Journal of Reliability and Statistical Studies; ISSN (Print): 0974-8024, (Online):2229-5666
Vol. 5, Issue 2 (2012): 01-10

A CLASS OF MODIFIED LINEAR REGRESSION ESTIMATORS FOR ESTIMATION OF FINITE POPULATION MEAN

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(Received April 17, 2012)

Abstract

In recent times, a large number of modified ratio estimators are introduced by assuming various population parameters are known. In the same direction we have suggested a class of modified linear regression estimators which are unbiased. We have derived their variances together with the values for which the proposed class of estimators perform better than the usual linear regression estimator and existing modified ratio type estimators. Further we have shown that the estimators from SRSWOR sample and the linear regression estimator are the particular cases of the proposed estimators. The performances of these proposed estimators are also assessed with that of linear regression estimator and some of the existing ratio type estimators for certain natural populations available in the literature. It is observed from the numerical comparisons that the proposed estimators perform better than the existing estimators and linear regression estimator.

Key Words: Auxiliary variable, Mean squared error, Modified ratio estimators, Simple random sampling

1. Introduction

Consider a finite population $U = \{U_1, U_2, \dots, U_N\}$ of N distinct and identifiable units. Let Y is a study variable with value Y_i measured on $U_i, i = 1,2,3, \dots, N$ giving a vector $Y = \{Y_1, Y_2, \dots, Y_N\}$. The problem is to estimate the population mean $\overline{Y}$ on the basis of a random sample selected from the population U. The simplest estimator of population mean is the sample mean obtained from simple random sampling without replacement, when there is no additional information available on the auxiliary variable. It is often be the case that an auxiliary variable X closely related to the study variable Y is available then one can improve the performance of the estimator of the study variable by using the known values of the population parameters of the auxiliary variable. That is, when the populations mean $\overline{X}$ of the auxiliary variable X is known, a number of estimators such as ratio, product and linear regression estimators are proposed in the literature. Theoretically, it has been established that, in general, the regression estimator is more efficient than the ratio and product estimators except when the regression line of the character under study on the auxiliary character passes through the neighbourhood of the origin. In this case the efficiency of these estimators is almost equal. However, due to the stronger intuitive appeal, statisticians are more inclined towards the use of ratio and regression estimators. Perhaps that is why an extensive work has been done in the direction of improving the performance of these estimators. Sen (1993) has presented the historical developments of the ratio method of estimation starting from the year 1662. The readers, who are interested in knowing more details on the chronological developments of the ratio methods of estimation and other related developments are referred to Sisodia and Dwivedi (1981), Sen (1993), Upadhyaya and Singh (1999), Singh (2003), Singh and Tailor (2003, 2005), Singh et al. (2004), Misra et al. (2009), Srivastava and Garg (2009), Tailor et al. (2012) and Subramani and Kumarapandiyan (2012a, 2012b, 2012c).

Before discussing further about the modified ratio estimators and the proposed modified linear regression estimators, the notations to be used in this paper are described below:

- N − Population size
- n − Sample size
- Y − Study variable
- X − Auxiliary variable
- $\bar{X}, \bar{Y}$ − Population means
- $\bar{x}, \bar{y}$ − Sample means
- x, y - Sample totals
- S_X, S_y − Population standard deviations
- C_X, C_y − Coefficient of variations
- ρ − Coefficient of correlation
- $\beta_1 = \frac{N\sum_{i=1}^{N}(X_i-\bar{X})^3}{(N-1)(N-2)S^3}$, Coefficient of skewness of the auxiliary variable
- $\beta_2 = \frac{N(N+1)\sum_{i=1}^{N}(X_i-\bar{X})^4}{(N-1)(N-2)(N-3)S^4} - \frac{3(N-1)^2}{(N-2)(N-3)}$, Coefficient of kurtosis of the auxiliary variable
- $B(.)$ − Bias of the estimator
- $MSE(.)$ − Mean squared error of the estimator
- $\hat{\bar{Y}}_i$-Existing modified ratio estimator of $\bar{Y}$
- $\hat{\bar{Y}}_{SK}$ - Proposed modified linear regression estimator of $\bar{Y}$
- $\hat{\bar{Y}}_{lr}$ − Linear regression estimator of $\bar{Y}$

