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SINGLE SAMPLING PLANS FOR VARIABLES INDEXED BY AQL AND AOQL UNDER EWMA MODEL

R. Sankle[1] and J. R. Singh[2]
School of Studies in Statistics, Vikram University, Ujjain, India
E Mail : [1]rajshreesankle@gmail.com; [2]jrsinghstat@gmail.com
School of Studies in Statistics,
Vikram University ,Ujjain (M.P.)

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Abstract

The objective of this paper is to provide expression for evaluating sample size(n), acceptance parameter(k), Average Outgoing Quality(AOQ) and Operating Characteristic(OC) function under Exponentially Weighted Moving Average (EWMA) model. The paper provides an investigation into the robustness of single sampling procedure indexed by Acceptance Quality Level(AQL) and Average Outgoing Quality Level(AOQL).

Key Words: Sampling Plans , EWMA model, AQL ,AOQL.

1. Introduction

The acceptance sampling plans is primarily concerned with the protection that a sampling inspection program gives a consumer against accepting individual bad lots and the incentive that may be given a supplier to produce acceptable quality through high rates of acceptance of good product and low rates of acceptance of poor product. An inspection programs is in which inspection itself plays an important role in affecting the final quality of outgoing product. In lot-by-lot sampling such schemes generally call for corrective inspection of rejected lots. In general such programs have the intention of correcting or eliminating through inspection, if necessary, a sufficient number of defective items to attain a specified quality objective. These are called “rectifying inspection” programs. A rectifying inspection scheme will be of interest to a manufacturer that wishes to know about the average quality of product that is likely to result at a given stage of manufacture from the combinations of production, sampling inspection, and rectification of rejected lots. Most rectifying inspection plans for lot-by-lot sampling call for 100 percent inspection of rejected lots. In rectifying inspection, 100 percent inspection is restricted to rejected lots and this will in most cases be a small percentage of all product submitted for inspection. The volume of inspection therefore, will likely be much less than that under final 100 percent inspection. We shall assume for simplicity that the 100 percent inspection of rejected product is perfect inspection. We shall also assume that defective items found during both sampling and 100 percent inspection are replaced by good ones. These assumptions will keep the analysis reasonably uncomplicated. With 100 percent inspection of rejected lots, two features of rectifying inspection become a principal importance. One relates to the average quality of the material turned out by the combination of sampling and 100 percent inspection. The other relates to the average amount of inspection required by the program. The variables sampling plans have the primary advantage that the same OC curve can be obtained with a smaller sample than is required by an attributes plan. The precise measurements required by a variables plan will probably cost more than the simple classification of items required by an attributes plan, but the reduction in sample size may more than offset this extra expense. Continuous sampling plans are used when the production is continuous and the formation of inspection lots for lot-by-lot inspection is impractical as in manufacturing industries like (i) confectionery and food industries (see, [2]) and (ii) ammunition loading and component manufacture (see, [1], [11] and [12]). For such production processes [7], devised a continuous sampling plan known as CSP-1. [8] studied the case in which rejected product was recycled (emptied and refilled) so that it would be sold in the primary market. Recently, in a related paper, [15] used Golhar's model to evaluate the economic effects of process variance reduction. The EWMA chart, introduced by [13], may be more difficult to interpret than an $\bar{X}$ chart but is more effective in detecting small shifts in the process mean(see, [9]), [14], [3],[4],[5] and [6]) and [10]. [9] points out that the EWMA chart for sample averages can be nicely graphed simultaneously with the Shewhart chart to enable easier interpretation.

Here we have given an expression for evaluating sample size, acceptance parameter, AOQ and OC function under EWMA model.

