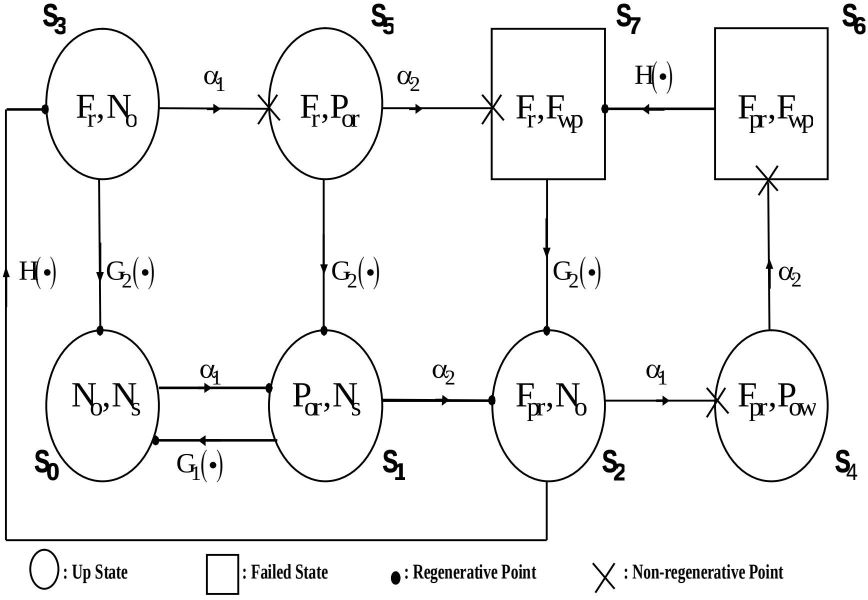

Fig.1

Using these symbols and keeping in view the assumptions stated in section-2, the possible states of the system are shown in transition diagram (Fig. 1). The epochs of transitions into the states S_4 from S_2, S_5 from S_3, S_6 from S_4 and S_7 from S_5 are non-regenerative while all the other entrance epochs into the states are regenerative.

4. Transition Probabilities

Let $X(t)$ be the state of the system at epoch t, then $\{X(t); t \geq 0\}$ constitutes a Markov-chain with state space E. The transition probability matrix of the embedded Markov chain is

$$P = \begin{bmatrix} p_{00} & p_{01} & p_{02} & p_{03} & p_{05} & p_{07} \\ p_{10} & p_{11} & p_{12} & p_{13} & p_{15} & p_{17} \\ p_{20} & p_{21} & p_{22} & p_{23} & p_{25}^{(4)} & p_{27}^{(4,6)} \\ p_{30} & p_{31}^{(5)} & p_{32}^{(5,7)} & p_{33} & p_{35} & p_{37} \\ p_{50} & p_{51} & p_{52}^{(7)} & p_{53} & p_{55} & p_{57} \\ p_{70} & p_{71} & p_{72} & p_{73} & p_{75} & p_{77} \end{bmatrix}$$

With non-zero elements-

$$p_{01} = 1, p_{10} = \tilde{G}_1(\alpha_2), \quad p_{12} = 1 - \tilde{G}_1(\alpha_2), \quad p_{23} = \tilde{H}(\alpha_1)$$

$$p_{25}^{(4)} = \frac{\alpha_1}{\alpha_1 - \alpha_2}\left[\tilde{H}(\alpha_2) - \tilde{H}(\alpha_1)\right],$$

$$p_{27}^{(4,6)} = \frac{1}{\alpha_1 - \alpha_2}\left[\alpha_1\left\{1 - \tilde{H}(\alpha_2)\right\} - \alpha_2\left\{1 - \tilde{H}(\alpha_1)\right\}\right]$$

$$p_{31}^{(5)} = \frac{\alpha_1}{\alpha_1 - \alpha_2}\left[\tilde{G}_2(\alpha_2) - \tilde{G}_2(\alpha_1)\right],$$

$$p_{32}^{(5,7)} = \frac{1}{\alpha_1 - \alpha_2}\left[\alpha_1\left\{1 - \tilde{G}_2(\alpha_2)\right\} - \alpha_2\left\{1 - \tilde{G}_2(\alpha_1)\right\}\right]$$

$$p_{30} = \tilde{G}_2(\alpha_1), \qquad p_{51} = \tilde{G}_2(\alpha_2), \qquad p_{52}^{(7)} = 1 - \tilde{G}_2(\alpha_2),$$

$$p_{72} = 1 \tag{1-12}$$

The other elements of t. p. m. will be zero.
It can be easily verified that

$$p_{01} = p_{72} = 1, \quad p_{10} + p_{12} = 1, \quad p_{23} + p_{25}^{(4)} + p_{27}^{(4,6)} = 1$$

$$p_{30} + p_{31}^{(5)} + p_{32}^{(5,7)} = 1, \qquad p_{51} + p_{52}^{(7)} = 1 \tag{13-17}$$

