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COST BENEFIT ANALYSIS OF A SYSTEM UNDER HEAD-OF-LINE REPAIR APPROACH USING GUMBEL-HOUGAARD FAMILY COPULA

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Abstract
This paper develops a new mathematical model of a system with a novel approach, which consists of two independent repairable subsystems. The model is analyzed under "Head-of-Line" repair policy considering two types of repair between two successive transitions at a stage contrast to the normal practice of assuming single type of transition in all states. Supplementary variable technique, Laplace transformation and Gumbel-Hougaard family copula techniques are applied to obtain the availability and cost analysis of the system. At last some numerical examples have been taken to illustrate the model.

Key Words: Cost benefit analysis; Maintenance; Reliability; Head-of-Line Repair; Complex Engineering Systems.

1. Introduction
The need of the hour demands such systems which are ever available and reliable. An organization wants to achieve the maximum customer satisfaction in order to sustain the competition. The management is highly concerned with the reliable operation of production systems. Reliability engineering has its own complexity due to due to involvement of rigorous statistics but at the same time it has a wide practical approach. According to a mature scientific theory, a probabilistic method which deals with uncertainty as it is random in nature but is of an ordered kind can be used to determine reliability. It can also be said that the reliability of a system is determined by the constituent subsystems and reliability of such systems is in turn determined by the associated components and their possible failure modes. This research is a step towards explaining the reliability application on a repairable system with three types of failure under 'head-of-line' repair policy and Gumbel-Hougaard family copula.

So in earlier research [1, 3, 5, 7, 18], different techniques have been applied to evaluate the reliability of distribution system, including distributed generation such as an analytical technique using the load duration curve, distributed processing technique, Characteristic function based approach for computing the probability distributers of reliability indices, probabilistic method for assessing the reliability and quantity of power supply to a customer, composite load point model, practical reliability assessment algorithm, validation method and impact of substation on distribution reliability respectively. Choi et al. [9] presents a new and practical approach in selecting a reasonable expansion plan prior to checking system stability and dynamics. The special protection system has been used to increase the transfer capability of the network. McCalley and Fu [16] elucidate the importance of developing a systematic and comprehensive reliability framework for the special protection system. Vanderperre [26] studied the system reliability when all distributions are general. Raghavendra et al. [19] introduced: distributed program reliability, distributed system reliability and Luo et al. [15] proposed two fault-tolerant techniques with fixed priority-based scheduling algorithms, for distributed systems. The heuristic approaches for computer communication networks and telecommunication networks developed by [10, 14] and High Performance Computing systems (HPC) by Gottumukkala [12].

Furthermore, a large number of researchers in the field of reliability have studied the repairable systems. Most of them [2, 4, 8, 11, 20, 22, 25] have concentrated their attention on system parameters like mean time to failure, reliability, availability and cost analysis with different types of failure and one type of repair. The authors [21, 24] have discussed various reliability measures of the system under 'preemptive-resume repair' and 'preemptive-repeat repair' discipline [6] with the help of copula discussed [17]. But they did not consider one of the important aspects of repair policies that the system can be analyzed with 'head of line repair' discipline. Although Kishore et al. [13] analyzed the system under 'head of line' repair policy, but they did not apply the concept of the copula.

2. Brief Introduction of Gumbel-Hougaard Family Copula

The family of copulas has been studied extensively by a number of authors including Nelsen [18]. The Gumbel-Hougaard family copula is as:

$$C_\theta(u_1,u_2)=\exp(-((-\log u_1)^\theta+(-\log u_2)^\theta)^{1/\theta}),\quad 1\le\theta\le\infty$$

For $\theta = 1$ the Gumbel-Hougaard copula models independence, for $\theta\to\infty$ it converges to comonotonicity.

