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A TWO NON-IDENTICAL UNIT STANDBY SYSTEM MODEL WITH REPAIR, INSPECTION AND POST-REPAIR UNDER CLASSICAL AND BAYESIAN VIEWPOINTS

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Abstract

In this paper some important measures of reliability characteristics of a two non-identical unit standby system model with repair, inspection and post repair are obtained using regenerative point technique. All time random variables are assumed to be independent and follow Weibull distribution. Monte Carlo simulation study is also carried out to illustrate the results for considered system model from Classical and Bayesian viewpoints.

Key Words: Mean time to system failure (MTSF), Fisher Information matrix and Non-Regenerative point.

1. Introduction

In order to improve the reliability or raise the availability and hence reduce the loss, a two-component redundant system is often employed. A two- component cold standby system with one repairman has been one of the classical models in the reliability theory. Under the condition that the operating time and the repair time of each component in the system both have exponentially distributed. Several studies including Dhillon and Yang (1992), Goel and Srivastava (1991), Mogha and Gupta (2002), Nakagawa and Osaki (1975), Singh and Mishra (1994) analyzed system models by using the concepts of warm standby with common cause failure and human error, correlated failures and repairs, two priority unit warm standby with preparation for repair, two unit priority standby with repair, two unit cold standby with two operating modes.

All the above studies assumed that each failed component after repair is “as good as new”. However, In real existing situations we observe that there are many sophisticated, costly equipments/units where it becomes necessary to inspect a repaired unit/equipment to check whether the repair done is satisfactory or not. The equipment may be sent for post-repair if repair is found unsatisfactory during inspection. Besides, these studies were mainly concerned to obtain reliability characteristics and not to estimate the parameters involved in the lifetime/repair time distributions of unit/system. Gupta and Pankaj (2012) analyzed two dissimilar unit cold standby system with Weibull failure and repair laws without estimating the parameter(s) involved in the life time/repair time distribution of system/ unit.

Keeping above idea in view, we, in the present paper analyze a two non-identical unit standby system model in which the first unit goes for repair, inspection and post repair whereas the second unit becomes as good as new after repair. Here the priority in operation is given to the first unit as it is highly sophisticated, costly unit which provides high quality product at low running cost. The second unit is ordinary unit which has high running cost. Priority in repair is given to second unit as repair of second unit is less time consuming and cheaper as compared to first unit. The purpose of the present paper is to analyze a two non-identical unit standby system model with repair, inspection and post repair by using Weibull distribution for both failure and repair times with different scale parameters and common shape parameter under classical and Bayesian setups. For a more concrete study the system model, a simulation study is also carried out.

The probability density function (p.d.f) of Weibull distribution is given by

$$f(t)=\theta p t^{p-1} \exp(-\theta t^{p}) \; ; \; t \geq 0 \text{ and } \theta, p > 0$$

The reliability/survival function and hazard (failure /repair) rate for Weibull distribution are respectively given by

$$R(t) = \exp(-\theta t^{p}); \; t \geq 0 \text{ and } \theta, p > 0$$

and

$$H(t) = \theta p t^{p-1}; \; t \geq 0 \text{ and } \theta, p > 0$$

It is important to note that p and θ are the shape and scale parameters respectively. If we put p=1 in the above p.d.f of Weibull distribution, it reduces to exponential distribution and for p=2, it reduces to Rayleigh distribution.

We evaluate the following reliability characteristics of interest to system designers as well as operating managers by using regenerative point technique.

(i) Steady-state transition probabilities and mean sojourn times in various states.
(ii) Pointwise availability of the system at time t and the steady state availability.
(iii) Reliability of the system and Mean time to system failure (MTSF).
(iv) Expected up time of the system and expected busy period of the repairman during (0, t).
(v) Expected profit incurred by the system in (0, t) and in the steady state.

Further, since life testing experiments are time consuming and as such the parameters representing the reliability characteristics of the system/unit are assumed to be random variables. Therefore, a simulation study is conducted for analyzing the considered system model both in classical and Bayesian set ups. The Monte Carlo simulation technique has been used in conducting the numerical study. In classical setup, the maximum likelihood (ML) estimates of the parameters involved in the model and reliability characteristics along with their standard errors (SE) and width of confidence intervals are obtained. In Bayesian setup, Bayes estimates of the parameters and reliability characteristics along with their posterior standard errors (PSE) and width of highest posterior density (HPD) intervals are computed. In the end, the comparative conclusions are drawn to judge the performances of the ML and Bayes estimates.

The rest of the paper is organized as follows: Section 2 deals with the system model description and assumptions. In Section 3, notations and states of the system model are given. In Section 4, transition probabilities and mean sojourn times in various states are considered. Section 5 deals with the analysis of various characteristics such as reliability, MTSF, Availability, Busy period and Profit function. In Section 6, Maximum likelihood estimation (MLE) and Bayes estimation of scale parameters, MTSF and Profit function are considered. Section 7 deals with the simulation study to examine the behavior of the estimates of parameters and reliability characteristics and finally in Section 8, concluding remarks are given on the basis of Tables 1-6 obtained under Section 7 and Figures 2-7.

2. System Model Description and Assumptions

(i) The system consists of two non-identical units (unit-1 and unit-2). Initially, system starts its operation from state S0 in which unit-1 is operative and unit-2 is kept in cold standby. Upon failure of an operative unit the cold standby unit becomes operative instantaneously.

(ii) Each unit has two modes- Normal (N) and total failure (F). After the repair of unit-1, it goes for inspection to decide whether the repair is perfect or not. If the repair of a unit is found to be perfect then the repaired unit becomes operational, otherwise it is sent for post repair. The probability of having a perfect repair is fixed. Unit-2 becomes as good as new after repair.

(iii) Upon failure of unit-1, unit-2 becomes operative instantaneously with a perfect switching device.

(iv) The second unit gets the priority in repair over the repair, inspection and post repair of unit-1.