5. Mean Sojourn Times

The mean sojourn time ψ_i in state S_i is defined as the expected time taken by the system in state S_i before transiting into any other state. If random variable U_i denotes the sojourn time in state S_i then

$$\psi_i = \int P\left[U_i > t\right]dt$$

Therefore, its values for various regenerative states are as follows:

$$\psi_0 = 1/\alpha_1, \quad \psi_1 = \left[1-\tilde{G}_1(\alpha_2)\right]/\alpha_2, \quad \psi_2 = \left[1-\tilde{H}(\alpha_1)\right]/\alpha_1$$

$$\psi_3 = \left[1-\tilde{G}_2(\alpha_1)\right]/\alpha_1, \qquad \psi_5 = \left[1-\tilde{G}_2(\alpha_2)\right]/\alpha_2,$$

$$\psi_7 = \int \bar{G}_2(t)dt = n \tag{18-23}$$

6. Analysis of Characteristics

a) Reliability of the system and MTSF

Let $R_i(t)$ be the probability that the system is operative during (0, t) given that at t=0 system starts from $S_i \in E$. To obtain it we assume the failed states S_6 and S_7 as absorbing. By simple probabilistic arguments, the value of $R_0(t)$ in terms of its Laplace Transform (L.T.) is given by

$$R_0^*(s) = \frac{N_1(s)}{D_1(s)} \tag{24}$$

Where,

$$N_1(s) = Z_0^*\left[1-q_{12}^*\left(q_{23}^*q_{31}^{(5)*} + q_{25}^{(4)*}q_{51}^*\right)\right] + q_{01}^*\left[Z_1^* + q_{12}^*\left(Z_2^* + q_{23}^*Z_3^* + q_{25}^*Z_5^*\right)\right]$$

$$D_1(s) = 1 - q_{12}^*\left(q_{23}^*q_{31}^{(5)*} + q_{25}^{(4)*}q_{51}^*\right) - q_{01}^*q_{10}^* - q_{01}^*q_{12}^*q_{23}^*q_{30}^*$$

And $Z_i^*(i=0,1,2,3)$ are the L. T. of

$$Z_0(t) = e^{-\alpha_1 t}, \quad Z_1(t) = e^{-\alpha_2 t}\bar{G}_1(t), \quad Z_2(t) = \frac{\bar{H}(t)\left[\alpha_1 e^{-\alpha_2 t} - \alpha_2 e^{-\alpha_1 t}\right]}{\alpha_1 - \alpha_2}$$

$$Z_3(t) = \frac{\bar{G}_2(t)\left[\alpha_1 e^{-\alpha_2 t} - \alpha_2 e^{-\alpha_1 t}\right]}{\alpha_1 - \alpha_2}, \quad Z_5(t) = e^{-\alpha_2 t}\bar{G}_2(t)$$

Taking the Inverse Laplace Transform of (24), one can get the reliability of the system when it starts from state S_0.

The MTSF is given by

$$E(T_0) = \int R_0(t)dt = \lim_{s\to 0} R_0^*(s) = N_1/D_1 \tag{25}$$

Where,

$$N_1 = \psi_0\left[1-p_{12}\left(p_{23}p_{31}^{(5)} + p_{25}^{(4)}p_{51}\right)\right] + \psi_1 + p_{12}\left(\theta_1 + p_{23}\theta_2 + p_{25}^{(4)}\psi_5\right)$$

$$D_1 = 1 - p_{12}\left(p_{23}p_{31}^{(5)} + p_{25}^{(4)}p_{51}\right) - p_{10} - p_{12}p_{23}p_{30}$$

Where, $\theta_1 = \int Z_2(t)dt$, $\theta_2 = \int Z_3(t)dt$

Availability Analysis

Let $A_i^n(t)$ and $A_i^p(t)$ be the respective probabilities that the system is operative in normal (N) mode and partial (P) mode at epoch t, when it initially starts from $S_i \in E$. Using the regenerative point technique and the tools of L. T., one can obtain the value of above two probabilities in terms of their L.T. i.e. $A_i^{n*}(s)$ and $A_i^{p*}(s)$.