3. Mathematical Model Details

A. Nomenclature

λ_A	Failure rate of both i.i.d. units for subsystem A.
λ_{Pj}, λ_C	Failure rates of subsystem B for partial and catastrophic failure respectively where $\lambda_{Pj}=\sum_{j=1}^{n}\lambda_{Pj}$ unless otherwise mentioned.
$\phi_i(r), S_i(r)$ i = A, r = x i = P, r = y i = C, r = z	Transition repair rate and probability density, elapsed repair time x, whereas repair of partial and catastrophic failure in subsystem B is completed in elapsed repair times y and z respectively where $\phi_{Pj}=\sum_{j=1}^{n}\phi_{Pj}$ unless otherwise mentioned.
$P_i(t)$	The probability that the system is in S_i a state at instant 't' for i =0 to 7.
$\overline{P}(s)$	Laplace transformation of $P(t)$.

u_1, u_2	Marginal distribution of random variables, where $u_1 = e^x$ and $u_2 = \phi_A(x)$.
$E_p(t)$	Expected profit during the interval (0, t].
K_1, K_2	Revenue per unit time and service cost per unit time respectively.

Letting $u_1 = e^x$ and $u_2 = \phi_A(x)$, the expression for joint probability (failed state S_4 to good state S_0) according to Gumbel-Hougaard family is given as: $\exp[x^\theta + \{\log\phi_A(x)\}^\theta]^{1/\theta}$.

B. Model Description and Assumptions

So, in contrast to the earlier research of Ram and Singh [23, 25], here authors have extended their research with 'head of line' repair policy. In this study the system considered consists of two independent repairable subsystems A and B in series (1-out-of-2:F) i.e. if any one of A or B fails, the complete system fails. Subsystem A has two identical units arranged in parallel redundancy (1-out-of-2:G), subsystem B has *n* unit in series (1-out-of-n:F) with two types of failure, viz., partial and catastrophic. The system is analyzed under head-of line repair policy i.e. the policy is first come first served. The present study has considered a model in which authors tried to address the problem where two different repair facilities are available between adjacent states S_4 and S_0 (where S_0 is the initial state when both the units are in good working condition and S_4 is the state when both the units have failed completely hence the whole system has failed completely). Here, the authors have considered a parallel redundant system with two independent and identically distributed units and three states: good, degraded and failed. The units can suffer from two types of failure: partial and catastrophic which brings a unit to degraded and failed state respectively. The catastrophic failure can occur in a degraded state too. The repair of partial failure is opportunistic, i.e. it would be repaired along with the catastrophic failure. The failure and repair times for the system follow exponential and general distribution respectively in general. However the repair from state S_4 to S_0 has two types namely exponential and general. The system is studied by using the supplementary variable technique, Laplace transformation and Gumbel-Hougaard family of copula to obtain various reliability measures such as transition state probabilities, steady state probability, mean time to failure availability and cost analysis. At last some particular cases of the system are taken to highlight the different possibilities.

The following assumptions are associated with the model:

(i). Initially the system is in good state.
(ii). Subsystem A (1-out-of-2:G) and B (1-out-of-n:) are in series.
(iii). Each unit of subsystem A has same constant failure rate and two states i.e. good, failed.
(iv). Partial failure brings subsystem B to degraded state and hence the whole system. Catastrophic failure can occur in this state too.
(v). Catastrophic failure breaks down subsystem B completely and hence the whole system.
(vi). Subsystem B has three states: good, degraded and failed.

(vii). All repairs follow the general time distribution except from S_4 to S_0 whereas all failures follow an exponential time distribution.
(viii). The system has only one repair facility except from S_4 to S_0 where it has got two.
(ix). After repairing system is as good as new. Repair never damages anything.
(x). Failed A-units are repaired only when the system is in complete failed state.
(xi). The repair of partial failure is undertaken along with the repair of catastrophic failure.
(xii). Subsystem A is undertaken under 'head-of-line repair' policy i.e. the repair of subsystem B is continued and repair of the subsystem A is entertained only when the repair of subsystem B is completed.
(xiii). The transition from the completely failed state S_4 to the initial state S_0 follows two different distributions.
(xiv). Joint probability distribution of repair rate from completely failed state S_4 to the initial state S_0 follows Gumbel- Hougaard family of Copula.

Figure 1 represents the state transition diagram of the system.