(v) The failure and repair time distributions of each unit are taken to be independent having the Weibull density with common shape parameter ‘p’ but different scale parameters α and β as follows:

$$f_i(t)=\alpha_i p t^{p-1}\exp(-\alpha_i t^p), t\geq 0 \text{ and } \alpha_i, p > 0, i=1,2$$

and

$$g_i(t)=\beta_i p t^{p-1}\exp(-\beta_i t^p), t\geq 0 \text{ and } \beta_i, p > 0, i=1,2$$

(vi) The inspection and post repair time distributions of unit-1 are taken to be independent having the Weibull density with common shape parameter ‘p’ but different scale parameters μ and λ as follows:

$$J(t)=\mu p t^{p-1}\exp(-\mu t^p), t\geq 0 \text{ and } \mu, p > 0$$

and

$$h(t)=\lambda p t^{p-1}\exp(-\lambda t^p), t\geq 0 \text{ and } \lambda, p > 0$$

(vii) The switching device is perfect and instantaneous.

(viii) A single repair facility is used to repair of both units and inspection and post repair of unit-2.
(ix) A repaired unit works as good as new.

3. Notations and States of the System

Notations

E : Set of regenerative states = $\{S_o, S_1, S_2, S_3, S_4, S_5. S_6\}$
α_i / β_i : Scale parameter of failure/repair time distribution for i^{th} (i=1,2) unit.
μ/λ : Scale parameter of inspection/post repair time distribution first unit.
p : Shape parameter of failure/repair time distribution of each unit.
hi(t) : failure rate of i^{th} (i=1, 2) unit when both the units are operative
$= \alpha_i pt^{p-1}$, α_i, p ,t > 0
$K_i(t)$: repair rate of i^{th} (i=1,2) unit
$= \beta_i pt^{p-1}$, β_i, p,t > 0
m(t) : inspection rate of first unit having the form
$= \mu pt^{p-1}$; μ, p, t > 0
e(t) : post repair rate of first unit having the form
$= \lambda pt^{p-1}$; λ, p, t > 0
a/b : Probabilities that the repair of unit-1 is perfect or imperfect (a+b=1)

$q_{ij}(\cdot), Q_{ij}(\cdot)$:p.d.f and cdf of one step or direct transition time from $S_i \epsilon E$ to $S_j \epsilon E$.

p_{ij} : Steady state transition probability from state S_i to S_j such that

$=\lim_{t\to\infty} Q_{ij}(t)$.

$p_{ij}^{(k)}$: Steady state transition probability from state S_i to S_j via S_k such that

$p_{ij}^{(k)} = \lim_{t\to\infty} Q_{ij}^{(k)}(t)$.

ψ_i : Mean sojourn time in regenerative state S_i i.e.

$=\int_0^\infty P[T_i > t]dt$

$R_i(t)$: Reliability of the system at time t when system starts from S_i.

$A_i(t)$: Probability that the system will be operative in state S_i at epoch t.

$B_i(t)$: Probability that the repairman will be busy in state S_i at epoch t.

$\mu_{up}(t)$: Expected up time of the system during interval (0, t) i.e.

$=\int_0^t A_0(u)du$.

$\mu_b(t)$: Expected busy period of repairman during interval (0, t) i.e.

$=\int_0^t B_0(u)du$.

$P(t)$: Profit incurred by the system during interval (0, t).

$*$: Symbol for Laplace Transform of a function i.e.

$q^{*}_{ij}=\int_0^{\infty} e^{-st} q_{ij}(t)dt.$

$\cdot$: Regenerative point.

$\times$: Non regenerative point.

Symbols for the States of the System

N_{10} : Unit-1 is in normal (N) mode and operative.
N_{20}, N_{2s} : Unit-2 is in N-mode and operative /cold standby.
F_{1r}, F_{1I}, F_{1pr} : Unit-1 is in F-mode and under repair/under inspection after repair/and post repair.
F_{2r} : Unit-2 is in F-mode and under repair.
F_{1wr}, F_{1wI}, F_{1wpr} : First Unit is in F-mode and waiting for repair, inspection and post repair respectively.

Considering the above symbols, we have the following states of the system.

Up States:	**Failed States:**
$S_0 \equiv (N_{10}, N_{2s})$	$S_2 \equiv (F_{1wr}, F_{2r})$
$S_1 \equiv (F_{1r}, N_{20})$	$S_4 \equiv (F_{1wI}, F_{2r})$
$S_3 \equiv (F_{1I}, N_{20})$	$S_6 \equiv (F_{1wpr}, F_{2r})$
$S_5 \equiv (F_{1pr}, N_{20})$	

Here all states are regenerative. The possible transitions between the states together with transition rates are shown in Fig.1.