The steady-state availability of the system is given by

$$A_0^n = \lim_{s\to 0} sA_0^{n*}(s) = N_2/D_2 \tag{26}$$

and

$$A_0^p = \lim_{s\to 0} sA_0^{p*}(s) = N_3/D_2 \tag{27}$$

Where,

$$N_2 = \psi_0\left[p_{23}p_{30} + p_{10}\left(p_{23}p_{31}^{(5)} + p_{25}^{(4)}p_{51}\right)\right] + p_{12}\left(\psi_2 + p_{23}\psi_3\right)$$

$$N_3 = \psi_1\left(1 - p_{27}^{(4,6)} - p_{23}p_{32}^{(5,7)} - p_{25}^{(4)}p_{52}^{(7)}\right) + p_{12}\left(\lambda_1 + p_{23}\lambda_2 + p_{25}^{(4)}\psi_5\right)$$

and

$$D_2 = \psi_0\left[p_{23}p_{30} + p_{10}\left(p_{23}p_{31}^{(5)} + p_{25}^{(4)}p_{51}\right)\right] + \psi_1\left(1 - p_{27}^{(4,6)} - p_{23}p_{32}^{(5,7)} - p_{25}^{(4)}p_{52}^{(7)}\right) + p_{12}(m+n) \tag{28}$$

Where,

$$\lambda_1 = \frac{\alpha_1}{\alpha_1 - \alpha_2}\int\left(e^{-\alpha_2 t} - e^{-\alpha_1 t}\right)\bar{H}(t)dt\,,$$

$$\lambda_2 = \frac{\alpha_1}{\alpha_1 - \alpha_2}\int\left(e^{-\alpha_2 t} - e^{-\alpha_1 t}\right)\bar{G}_2(t)dt$$

The expected up (operative) times of the system in N-mode and P-mode during (0, t) are given by

$$\mu_{up}^n(t) = \int_0^t A_0^n(u)du \quad \text{and} \quad \mu_{up}^p(t) = \int_0^t A_0^p(u)du \tag{29-30}$$

So that

$$\mu_{up}^{n*}(s) = A_0^{n*}(s)/s \quad \text{and} \quad \mu_{up}^{p*}(s) = A_0^{p*}(s)/s \tag{31-32}$$

b) Busy Period Analysis

Let $B_i^p(t)$, $B_i^{pr}(t)$ and $B_i^f(t)$ be the respective probabilities that the repairman is busy in the repair of partially failed unit, preparation for repair of a totally failed unit and repair of a totally failed unit at epoch t, when the system initially starts operation from state $S_i \in E$. Using the regenerative point technique and the tools of L.T., one can obtain the values of above three probabilities in terms of their L. T. i.e. $B_i^{p*}(s)$, $B_i^{pr*}(s)$ and $B_i^{f*}(s)$.

The steady state results for the above three probabilities are given by

$$B_o^p = \lim_{s\to 0} sB_0^{p*}(s) = N_4/D_2 \qquad (33)$$

Similarly,

$$B_0^{pr} = N_5/D_2 \qquad \text{and} \qquad B_0^f = N_6/D_2 \qquad (34\text{-}35)$$

Where,

$$N_4 = \psi_1\left(1 - p_{27}^{(4,6)} - p_{23}p_{32}^{(5,7)} - p_{25}^{(4)}p_{52}^{(7)}\right)$$

$$N_5 = mp_{12} \qquad \text{and} \qquad N_6 = np_{12}$$

and D_2 is same as expressed by equation (28).

The expected busy period of repairman due to repair of partially failed unit, preparation for repair of a totally failed unit and repair of a totally failed unit during time interval (0,t) are respectively given by

$$\mu_b^p(t) = \int_0^t B_0^p(u)du, \quad \mu_b^{pr}(t) = \int_0^t B_0^{pr}(u)du, \quad \mu_b^f(t) = \int_0^t B_0^f(u)du \qquad (36\text{-}38)$$

So that,

$$\mu_b^{p*}(s) = B_0^{p*}(s)/s, \qquad \mu_b^{pr*}(s) = B_0^{pr*}(s)/s,$$

$$\mu_b^{f*}(s) = B_0^{f*}(s)/s \qquad (39\text{-}41)$$

7. Cost Benefit Analysis

We are now in the position to obtain the profit function by considering mean up time of the system during (0, t), expected busy period of repairman in repair of partial failed unit, preparation for repair a totally failed unit and repair of a totally failed unit during (0, t).

Let us suppose

K_0 =revenue per-unit time by the system when it is operative in N-mode.

K_1 =revenue per-unit time by the system when it is operative in P-mode.

K_2 =cost per-unit time when repairman is busy in repairing of a partially failed unit.

K_3 =cost per-unit time when the repairman is busy in the preparation for repair of totally failed unit.

K_4 =cost per-unit time when repairman is busy in repairing of a totally failed unit.

Now, the net expected profit incurred in time interval (0, t) is given by-

$$P_0(t) = K_0\mu_{up}^{n}(t) + K_1\mu_{up}^{p}(t) - K_2\mu_b^{p}(t) - K_3\mu_b^{pr}(t) - K_4\mu_b^{f}(t) \qquad (42)$$

The expected profit per-unit time in steady state is

$$P_0 = \lim_{t\to\infty} P_0(t)/t = \lim_{s\to 0} s^2 P_0^*(s)$$

$$= K_0 A_0^n + K_1 A_0^p - K_2 B_0^p - K_3 B_0^{pr} - K_4 B_0^f \qquad (43)$$

8. Case Studies

The system model has wide applicability for various forms of p.d.f.s of repair times of P-unit, F-unit and preparation time for repair of F-unit. As an illustration, we consider the following two cases to obtain the measures of system effectiveness obtained in earlier sections.