2. Model Description

For the EWMA model the sample statistic is a weighted average of the current observation Z_t and all previous observations with the current observation receiving the most weight(λ), i.e.,

$$X_t = \lambda Z_t + (1-\lambda)X_{t-1}, \quad 0 < \lambda < 1 \tag{2.1}$$

where $X_0 = \mu$. As in moving range process, the Z_t are independent but the sample statistics are autocorrelated. Note, though, the degree of autocorrelation is greater in the EWMA model than in moving range process because each sample statistic depends on all previous observations not only the past *n* observations. The mean and variance of X_t and the asymptotic variance as t $\rightarrow \infty$ are

$$E(X_t) = \mu$$
$$Var(X_t) = \sigma^2 \left\{\frac{\lambda}{(2-\lambda)}\right\}(1-(1-\lambda)^{2t})$$

$$\lim_{t\to\infty} Var(X_t) = \sigma^2 \left\{\frac{\lambda}{(2-\lambda)}\right\} = \sigma^2 T^2, \tag{2.2}$$

where $T^2 = \frac{\lambda}{(2-\lambda)}$

When the weight λ in equation (2.1) is set equal to 0.2 the asymptotic standard of X_t is $\left(\frac{1}{3\sigma}\right)$.A variables sampling plan is usually used with a single characteristic which is measurable on a continuous scale and distributed normally with mean μ and standard deviation σ. We assume that the purpose of inspection is to control the fraction of nonconforming items, and that there is both specification limits L or U (lower or upper) which is used for determining whether a unit is conforming. The criterion for the σ - known method is accept the lot if

$$\bar{x} + k\sigma \leq U, \tag{2.3}$$

where $\bar{x}$ is the sample mean of sample size n. When used with a lower specification limit L, the inequalities are reversed for the purpose of acceptance, i.e., accept if

$$\bar{x} - k\sigma \geq L, \quad (2.4)$$

The fraction nonconforming in a given lot will be

$$\Phi(-K_p) = p \quad (2.5)$$

with $K_p = \frac{U-\mu}{\sigma}$ (2.6)

$$\Phi(y) = \int_{-\infty}^{y} \left(\frac{1}{\sqrt{2\pi}}\right) exp\left(-\frac{1}{2}z^2\right) dz$$

The probability of acceptance for fraction nonconforming under EWMA model will be,

$$P_a(p) = \Phi(w) \quad (2.7)$$

with $w = (K_p - k)\frac{\sqrt{n}}{T}$ (2.8)

If the quality of the accepted lot is p and all nonconforming units found in the rejected lots are replaced by conforming units in a rectifying inspection scheme. The determination of AOQ function is straight forward process which involves the use of the well known formula,

$$AOQ = \left(1 - \frac{n}{N}\right) pP_a(p) \quad (2.9)$$

where $P_a(p)$ is the OC function corresponding to fraction defective p. The equation (2.9) can be approximated as,

$$AOQ = pP_a(p) \quad (2.10)$$

If p_m is the proportion nonconforming at which AOQ is maximum, then AOQL under EWMA model is,

$$AOQL = p_m P_a(p_m) \quad (2.11)$$

In single sampling plan p_1 is the AQL (percent nonconforming) that is considered acceptable, called the Producer's Quality Level (PQL), and α is the probability that a lot of quality p_1 will be rejected (producer's risk). The p_2 is the Consumer's Quality Level (unacceptable level) and β is the consumer's risk or the probability that a product of nonconforming quality level p_2 is accepted. If p_1 is prescribed, then the corresponding value of K_{AQL} or K_1 will be fixed and if $P_a(p_1)$ is fixed at 95%, then

$$w_{AQL} = w_1 = 1.645$$

Hence, we have

$$1.645 = (K_1 - k)\frac{\sqrt{n}}{T}, \quad (2.12)$$

so that for a given AQL, k is determined by sample size n.

3. Numerical Illustration and Results

The plan parameters of single sampling plans under EWMA model is presented in Table-1. For example, if the AQL is fixed at 1% and AOQL is fixed at 1.25 % and λ = 0.2, 0.4, 0.6 and 1, Table-1 yields n = 5, 7, 10 and 25 and k = 2.47, 2.27, 2.14 and 1.994 respectively. It shows that under EWMA model the sample sizes is reduced to get the OC function along with the acceptance parameter is increases. Let there exist an upper specification limit U = 10.0 and a unit for which the quality characteristic $x > U$ is considered as nonconforming. In such a case Table-2 shows that the performance characteristic of the plan with above (n , k) values under rectifying inspection scheme. If λ = 1 in EWMA model, the true process average quality is operating at AQL (μ = 5.35), then 95% of the lots submitted will be accepted during the sampling inspection stage itself and only 5% of the rejected lots will be rectified by replacing nonconforming units with conforming units.