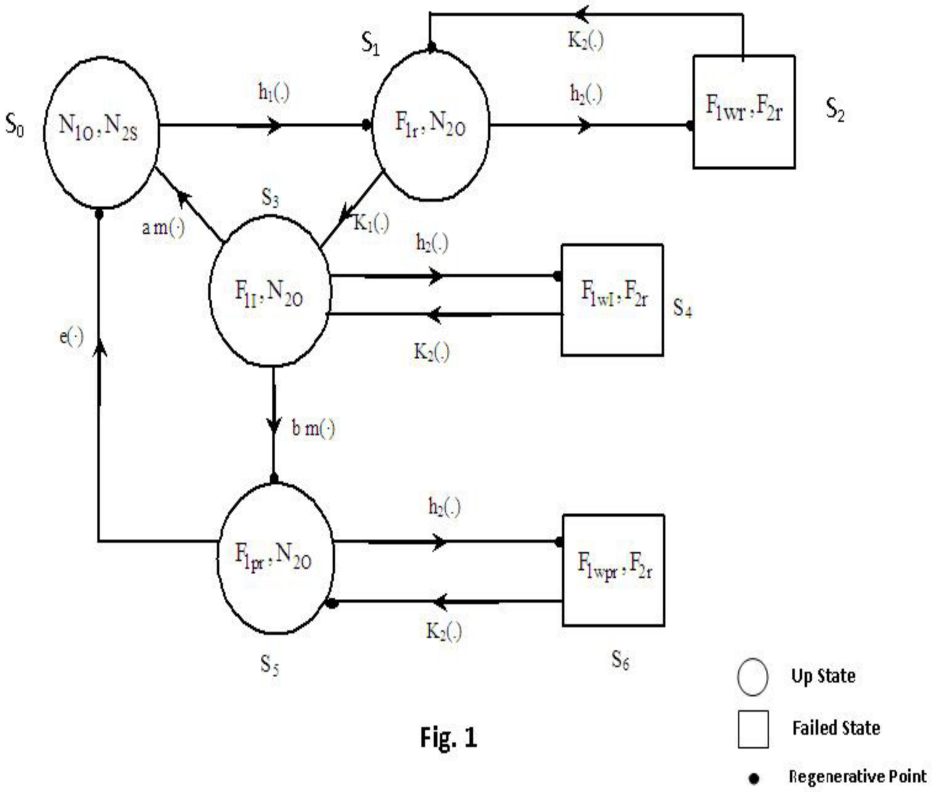

Fig. 1

4. Transition Probabilities and Sojourn Times

The non -zero elements p_{ij} of transition probability matrix (t.p.m) for the system model are as follows:

The steady state transition probabilities can be obtained by using the results,

$$p_{ij} = \lim_{t \to \infty} Q_{ij}(t)$$

$$p_{01} = \int \alpha_1 p t^{p-1} e^{-\alpha_1 t^p} dt = 1$$

Similarly,

$$p_{12}=\frac{\alpha_2}{\alpha_2+\beta_1};p_{13}=\frac{\beta_1}{\beta_1+\alpha_2};p_{21}=1;p_{30}=\frac{a\mu}{\mu+\alpha_2};p_{34}=\frac{\alpha_2}{\mu+\alpha_2};p_{35}=\frac{b\mu}{\mu+\alpha_2}$$

$$p_{43}=1;p_{50}=\frac{\lambda}{\lambda+\alpha_2};p_{56}=\frac{\alpha_2}{\lambda+\alpha_2};p_{65}=1$$

It can be easily verified that

$$\begin{aligned} &p_{01}=1\\ &p_{12}+p_{13}=1\\ &p_{21}=1\\ &p_{30}+p_{34}+p_{35}=1\\ &p_{43}=1\\ &p_{50}+p_{56}=1\\ &p_{65}=1 \end{aligned} \tag{1-7}$$

The limits of integration are 0 to ∞ whenever not mentioned.

Mean Sojourn Times

If T_i is the sojourn time in state S_i, then mean sojourn time in state S_i is given by,

$$\psi_0=\int P(T_i>t)dt$$

Therefore, the mean sojourn times for various states are as follows:

$$\psi_0=\int e^{-\alpha_1 t^p}dt=\frac{\Gamma(1+\frac{1}{p})}{(\alpha_1)^{1/p}}$$

$$\psi_1=\int e^{-\beta_1 t^p}e^{-\alpha_2 t^p}dt=\int e^{-(\beta_1+\alpha_2)t^p}dt=\frac{\Gamma(1+\frac{1}{p})}{(\beta_1+\alpha_2)^{1/p}}$$

$$\psi_2=\int e^{-\beta_2 t^p}dt=\frac{\Gamma(1+\frac{1}{p})}{(\beta_2)^{1/p}}$$

$$\psi_3 = \int e^{-\alpha_2 t^p} e^{-\mu t^p} dt = \int e^{-(\alpha_2+\mu)t^p} dt = \frac{\Gamma(1+\frac{1}{p})}{(\alpha_2+\mu)^{1/p}}$$

$$\psi_4 = \int e^{-\beta_2 t^p} dt = \frac{\Gamma(1+\frac{1}{p})}{(\beta_2)^{1/p}}$$

$$\psi_5 = \int e^{-\alpha_2 t^p} e^{-\lambda t^p} dt = \int e^{-(\alpha_2+\eta)t^p} dt = \frac{\Gamma(1+\frac{1}{p})}{(\alpha_2+\lambda)^{1/p}}$$

$$\psi_6 = \int e^{-\beta_2 t^p} dt = \frac{\Gamma(1+\frac{1}{p})}{(\beta_2)^{1/p}}$$

(8-14)

5. Analysis of Characteristics

5.1 Reliability and Mean Time to System Failure (MTSF)

Let $R_i(t)$ be the probability that the system is operative during (0,t) given that at t=0, it starts from state $S_i \in E$ .Using the regenerative point technique, reliability of the system when it starts from state S_0 ,in terms of its Laplace transform (i.e. the value of $R_0(t)$) is given by

$$R_0^*(s) = \frac{N_1(s)}{D_1(s)} = \frac{Z_0^* + q_{01}^*[Z_1^* + q_{13}^*(Z_3^* + q_{35}^* Z_5^*)]}{1 - q_{01}^*[q_{13}^*(q_{30}^* + q_{35}^* q_{50}^*)]} \tag{15}$$

Where

$Z_0^*(s), Z_1^*(s), Z_3^*(s)$ and $Z_5^*(s)$ are the Laplace transforms of $Z_0(t), Z_1(t), Z_3(t)$ and $Z_5(t)$, given by

$$Z_0^*(0) = \int e^{-\alpha_1 t^p} dt = \psi_0$$

Similarly,

$$Z_1^*(0) = \psi_1,\ Z_3^*(0) = \psi_3,\ Z_5^*(0) = \psi_5$$

Taking the inverse Laplace transform (ILT) of eq. (15), one can get the reliability of the system when it starts from state S_0.