Case I: When the repair time of partially failed unit, preparation time for repair of a totally failed unit and repair time of a totally failed unit follow Lindley distributions with p.d.f. as follows-

$$g_1(t) = \frac{\mu_1^{\,2}}{(1+\mu_1)}(1+t)e^{-\mu_1 t}, \qquad h(t) = \frac{\eta^2}{(1+\eta)}(1+t)e^{-\eta t},$$

$$g_2(t) = \frac{\mu_2^{\,2}}{(1+\mu_2)}(1+t)e^{-\mu_2 t}$$

The Laplace Transforms of above three density functions and Laplace Stieltjes Transforms of corresponding c.d.f.'s are as given below-

$$g_1^*(s) = \tilde{G}_1(s) = (s+\mu_1+1)\mu_1^{\,2} \big/ (1+\mu_1)(s+\mu_1)^2$$

$$h^*(s) = \tilde{H}(s) = (s+\eta+1)\eta^2 \big/ (1+\eta)(s+\eta)^2$$

$$g_2^*(s) = \tilde{G}_2(s) = (s+\mu_2+1)\mu_2^{\,2} \big/ (1+\mu_2)(s+\mu_2)^2$$

Here $\tilde{G}_1(s)$, $\tilde{H}(s)$ and $\tilde{G}_2(s)$ are the Laplace Stieltjes Transforms of the c.d.f. $G_1(t), H(t)$ and $G_2(t)$ corresponding to the p.d.f. $g_1(t)$, $h(t)$ and $g_2(t)$.

In view of above we have the following changes in results (2-11) and (19-22)-

$$p_{10} = \left(1+\frac{\alpha_2}{1+\mu_1}\right)\left(\frac{\mu_1}{\alpha_2+\mu_1}\right)^2, \quad p_{12} = 1-\left(1+\frac{\alpha_2}{1+\mu_1}\right)\left(\frac{\mu_1}{\alpha_2+\mu_1}\right)^2,$$

$$p_{23} = \left(1+\frac{\alpha_1}{1+\eta}\right)\left(\frac{\eta}{\alpha_1+\eta}\right)^2$$

$$p_{25}^{(4)} = \frac{\alpha_1}{\alpha_1-\alpha_2}\left[\left(1+\frac{\alpha_2}{1+\eta}\right)\left(\frac{\eta}{\alpha_2+\eta}\right)^2 - \left(1+\frac{\alpha_1}{1+\eta}\right)\left(\frac{\eta}{\alpha_1+\eta}\right)^2\right]$$

$$p_{27}^{(4,6)} = \frac{1}{\alpha_1 - \alpha_2}\left[\alpha_1\left\{1-\left(1+\frac{\alpha_2}{1+\eta}\right)\left(\frac{\eta}{\alpha_2+\eta}\right)^2\right\} - \alpha_2\left\{1-\left(1+\frac{\alpha_1}{1+\eta}\right)\left(\frac{\eta}{\alpha_1+\eta}\right)^2\right\}\right]$$

$$p_{31}^{(5)} = \frac{\alpha_1}{\alpha_1 - \alpha_2}\left[\left(1+\frac{\alpha_2}{1+\mu_2}\right)\left(\frac{\mu_2}{\alpha_2+\mu_2}\right)^2 - \left(1+\frac{\alpha_1}{1+\mu_2}\right)\left(\frac{\mu_2}{\alpha_1+\mu_2}\right)^2\right]$$

$$p_{32}^{(5,7)} = \frac{1}{\alpha_1 - \alpha_2}\left[\alpha_1\left\{1-\left(1+\frac{\alpha_2}{1+\mu_2}\right)\left(\frac{\mu_2}{\alpha_2+\mu_2}\right)^2\right\} - \alpha_2\left\{1-\left(1+\frac{\alpha_1}{1+\mu_2}\right)\left(\frac{\mu_2}{\alpha_1+\mu_2}\right)^2\right\}\right]$$

$$p_{30} = \left(1+\frac{\alpha_1}{1+\mu_2}\right)\left(\frac{\mu_2}{\alpha_1+\mu_2}\right)^2, \qquad p_{51} = \left(1+\frac{\alpha_2}{1+\mu_2}\right)\left(\frac{\mu_2}{\alpha_2+\mu_2}\right)^2$$

$$p_{52}^{(7)} = 1-\left(1+\frac{\alpha_2}{1+\mu_2}\right)\left(\frac{\mu_2}{\alpha_2+\mu_2}\right)^2,$$