The classical Ratio estimator for the population mean $\bar{Y}$ of the study variable Y is defined as

$$\hat{\bar{Y}}_R = \frac{\bar{y}}{\bar{x}}\bar{X} = \hat{R}\bar{X} \quad (1.1)$$

where $\hat{R} = \frac{\bar{y}}{\bar{x}} = \frac{y}{x}$ is the estimate of $R = \frac{\bar{Y}}{\bar{X}} = \frac{Y}{X}$. The bias and mean squared error of $\hat{\bar{Y}}_R$ to the first degree of approximation are given below:

$$B\left(\hat{\bar{Y}}_R\right) = \frac{(1-f)}{n}\bar{Y}\left(C_x^2 - C_x C_y \rho\right) \quad (1.2)$$

$$MSE\left(\hat{\bar{Y}}_R\right) = \frac{(1-f)}{n}\bar{Y}^2\left(C_y^2 + C_x^2 - 2C_x C_y \rho\right) \quad (1.3)$$

The usual linear regression estimator and its variance are given as

$$\hat{\bar{Y}}_{lr} = \bar{y} + \hat{\beta}(\bar{X} - \bar{x}) \quad (1.4)$$

$$V\left(\hat{\bar{Y}}_{lr}\right) = \frac{(1-f)}{n}S_y^2(1-\rho^2) \quad (1.5)$$

where $\hat{\beta}$ is the sample regression coefficient of Y on X.

The Ratio estimator given in (1.1) is used for improving the precision of estimator of the population mean compared to simple random sample estimator when there is a positive correlation between X and Y. Further improvements are achieved by introducing a large number of modified ratio type estimators with known Co-efficient of Variation, Co-efficient of Kurtosis, Co-efficient of Skewness, Population Correlation Coefficient, Population Median and their linear combinations. For a more detailed discussion on the ratio estimator, linear regression estimator and their modifications one may refer to Murthy (1967), Cochran (1977), Bhushan et al. (2008), Banerjie and Tiwari (2011), Al-Jararha and Al-Haj Ebrahem (2012), Singh et al. (2012) and Subramani and Kumarapandiyan (2012). The other important and related works are due to Kadilar and Cingi (2004, 2006), Koyuncu and Kadilar (2009), Yan and Tian (2010) and Subramani and Kumarapandiyan (2012d). A list of modified ratio estimators to be compared with proposed estimators are given in Table 1.1 together with their mean squared errors and constants.