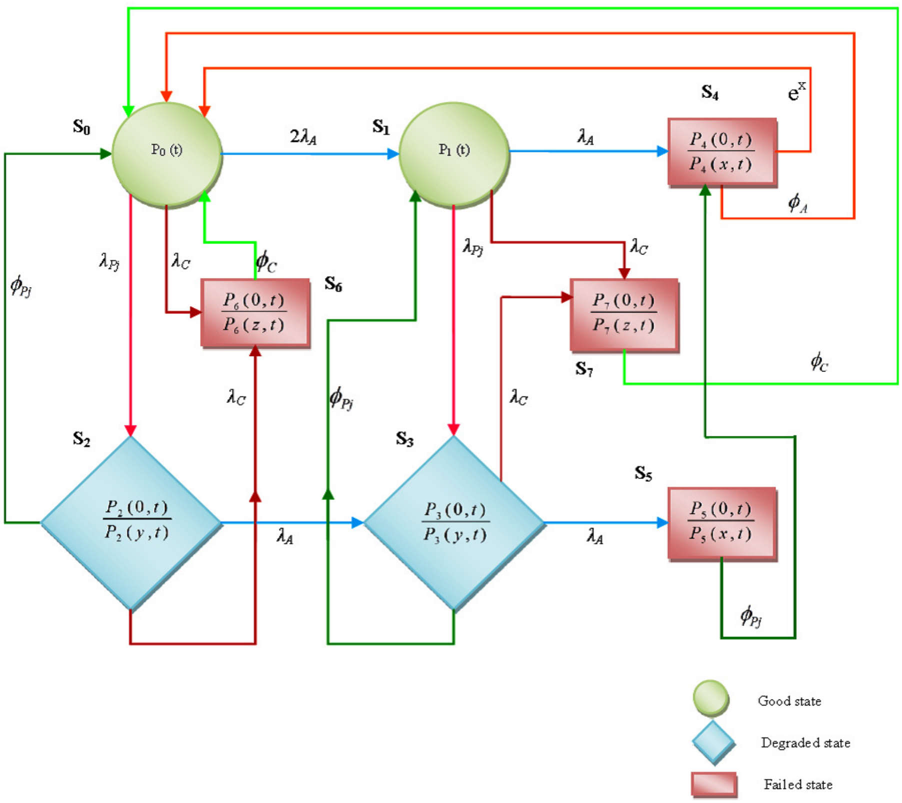

Fig. 1: State Transition Diagram

C. Formulation and Solution of Mathematical Model

By the probability of the considerations and continuity arguments we can obtain the following set of difference-differential equations governing the present mathematical model:

$$\left[\frac{\partial}{\partial t}+2\lambda_A+\lambda_{Pj}+\lambda_C\right]P_0(t)=\int_0^\infty P_2(y,t)\phi_{Pj}(y)dy+\int_0^\infty P_4(x,t)\exp[x^\theta+\{\log\phi_A(x)\}^\theta]^{1/\theta}dx$$
$$+\int_0^\infty P_6(z,t)\phi_C(z)dz+\int_0^\infty P_7(z,t)\phi_C(z)dz \quad (1)$$

$$\left[\frac{\partial}{\partial t}+\lambda_A+\lambda_{Pj}+\lambda_C\right]P_1(t)=2\lambda_A P_0(t)+\int_0^\infty P_3(y,t)\phi_{Pj}(y)dy \quad (2)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial y}+2\lambda_A+\lambda_C+\phi_{Pj}(y)\right]P_2(y,t)=0 \quad (3)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial y}+\lambda_A+\lambda_C+\phi_{Pj}(y)\right]P_3(y,t)=2\lambda_A P_2(y,t)$$
(4)

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\exp[x^\theta+\{\log\phi_A(x)\}^\theta]^{1/\theta}\right]P_4(x,t)=0 \quad (5)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial y}+\phi_{Pj}(y)\right]P_5(y,t)=\lambda_A P_3(y,t) \quad (6)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial z}+\phi_C(z)\right]P_6(z,t)=0 \quad (7)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial z}+\phi_C(z)\right]P_7(z,t)=0 \quad (8)$$