If we consider EWMA model, 5% of the lot will be rejected when the sample sizes are n = 10, 7 and 5 when λ = 0.6, 0.4 and 0.2 respectively. Therefore, we conclude that EWMA model quickly response the acceptance of a lot with greater probability of acceptance. The assumption underlying the AOQL principle is that the homogeneity in the qualities of individual lots is unimportant and only the average quality matters. From Table-3 at AQL of 0.25 %, and AOQL of 1.25 % and λ = 0.2, 0.4, 0.6 and 1.0, then $P_a(p_m)$ is 0.233, 0.264, 0.262 and 0.338 respectively. These values shows that under EWMA model at $p = p_m$ the rate of acceptance of a lot is low therefore, the producer feel pressure for the improvement of the submitted lot quality, which is clear from Figure-1 and Figure-2, which gives the OC and AOQ curves. A very large initial data set is required before the properties of EWMA sampling plan indexed by AQL and AOQL, with estimated process parameters are similar to those calculated under the assumption that parameters are known.

λ	AOQL(%)	AQL(%)											
		0.040	0.065	0.100	0.150	0.250	0.400	0.650	1.000	1.500	2.500	4.000	6.500
0.2	0.040	9,3.467	15,3.376	29,3.301									
	0.080	6,3.429	9,3.334	13,3.252	24,3.178								
	0.125	5,3.393	6,3.296	8,3.210	12,3.130	25,3.034							
	0.200	4,3.355	4,3.257	5,3.168	7,3.085	11,2.981	23,2.889						
	0.320	3,3.315	3,3.216	4,3.126	5,3.041	7,2.932	10,2.833	22,2.735					
	0.500	2,3.275	3,3.175	3,3.085	4,2.998	5,2.887	6,2.784	10,2.678	19,2.587				
	0.800	2,3.241	2,3.130	2,3.039	3,2.951	3,2.839	4,2.733	6,2.622	8,2.524	15,2.434			
	1.250		2,3.083	2,2.991	2,2.903	2,2.790	3,2.683	4,2.570	5,2.468	7,2.371	15,2.253		
	2.000				2,2.848	2,2.734	2,2.626	3,2.511	3,2.407	4,2.307	7,2.180	13,2.066	
	3.200						2,2.562	2,2.447	2,2.342	3,2.240	4,2.108	6,1.985	12,1.860
	5.000								2,2.272	2,2.170	2,2.036	3,1.909	5,1.774
	8.000										2,1.948	2,1.820	3,1.681
0.4	0.040	14,3.317	27,3.253										
	0.080	8,3.255	13,3.180	24,3.121	91,2.992								
	0.125	6,3.198	8,3.118	12,3.050	20,2.992								
	0.200	4,3.139	5,3.055	7,2.982	10,2.916	20,2.840							
	0.320	3,3.078	4,2.992	5,2.916	6,2.845	10,2.758	18,2.686						
	0.500	2,3.018	3,2.930	4,2.852	4,2.778	6,2.686	9,2.603	18,2.528					
	0.800	2,2.950	2,2.861	3,2.781	3,2.706	4,2.610	5,2.522	8,2.434	15,2.363	57,2.324			
	1.250		2,2.790	2,2.710	2,2.633	3,2.535	4,2.444	5,2.350	7,2.269	12,2.199	92,2.156		
	2.000			2,2.626	2,2.549	2,2.450	2,2.357	3,2.259	4,2.173	6,2.093	12,2.001		
	3.200					2,2.354	2,2.261	2,2.161	3,2.072	3,1.987	5,1.882	10,1.793	
	5.000							2,2.057	2,1.966	2,1.879	3,1.769	5,1.668	9,1.572
	8.000										2,1.636	2,1.529	4,1.420
0.6	0.040	19,3.221	59,3.195										
	0.080	10,3.134	18,3.082	47,3.057									
	0.125	7,3.056	10,2.995	16,2.948	38,2.921								
	0.200	5,2.976	6,2.909	9,2.852	14,2.805	40,2.771							
	0.320	4,2.895	4,2.824	6,2.762	8,2.706	13,2.645	37,2.614						
	0.500	3,2.815	3,2.741	4,2.676	5,2.616	7,2.544	13,2.488	39,2.458					
	0.800	2,2.725	2,2.649	3,2.582	3,2.519	5,2.441	7,2.373	11,2.313	30,2.284				
	1.250		2,2.555	2,2.487	3,2.422	3,2.341	4,2.267	6,2.196	10,2.141	23,2.109			
	2.000			2,2.377	2,2.311	2,2.228	3,2.151	4,2.073	5,2.007	8,1.952	23,1.911		
	3.200					2,2.103	2,2.024	2,1.943	3,1.872	4,1.807	7,1.735	20,1.698	
	5.000							2,1.805	2,1.732	3,1.663	4,1.579	6,1.511	19,1.477
	8.000									2,1.490	2,1.401	3,1.322	5,1.250
1	0.040	47,3.084											
	0.080	17,2.900	46,2.972										
	0.125	9,2.754	16,2.807	40,2.831									
	0.200	6,2.608	9,2.661	14,2.652	30,2.670								
	0.320	4,2.464	6,2.523	8,2.500	12,2.489	31,2.513							
	0.500	3,2.324	4,2.392	5,2.361	7,2.338	12,2.327	31,2.355						
	0.800	2,2.168	3,2.249	4,2.213	4,2.183	6,2.154	11,2.145	29,2.178					
	1.250		2,2.104	3,2.065	3,2.031	4,1.994	6,1.968	10,1.961	25,1.994				
	2.000			2,2.006	2,1.860	3,1.817	4,1.783	5,1.748	8,1.741	18,1.788			
	3.200					2,1.623	2,1.585	3,1.551	4,1.529	6,1.522	17,1.561		
	5.000							2,1.337	3,1.308	3,1.288	6,1.284	15,1.332	
	8.000								2,1.044	2,1.019	3,1.998	5,0.999	13,1.060