The mean time to system failure (MTSF) can be obtained by using the well known formula-

MTSF= $E(T_0) = \lim_{s\to 0} R_0^*(s) = \dfrac{N_1(s)}{D_1(s)} = \dfrac{N_1(0)}{D_1(0)} = \dfrac{N_1}{D_1}$ (16)

Now using the results $q_{ij}^*(0) = p_{ij}$ and $Z_i^*(0) = \psi_i$, we get

$$N_1 = \psi_0 + \psi_1 + p_{13}(\psi_3 + p_{35}\psi_5) = \frac{\Gamma(1+\frac{1}{p})}{(\alpha_1)^{1/p}} + \frac{\Gamma(1+\frac{1}{p})}{(\beta_1+\alpha_2)^{1/p}} + \frac{\beta_1}{\beta_1+\alpha_2}\left(\frac{\Gamma(1+\frac{1}{p})}{(\alpha_2+\mu)^{1/p}} + \frac{b\mu}{\mu+\alpha_2} * \frac{\Gamma(1+\frac{1}{p})}{(\alpha_2+\lambda)^{1/p}}\right)$$

$$D_1 = 1 - p_{13}(p_{30} + p_{35}p_{50}) = 1 - \frac{\beta_1}{\beta_1+\alpha_2}\left(\frac{a\mu}{\mu+\alpha_2} + \frac{b\mu}{\mu+\alpha_2} . \frac{\lambda}{\lambda+\alpha_2}\right)$$

5.2 Availability Analysis

Let us define $A_i(t)$ as the probability that the system is up at time t when initially it starts from state $S_i \in E$. Using the technique of Laplace transform, one can obtain the value of $A_0(t)$ in terms of its L.T; i.e. $A_0^*(s)$ .Now the steady state availability (probability that in the long run the system will be operative) of the system when it starts from state S_0 is given by

$$A_0 = \lim_{t\to\infty} A_0(t)$$

$$= \lim_{s\to 0} s\, A_0^*(s) = \frac{N_2}{D_2} \qquad (17)$$

Where,

$$N_2 = [\psi_0 p_{13} + \psi_1]p_{50}(1 - p_{34}) + [\psi_3 p_{50} + p_{35}\psi_5]p_{13}$$

$$= \left[\frac{\Gamma(1+\frac{1}{p})}{(\alpha_1)^{1/p}} * \frac{\beta_1}{\beta_1+\alpha_2} + \frac{\Gamma(1+\frac{1}{p})}{(\beta_1+\alpha_2)^{1/p}}\right] \cdot \frac{\lambda}{\lambda+\alpha_2} \cdot \left(1 - \frac{b\mu}{\mu+\alpha_2}\right) +$$

$$\left[\frac{\Gamma(1+\frac{1}{p})}{(\alpha_2+\mu)^{1/p}} \cdot \frac{\lambda}{\lambda+\alpha_2} + \frac{b\mu}{\mu+\alpha_2} \cdot \frac{\Gamma(1+\frac{1}{p})}{(\alpha_2+\lambda)^{1/p}}\right] \cdot \frac{\beta_1}{\beta_1+\alpha_2}$$

$$D_2 = p_{13}(1-p_{34})\psi_0 + p_{50}(1-p_{34})\psi_1 + p_{12}(1-p_{34})p_{50}\psi_2$$
$$+p_{13}p_{50}\psi_3 + p_{13}p_{34}p_{50}\psi_4 + p_{13}p_{35}\psi_5 + p_{13}p_{35}p_{56}\psi_6$$
$$= \frac{\beta_1}{\beta_1+\alpha_2}\cdot(1-\frac{\alpha_2}{\mu+\alpha_2})\cdot\frac{\Gamma(1+\frac{1}{p})}{(\alpha_1)^{1/p}} + \frac{\lambda}{\lambda+\alpha_2}\cdot(1-\frac{\alpha_2}{\mu+\alpha_2})\cdot\frac{\Gamma(1+\frac{1}{p})}{(\beta_1+\alpha_2)^{1/p}} +$$
$$\frac{\alpha_2}{\alpha_2+\beta_1}\cdot(1-\frac{\alpha_2}{\mu+\alpha_2})\cdot\frac{\lambda}{\lambda+\alpha_2}\cdot\frac{\Gamma(1+\frac{1}{p})}{(\beta_2)^{1/p}} + \frac{\beta_1}{\beta_1+\alpha_2}\cdot\frac{\lambda}{\lambda+\alpha_2}*\frac{\Gamma(1+\frac{1}{p})}{(\alpha_2+\mu)^{1/p}} +$$
$$\frac{\beta_1}{\beta_1+\alpha_2}\cdot\frac{\lambda}{\lambda+\alpha_2}\cdot\frac{\alpha_2}{\mu+\alpha_2}\cdot\frac{\Gamma(1+\frac{1}{p})}{(\beta_2)^{1/p}} + \frac{\beta_1}{\beta_1+\alpha_2}\cdot\frac{b\mu}{\mu+\alpha_2}\cdot\frac{\Gamma(1+\frac{1}{p})}{(\alpha_2+\lambda)^{1/p}} +$$
$$\frac{\beta_1}{\beta_1+\alpha_2}\cdot\frac{b\mu}{\mu+\alpha_2}\cdot\frac{\alpha_2}{\lambda+\alpha_2}\cdot\frac{\Gamma(1+\frac{1}{p})}{(\beta_2)^{1/p}}$$

The expected up time of the system during (0, t) is given by

$$\mu_{up}(t) = \int_0^t A_0(u)du \quad \text{So that,} \quad \mu^*_{up}(s) = \frac{A^*_0(s)}{s} \tag{18}$$

5.3 Busy Period Analysis

Let us define $B_i^r(t)$, $B_i^I(t)$ and $B_i^{pr}(t)$ as the probabilities that the repairman is busy in repair, inspection after repair of unit-1, post repair after the inspection of repaired unit-1 at epoch t when the system starts from state $S_i \in E$. Using the probabilistic arguments one can obtain the values of $B_0^r(t)$, $B_0^I(t)$ and $B_0^{pr}(t)$ in terms of their Laplace transforms i.e. $B_0^{r*}(s)$, $B_0^{I*}(s)$ and $B_0^{pr*}(s)$.

