Journal of Reliability and Statistical Studies; ISSN (Print): 0974-8024, (Online):2229-5666
Vol. 5, Issue 2 (2012): 33-43

AVAILABILITY ASSESSMENT FOR AGING REFRIGERATION SYSTEM BY USING L_Z-TRANSFORM

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(Received May 31, 2012)

Abstract

This paper presents a case study of L_z-transform method application to calculation of availability for complex aging refrigeration system. The system and its components can have different performance levels ranging from perfect functioning to complete failure and, so it is treated as a multi-state system. Straightforward Markov method applied to solve the problem will require building of the system model with numerous numbers of states and solving a corresponding system of differential equations. L_z-transform method, which is used for computation of availability for such refrigeration system, drastically simplified the solution.

Key Words: availability, multi-state system, L_z-transform method, universal generating function

1. Introduction

Modern supermarkets suffer serious financial losses because of problems with unreliability of their refrigeration systems. A typical supermarket may contain more than one hundred individual refrigerated cabinets, cold store rooms and items of plant machinery which interact as part of a complex integrated refrigeration system within the store [1]-[3]. Things very often go wrong with individual units (icing up of components, electrical or mechanical failure, and so forth…) or with components which serve a network of units (coolant tanks, pumps, compressors, and so on).

Due to the system's nature, a fault in a single unit or item of machinery usually has no complete failure effect on the entire store, only decrease of system cool capacity. Partial failure of compressor or axial condenser blower leads to partial system failure (reduction of output cooling capacity) as well as to complete failures of the system. So, the refrigeration system can be treated as multi-state system (MSS), where components and entire system in general case have an arbitrary finite number of states corresponding to the different performance rates. The performance rate (cold capacity) of the system at any instant t is interpreted as a discrete-state continuous-time stochastic process $G(t)$ [4]. In real refrigeration systems we often have aging components and so, a reliability model in general case will be non-homogeneous Markov model where some transition rates (intensities) are time-dependent. The model is enough complex - even in relatively simple cases it has about half hundred states. So, it is rather difficult to build the model and to solve the corresponding system of differential equations by using straightforward Markov method.

During last years a specific approach called the universal generating function (UGF) technique has been widely applied to MSS reliability analysis [5], [6]. The approach was primarily introduced by Ushakov [7], [8]. The UGF technique allows one to algebraically find the entire MSS performance distribution through the performance distributions of its elements. However, the main restriction of this powerful technique is that theoretically it may be only applied to random variables and, so, concerning MSS reliability, it operates with only steady-states performance distributions. For example, it means that UGF technique cannot be used, when an aging system is under consideration.

In order to extend the UGF technique application to dynamic MSS reliability analysis a special transform was introduced [9] for a discrete-states continuous-time Markov process that is called L_z-transform. This transform is similar to UGF for discrete random variable in sense that Ushakov's operator Ω_f [10] can be applied. In [9] were proven an existence and a uniqueness of L_z-transform and was shown that in reliability context L_z-transform may be applied to an aging system and to a system at burn-in period as well as to a system with constant failure and repair rates. The unique condition that should be fulfilled is a continuity of transitions intensities of the corresponding Markov process.

In the presented article the L_z-transform application to real case study of aging MSS - industrial refrigeration system - is considered. L_z-transform is applied to real supermarket aging refrigeration system that is functioning under minimal repair and its availability was analyzed. It was shown that L_z-transform application drastically simplifies a reliability computation for such a system compared with the ordinary Markov method.

2. Reliability Model for the Refrigeration System

The most commonly used refrigeration system for supermarkets today is the multiplex direct expansion system. Heat rejection is usually done with compressors and air-cooled condensers with simultaneously working axial blowers. All display cases and cold store rooms use direct expansion air-refrigerant coils that are connected to the system compressors in a remote machine room located in the back or on the roof of the store.

According to Annex 26 of International Energy Agency [2] typical refrigeration system includes 4 basic elements: compressor, evaporators, condensers and thermal expansion valves. From the reliability point of view we are interested only in two elements: compressor and condensers. The rest of elements have very high reliability [3] and almost do not influence on the refrigeration system's availability.

In the case of middle and big size supermarket commercial refrigeration system multiple compressors are installed in parallel and can be selected and cycled as needed to meet the refrigeration load [4].