Table-1: Single Sampling Plans for Variables Indexed by AQL and AOQL under EWMA Model

λ	μ	v'	p(%)	w	Pa	AOQ
0.2	4.5736	2.7132	0.33	1.6450	0.9500	0.3165
	4.6000	2.7000	0.35	1.5565	0.9402	0.3260
	4.8000	2.6000	0.47	0.8856	0.8121	0.3785
	5.0000	2.5000	0.62	0.2147	0.5850	0.3633
	5.2000	2.4000	0.82	-0.4562	0.3241	0.2657
	5.4000	2.3000	1.07	-1.1271	0.1299	0.1393
	5.6000	2.2000	1.39	-1.7980	0.0361	0.0502
	5.8000	2.1000	1.79	-2.4689	0.0068	0.0121
	6.0000	2.0000	2.28	-3.1398	0.0008	0.0019
0.4	4.8402	2.5799	0.49	1.6450	0.9500	0.4695
	5.0000	2.5000	0.62	1.2223	0.8892	0.5522
	5.2000	2.4000	0.82	0.6932	0.7559	0.6197
	5.4000	2.3000	1.07	0.1640	0.5651	0.6061
	5.6000	2.2000	1.39	-0.3651	0.3575	0.4971
	5.8000	2.1000	1.79	-0.8943	0.1856	0.3315
	6.0000	2.0000	2.28	-1.4234	0.0773	0.1759
	6.2000	1.9000	2.87	-1.9526	0.0254	0.0730
	6.4000	1.8000	3.59	-2.4817	0.0065	0.0235
0.6	5.0369	2.4815	0.65	1.6450	0.9500	0.6214
	5.2000	2.4000	0.82	1.2511	0.8946	0.7333
	5.4000	2.3000	1.07	0.7680	0.7788	0.8352
	5.6000	2.2000	1.39	0.2850	0.6122	0.8511
	5.8000	2.1000	1.79	-0.1980	0.4215	0.7530
	6.0000	2.0000	2.28	-0.6811	0.2479	0.5640
	6.2000	1.9000	2.87	-1.1641	0.1222	0.3509
	6.4000	1.8000	3.59	-1.6472	0.0498	0.1788
	6.6000	1.7000	4.46	-2.1302	0.0166	0.0739
1	5.3540	2.3230	1.01	1.6450	0.9500	0.9585
	5.4000	2.3000	1.07	1.5300	0.9370	1.0048
	5.6000	2.2000	1.39	1.0300	0.8485	1.1797
	5.8000	2.1000	1.79	0.5300	0.7019	1.2540
	6.0000	2.0000	2.28	0.0300	0.5120	1.1647
	6.2000	1.9000	2.87	-0.4700	0.3192	0.9166
	6.4000	1.8000	3.59	-0.9700	0.1660	0.5965
	6.6000	1.7000	4.46	-1.4700	0.0708	0.3154
	6.8000	1.6000	5.48	-1.9700	0.0244	0.1338