In the long run, the probabilities that the repairman will be busy in repair, inspection after repair of unit-1 and post repair of unit-1and replacement of failed unit respectively are given by

$$B_0^r = \frac{N_3}{D_2}$$
$$B_0^I = \frac{N_4}{D_2}$$
$$B_0^{pr} = \frac{N_4}{D_2} \tag{19-21}$$

Where,

$$N_3 = (1-p_{34}p_{43})[(\psi_1 + p_{12}\psi_2)p_{50} + p_{13}\psi_6]$$

$$= (1-\frac{\alpha_2}{\mu+\alpha_2})\cdot[(\frac{\Gamma(1+\frac{1}{p})}{(\beta_1+\alpha_2)^{1/p}} + \frac{\alpha_2}{\alpha_2+\beta_1}\cdot\frac{\Gamma(1+\frac{1}{p})}{(\beta_2)^{1/p}})\cdot\frac{\lambda}{\lambda+\alpha_2} + \frac{\beta_1}{\beta_1+\alpha_2}\cdot\frac{\Gamma(1+\frac{1}{p})}{(\beta_2)^{1/p}}]$$

$$N_4 = (1-p_{56})p_{13}\psi_3$$

$$= (1-\frac{\alpha_2}{\lambda+\alpha_2})\cdot\frac{\beta_1}{\beta_1+\alpha_2}\cdot\frac{\Gamma(1+\frac{1}{p})}{(\alpha_2+\mu)^{1/p}}$$

$$N_5 = p_{13}p_{35}\psi_5$$

$$= \frac{\beta_1}{\beta_1+\alpha_2}\cdot\frac{b\mu}{\mu+\alpha_2}\cdot\frac{\Gamma(1+\frac{1}{p})}{(\alpha_2+\lambda)^{1/p}}$$

and D_2 is same as given in availability analysis.

The expected busy periods of the repairman in repair, inspection & post repair during (0, t) respectively are given by

$$\mu_b^r(t) = \int_0^t B_0^r(u)\,du\,,\ \mu_b^I(t) = \int_0^t B_0^I(u)\,du \ \text{ and } \ \mu_b^{pr}(t) = \int_0^t B_0^{pr}(u)\,du\,,$$

So that,

$$\mu_b^{r*}(s) = \frac{B_0^{r*}(s)}{s}\ ,\mu_b^{I*}(s) = \frac{B_0^{I*}(s)}{s} \ \text{ and } \ \mu_b^{pr*}(s) = \frac{B_0^{pr*}(s)}{s} \qquad (22\text{-}24)$$

5.4 Profit Function Analysis

Let us define

C_1 = revenue per-unit up time (in Rs.) of the system.

C_2 = cost per unit time (in Rs.) when the repairman is busy in repairing of either failed unit.

C_3 = cost per unit time (in Rs.) for which the repairman is busy in the Inspection of first repaired unit.

C_4 = cost per unit time (in Rs.) for which the repairman is busy in the post Repair of first unit after the inspection.

Then, expected total profit incurred in time interval (0, t) is

P(t) = Expected total revenue in (0, t) – Expected cost of repair in (0, t) – Expected cost of Inspection in (0, t) – Expected cost of post repair in (0, t)

$$= C_1\,\mu_{up}(t) - C_2\,\mu_b^r(t) - C_3\,\mu_b^I(t) - C_4\,\mu_b^{pr}(t) \qquad (25)$$

The expected total cost per-unit time in steady state is given by

$$= C_1A_0 - C_2B_0^r - C_3B_0^I - C_4B_0^{pr} \tag{26}$$

Where A_0, B_0^r, B_0^I and B_0^{pr} have been already defined.

6. Estimation of Parameters, MTSF and Profit Function

6.1 Classical Estimation

Suppose the shape parameter p is known and the scale parameters $\alpha_1,\alpha_2,\beta_1,\beta_2,\mu,\lambda$ involved in the distributions are assumed to be unknown and follow the following (prior) distributions:

$$\begin{aligned} &\alpha_1 \sim \text{Gamma}(a_1,b_1)\\ &\alpha_2 \sim \text{Gamma}(a_2,b_2)\\ &\beta_1 \sim \text{Gamma}(a_3,b_3)\\ &\beta_2 \sim \text{Gamma}(a_4,b_4)\\ &\mu \sim \text{Gamma}(a_5,b_5)\\ &\lambda \sim \text{Gamma}(a_6,b_6) \end{aligned} \tag{27-32}$$

Here, a_i and b_i (i = 1, 2, 3, 4, 5, 6) respectively denote the scale and shape parameters.

6.1.1 ML Estimation

The failure, repair, inspection and post repair times of units of system are assumed to be independently Weibull distributed random variables with failure rates $h_1(.)$, $h_2(.)$, repair rates $K_1(.)$, $K_2(.)$, inspection rate m(.) and post repair rate e(.) respectively.

Where

$h_i(t) = \alpha_i p t^{p-1}$, $K_i(t) = \beta_i p t^{p-1}$, $t \geq 0$ and α_i, β_i, $p > 0$ and i=1, 2

$m(t) = \mu p t^{p-1}$; $\mu, p, t > 0$

$e(t) = \lambda p t^{p-1}$; $\lambda, p, t > 0$

Here α_i, β_i, μ, λ are scale parameters and p is the shape parameter.