In some refrigeration systems it is possible to use one compressor with "unloading mechanism", that could support several levels of refrigeration capacity (performance) depended on demanded refrigeration load level. The working principle is as follows: the intake valve of the cylinder, which response on the definite performance level, will be opened forcibly in case of decreasing the refrigeration load and stop heat rejection. Compressor unloading may be produced by pneumatic, hydraulic and electric modes. This leads to separation the compressor elements to independent ones (cylinders, their failures influence on performance reduction) and common part (crank mechanism and electric engine, which failure leads to failure of the whole compressor).

In our case of the supermarket commercial refrigeration system it is used compressor that could support 5 levels of refrigeration capacity (performance) depended on demanded refrigeration level. Using multi-level compressor provides a capacity control, since the compressor can be cycled as needed to meet the refrigeration load. Principal scheme of the refrigeration system is presented on the Figure 1.

So, we consider here a refrigeration system, consists of 3 elements: the first element is 5-levels compressor, situated in the machine room and the second and the third ones are connected in parallel blocks of condensers with axial blowers. Structure system's scheme and state-transition diagrams of the elements are presented in the Figure 2.

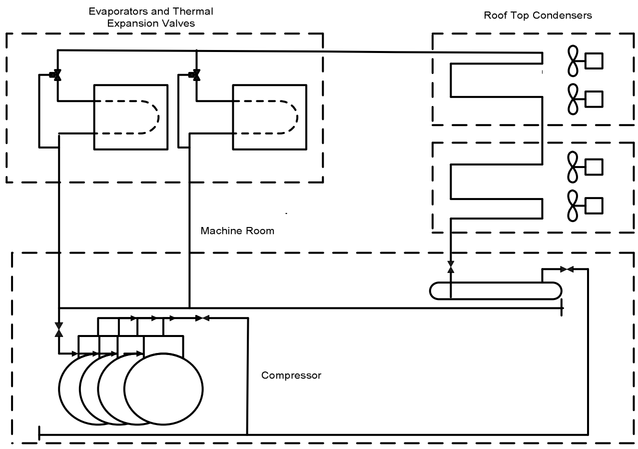

Fig. 1: Principal scheme of the refrigeration system

The performance of the elements is measured by their produced cold capacity (BTU per year). Times to failures and times to repairs are distributed exponentially for all elements. Elements are repairable. It is possible only minimal repair. All elements are multi-state elements with minor and major failures and repairs.

The first element – compressor - can be in one of five states: a state of total failure corresponding to a capacity of 0, states of partial failures corresponding to capacities of $3 \cdot 10^9$, $6 \cdot 10^9$, $9 \cdot 10^9$ BTU per year and a fully operational state with a capacity of $12 \cdot 10^9$ BTU per year. For simplification we will present system capacity in 10^9 BTU per year units. Therefore,

$$G_1(t) \in \{g_{11}, g_{12}, g_{13}, g_{14}, g_{15}\} = \{0,3,6,9,12\}. \quad (1)$$

The failure rate of mechanical part of compressors is increasing function $\lambda^C(t) = 1 + 0.33t$ year^{-1} and the failure rate of electrical part is $\lambda^{CT} = 0.4$ year^{-1}. The repair rates of the first element are $\mu^C = 200$ year^{-1} and $\mu^{CT} = 12$ year^{-1}.