Table-2: Performance Characteristics of the Variables Plan under EWMA Model
For AQL=.01, AOQL=.0125, U=10, S.D=2

λ	AOQL(%)	AQL(%)											
		0.040	0.065	0.100	0.150	0.250	0.400	0.650	1.000	1.500	2.500	4.000	6.500
0.2	0.050	0.346	0.457	0.605									
	0.080	0.276	0.352	0.448	0.579								
	0.125	0.230	0.285	0.353	0.440	0.607							
	0.200	0.196	0.237	0.285	0.346	0.454	0.609						
	0.320	0.174	0.204	0.240	0.283	0.358	0.457	0.617					
	0.500	0.160	0.184	0.211	0.244	0.298	0.367	0.471	0.612				
	0.800	0.152	0.171	0.192	0.217	0.257	0.306	0.376	0.465	0.589			
	1.250		0.166	0.183	0.202	0.233	0.269	0.320	0.381	0.461	0.617		
	2.000				0.196	0.220	0.247	0.284	0.327	0.381	0.477	0.619	
	3.200						0.239	0.266	0.297	0.335	0.399	0.486	0.629
	5.000								0.287	0.314	0.359	0.417	0.505
	8.000										0.345	0.383	0.439
0.4	0.050	0.435	0.594										
	0.080	0.338	0.442	0.581	0.798								
	0.125	0.275	0.348	0.440	0.565								
	0.200	0.229	0.282	0.345	0.427	0.582							
	0.320	0.199	0.238	0.283	0.340	0.441	0.584						
	0.500	0.179	0.210	0.244	0.285	0.356	0.451	0.601					
	0.800	0.168	0.191	0.217	0.247	0.298	0.363	0.459	0.589	0.799			
	1.250		0.182	0.202	0.226	0.264	0.311	0.377	0.461	0.577	0.856		
	2.000			0.196	0.214	0.243	0.277	0.324	0.381	0.454	0.595		
	3.200					0.236	0.261	0.295	0.335	0.384	0.472	0.599	
	5.000							0.285	0.314	0.348	0.408	0.488	0.619
	8.000										0.377	0.428	0.505
0.6	0.050	0.514	0.730										
	0.080	0.390	0.521	0.707									
	0.125	0.312	0.402	0.518	0.687								
	0.200	0.257	0.319	0.397	0.499	0.708							
	0.320	0.219	0.265	0.319	0.388	0.515	0.709						
	0.500	0.195	0.230	0.270	0.319	0.406	0.525	0.733					
	0.800	0.180	0.206	0.237	0.272	0.333	0.412	0.533	0.712				
	1.250		0.194	0.217	0.244	0.262	0.345	0.426	0.533	0.690			
	2.000			0.208	0.229	0.249	0.302	0.357	0.427	0.519	0.714		
	3.200						0.279	0.319	0.366	0.426	0.537	0.716	
	5.000							0.303	0.336	0.377	0.449	0.552	0.739
	8.000									0.356	0.404	0.466	0.567
1	0.050	0.700											
	0.080	0.501	0.727										
	0.125	0.389	0.515	0.700									
	0.200	0.311	0.395	0.505	0.663								
	0.320	0.258	0.318	0.392	0.489	0.696							
	0.500	0.226	0.270	0.322	0.389	0.510	0.714						
	0.800	0.203	0.236	0.274	0.321	0.402	0.514	0.719					
	1.250		0.227	0.246	0.281	0.338	0.413	0.530	0.702				
	2.000			0.104	0.255	0.297	0.349	0.425	0.524	0.672			
	3.200					0.276	0.313	0.365	0.428	0.514	0.696		
	5.000							0.336	0.379	0.435	0.540	0.714	
	8.000								0.358	0.395	0.460	0.553	0.720