$$\psi_1 = \frac{1}{\alpha_2}\left[1-\left(1+\frac{\alpha_2}{(1+\mu_1)}\right)\left(\frac{\mu_1}{\alpha_2+\mu_1}\right)^2\right], \quad \psi_2 = \frac{1}{\alpha_1}\left[1-\left(1+\frac{\alpha_1}{(1+\eta)}\right)\left(\frac{\eta}{\alpha_1+\eta}\right)^2\right]$$

$$\psi_3 = \frac{1}{\alpha_1}\left[1-\left(1+\frac{\alpha_1}{(1+\mu_2)}\right)\left(\frac{\mu_2}{\alpha_1+\mu_2}\right)^2\right], \quad \psi_5 = \frac{1}{\alpha_2}\left[1-\left(1+\frac{\alpha_2}{(1+\mu_2)}\right)\left(\frac{\mu_2}{\alpha_2+\mu_2}\right)^2\right]$$

$$\psi_2 = \frac{1}{\alpha_1}\left[1-\left(1+\frac{\alpha_1}{(1+\eta)}\right)\left(\frac{\eta}{\alpha_1+\eta}\right)^2\right]$$

$$\theta_1 = \frac{1}{\alpha_1 - \alpha_2}\left[\frac{\alpha_1}{\alpha_2}\left\{1-\left(1+\frac{\alpha_2}{1+\eta}\right)\left(\frac{\eta}{\alpha_2+\eta}\right)^2\right\} - \frac{\alpha_2}{\alpha_1}\left\{1-\left(1+\frac{\alpha_1}{1+\eta}\right)\left(\frac{\eta}{\alpha_1+\eta}\right)^2\right\}\right]$$

$$\theta_2 = \frac{1}{\alpha_1 - \alpha_2}\left[\frac{\alpha_1}{\alpha_2}\left\{1-\left(1+\frac{\alpha_2}{1+\mu_2}\right)\left(\frac{\mu_2}{\alpha_2+\mu_2}\right)^2\right\} - \frac{\alpha_2}{\alpha_1}\left\{1-\left(1+\frac{\alpha_1}{1+\mu_2}\right)\left(\frac{\mu_2}{\alpha_1+\mu_2}\right)^2\right\}\right]$$

$$\lambda_1 = \frac{\alpha_1}{\alpha_1 - \alpha_2}\left[\frac{1}{\alpha_2}\left\{1-\left(1+\frac{\alpha_2}{1+\eta}\right)\left(\frac{\eta}{\alpha_2+\eta}\right)^2\right\} - \frac{1}{\alpha_1}\left\{1-\left(1+\frac{\alpha_1}{1+\eta}\right)\left(\frac{\eta}{\alpha_1+\eta}\right)^2\right\}\right]$$

$$\lambda_2 = \frac{\alpha_1}{\alpha_1 - \alpha_2}\left[\frac{1}{\alpha_2}\left\{1-\left(1+\frac{\alpha_2}{1+\mu_2}\right)\left(\frac{\mu_2}{\alpha_2+\mu_2}\right)^2\right\} - \frac{1}{\alpha_1}\left\{1-\left(1+\frac{\alpha_1}{1+\mu_2}\right)\left(\frac{\mu_2}{\alpha_1+\mu_2}\right)^2\right\}\right]$$

The values of n and m will be as follows-

$$n = \frac{\mu_2 + 2}{\mu_2(1+\mu_2)}, \qquad m = \frac{\eta + 2}{\eta(1+\eta)}$$

Case II: When the repair time of partially failed unit, preparation time for repair and repair time of failed unit follows exponential distribution with p.d.f.s as follows-

$g_1(t) = \mu_1 e^{-\mu_1 t}$, $h(t) = \eta e^{-\eta t}$, $g_2(t) = \mu_2 e^{-\mu_2 t}$

The Laplace Transforms of above three density functions are as given below.

$g_1^*(s) = \tilde{G}_1(s) = \mu_1/(s+\mu_1)$, $h^*(s) = \tilde{H}(s) = \eta/(s+\eta)$,

$g_2^*(s) = \tilde{G}_2(s) = \mu_2/(s+\mu_2)$

Here $\tilde{G}_1(s)$, $\tilde{H}(s)$ and $\tilde{G}_2(s)$ are the Laplace-Stieltjes Transforms of the c.d.f. $G_1(t)$, $H(t)$ and $G_2(t)$ corresponding to the p.d.f. $g_1(t)$, $h(t)$ and $g_2(t)$.