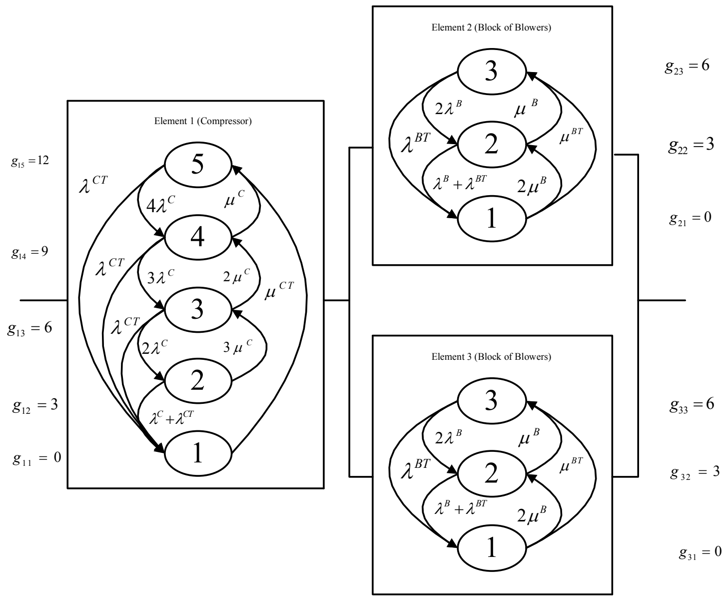

Fig. 2: Structure scheme and state-transition diagram of the elements of the system

The second and third elements - blowers - can be in one of three states: a state of total failure corresponding to a capacity of 0, state of partial failure corresponding to capacity of $3 \cdot 10^9$ BTU per year and a fully operational state with a capacity of $6 \cdot 10^9$ BTU per year. Therefore,

$$G_2(t) \in \{g_{21}, g_{22}, g_{23}\} = \{0,3,6\},$$
$$G_3(t) \in \{g_{31}, g_{32}, g_{33}\} = \{0,3,6\}. \quad (2)$$

The failure rates and repair rates corresponding to these elements are

$\lambda^B = 10\text{year}^{-1}$, $\lambda^{BT} = 0.1\text{year}^{-1}$, $\mu^B = 365\text{year}^{-1}$, $\mu^{BT} = 12\text{year}^{-1}$.

The MSS structure function is:

$$G_s(t) = f(G_1(t), G_2(t), G_3(t)) = \min\{G_1(t), G_2(t) + G_3(t)\}. \quad (3)$$

For calculation of supermarket's refrigeration system parameters in according to [1] in the paper is using constant level of external demand, defined by store size, that is not varied from season to season. The demand value is defined only by the supermarket's store size. Usually constant demand $w_1 = 7 \cdot 10^9$ BTU per year is for middle supermarkets and $w_2 = 10 \cdot 10^9$ BTU per year for big supermarkets.

3. Brief Description of the L_z-transform Method

In this paper L_z-transform method is applied for calculation of availability for complex aging supermarket refrigeration system functioning under constant demand. The method was introduced in [9] where one can find its detailed description and corresponding mathematical proofs. Shortly, the method description is as follows.

We consider a discrete-state continuous-time (DSCT) Markov process [11] $X(t) \in \{x_1, ..., x_K\}$, which has *K* possible states *i*, (*i*=1,…,*K*) where performance level associated with any state *i* is x_i. This Markov process is completely defined by set of possible states $\mathbf{x} = \{x_1, x_2, ..., x_n\}$, transitions intensities matrix depending on time $A = (a_{ij}(t)), i, j = 1, 2, ..., K$ and by initial states probability distribution that can be presented by corresponding set

$$\mathbf{p}_0 = [p_{10} = \Pr\{X(0) = x_1\}, ..., p_{K0} = \Pr\{X(0) = x_K\}]. \tag{4}$$

In according to [9] L_z-transform of a DSCT Markov process $X(t)$ is a function defined as follows

$$L_z\{X(t)\} = \sum_{i=1}^{K} p_i(t) z^{g_i} \tag{5}$$

where $p_i(t)$ is a probability that the process is in state *i* at time instant $t \geq 0$ for a given initial states probability distribution $\mathbf{p}_0$ and *z* in general case is a complex variable.

In general case any element *j* in MSS can have k_j different states corresponding to different performance, represented by the set $\mathbf{g}_j = \{g_{j1}, ..., g_{jk_j}\}$, where g_{ji} is the performance rate of element *j* in the state *i*, $i \in \{1, 2, ..., k_j\}$, and $j \in \{1, ...n\}$, where *n* is the number of elements in the MSS.