Boundary conditions

$$P_2(0,t)=\lambda_{Pj}P_0(t) \quad (9)$$

$$P_3(0,t)=\lambda_{Pj}P_1(t) \quad (10)$$

$$P_4(0,t)=\lambda_A P_1(t)+\int_0^\infty P_5(y,t)\phi_{Pj}(y)dy \quad (11)$$

$$P_5(0,t)=0 \quad (12)$$

$$P_6(0,t)=\lambda_C P_0(t)+\lambda_C P_2(t) \quad (13)$$

$$P_7(0,t)=\lambda_C P_1(t)+\lambda_C P_3(t) \quad (14)$$

Initial condition

$P_0(0)=1$ and other state probabilities are zero at t = 0 (15)

Taking Laplace transformation of equations (1-14) and using equation (15), we obtain

$$\left[s+2\lambda_A+\lambda_{Pj}+\lambda_C\right]\overline{P}_0(s)=1+\int_0^\infty \overline{P}_2(y,s)\phi_{Pj}(y)dy+\int_0^\infty \overline{P}_4(x,s)\exp[x^\theta+\{\log\phi_A(x)\}^\theta]^{1/\theta}dx$$
$$+\int_0^\infty \overline{P}_6(z,s)\phi_C(z)dz+\int_0^\infty \overline{P}_7(z,s)\phi_C(z)dz \quad (16)$$

$$\left[s+\lambda_A+\lambda_{Pj}+\lambda_C\right]\overline{P}_1(s)=2\lambda_A\overline{P}_0(s)+\int_0^\infty \overline{P}_3(y,s)\phi_{Pj}(y)dy \quad (17)$$

$$\left[s+\frac{\partial}{\partial y}+2\lambda_A+\lambda_C+\phi_{Pj}(y)\right]\overline{P}_2(y,s)=0 \quad (18)$$

$$\left[s+\frac{\partial}{\partial y}+\lambda_A+\lambda_C+\phi_{Pj}(y)\right]\overline{P}_3(y,s)=2\lambda_A\overline{P}_2(y,s) \tag{19}$$

$$\left[s+\frac{\partial}{\partial x}+\exp[x^{\theta}+\{\log\phi_A(x)\}^{\theta}]^{1/\theta}\right]\overline{P}_4(x,s)=0 \tag{20}$$

$$\left[s+\frac{\partial}{\partial y}+\phi_{Pj}(y)\right]\overline{P}_5(y,s)=\lambda_A\overline{P}_3(y,s) \tag{21}$$

$$\left[s+\frac{\partial}{\partial z}+\phi_C(z)\right]\overline{P}_6(z,s)=0 \tag{22}$$

$$\left[s+\frac{\partial}{\partial z}+\phi_C(z)\right]\overline{P}_7(z,s)=0 \tag{23}$$

$$\overline{P}_2(0,s)=\lambda_P\overline{P}_0(s) \tag{24}$$

$$\overline{P}_3(0,s)=\lambda_{Pj}\overline{P}_1(s) \tag{25}$$

$$\overline{P}_4(0,s)=\lambda_A\overline{P}_1(s)+\int_0^{\infty}P_5(y,s)\phi_{Pj}(y)dy \tag{26}$$

$$\overline{P}_5(0,s)=0 \tag{27}$$

$$\overline{P}_6(0,s)=\lambda_C\overline{P}_0(s)+\lambda_C\overline{P}_2(s) \tag{28}$$

$$\overline{P}_7(0,s)=\lambda_C\overline{P}_1(s)+\lambda_C\overline{P}_3(s) \tag{29}$$