Let

$$\underset{\sim}{X}_1=(x_{11},x_{12},\ldots\ldots,x_{1n_1}),\underset{\sim}{X}_2=(x_{21},x_{22},\ldots\ldots,x_{2n_2}),\ \underset{\sim}{X}_3=(x_{31},x_{32},\ldots\ldots,x_{3n_3}),\underset{\sim}{X}_4=(x_{41},x_{42},\ldots\ldots,x_{4n_4}),$$
$$\underset{\sim}{X}_5=(x_{51},x_{52},\ldots\ldots,x_{5n_5}),\underset{\sim}{X}_6=(x_{61},x_{62},\ldots\ldots,x_{6n_6})$$

be six independent random samples of size n_i (i=1,2,3,4,5,6) drawn from Weibull distribution with failure rates h_1 (.), h_2 (.), repair rates k_1 (.), k_2 (.),inspection rate m(.) and post repair rate e(.) respectively.

The likelihood function of the combined sample is

$$L\left(\underset{\sim}{X}_1,\underset{\sim}{X}_2,\underset{\sim}{X}_3,\underset{\sim}{X}_4,\underset{\sim}{X}_5,\underset{\sim}{X}_6 \middle| \alpha_1,\alpha_2,\beta_1,\beta_2,\mu,\lambda\right)=\alpha_1^{\,n_1}\alpha_2^{\,n_2}\beta_1^{\,n_3}\beta_2^{\,n_4}\mu^{n_5}\lambda^{n_6}\,p^{n_1+n_2+n_3+n_4+n_5+n_6}$$
$$Z_1 Z_2 Z_3 Z_4 Z_5 Z_6\, e^{-(\alpha_1 W_1+\alpha_2 W_2+\beta_1 W_3+\beta_2 W_4+\mu W_5+\lambda W_6)} \tag{33}$$

Where

$$W_i=\sum_{j=1}^{n_i} x_{ij}^{\,p} \text{ and } Z_i=\prod_{j=1}^{n_i} x_{ij}^{\,p-1} \quad ; \quad i=1,2,3,4,5,6$$

By using usual maximization likelihood approach, the M.L. estimates say($\hat{\alpha}_1,\hat{\alpha}_2,\hat{\beta}_1,\hat{\beta}_2,\hat{\mu},\hat{\lambda}$) of the parameters ($\alpha_1,\alpha_2,\beta_1,\beta_2,\mu,\lambda$) are

$$\hat{\alpha}_1=n_1/W_1$$
$$\hat{\alpha}_2=n_2/W_2$$
$$\hat{\beta}_1=n_3/W_3$$
$$\hat{\beta}_2=n_4/W_4$$
$$\hat{\mu}=n_5/W_5$$
$$\hat{\lambda}=n_6/W_6$$

Now, using the invariance property of ML estimates, the estimates of the MTSF and profit function, say, $\hat{M}$ and $\hat{P}$ can be obtained. The asymptotic sampling distribution of

$$\begin{pmatrix} \hat{\alpha}_1-\alpha_1 \\ \hat{\alpha}_2-\alpha_2 \\ \hat{\beta}_1-\beta_1 \\ \hat{\beta}_2-\beta_2 \\ \hat{\mu}-\mu \\ \hat{\lambda}-\lambda \end{pmatrix} \sim N_6(0,I^{-1}),$$

where I denotes the Fisher information matrix with diagonal elements

$I_{11}=\dfrac{n_1}{\alpha_1^2}$, $I_{22}=\dfrac{n_2}{\alpha_2^2}$, $I_{33}=\dfrac{n_3}{\beta_1^2}$, $I_{44}=\dfrac{n_4}{\beta_2^2}$, $I_{55}=\dfrac{n_5}{\mu^2}$, $I_{66}=\dfrac{n_6}{\lambda^2}$ and non diagonal elements are all zero.

Also, the asymptotic distribution of $\left(\hat{M}-M\right)\sim N_6\left(0,A'I^{-1}A\right)$ and $\left(\hat{P}-P\right)\sim N_6\left(0,B'I^{-1}B\right)$, where

$$A'=\left(\frac{\partial M}{\partial \alpha_1},\frac{\partial M}{\partial \alpha_2},\frac{\partial M}{\partial \beta_1},\frac{\partial M}{\partial \beta_2},\frac{\partial M}{\partial \mu},\frac{\partial M}{\partial \lambda}\right)', B'=\left(\frac{\partial P}{\partial \alpha_1},\frac{\partial P}{\partial \alpha_2},\frac{\partial P}{\partial \beta_1},\frac{\partial P}{\partial \beta_2},\frac{\partial p}{\partial \mu},\frac{\partial P}{\partial \lambda}\right)$$

6.1.2 Bayesian Estimation

Using the likelihood function in (33) and prior distribution of $\alpha_1,\alpha_2,\beta_1,\beta_2,\mu,\lambda$ in (27-32) the posterior distributions of these parameters are obtained as follows:

$$\begin{aligned}
&\alpha_1 \mid \underset{\sim}{X}_1 \sim \text{Gamma}(n_1+a_1, b_1+W_1)\\
&\alpha_2 \mid \underset{\sim}{X}_2 \sim \text{Gamma}(n_2+a_2, b_2+W_2)\\
&\beta_1 \mid \underset{\sim}{X}_3 \sim \text{Gamma}(n_3+a_3, b_3+W_3)\\
&\beta_2 \mid \underset{\sim}{X}_4 \sim \text{Gamma}(n_4+a_4, b_4+W_4)\\
&\mu \mid \underset{\sim}{X}_5 \sim \text{Gamma}(n_5+a_5, b_5+W_5)\\
&\lambda \mid \underset{\sim}{X}_6 \sim \text{Gamma}(n_6+a_6, b_6+W_6)
\end{aligned} \qquad (34\text{-}39)$$

For obtaining Bayes estimates and width of HPD intervals of the parameters, we generated observations from the above posteriors distributions. For obtaining Bayesian estimation and width of HPD intervals of MTSF and Profit function, we substituted the above draws directly in the equations (16) and (25). Assuming square error loss function, the sample means of the respective draws are taken as the Bayes estimates of the parameter and reliability characteristics. For obtaining width of HPD intervals, 'boa' package of R-software has been used.