At first stage a Markov model of stochastic process should be built for each multi-state element in MSS. Based on this model state probabilities

$$p_{ji}(t) = \Pr\{G_j(t) = g_{ji}\},\ i \in \{1, ..., k_j\} \tag{6}$$

for every MSS's element can be obtained as a solution of the corresponding system of differential equations under a given initial conditions.
Then individual L_z-transform for each element *j* should be found

$$L_z\{G_j(t)\} = \sum_{i=1}^{k_j} p_{ji}(t) z^{g_{ji}},\ j = 1, ..., n \tag{7}$$

At the next stage L_z-transform of the output stochastic process for the entire MSS should be defined based on previously determined L_z-transform for each element *j* and system structure function *f*, which produces the output stochastic process of the entire MSS based on stochastic processes of all MSS's elements:

$$G(t) = f(G_1(t),...,G_n(t)) \tag{8}$$

In [9] it was shown that in order to find L_z-transform of the resulting DSCT Markov process *G*(*t*), which is the single-valued function $G(t) = f(G_1(t),...,G_n(t))$ of *n* independent DSCT Markov processes $G_j(t)$, j=1,…,*n*, one can apply Ushakov's Universal Generating Operator (UGO) to all individual L_z-transforms $L_z\left\{G_j\left(t\right)\right\}$ over all time points $t \geq 0$

$$L_z\{G(t)\} = \Omega_f\left\{L_z[G_1(t)],...,L_z[G_n(t)]\right\} \tag{9}$$

So, by using Ushakov's operator Ω_f over all L_z-transforms of individual elements one can obtain the resulting L_z-transform $L_z\{G(t)\}$ associated with output performance stochastic process *G*(*t*) of the entire MSS.

The technique of Ushakov's operator applying is well established for many different structure functions *f* [5]. By using this technique the computational burden (9) decreasing drastically.

The resulting L_z-transform $L_z\{G(t)\}$ is associated with the output performance stochastic process for the entire MSS. In according to [9] MSS reliability measures can be easily derived from the resulting L_z-transform $L_z\{G(t)\}$.

If L_z-transform

$$L_z\{G(t)\} = \sum_{k=1}^{K} p_i(t) z^{g_k} \tag{10}$$

of the entire MSS's output stochastic process $G(t) \in \{g_1,...,g_K\}$ is known, then important system's reliability measures can be easily found.

For example, the system availability for the constant demand level *w* at instant $t \geq 0$

$$A(t) = \sum_{g \geq w} p_i(t) \tag{11}$$

In other words, in order to find MSS's instantaneous availability one should summarize all probabilities in L_z-transform from terms where powers of *z* are greater or equal to demand *w*.

4. Availability Calculation for the Supermarket Refrigeration System

Applying the procedure, described above we proceed as follows.

According to the Markov method we build the following systems of differential equations for each element (using state-transitions diagrams presented in Fig. 2):

For the first element:

$$\begin{cases} \frac{dp_{11}(t)}{dt} = -\mu^{CT} p_{11}(t) + \left(\lambda^{C} + \lambda^{CT}\right)(t) p_{12}(t) + \lambda^{CT}\left[p_{13}(t) + p_{14}(t) + p_{15}(t)\right] \\ \frac{dp_{12}(t)}{dt} = -\left[\lambda^{C}(t) + \lambda^{CT} + 3\mu^{C}\right] p_{12}(t) + 2\lambda^{C}(t) p_{13}(t) \\ \frac{dp_{13}(t)}{dt} = 3\mu^{C} p_{12}(t) - \left[2\lambda^{C}(t) + \lambda^{CT} + 2\mu^{C}\right] p_{13}(t) + 3\lambda^{C}(t) p_{14}(t) \\ \frac{dp_{14}(t)}{dt} = 2\mu^{C} p_{13}(t) - \left[3\lambda^{C}(t) + \lambda^{CT} + \mu^{C}\right] p_{14}(t) + 4\lambda^{C}(t) p_{15}(t) \\ \frac{dp_{15}(t)}{dt} = \mu^{CT} p_{11}(t) + \mu^{C} p_{14}(t) - \left[4\lambda^{C}(t) + \lambda^{CT}\right] p_{i5}(t). \end{cases} \tag{12}$$

Initial conditions are: $p_{11}(0) = p_{12}(0) = p_{13}(0) = p_{14}(0) = 0;\; p_{15}(0) = 1.$