Solving (16-23) with the help of (24-29), one can get

$$\overline{P}_0(s)=\frac{D(s)}{T(s)} \tag{30}$$

$$\overline{P}_1(s)=\frac{C(s)}{T(s)} \tag{31}$$

$$\overline{P}_2(s)=\lambda_{Pj}\,\delta_{Pj}(s+2\lambda_A+\lambda_C)\overline{P}_0(s) \tag{32}$$

$$\begin{aligned}\overline{P}_3(s)=&2\lambda_{Pj}[\delta_{Pj}(s+\lambda_A+\lambda_C)-\delta_{Pj}(s+2\lambda_A+\lambda_C)]\\ &\times\overline{P}_0(s)+\lambda_{Pj}\delta_{Pj}(s+\lambda_A+\lambda_C)\overline{P}_1(s)\end{aligned} \tag{33}$$

$$\overline{P}_4(s)=\delta_A(s)[E(s)\overline{P}_0(s)+\{\lambda_A+F(s)\}\overline{P}_1(s)] \tag{34}$$

$$\begin{aligned}\overline{P}_5(s)=&2\lambda_A\lambda_{Pj}\left[\frac{\delta_{Pj}(s)-r_{Pj}(s+\lambda_A+\lambda_C)}{\lambda_A+\lambda_C}+\frac{\delta_{Pj}(s+2\lambda_A+\lambda_C)-\delta_{Pj}(s)}{2\lambda_A+\lambda_C}\right]\overline{P}_0(s)\\ &+\frac{\lambda_A\lambda_{Pj}}{\lambda_A+\lambda_C}\left[\delta_{Pj}(s)-\delta_{Pj}(s+\lambda_A+\lambda_C)\right]\overline{P}_1(s)\end{aligned} \tag{35}$$

$$\overline{P}_6(s)=\lambda_C\delta_C(s)[1+\lambda_{Pj}\,\delta_{Pj}(s+2\lambda_A+\lambda_C)]\overline{P}_0(s) \tag{36}$$

$$\begin{aligned}\overline{P}_7(s)=&\lambda_C\delta_C(s)[2\lambda_{Pj}\{\delta_{Pj}(s+\lambda_A+\lambda_C)-\delta_{Pj}(s+2\lambda_A+\lambda_C)\}\overline{P}_0(s)\\ &+\{1+\lambda_{Pj}\delta_{Pj}(s+\lambda_A+\lambda_C)\}\ \overline{P}_1(s)]\end{aligned} \tag{37}$$

where

$$D(s) = s + \lambda_A + \lambda_{Pj} + \lambda_C - \lambda_{Pj}\overline{S}_{Pj}(s + \lambda_A + \lambda_C),$$

$$C(s) = 2\lambda_A + 2\lambda_{Pj}[\overline{S}_{Pj}(s + \lambda_A + \lambda_C) - \overline{S}_{Pj}(s + 2\lambda_A + \lambda_C)],$$

$$A(s) = s + 2\lambda_A + \lambda_{Pj} + \lambda_C - \lambda_{Pj}\overline{S}_{Pj}(s + 2\lambda_A + \lambda_C) - \overline{S}_A(s)E(s)$$
$$- \lambda_C \overline{S}_C(s)[1 - \lambda_{Pj}\delta_{Pj}(s + 2\lambda_A + \lambda_C) + 2\lambda_{Pj}\delta_{Pj}(s + \lambda_A + \lambda_C)],$$

$$B(s) = \overline{S}_A(s)[\lambda_A + F(s)] + \lambda_C\overline{S}_C(s)[1 + \lambda_{Pj}\delta_{Pj}(s + \lambda_A + \lambda_C)],$$

$$T(s) = A(s)D(S) - B(s)C(s),$$

$$E(s) = 2\lambda_A\lambda_{Pj}\left[\frac{\overline{S}_{Pj}(s) - \overline{S}_{Pj}(s + \lambda_A + \lambda_C)}{\lambda_A + \lambda_C} + \frac{\overline{S}_{Pj}(s + 2\lambda_A + \lambda_C) - \overline{S}_{Pj}(s)}{2\lambda_A + \lambda_C}\right],$$

$$F(s) = \lambda_A\lambda_{Pj}\left[\frac{\overline{S}_{Pj}(s) - \overline{S}_{Pj}(s + \lambda_A + \lambda_C)}{\lambda_A + \lambda_C}\right], \ \delta_i(s) = \frac{[1 - \overline{S}_i(s)]}{s}$$