Table-3 : $P_a(p_m)$ Values of Known Sigma Plans under EWMA Model

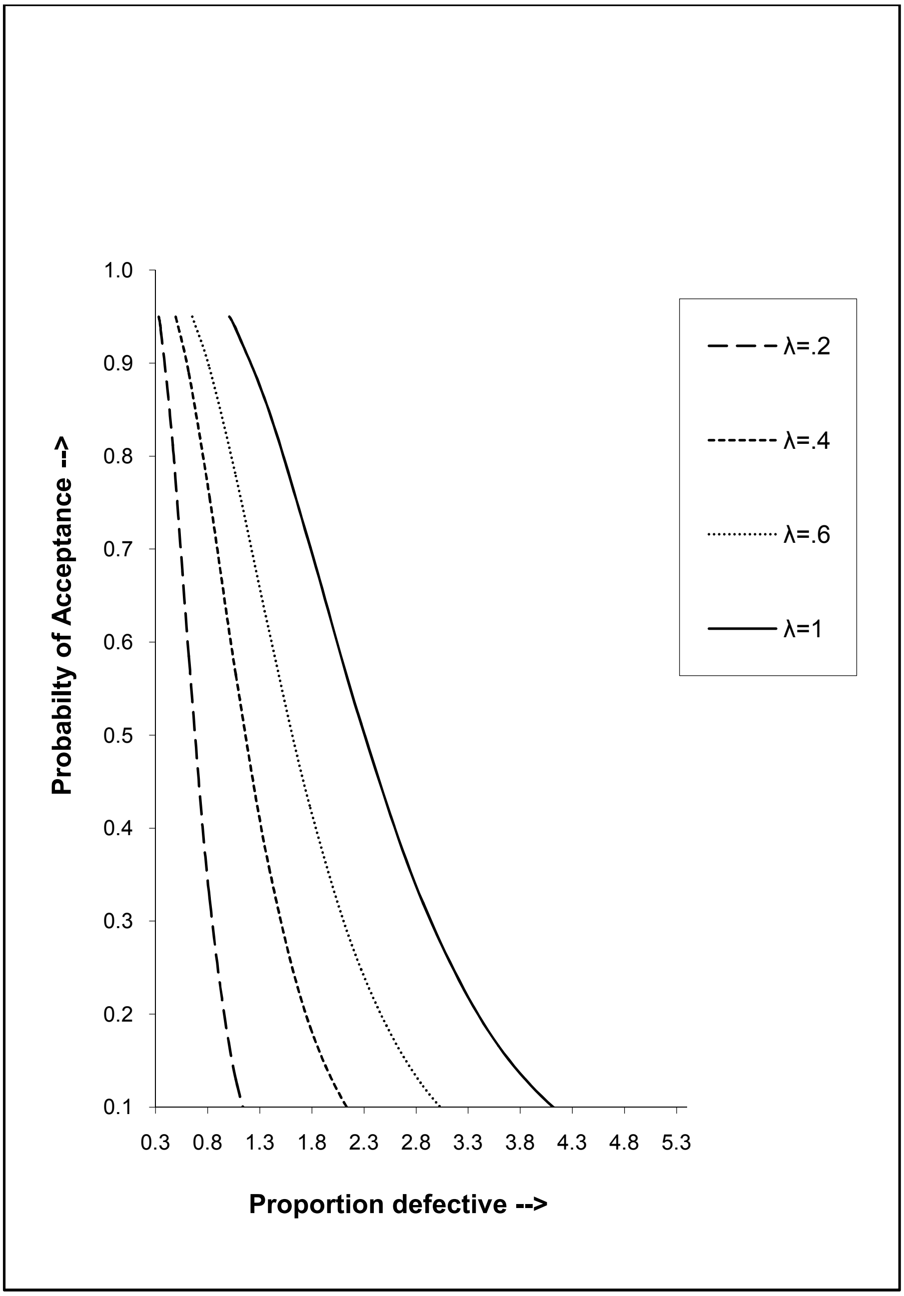

Fig. 1: Operating Characteristic Curves under EWMA Model

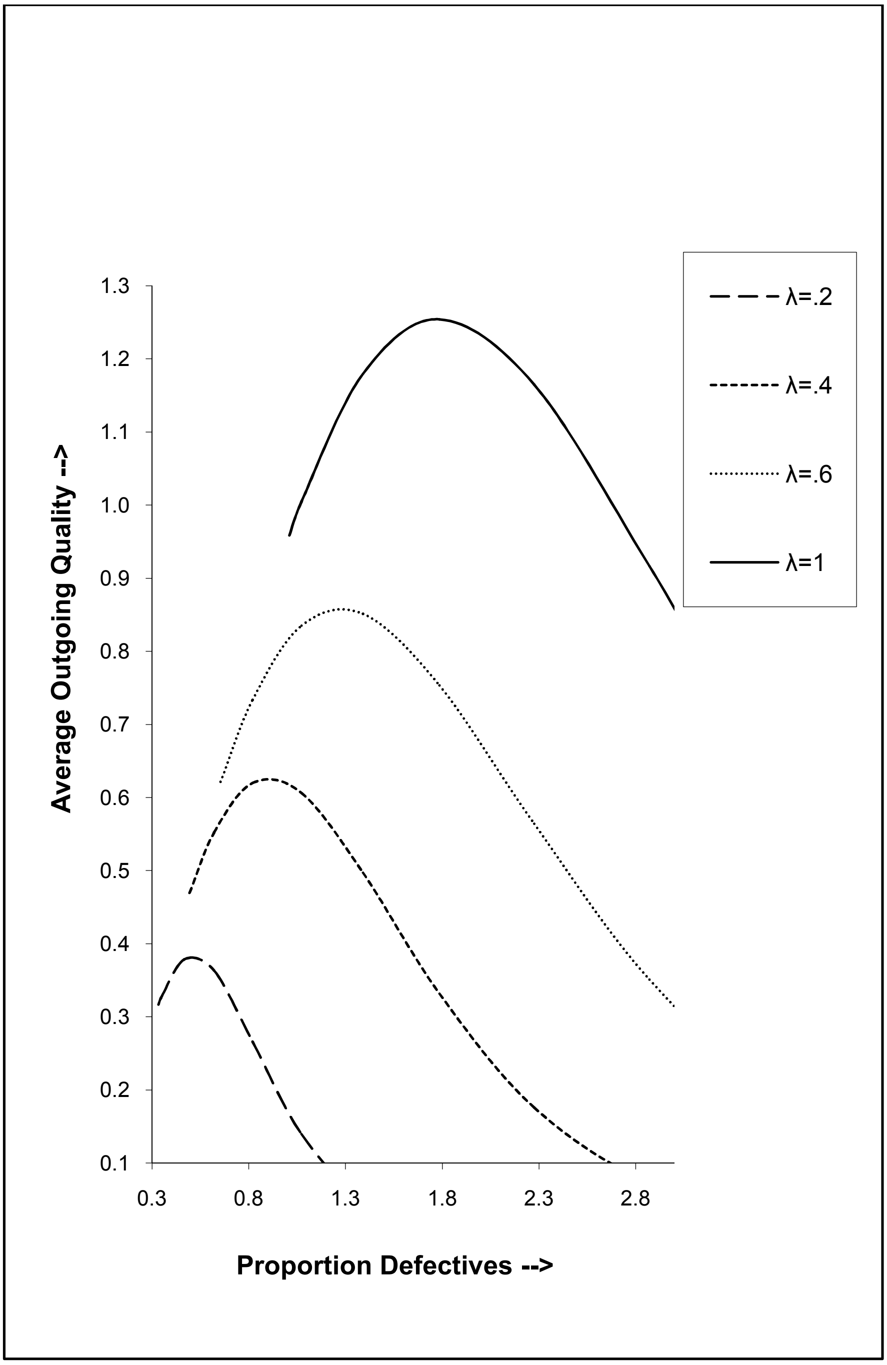

Fig.2: Average Outgoing Quality Curves under EWMA Model

In many practical applications very large data sets may not be available, thus there is a need for more research and development work on EWMA model which are not highly sensitive to model misspecification and to error in parameter estimates. Our results indicate that good choices for different λ depends on the number of variables in the sampling scheme.

References

1. Banzhaf, R.A., and Brugger, R.M. (1970). Review of standards and specification: MIL-STD-1235 (ORD), Single and multilevel continuous sampling procedures and Tables for inspection by attributes, Journal of Quality Technology, 2, p. 41-53.
2. Chiu, W.K. and Wetherill, G.B. (1975). Quality control practice. Int. J. Prod. Res, 13, p. 175-182.