For the second element:

$$\begin{cases} \frac{dp_{21}(t)}{dt} = -\left(2\mu^{B} + \mu^{BT}\right) p_{21}(t) + \lambda^{B} p_{22}(t) + \lambda^{BT} p_{23}(t), \\ \frac{dp_{22}(t)}{dt} = 2\mu^{B} p_{21}(t) - (\lambda^{B} + \mu^{B}) p_{22}(t) + 2\lambda^{B} p_{23}(t), \\ \frac{dp_{23}(t)}{dt} = \mu^{BT} p_{21}(t) + \mu^{B} p_{22}(t) - \left(2\lambda^{B} + \lambda^{BT}\right) p_{23}(t). \end{cases} \tag{13}$$

Initial conditions are: $p_{21}(0) = p_{22}(0) = 0;\; p_{23}(0) = 1.$

For the third element:

$$\begin{cases} \frac{dp_{31}(t)}{dt} = -\left(2\mu^{B} + \mu^{BT}\right) p_{31}(t) + \lambda^{B} p_{32}(t) + \lambda^{BT} p_{33}(t), \\ \frac{dp_{32}(t)}{dt} = 2\mu^{B} p_{31}(t) - (\lambda^{B} + \mu^{B}) p_{32}(t) + 2\lambda^{B} p_{33}(t), \\ \frac{dp_{33}(t)}{dt} = \mu^{BT} p_{31}(t) + \mu^{B} p_{32}(t) - \left(2\lambda^{B} + \lambda^{BT}\right) p_{33}(t). \end{cases} \tag{14}$$

Initial conditions are: $p_{31}(0) = p_{32}(0) = 0;\; p_{33}(0) = 1.$

A closed form solution or numerical solution for probabilities $p_{ij}(t)$ can be obtained for each of these 3 systems of differential equations using MATLAB®. Therefore, one obtains 3 following output performance stochastic processes:

Element 1:

$$\begin{cases} \mathbf{g}_1 = \{g_{11}, g_{12}, g_{13}, g_{14}, g_{15}\} = \{0, 3, 6, 9, 12\}, \\ \mathbf{p}_1(t) = \{p_{11}(t), p_{12}(t), p_{13}(t), p_{14}(t), p_{15}(t)\}; \end{cases}$$

Element 2:

$$\begin{cases} \mathbf{g}_2 = \{g_{21}, g_{22}, g_{23}\} = \{0,3,6\}, \\ \mathbf{p}_2(t) = \{p_{21}(t), p_{22}(t), p_{23}(t)\}, \end{cases}$$

Element 3:

$$\begin{cases} \mathbf{g}_3 = \{g_{31}, g_{32}, g_{33}\} = \{0,3,6\}, \\ \mathbf{p}_3(t) = \{p_{31}(t), p_{32}(t), p_{33}(t)\}. \end{cases}$$

Having the sets $g_i, \mathbf{p}_j(t), j = 1,2,3$ one can define for each individual element j L_z-transforms associated with the element's output performance stochastic process:

$$\begin{aligned} L_z\{G_1(t)\} &= p_{11}(t)z^{g_{11}} + p_{12}(t)z^{g_{12}} + p_{13}(t)z^{g_{13}} + p_{14}(t)z^{g_{14}} + p_{15}(t)z^{g_{15}} \\ &= p_{11}(t)z^0 + p_{12}(t)z^3 + p_{13}(t)z^6 + p_{14}(t)z^9 + p_{15}(t)z^{12}, \\ L_z\{G_2(t)\} &= p_{21}(t)z^{g_{21}} + p_{22}(t)z^{g_{22}} + p_{23}(t)z^{g_{23}} = p_{21}(t)z^0 + p_{22}(t)z^3 + p_{23}(t)z^6, \\ L_z\{G_3(t)\} &= p_{31}(t)z^{g_{31}} + p_{32}(t)z^{g_{32}} + p_{33}(t)z^{g_{33}} = p_{31}(t)z^0 + p_{32}(t)z^3 + p_{33}(t)z^6. \end{aligned} \tag{15}$$