The Laplace transformations of the probabilities that the system is in up (i.e. either good or degraded state) and failed state at any time are as follows:

$$\begin{aligned}\overline{P}_{\text{up}}(s) &= \overline{P}_0(s) + \overline{P}_1(s) + \overline{P}_2(s) + \overline{P}_3(s)\\ &= [1 + \lambda_{Pj}\,\delta_{Pj}(s + 2\lambda_A + \lambda_C) + 2\lambda_{Pj}\{\delta_{Pj}(s + \lambda_A + \lambda_C)\\ &\quad -\delta_{Pj}(s + 2\lambda_A + \lambda_C)\}]\overline{P}_0(s) + [1 + \lambda_{Pj}\,\delta_{Pj}(s + \lambda_A + \lambda_C)]\overline{P}_1(s)\end{aligned} \tag{38}$$

$$\begin{aligned}\overline{P}_{\text{failed}}(s) &= \overline{P}_4(s) + \overline{P}_5(s) + \overline{P}_6(s) + \overline{P}_7(s)\\ &= \left[\delta_A(s)E(s) + 2\lambda_A\lambda_{Pj}\left\{\begin{array}{l}\dfrac{\delta_{Pj}(s) - \delta_{Pj}(s + \lambda_A + \lambda_C)}{\lambda_A + \lambda_C}\\ + \dfrac{\delta_{Pj}(s + 2\lambda_A + \lambda_C) - \delta_{Pj}(s)}{2\lambda_A + \lambda_C}\end{array}\right\}\right.\\ &\quad \left. + \delta_C(s)\lambda_C\{1 - \lambda_P\delta_{Pj}(s + 2\lambda_A + \lambda_C) + 2\lambda_{Pj}\delta_{Pj}(s + \lambda_A + \lambda_C)\}\right]\overline{P}_0(s)\\ &\quad + \left[\begin{array}{l}\delta_A(s)\{\lambda_A + F(s)\} + \lambda_A\lambda_{Pj}\left\{\dfrac{\delta_{Pj}(s) - \delta_{Pj}(s + \lambda_A + \lambda_C)}{\lambda_A + \lambda_C}\right\}\\ + \delta_C(s)\lambda_C\{1 + \lambda_{Pj}\delta_{Pj}(s + \lambda_A + \lambda_C)\}\end{array}\right]\overline{P}_1(s)\end{aligned} \tag{39}$$

4. Particular Cases

A. When a catastrophic failure does not occur in subsystem B

The result can be derived by putting λ_C=0 in equation (30) through equation (37), Laplace transformation of various state probabilities is as follows:

$$\overline{P}_0(s) = \frac{D_1(s)}{T_1(s)} \tag{40}$$

$$\overline{P}_1(s) = \frac{C_1(s)}{T_1(s)} \tag{41}$$

$$\overline{P}_2(s) = \lambda_{Pj}\,\delta_{Pj}(s+2\lambda_A)\overline{P}_0(s) \tag{42}$$

$$\overline{P}_3(s) = 2\lambda_{Pj}[\delta_{Pj}(s+\lambda_A) - \delta_{Pj}(s+2\lambda_A)]\,\overline{P}_0(s) + \lambda_{Pj}\delta_{Pj}(s+\lambda_A)\overline{P}_1(s) \tag{43}$$

$$\overline{P}_4(s) = \delta_A(s)[E_1(s)\overline{P}_0(s) + \{\lambda_A + F_1(s)\}\overline{P}_1(s)] \tag{44}$$

$$\overline{P}_5(s) = \lambda_{Pj}[\delta_{Pj}(s) - 2\delta_{Pj}(s+\lambda_A) + \delta_{Pj}(s+2\lambda_A)]\overline{P}_0(s)$$
$$+\lambda_{Pj}[\delta_{Pj}(s) - r_{Pj}(s+\lambda_A)]\overline{P}_1(s) \tag{45}$$