In view of above the changed values of transition probabilities, mean sojourn times, n and m are given below-

$$p_{10} = \frac{\mu_1}{\mu_1 + \alpha_2}, \qquad p_{12} = 1 - \frac{\mu_1}{\mu_1 + \alpha_2}, \qquad p_{23} = \frac{\eta}{\eta + \alpha_1}$$

$$p_{25}^{(4)} = \frac{\alpha_1}{\alpha_1 - \alpha_2}\left[\frac{\eta}{\eta + \alpha_2} - \frac{\eta}{\eta + \alpha_1}\right],$$

$$p_{27}^{(4,6)} = \frac{1}{\alpha_1 - \alpha_2}\left[\alpha_1\left\{1 - \frac{\eta}{\eta + \alpha_2}\right\} - \alpha_2\left\{1 - \frac{\eta}{\eta + \alpha_1}\right\}\right],$$

$$p_{31}^{(5)} = \frac{\alpha_1}{\alpha_1 - \alpha_2}\left[\frac{\mu_2}{\mu_2 + \alpha_2} - \frac{\mu_2}{\mu_2 + \alpha_1}\right],$$

$$p_{32}^{(5,7)} = \frac{1}{\alpha_1 - \alpha_2}\left[\alpha_1\left\{1 - \frac{\mu_2}{\mu_2 + \alpha_2}\right\} - \alpha_2\left\{1 - \frac{\mu_2}{\mu_2 + \alpha_1}\right\}\right],$$

$$p_{30} = \frac{\mu_2}{\mu_2 + \alpha_1}, \qquad p_{51} = \frac{\mu_2}{\mu_2 + \alpha_2}, \qquad p_{52}^{(7)} = 1 - \frac{\mu_2}{\mu_2 + \alpha_2}$$

$$\psi_1 = \frac{1}{\alpha_2}\left[1 - \frac{\mu_1}{\mu_1 + \alpha_2}\right], \qquad \psi_2 = \frac{1}{\alpha_1}\left[1 - \frac{\eta}{\eta + \alpha_1}\right]$$

$$\psi_3 = \frac{1}{\alpha_1}\left[1 - \frac{\mu_2}{\mu_2 + \alpha_1}\right], \qquad \psi_5 = \frac{1}{\alpha_2}\left[1 - \frac{\mu_2}{\mu_2 + \alpha_2}\right]$$

$$\theta_1 = \frac{1}{\alpha_1 - \alpha_2}\left[\frac{\alpha_1}{\alpha_2}\left\{1 - \frac{\eta}{\eta + \alpha_2}\right\} - \frac{\alpha_2}{\alpha_1}\left\{1 - \frac{\eta}{\eta + \alpha_1}\right\}\right]$$

$$\theta_2 = \frac{1}{\alpha_1 - \alpha_2}\left[\frac{\alpha_1}{\alpha_2}\left\{1 - \frac{\mu_2}{\mu_2 + \alpha_2}\right\} - \frac{\alpha_2}{\alpha_1}\left\{1 - \frac{\mu_2}{\mu_2 + \alpha_1}\right\}\right]$$

$$\lambda_1 = \frac{\alpha_1}{\alpha_1 - \alpha_2}\left[\frac{1}{\alpha_2}\left\{1 - \frac{\eta}{\eta + \alpha_2}\right\} - \frac{1}{\alpha_1}\left\{1 - \frac{\eta}{\eta + \alpha_1}\right\}\right]$$

$$\lambda_2 = \frac{\alpha_1}{\alpha_1 - \alpha_2}\left[\frac{1}{\alpha_2}\left\{1 - \frac{\mu_2}{\mu_2 + \alpha_2}\right\} - \frac{1}{\alpha_1}\left\{1 - \frac{\mu_2}{\mu_2 + \alpha_1}\right\}\right]$$

$n = 1/\mu_2$, $m = 1/\eta$

9. Graphical Representation and Conclusions

The curves for MTSF and profit functions are drown for the two particular cases I and II in respect of different parameters.

In case-I, when repair time follows lindley distribution,

Figs. 2 and 3 depict the variations in MTSF and profit function with respect to failure parameter α_1 (of an operating unit) for different value of failure parameter α_2 (of a partially failed unit) and η (preparation time of a totally failed unit). We may clearly observe from Fig.2 that MTSF decreases uniformly as the values of α_1 increase. It also reveals that MTSF decreases with the increase in α_2 and increase with the increase in η.

Similarly, Figure 3 reveals the variations in profit with respect to α_1 for varying values of α_2 and η when the values of other parameters are kept fix as K_0=300, K_1=150, K_2=250, K_3=150 and K_4=400. From this Figure it is clearly observed from dotted curves that system is profitable only if α_1 is less than 0.055, 0.06 and 0.075 for α_2=0.5, 0.7 and 0.9 respectively for fixed value of η=0.03 and from smooth curves we conclude that system is profitable only if α_1 is less than 0.125, 0.15 and 0.20 for α_2=0.5, 0.7 and 0.9 respectively for fixed value of η=0.05.

In case-II, when repair time follows exponential distribution

Figs. 4 and 5 depict the variations in MTSF and profit function with respect to failure parameter α_1 for different values of α_2 and η. We may clearly reveal from Fig.4 that MTSF decreases as the values of α_1 increase. It is also pointed out that MTSF decreases with the increase in α_2 and increase with the increases in η.