3. Crowder, S.V. (1987a). A Simple Method for Studying Run-Length Distributions of Exponentially Weighted Moving Average Charts. Technometrics, 29, p. 401-407.
4. Crowder, S.V. (1987b). Computation of ARL for Combined Individual Measurement and Moving Range Charts, Journal of Quality Technology, 19, p. 98-102.
5. Crowder, S.V. (1987c). Average Run Lengths of Exponentially Weighted Moving Average Control Charts, Journal of Quality Technology, 19, p. 161-164.
6. Crowder, S.V. (1989). Design of Exponentially Weighted Moving Average Schemes, Journal of Quality Technology, 21, p. 155-162.
7. Dodge, H.F. (1943). Sampling inspection plan for continuous production. The Annals to Mathematical Statistics, 14, p. 264 -279.
8. Golhar, D.Y. (1987). Determining the Best Mean Contents for a Canning Problem, Journal of Quality Technology, 19, p. 82-84.
9. Hunter, J.S. (1986). The Exponentially Weighted Moving Average, Journal of Quality Technology, 18, p. 203-210.
10. Lucas, J.M. and Saccucci, M S. (1990). Exponentially Weighted Moving Average Control Schemes: Properties and Enhancements. Technometrics, 32, p. 1-12.
11. Military Handbooks (1958). Multi-level Continuous sampling procedures and Tables for inspection by attributes. Handbook (Interim) H 106, office of the Assistant Secretary of Defense (Supply and Logistic), Washington, D.C.
12. Military Handbooks (1959). Single-level Continuous sampling procedures and Tables for inspection by attributes. Handbook (Interim) H 107, office of the Assistant Secretary of Defense (Supply and Logistic), Washington, D.C.
13. Roberts, S.W. (1959). Control Chart Test Based on Geometric Moving Averages. Technometrics, 1, p. 239-250.
14. Robinson, P.B. and HO, T.Y. (1978). Average Run Length of Geometric Moving Average Charts by Numerical Methods. Technometrics, 20, p.85-93.
15. Schmidt, R. L. and Pfeifer, P.E. (1989). An Economic Evaluation of Improvement in Process Capability for Single level Canning Problem, Journal of Quality Technology, 21, p. 16-19.