$$\overline{P}_6(s) = 0 \tag{46}$$

$$\overline{P}_7(s) = 0 \tag{47}$$

where

$$D_1(s) = s + \lambda_A + \lambda_{Pj} - \lambda_{Pj}\overline{S}_{Pj}(s+\lambda_A)\,,$$

$$C_1(s) = 2\lambda_A + 2\lambda_{Pj}[\overline{S}_{Pj}(s+\lambda_A) - \overline{S}_{Pj}(s+2\lambda_A)]\,,$$

$$A_1(s) = s + 2\lambda_A + \lambda_{Pj} - \lambda_{Pj}\overline{S}_{Pj}(s+2\lambda_A) - \overline{S}_A(s)E_1(s)\,,$$

$$B_1(s) = \overline{S}_A(s)[\lambda_A + F_1(s)]\,,$$

$$T_1(s) = A_1(s)D_1(S) - B_1(s)C_1(s)\,,$$

$$E_1(s) = \lambda_{Pj}[\overline{S}_{Pj}(s) - 2\overline{S}_{Pj}(s+\lambda_A) + \overline{S}_{Pj}(s+2\lambda_A)]\,,$$

$$F_1(s) = \lambda_{Pj}[\overline{S}_{Pj}(s) - \overline{S}_{Pj}(s+\lambda_A)]\,,$$

B. When repair follows an exponential distribution

Setting $\overline{S}_A(s) = \dfrac{\exp[x^\theta + \{\log\phi_A(x)\}^\theta]^{1/\theta}}{s + \exp[x^\theta + \{\log\phi_A(x)\}^\theta]^{1/\theta}}$, $\overline{S}_{Pj}(s) = \dfrac{\phi_{Pj}}{s+\phi_{Pj}}$, $\overline{S}_C(s) = \dfrac{\phi_C}{s+\phi_C}$

in equation (30) through equation (37), the Laplace transformations of various state probabilities are as follows:

$$\overline{P}_0(s) = \frac{D_2(s)}{T_2(s)} \tag{48}$$

$$\overline{P}_1(s) = \frac{C_2(s)}{T_2(s)} \tag{49}$$

$$\overline{P}_2(s) = \left[\frac{\lambda_{Pj}}{s + 2\lambda_A + \lambda_C + \phi_{Pj}}\right]\overline{P}_0(s) \tag{50}$$

$$\overline{P}_3(s) = \left[\frac{2\lambda_{Pj}\lambda_A}{(s+\lambda_A+\lambda_C+\phi_{Pj})(s+2\lambda_A+\lambda_C+\phi_{Pj})}\right]\overline{P}_0(s) + \left[\frac{\lambda_{Pj}}{s+\lambda_A+\lambda_C+\phi_{Pj}}\right]\overline{P}_1(s) \tag{51}$$

$$\overline{P}_4(s) = \frac{1}{s + \exp[x^\theta + \{\log\phi_A(x)\}^\theta]^{1/\theta}}[E_2(s)\overline{P}_0(s) + \{\lambda_A + F_2(s)\}\overline{P}_1(s)] \tag{52}$$

$$\overline{P}_5(s) = \frac{\lambda_A\lambda_{Pj}}{(s+\phi_{Pj})(s+\lambda_A+\lambda_C+\phi_{Pj})}\left[\frac{2\lambda_A}{s+2\lambda_A+\lambda_C+\phi_{Pj}}\overline{P}_0(s) + \overline{P}_1(s)\right] \tag{53}$$

$$\bar{P}_6(s)=\frac{\lambda_C}{s+\phi_C}\left\{1+\frac{\lambda_{Pj}}{s+2\lambda_A+\lambda_C+\phi_{Pj}}\right\}\bar{P}_0(s)$$

(54)

$$\bar{P}_7(s)=\frac{\lambda_C}{s+\phi_C}\left[\left\{\frac{2\lambda_{Pj}\lambda_A}{(s+\lambda_A+\lambda_C+\phi_{Pj})(s+2\lambda_A+\lambda_C+\phi_{Pj})}\right\}\bar{P}_0(s)+\left\{1+\frac{\lambda_{Pj}}{s+\lambda_A+\lambda_C+\phi_{Pj}}\right\}\bar{P}_1(s)\right] \quad (55)$$