Similarly, Fig. 5, shows the variations in profit with respect to α_1 for varying values of α_2 and η when the values of other parameters are kept fix as K_0=15, K_1=10, K_2=100, K_3=50 and K_4=150. From Fig. 4 it is observed from dotted curves that system is profitable only if α_1 is less than 0.011, 0.15 and 0.24 for α_2=0.3, 0.4 and 0.5 respectively for fixed value of η=0.05 and from smooth curves we conclude that system is profitable only if α_1 is less than 0.18, 0.26 and 0.50 for α_2=0.3, 0.4 and 0.5 respectively for fixed value of η=0.6.

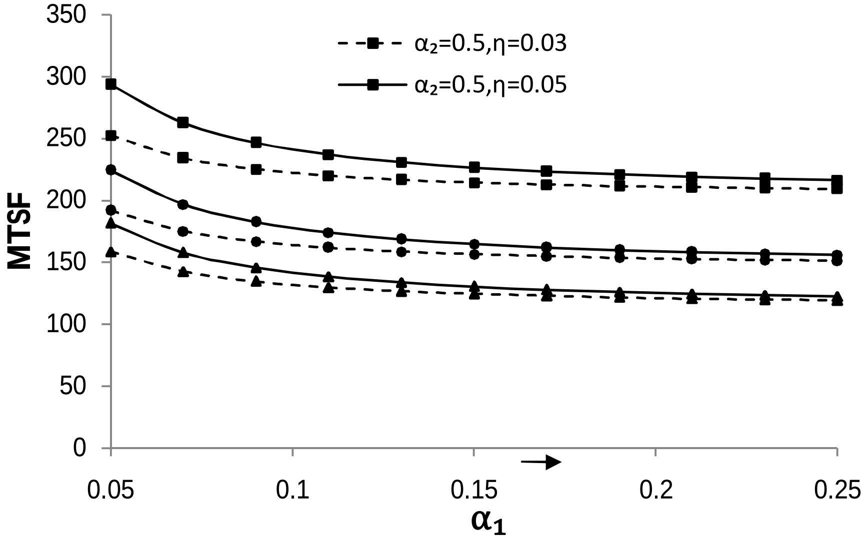

Fig.2: Behavior of MTSF for particular case-1 with respect to α_1, α_2 and η

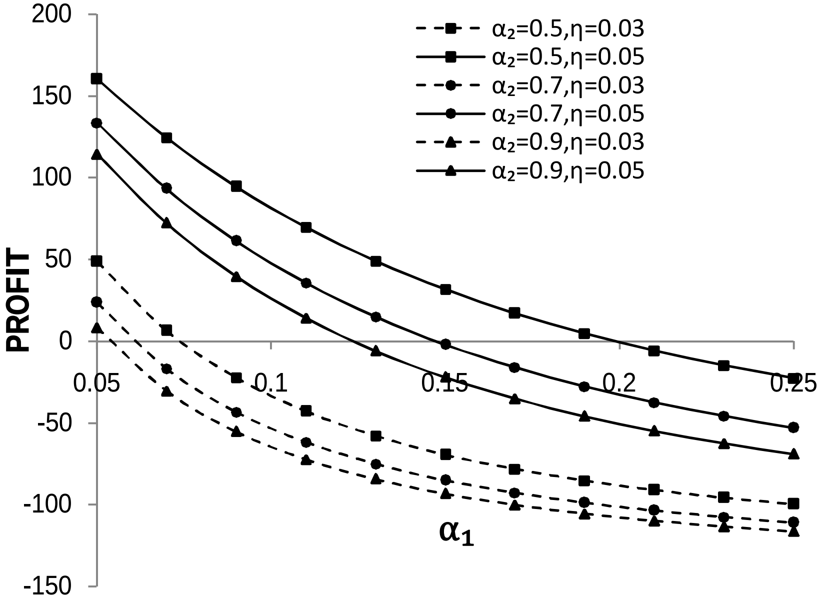

Fig. 3: Behavior of profit for particular case-1 with respect to α_1, α_2 and η

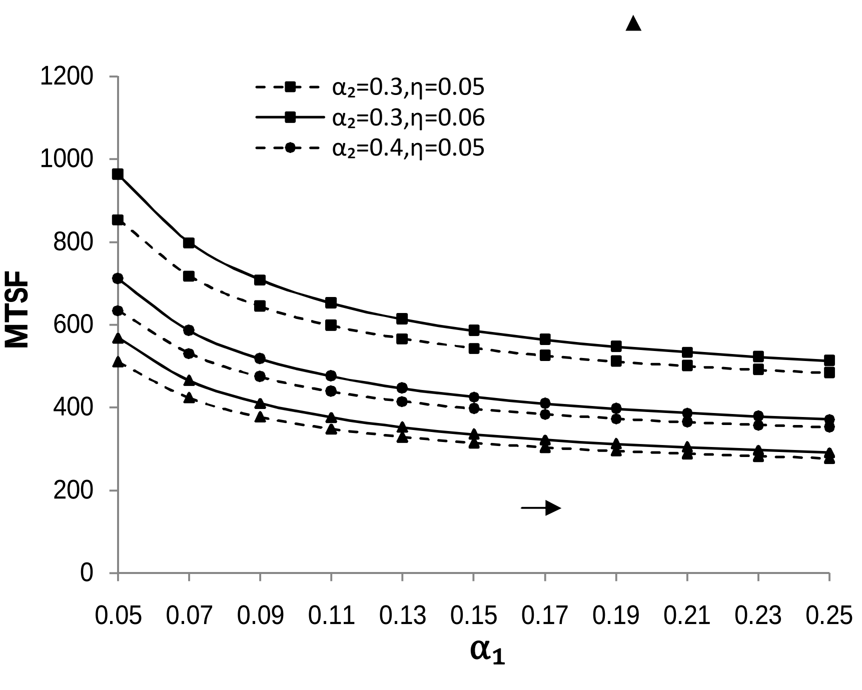

Fig. 4: Behavior of MTSF for particular case-2 with respect to α_1, α_2 and η

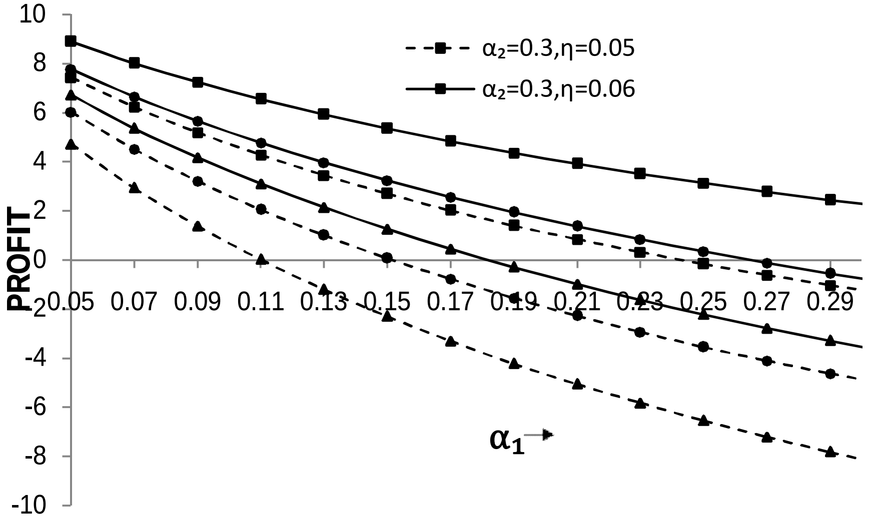

Fig. 5: Behavior of profit for particular case-2 with respect to α_1, α_2 and η

References:

1. Goel, L.R. and P. Gupta, "Stochastic analysis of a deteriorating standby system with three modes and inspection", Microelectron reliability, Vol-25, No. 2, pp. 285-290 (1985).
2. Goel, L.R. and Rakesh Gupta, "Analysis of a two unit cold standby system with three modes and imperfect switching device", Microelectron reliability, Vol-24, No.3, pp. 425-429 (1984).