Using the composition operator Ω_{fpar} for pairs of MSS elements 2 and 3, connected in parallel, one obtains the L_z-transform $L_z\{G_{2,3}(t)\}$ for the elements connected in parallel, where the powers of z are found as sum of powers of corresponding terms:

$$\begin{aligned} L_z\{G_{2,3}(t)\} &= \Omega_{fpar}\left(G_2(t), G_3(t)\right) = \\ &= \Omega_{f_{par}}\left(p_{21}(t)z^0 + p_{22}(t)z^3 + p_{23}(t)z^6, p_{31}(t)z^0 + p_{32}(t)z^3 + p_{33}(t)z^6\right) \\ &= p_{21}(t)p_{31}(t)z^0 + \left(p_{21}(t)p_{32}(t) + p_{22}(t)p_{31}(t)\right)z^3 \\ &+ \left(p_{21}(t)p_{33}(t) + p_{22}(t)p_{32}(t) + p_{23}(t)p_{31}(t)\right)z^6 \\ &+ \left(p_{22}(t)p_{33}(t) + p_{23}(t)p_{32}(t)\right)z^9 + p_{23}(t)p_{33}(t)z^{12}. \end{aligned} \tag{16}$$

Using the composition operator Ω_{fser} for element 1, which is connected in series with subsystem of two connected in parallel elements 2 and 3, one obtains the L_z-transform $L_z\{G(t)\}$, where the powers of z are found as minimum of powers of corresponding terms:

$$\begin{aligned} L_z\{G(t)\} &= \Omega_{fser}\left(G_1(t), G_{2,3}(t)\right) \\ &= p_{21}(t)p_{31}(t)z^0 + \left(p_{21}(t)p_{32}(t) + p_{22}(t)p_{31}(t)\right)z^3 \\ &+ \left(p_{21}(t)p_{33}(t) + p_{22}(t)p_{32}(t) + p_{23}(t)p_{31}(t)\right)z^6 \\ &+ \left(p_{22}(t)p_{33}(t) + p_{23}(t)p_{32}(t)\right)z^9 + p_{23}(t)p_{33}(t)z^{12}\Big). \end{aligned} \tag{17}$$

Taking into account that $p_{11}(t)+p_{12}(t)+p_{13}(t)+p_{14}(t)+p_{15}(t)=1$, $p_{21}(t)+p_{22}(t)+p_{23}(t)=1$ and $p_{31}(t)+p_{32}(t)+p_{33}(t)=1$ one can simplify the expression for $L_z\{G(t)\}$ and obtain the resulting L_z-transform in the following form

$$\begin{aligned}
L_z\{G(t)\} &= \left[p_{11}(t)+\left(1-p_{11}(t)\right)p_{21}(t)p_{31}(t)\right]z^0 \\
&+\left[p_{12}(t)\left(1-p_{21}(t)\right)+\left(1-p_{11}(t)\right)p_{21}(t)p_{32}(t)+\right. \\
&\left.\left(1-p_{11}(t)-p_{12}(t)\right)p_{22}(t)p_{31}(t)+p_{12}(t)p_{21}(t)p_{32}(t)\right]z^3 \\
&+\left[\left(1-p_{11}(t)-p_{12}(t)\right)\cdot\left(p_{21}(t)p_{33}(t)+p_{22}(t)p_{32}(t)+p_{23}(t)p_{31}(t)\right)+p_{13}(t)p_{22}(t)p_{33}(t)\right]z^6 \\
&+\left[\left(p_{14}(t)+p_{15}(t)\right)\left(p_{22}(t)p_{33}(t)+p_{23}(t)p_{32}(t)\right)+p_{14}(t)p_{23}(t)p_{33}(t)\right]z^9 \\
&+\left[p_{15}(t)p_{23}(t)p_{33}(t)\right]z^{12}.
\end{aligned} \tag{18}$$

And finally

$$L_z\{G(t)\}=\sum_{i=1}^{5}p_i(t)z^{g_i} \tag{19}$$

where

$g_1=0,$ $\quad p_1(t)=p_{11}(t)+\left(1-p_{11}(t)\right)p_{21}(t)p_{31}(t)$

$g_2=3\cdot 10^9$ BTU/year, $\quad p_2(t)=p_{12}(t)\left(1-p_{21}(t)\right)+\left(1-p_{11}(t)\right)p_{21}(t)p_{32}(t)$
$+\left(1-p_{11}(t)-p_{12}(t)\right)p_{22}(t)p_{31}(t)+p_{12}(t)p_{21}(t)p_{32}(t),$