7. Simulation Study

A simulation study is carried out to examine the behavior of the estimates of parameters and reliability characteristics. . For comparing the performances of MLE and Bayes estimates, the Standard Error (SE)/Posterior Standard Error (PSE) and width of Confidence/HPD intervals are computed and are given in Tables 1-6.

Samples of sizes $n_1 = n_2 = n_3 = n_4 = n_5 = n_6 = 180$ have been drawn from the six considered distributions by assuming various values of the parameters as given in Tables 1-6. The number of repetitions used is 10000. All calculations are performed on R.2.14.2.

For a more concrete study of the system behavior, we also plot curves for MTSF and Profit function w.r.t. failure rate α_1 for different values of repair rate β_1=0.4,0.5,0.6, while the other parameters are kept fixed as β_2=.2, α_2=.6, μ_1=.2, λ=.5, C_1=2500; C_2=800; C_3=100; C_4=50; a=.5; b=.5, p=1.0.

8. Concluding Remarks

- From Figs 2-4, it is observed that MTSF decreases as failure rate α_1 increases while it increases as repair rate β_1 increases. Same trends for profit function are also observed from Figs 5-7.

- From Tables 1-6, it is also observed that for fixed β_1 and varying α_1, Bayes estimates of MTSF and profit function perform well as compared to their MLEs as they have lesser PSE than that of MLEs. Also width of HPD intervals is more conservative as compared to the width of confidence intervals.
- Hence, from the above discussion we conclude that Bayes approach is better than Classical approach for estimating the MTSF and profit function for the considered model.

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α_1	True. MTSF	ML.MTSF	SE	C.I.	Gamma-Bayes. MTSF	PSE	HPD Interval
0.1	12.451	12.994	0.836	3.277	12.135	0.823	3.193
0.2	7.059	7.314	0.438	1.717	6.659	0.42	1.643
0.3	5.261	5.42	0.311	1.219	4.834	0.288	1.13
0.4	4.363	4.473	0.25	0.98	3.922	0.223	0.875
0.5	3.824	3.905	0.216	0.847	3.375	0.185	0.723
0.6	3.464	3.526	0.194	0.76	3.01	0.161	0.623
0.7	3.207	3.256	0.18	0.706	2.75	0.143	0.556
0.8	3.015	3.053	0.169	0.662	2.555	0.131	0.508
0.9	2.865	2.895	0.162	0.635	2.404	0.122	0.473
1.0	2.745	2.769	0.156	0.612	2.282	0.115	0.445

Table-1: The values of MTSF for fixed β_1=.4 and varying α_1

α1	True.MTSF	ML.MTSF	SE	C.I.	Gamma-Bayes. MTSF	PSE	HPD Interval
0.1	12.568	13.114	0.848	3.324	12.245	0.833	3.234
0.2	7.117	7.374	0.445	1.744	6.714	0.426	1.665
0.3	5.3	5.46	0.315	1.235	4.871	0.292	1.144
0.4	4.392	4.503	.254	0.996	3.95	0.226	0.885
0.5	3.847	3.929	0.219	0.858	3.397	0.188	0.732
0.6	3.483	3.547	0.197	0.772	3.029	0.163	0.63
0.7	3.224	3.273	0.182	0.713	2.766	0.145	0.563
0.8	3.029	3.068	0.171	0.67	2.569	0.133	0.515
0.9	2.878	2.909	0.164	0.643	2.416	0.123	0.478
1.0	2.757	2.781	0.158	0.619	2.294	0.116	0.449

Table-2: The values of MTSF for fixed β_1=.5 and varying α_1

α_1	True.MTSF	ML.MTSF	SE	C.I.	Gamma-Bayes.MTSF	PSE	HPD Interval
0.1	12.667	13.216	0.858	3.363	12.326	0.841	3.261
0.2	7.167	7.425	0.45	1.764	6.755	0.43	1.68
0.3	5.333	5.494	0.319	1.25	4.898	0.294	1.155
0.4	4.417	4.529	0.257	1.007	3.97	0.228	0.893
0.5	3.867	3.95	0.221	0.866	3.413	0.189	0.739
0.6	3.5	3.564	0.199	0.78	3.042	0.164	0.635
0.7	3.238	3.288	0.184	0.721	2.778	0.146	0.568
0.8	3.042	3.081	0.173	0.678	2.579	0.134	0.519
0.9	2.889	2.92	0.165	0.647	2.425	0.124	0.482
1.0	2.767	2.792	0.159	0.623	2.302	0.117	0.453

Table-3: The values of MTSF for fixed β_1=.6 and varying α_1

α_1	True.profit	ML.profit	SE	C.I.	Gamma-Bayes.profit	PSE	HPD Interval
0.1	3709.091	3774.547	495.887	1943.877	3221.868	353.727	1374.642
0.2	3034.091	3067.671	431.279	1690.614	2519.426	278.771	1083.3
0.3	2737.879	2752.986	442.687	1735.333	2255.966	272.225	1068.162
0.4	2536.364	2536.348	479.407	1879.275	2102.285	285.922	1116.85
0.5	2372.727	2358.929	529.199	2074.46	1992.555	309.3	1207.349
0.6	2228.03	2201.119	586.816	2300.319	1904.842	338.216	1321.369
0.7	2094.156	2054.515	649.476	2545.946	1829.754	370.584	1447.523
0.8	1967.045	1914.914	715.546	2804.94	1762.601	405.213	1582.455
0.9	1844.444	1779.983	784.013	3073.331	1700.783	441.375	1724.584
1.0	1725	1648.319	854.221	3348.546	1642.747	478.597	1865.923