Estimators	Mean squared errors $MSE(.)$	Constants R_i
$\hat{\bar{Y}}_1 = \frac{\bar{y} + b(\bar{X} - \bar{x})}{\bar{x}} \bar{X}$ Kadilar and Cingi (2004)	$\frac{(1-f)}{n}\left(R_1^2 S_x^2 + S_y^2(1-\rho^2)\right)$	$R_1 = \frac{\bar{Y}}{\bar{X}}$
$\hat{\bar{Y}}_2 = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x} + C_x)}(\bar{X} + C_x)$ Kadilar and Cingi (2004)	$\frac{(1-f)}{n}\left(R_2^2 S_x^2 + S_y^2(1-\rho^2)\right)$	$R_2 = \frac{\bar{Y}}{\bar{X} + C_x}$
$\hat{\bar{Y}}_3 = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x} + \beta_2)}(\bar{X} + \beta_2)$ Kadilar and Cingi (2004)	$\frac{(1-f)}{n}\left(R_3^2 S_x^2 + S_y^2(1-\rho^2)\right)$	$R_3 = \frac{\bar{Y}}{\bar{X} + \beta_2}$
$\hat{\bar{Y}}_4 = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x} C_x + \beta_2)}(\bar{X} C_x + \beta_2)$ Kadilar and Cingi (2004)	$\frac{(1-f)}{n}\left(R_4^2 S_x^2 + S_y^2(1-\rho^2)\right)$	$R_4 = \frac{\bar{Y} C_x}{\bar{X} C_x + \beta_2}$
$\hat{\bar{Y}}_5 = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x} + \beta_1)}(\bar{X} + \beta_1)$ Yan and Tian (2010)	$\frac{(1-f)}{n}\left(R_5^2 S_x^2 + S_y^2(1-\rho^2)\right)$	$R_5 = \frac{\bar{Y}}{\bar{X} + \beta_1}$
$\hat{\bar{Y}}_6 = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x} \beta_1 + \beta_2)}(\bar{X} \beta_1 + \beta_2)$ Yan and Tian (2010)	$\frac{(1-f)}{n}\left(R_6^2 S_x^2 + S_y^2(1-\rho^2)\right)$	$R_6 = \frac{\bar{Y} \beta_1}{\bar{X} \beta_1 + \beta_2}$
$\hat{\bar{Y}}_7 = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x} + \rho)}(\bar{X} + \rho)$ Kadilar and Cingi (2006)	$\frac{(1-f)}{n}\left(R_7^2 S_x^2 + S_y^2(1-\rho^2)\right)$	$R_7 = \frac{\bar{Y}}{\bar{X} + \rho}$
$\hat{\bar{Y}}_8 = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x} C_x + \rho)}(\bar{X} C_x + \rho)$ Kadilar and Cingi (2006)	$\frac{(1-f)}{n}\left(R_8^2 S_x^2 + S_y^2(1-\rho^2)\right)$	$R_8 = \frac{\bar{Y} C_x}{\bar{X} C_x + \rho}$
$\hat{\bar{Y}}_9 = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x} \rho + C_x)}(\bar{X} \rho + C_x)$ Kadilar and Cingi (2006)	$\frac{(1-f)}{n}\left(R_9^2 S_x^2 + S_y^2(1-\rho^2)\right)$	$R_9 = \frac{\bar{Y} \rho}{\bar{X} \rho + C_x}$
$\hat{\bar{Y}}_{10} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x} \rho + \beta_2)}(\bar{X} \rho + \beta_2)$ Kadilar and Cingi (2006)	$\frac{(1-f)}{n}\left(R_{10}^2 S_x^2 + S_y^2(1-\rho^2)\right)$	$R_{10} = \frac{\bar{Y} \rho}{\bar{X} \rho + \beta_2}$

Table 1.1: Modified ratio type estimators with their mean squared errors and constants

The modified ratio estimators given in Table 1.1are biased but have minimum mean squared errors compared to the usual ratio estimator. The list of estimators given above uses the known values of the parameters like $\overline{X}$, C_x, β_1, β_2, ρ and their linear combinations. It is a well known result that the regression estimator is more efficient than the ratio estimator when the regression line does not pass through the origin. These points have motivated us to introduce a class of modified linear regression type estimators which perform better than the existing modified ratio estimators. The usual linear regression estimator and the estimator based on SRSWOR sample are the particular cases of the proposed estimators. Hence the proposed estimators can be called as a class of modified linear regression type estimators.

2. A Class of Proposed Modified Linear Regression Type Estimators

As we stated earlier one can always improve the performance of the estimator of the study variable by using the values of the known values of the population parameters of the auxiliary variable, which are positively correlated with that of study variable. In this section we have defined a class of modified linear regression estimators.

The proposed class of modified linear regression estimators for population mean $\overline{Y}$ is

$$\hat{\overline{Y}}_{SK} = \alpha\frac{S_y}{C_y} + (1-\alpha)\left(\bar{y} - \frac{b_{yx}}{\rho}(\bar{x} - \overline{X})\right) \tag{2.1}$$

Where $e_1 = \frac{\bar{x}-\overline{X}}{\overline{X}}$, $b_{yx} = \frac{S_{yx}}{S_x^2}$ and α is a suitably chosen scalar.