3. Goel, L.R., Rakesh Gupta and P.K. Tyagi "Analysis of a two- unit standby system with preparation time and correlated failures and repairs", Microelectron reliability, Vol-35, No. 8, pp. 1163-1165 (1995).
4. Gupta, R. and Ram Krishan, "On profit comparison of two stochastic models each pertaining to two unit standby system with fixed preparation time and hyper-exponential repair time distribution", Int. J. of Systems Science, 30(12), pp.1309-1317 (1999).
5. Gupta, S.M., N.K. Jaiswal and L.R. Goel, "Reliability analysis of a two-unit cold standby redundant system with two operating modes", Microelectron reliability, Vol-22, No. 4, pp. 747-758 (1982).
6. Gupta, S.M., N.K. Jaiswal and L.R. Goel, "Stochastic behavior of a standby redundant system with three modes", Microelectron reliability, Vol-23, No. 2, pp. 329-331 (1983).
7. Mogha, A.K. and A.K. Gupta, "A two priority unit warm standby system model with preparation for repair", Aligarh J of Statistics, 22, 73-90 (2002).

8. Mokaddis, G.S. and M.L. Tawfek, “Some characteristics of a two-dissimilar unit cold standby redundant system with three modes”, Microelectron reliability, Vol-36, No. 4, pp. 497-503 (1996).
9. Singh, S.K. and B. Srinivasu, “Stochastic analysis of a two unit cold standby system with preparation time for repair”, Microelectron reliability, Vol-27, pp. 50-60 (1987).
10. Tuteja, R.K. and S.C. Malik, “Reliability and profit analysis of two single unit models with three modes and different repair policies of repairman who appears and disappears randomly”, Microelectron reliability, Vol-32, No. 3, pp. 351-356 (1992).