Table-4: The values of profit for fixed β_1=.4 and varying α_1

α_1	True. profit	ML. profit	SE	C.I.	Gamma-Bayes. profit	PSE	HPD Interval
0.1	4077.479	4157.894	515.518	2020.831	3490.756	368.171	1432.266
0.2	3397.727	3442.821	438.089	1717.309	2756.128	283.413	1104.044
0.3	3148.416	3173.883	442.154	1733.244	2509.932	272.443	1070.05
0.4	3006.715	3016.48	472.354	1851.628	2385.845	282.789	1105.518
0.5	2908.058	2903.69	515.904	2022.344	2310.604	303.193	1185.388
0.6	2830.923	2813.207	567.476	2224.506	2259.787	329.332	1287.157
0.7	2766.086	2735.47	624.256	2447.084	2222.929	359.05	1401.413
0.8	2708.936	2665.7	684.586	2683.577	2194.798	391.123	1526.627
0.9	2656.91	2601.242	747.43	2929.926	2172.487	424.798	1655.543
1.0	2608.471	2540.501	812.109	3183.467	2154.252	459.589	1793.682

Table-5: The values of profit for fixed β_1=.5 and varying α_1

α_1	True. Profit	ML. profit	SE	C.I.	Gamma-Bayes. profit	PSE	HPD Interval
0.1	4372.348	4465.725	530.408	2079.199	3680.086	378.007	1474.73
0.2	3676.515	3731.572	443.598	1738.904	2917.265	286.654	1116.244
0.3	3454.167	3488.595	442.934	1736.301	2678.602	272.963	1070.515
0.4	3350.189	3368.411	469.195	1839.244	2570.96	281.406	1103.539
0.5	3293.561	3297.346	509.064	1995.531	2515.705	300.196	1173.141
0.6	3260.606	3250.839	557.104	2183.848	2486.622	324.862	1270.968
0.7	3241.18	3218.366	610.466	2393.027	2472.471	353.194	1378.175
0.8	3230.208	3194.664	667.47	2616.482	2467.63	383.937	1499.417
0.9	3224.874	3176.809	727.063	2850.087	2458.97	416.326	1621.464
1.0	3223.485	3163.048	788.55	3091.116	2430.614	449.863	1753.025

Table-6: The values of profit for fixed β_1=.6 and varying α_1

Fig.-2 : Plot of MTSF for fixed β_1 = 0.4 and varying α_1

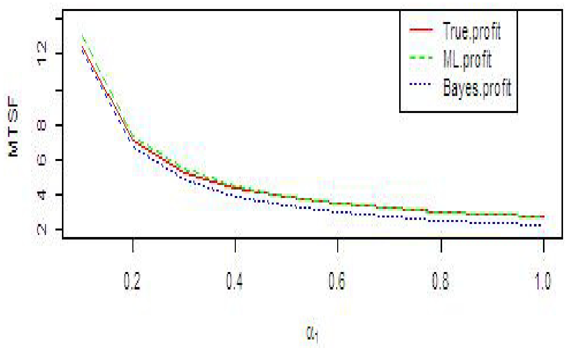

Fig.-3 : Plot of MTSF for fixed β_1 = 0.5 and varying α_1

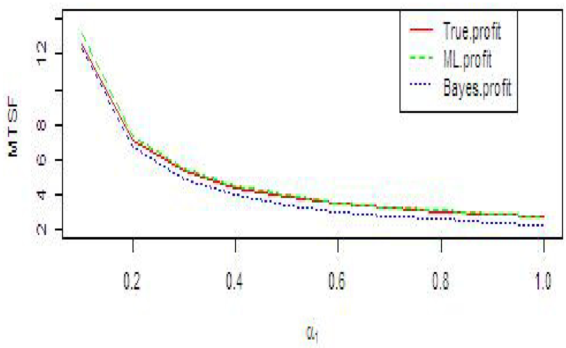

Fig.-4 : Plot of MTSF for fixed β_1 = 0.6 and varying α_1

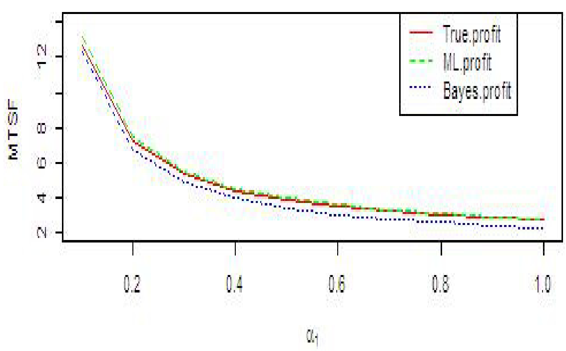

Fig.-5 : Plot of Profit for fixed β_1 = 0.4 and varying α_1

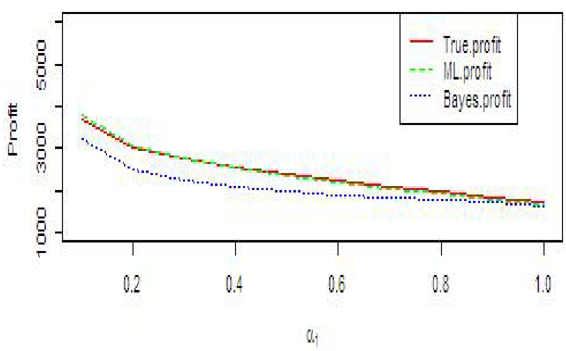

Fig.-6 : Plot of Profit for fixed β_1 = 0.5 and varying α_1

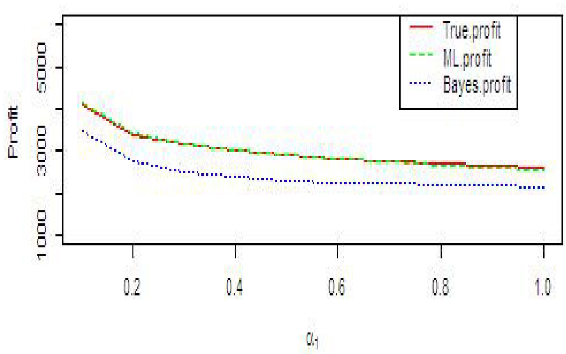

Fig.-7 : Plot of Profit for fixed β_1 = 0.6 and varying α_1