For the sake of convenience to the readers and to derive the variance expressions, the proposed estimators given in (2.1) can be written in a more compact form as given below:

$$\hat{\overline{Y}}_{SK} = \bar{y} - S_y\left(\frac{\alpha}{C_y}e_0 + \frac{(1-\alpha)}{C_x}e_1\right) \tag{2.2}$$

Where $e_0 = \frac{\bar{y}-\overline{Y}}{\overline{Y}}$ and S_y and C_y are respectively the population variance and coefficient of variation of the study variable $Y$and C_x is the co-efficient of variation of the auxiliary variable X. It is reasonable to assume that the values of S_y and C_y are known. Further we can write $\bar{y} = \overline{Y}(1+e_0)$ and $\bar{x} = \overline{X}(1+e_1)$ such that

$$\left.\begin{aligned} &E[e_0] = E[e_1] = 0 \\ &E[e_0^2] = \frac{(1-f)}{n}C_y^2 \\ &E[e_1^2] = \frac{(1-f)}{n}C_x^2, \\ &E[e_0e_1] = \frac{(1-f)}{n}\rho C_y C_x \text{ and } f = \frac{n}{N} \end{aligned}\right\} \tag{2.3}$$

Taking expectation on both sides of equation (2.2), the expected values of the proposed estimators are obtained as

$$E\left[\hat{\overline{Y}}_{SK}\right] = E\left[\bar{y} - S_y\left(\frac{\alpha}{C_y}e_0 + \frac{(1-\alpha)}{C_x}e_1\right)\right] = \overline{Y} \tag{2.4}$$

Since $E[e_0] = E[e_1] = 0$, the proposed estimators are unbiased modified linear regression estimators. The corresponding variances of the proposed estimators are as given below

$$V\left(\hat{\overline{Y}}_{SK}\right) = \frac{2(1-f)}{n}S_y^2(1-\alpha)^2(1-\rho) \tag{2.5}$$

Particular cases:

(i) At $\alpha = 1 \pm \sqrt{\left(\frac{1}{2(1-\rho)}\right)}$, variance of the proposed estimator is
$V\left(\hat{\bar{Y}}_{SK}\right) = \frac{(1-f)}{n} S_y^2$, which is equal to the variance of SRSWOR mean.

(ii) At $\alpha = 1 \pm \sqrt{\left(\frac{(1+\rho)}{2}\right)}$, variance of the proposed estimator is
$V\left(\hat{\bar{Y}}_{SK}\right) = \frac{(1-f)}{n} S_y^2(1-\rho^2)$, which is equal to the variance of usual linear regression estimator.

3. Efficiency of the Proposed Estimators

The mean squared errors and the constants of the ten modified ratio estimators $\hat{\bar{Y}}_1$ to $\hat{\bar{Y}}_{10}$ listed in the table 1.1 are grouped into a single class, which will be very much useful for comparing with that of proposed estimators are given below:

$$MSE\left(\hat{\bar{Y}}_i\right) = \frac{(1-f)}{n}\left(R_i^2 S_x^2 + S_y^2(1-\rho^2)\right) \quad i = 1, 2, 3, 4, 5, 6, 7, 8, 9 \text{ and } 10 \tag{3.1}$$

Where $R_1 = \frac{\bar{Y}}{\bar{X}}$, $R_2 = \frac{\bar{Y}}{\bar{X}+C_x}$, $R_3 = \frac{\bar{Y}}{\bar{X}+\beta_2}$, $R_4 = \frac{\bar{Y}C_x}{\bar{X}C_x+\beta_2}$, $R_5 = \frac{\bar{Y}}{\bar{X}+\beta_1}$, $R_6 = \frac{\bar{Y}\beta_1}{\bar{X}\beta_1+\beta_2}$, $R_7 = \frac{\bar{Y}}{\bar{X}+\rho}$, $R_8 = \frac{\bar{Y}C_x}{\bar{X}C_x+\rho}$, $R_9 = \frac{\bar{Y}\rho}{\bar{X}\rho+C_x}$ and $R_{10} = \frac{\bar{Y}\rho}{\bar{X}\rho+\beta_2}$