$g_3=6\cdot 10^9$ BTU/year $\quad p_3(t)=\left(1-p_{11}(t)-p_{12}(t)\right)$
$\cdot\left(p_{21}(t)p_{33}(t)+p_{22}(t)p_{32}(t)+p_{23}(t)p_{31}(t)\right)$
$+p_{13}(t)p_{22}(t)p_{33}(t),$

$g_4=9\cdot 10^9$ BTU/year, $\quad p_4(t)=\left(p_{14}(t)+p_{15}(t)\right)\left(p_{22}(t)p_{33}(t)+p_{23}(t)p_{32}(t)\right)$
$+p_{15}(t)p_{23}(t)p_{33}(t),$

$g_5=12\cdot 10^9$ BTU/year, $\quad p_5(t)=p_{15}(t)p_{23}(t)p_{33}(t)$

These two sets $\mathbf{g}=\{g_1,g_2,g_3,g_4,g_5\}$ and $\mathbf{p}(t)=\{p_1(t),p_2(t),p_3(t),p_4(t),p_5(t)\}$ define capacities and states probabilities of output performance stochastic process for the entire MSS.

Based on the resulting L_z-transform $L_z\{G(t)\}$ of the entire MSS, one can obtain the MSS instantaneous availability for the middle size supermarket with constant demand level $w_1=7\cdot 10^9$ BTU per year by using expression (11)

$$A_1(t)=\sum_{g\geq w_1}p_i(t)=p_4(t)+p_5(t) \tag{20}$$

For the big size supermarket with demand level $w_2 = 10 \cdot 10^9$ one will obtain the following

$$A_2(t) = \sum_{g \geq w_2} p_i(t) = p_5(t) \tag{21}$$

Calculated MSS instantaneous availability $A(t)$ is presented on the Figure 3. The curves in this figure show for aging system its availability decreasing over time.

The calculation results present availability analysis for middle and big supermarkets: 3500 m^2 store size with demand $w_1 = 7 \cdot 10^9$ BTU per year (dotted line) and 5000 m^2 store size with demand $w_2 = 10 \cdot 10^9$ BTU per year (solid line). The required availability level is 0.8 and presented by bold line.

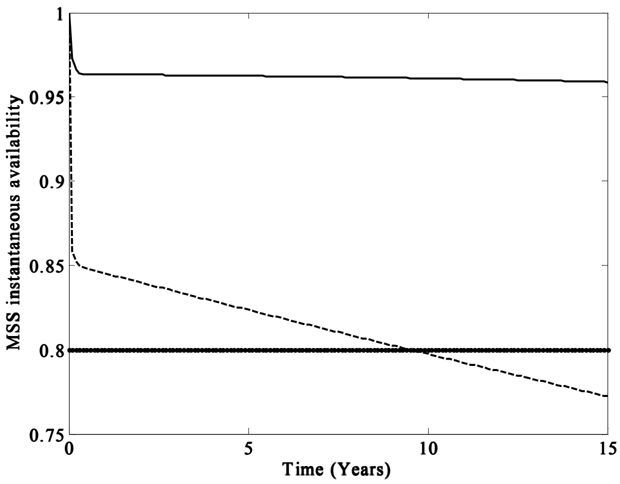

Fig. 3: MSS instantaneous availability for different demand levels (dotted line $w_1 = 7 \cdot 10^9$ BTU per year, solid line $w_2 = 10 \cdot 10^9$ BTU per year, bold line - required availability level)

As one can see from the calculation results presented on Figure 3, in order to provide the required availability level, replacement of aging mechanical part of compressors should be performed for the big supermarket after 9.5 years. For middle supermarket the refrigeration system's exploitation can be continued up to 15 years, when usually (according the specification) the whole system replacement should be performed.

5. Conclusion

In the presented article the L_z -transform application to real case study of aging MSS - industrial refrigeration system - is described. The method proved to be very effective for real world problem in such a case.

It was demonstrated that the L_z-transform method is well formalized and suitable for practical application in reliability engineering for real-world MSSs analysis. It supports the engineering decision making and determines different system structures providing a required reliability/availability level for complex multi-state aging systems. The method provides drastically decreasing of computational burden compared with straightforward Markov method where the model with 45 states should be built and solved.

Acknowledgments

This work was partially supported by the Internal Funding Program of the Shamoon College of Engineering (SCE).

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