As mentioned earlier, the variance of usual linear regression estimator is given as

$$V\left(\hat{\bar{Y}}_{lr}\right) = \frac{(1-f)}{n} S_y^2(1-\rho^2) \tag{3.2}$$

The variance of the proposed estimators are given as

$$V\left(\hat{\bar{Y}}_{SK}\right) = \frac{2(1-f)}{n} S_y^2(1-\alpha)^2(1-\rho) \tag{3.3}$$

At various values of α, variances of the proposed estimators are listed in the following table:

α	$V\left(\hat{\bar{Y}}_{SK}\right)$
$\alpha_1 = 1 \pm \sqrt{\frac{1}{2}}$	$\frac{(1-f)}{n} S_y^2 (1-\rho)$
$\alpha_2 = 1 \pm \frac{1}{S_y}\sqrt{\frac{1}{2}\left(\frac{n}{(1-f)}\right)}$	$(1-\rho)$
$\alpha_3 = 1 \pm \frac{1}{S_y}\sqrt{\frac{1}{2}}$	$\frac{(1-f)}{n} (1-\rho)$

Table-3.1: **Variances of the proposed estimators at various values of α**

From the expressions given in (3.1) and (3.3), we have derived the conditions in terms of α and correlation coefficient ρ for which the proposed estimators $\hat{\bar{Y}}_{SK}$ are more efficient than that of existing modified ratio type estimators $\hat{\bar{Y}}_i$; $i = 1, 2, 3, 4, 5, 6, 7, 8, 9$ and 10 listed in Table 1.1 and are given below:

That is, $V\left(\hat{\bar{Y}}_{SK}\right) < MSE\left(\hat{\bar{Y}}_i\right)$

$$\text{if } 1 - \sqrt{\frac{1}{2}\left[(1+\rho) + \frac{R_i S_x^2}{(1-\rho) S_y^2}\right]} < \alpha < 1 + \sqrt{\frac{1}{2}\left[(1+\rho) + \frac{R_i S_x^2}{(1-\rho) S_y^2}\right]} \text{ or} \quad (3.4)$$

$$\rho < (1-\alpha)^2 + \sqrt{\frac{R_i^2 S_x^2}{S_y^2} + [1-(1-\alpha)^2]^2}; \; i = 1, 2, 3, 4, 5, 6, 7, 8, 9 \text{ and } 10 \quad (3.5)$$

From the expressions given in (3.2) and (3.3) we have derived the conditions for which the proposed estimators $\hat{\bar{Y}}_{SK}$ are more efficient than the usual linear regression estimator as given below:

$$\text{That is, } V\left(\hat{\bar{Y}}_{SK}\right) < V\left(\hat{\bar{Y}}_{lr}\right) \text{ if } 1 - \sqrt{\frac{1}{2}(1+\rho)} < \alpha < 1 + \sqrt{\frac{1}{2}(1+\rho)} \quad (3.6)$$

4. Empirical Study

The performances of the proposed modified linear regression estimators are assessed and compared to that of the modified ratio estimators listed in Table 1.1 based on for six natural populations. The populations 1 and 2 are taken from Murthy (1967) in page 228, population 3 is taken from Murthy (1967) in page 422, population 4 is taken from Murthy (1967) in page 178, population 5 is taken from Cochran (1977) in page 325 and population 6 is taken from Koyuncu and Kadilar (2009). The population parameters and the constants obtained from the above data are given below:

Population-1: Murthy (1967) page 228

X= Fixed Capital Y= Output for 80 factories in a region

$N = 80$	$n = 20$	$\bar{Y} = 51.8264$	$\bar{X} = 11.2646$
$\rho = 0.9413$	$S_y = 18.3569$	$C_y = 0.3542$	$S_x = 8.4563$
$C_x = 0.7507$	$\beta_2 = -0.06339$	$\beta_1 = 1.05$	

Population-2: Murthy (1967) page 228

X = Data on number of workers Y= Output for 80 factories in a region

$N = 80$	$n = 20$	$\bar{Y} = 51.8264$	$\bar{X} = 2.8513$
$\rho = 0.9150$	$S_y = 18.3569$	$C_y = 0.3542$	$S_x = 2.7042$
$C_x = 0.9484$	$\beta_2 = 1.3005$	$\beta_1 = 0.6978$	

Population-3: Murthy (1967) page 422

Number of Cattle for 24 sample villages X- Census Y- Survey

$N = 24$	$n = 12$	$\bar{Y} = 568.5833$	$\bar{X} = 568.25$
$\rho = 0.9589$	$S_y = 506.1174$	$C_y = 0.8901$	$S_x = 528.0501$
$C_x = 0.9293$	$\beta_2 = 2.2559$	$\beta_1 = 1.6651$	

Population-4: Murthy (1967) page 178

X- Geographical area Y- Area under winter paddy

$N = 108$	$n = 16$	$\bar{Y} = 172.3704$	$\bar{X} = 461.3981$
$\rho = 0.7896$	$S_y = 134.3567$	$C_y = 0.7795$	$S_x = 318.5022$
$C_x = 0.6903$	$\beta_2 = 1.6307$	$\beta_1 = 1.3612$	

Population-5: Cochran (1977) page 325
X =Number of rooms and Y=Number of persons

N = 10	n = 4	$\bar{Y}$ = 101.1	$\bar{X}$ = 58.8
ρ = 0.6515	S_y = 14.6523	C_y = 0.1449	S_x = 7.5339
C_x= 0.1281	β_2=-0.3814	β_1 = 0.5764	

Population-6: Koyuncu and Kadilar (2009)
X =Number of students and Y=Number of teachers

N = 923	n = 180	$\bar{Y}$ = 436.4345	$\bar{X}$ = 11440.498
ρ = 0.9543	S_y = 749.9394	C_y = 1.7183	S_x = 21331.131
C_x= 1.8645	β_2=18.7208	β_1 = 3.9365	

The constants and mean squared errors of the existing modified ratio estimators and the variance of the proposed estimators (at different values of α) and linear regression estimators for the six populations discussed above are given in the Tables 4.1 and Table 4.2 respectively.

Existing Modified Ratio Estimator	Constants of the estimators					
	Populations					
	1	2	3	4	5	6
$\widehat{\bar{Y}}_1$ Kadilar and Cingi (2004)	4.6008	18.1764	1.0005	0.3735	1.7193	0.0381
$\widehat{\bar{Y}}_2$ Kadilar and Cingi (2004)	4.3133	13.6396	0.9989	0.3730	1.7156	0.0381
$\widehat{\bar{Y}}_3$ Kadilar and Cingi (2004)	4.6268	12.4828	0.9966	0.3722	1.7306	0.0380
$\widehat{\bar{Y}}_4$ Kadilar and Cingi (2004)	4.6355	12.2737	0.9963	0.3716	1.8110	0.0381
$\widehat{\bar{Y}}_5$ Yan and Tian (2010)	4.2085	14.6026	0.9976	0.3725	1.7026	0.0381
$\widehat{\bar{Y}}_6$ Yan and Tian (2010)	4.6256	10.9917	0.9982	0.3726	1.7389	0.0381
$\widehat{\bar{Y}}_7$ Kadilar and Cingi (2006)	4.2460	13.7605	0.9989	0.3729	1.7005	0.0381
$\widehat{\bar{Y}}_8$ Kadilar and Cingi (2006)	4.1399	13.5810	0.9987	0.3726	1.5825	0.0381
$\widehat{\bar{Y}}_9$ Kadilar and Cingi (2006)	4.2966	13.3305	0.9988	0.3728	1.7136	0.0381
$\widehat{\bar{Y}}_{10}$ Kadilar and Cingi (2006)	4.6284	12.1299	0.9964	0.3719	1.7366	0.0380

Table 4.1: Constants of the existing modified ratio estimators

Estimator	Mean squared errors of estimators					
	Populations					
	1	2	3	4	5	6
$\widehat{\bar{Y}}_1$ Kadilar and Cingi (2004)	58.2026	92.6563	12491.1400	1115.6570	43.7044	3185.9900
$\widehat{\bar{Y}}_2$ Kadilar and Cingi (2004)	51.3313	53.0736	12453.1900	1113.4060	43.5951	3185.0250
$\widehat{\bar{Y}}_3$ Kadilar and Cingi (2004)	58.8469	44.7874	12399.3300	1110.3570	44.0341	3176.3220

$\hat{\bar{Y}}_4$ Kadilar and Cingi (2004)		59.0633	43.3674	12392.3900	1107.9970	46.4609	3180.7980
$\hat{\bar{Y}}_5$ Yan and Tian (2010)		48.9356	60.5325	12423.2700	1111.2290	43.2181	3183.9530
$\hat{\bar{Y}}_6$ Yan and Tian (2010)		58.8160	35.1887	12435.8700	1111.7580	44.2806	3183.5290
$\hat{\bar{Y}}_7$ Kadilar and Cingi (2006)		49.7853	53.9825	12451.9800	1113.0830	43.1558	3185.4960
$\hat{\bar{Y}}_8$ Kadilar and Cingi (2006)		47.4010	52.6365	12449.0100	1111.9330	39.8564	3185.7250
$\hat{\bar{Y}}_9$ Kadilar and Cingi (2006)		50.9448	50.7876	12451.5600	1112.8080	43.5369	3184.9780
$\hat{\bar{Y}}_{10}$ Kadilar and Cingi (2006)		58.8875	42.4051	12395.4200	1108.9540	44.2132	3175.8600
$\hat{\bar{Y}}_{lr}$ (Linear Regression Estimator)		1.4399	2.0569	859.2285	361.9124	18.5273	224.6250
$\hat{\bar{Y}}_{SK}$	$at\ \alpha_1$	0.7418	1.0741	438.6280	202.2309	11.2185	114.9389
	$at\ \alpha_2$	0.0587	0.0850	0.0411	0.2104	0.3485	0.0457
	$at\ \alpha_3$	0.0022	0.0031	0.0017	0.0112	0.0523	0.0002

Table 4.2: Mean squared errors of the existing modified ratio estimators, linear regression and proposed estimators

From the values of Table 4.2, it is observed that the variances of the proposed estimators are less than the mean squared errors of all the 10 existing modified ratio estimators and the variance of the linear regression estimator. That is, the proposed modified linear regression estimators are performing better than the existing modified ratio type estimators as well as usual linear regression estimator.

5. Conclusion

In this paper we have suggested a class of modified linear regression estimators and also obtained their variances. We have also derived the conditions for which proposed estimators perform better than the existing modified ratio estimators and linear regression estimator. Further we have also assessed the performances of the proposed estimators with that of the existing modified ratio estimators and the linear regression estimator for certain known populations. From the numerical comparisons, we have observed that the proposed estimators are performed better than the other existing modified ratio estimators and linear regression estimator and hence we recommend the proposed estimators for the use of practical applications.

Acknowledgement

The authors are thankful to the referees whose constructive comments led to improvement in the paper. The authors wish to record their gratitude and thanks for the financial assistance received from UGC-MRP and Department of Science and Technology (DST), New Delhi.

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