Where

$$D_2(s)=(s+\lambda_A+\lambda_C)\left[1+\frac{\lambda_{Pj}}{(s+\lambda_A+\lambda_C+\phi_{Pj})}\right],$$

$$C_2(s)=2\lambda_A\left[1+\frac{\lambda_{Pj}\phi_{Pj}}{(s+\lambda_A+\lambda_C+\phi_{Pj})(s+2\lambda_A+\lambda_C+\phi_{Pj})}\right],$$

$$A_2(s)=(s+2\lambda_A+\lambda_C)\left[1+\frac{\lambda_{Pj}}{(s+2\lambda_A+\lambda_C+\phi_{Pj})}\right]-\frac{\exp[x^\theta+\{\log\phi_A(x)\}^\theta]^{1/\theta}E_2(s)}{s+\exp[x^\theta+\{\log\phi_A(x)\}^\theta]^{1/\theta}}-\frac{\lambda_C\phi_C}{s+\phi_C}\left[1-\frac{\lambda_{Pj}}{(s+2\lambda_A+\lambda_C+\phi_{Pj})}+\frac{2\lambda_{Pj}}{(s+\lambda_A+\lambda_C+\phi_{Pj})}\right],$$

$$B_2(s)=\frac{\exp[x^\theta+\{\log\phi_A(x)\}^\theta]^{1/\theta}}{s+\exp[x^\theta+\{\log\phi_A(x)\}^\theta]^{1/\theta}}[\lambda_A+F_2(s)]+\frac{\lambda_C\phi_C}{s+\phi_C}\left[1+\frac{\lambda_{Pj}}{(s+\lambda_A+\lambda_C+\phi_{Pj})}\right]$$

$$T_2(s)=A_2(s)D_2(S)-B_2(s)C_2(s)$$

$$E_2(s)=\frac{2\lambda_A^2\lambda_{Pj}\phi_{Pj}}{(s+\phi_{Pj})(s+\lambda_A+\lambda_C+\phi_{Pj})(s+2\lambda_A+\lambda_C+\phi_{Pj})}$$

$$F_2(s)=\frac{\lambda_A\lambda_{Pj}\phi_{Pj}}{(s+\phi_{Pj})(s+\lambda_A+\lambda_C+\phi_{Pj})}$$

5. Numerical computations

Assuming that when repair follows an exponential time distribution, we have

$$\bar{S}_A(s)=\frac{\exp[x^\theta+\{\log\phi_A(x)\}^\theta]^{1/\theta}}{s+\exp[x^\theta+\{\log\phi_A(x)\}^\theta]^{1/\theta}},\ \bar{S}_P(s)=\frac{\phi_{Pj}}{s+\phi_{Pj}},\ \bar{S}_C(s)=\frac{\phi_C}{s+\phi_C}$$

A. Availability Analysis

(a) Let the failure rates of subsystems A and B for partial and catastrophic failure be $\lambda_A=\lambda_{Pj}=\lambda_C=0.01$, $x=1,\theta=1,\phi_A=\phi_{Pj}=\phi_C=1$ in equation (38) and taking the inverse Laplace transform, one can get

$$P_{up}(t) = -0.00002847737880\, e^{(-2.718123048t)} + 0.004704285734\, e^{(-1.045862357t)} + 0.005197005350\, e^{(-1.017069307t)} \cos(0.006154998102t) + 0.03306981389\, e^{(-1.017069307t)} \sin(0.006154998102t) + 0.0018764303\, e^{(-0.04007598092t)} + 0.9882507561 \quad (56)$$

(b) Setting parameters as: $\lambda_A = 0.1$, $\lambda_{Pj} = 0.2$, $\lambda_C = 0.3$, mean time to repair and $x = 1, \theta = 1, \phi_A = \phi_{Pj} = \phi_C = 1$ in (38) and taking inverse Laplace transform, we have

$$P_{up}(t) = -0.01066101081 e^{(-2.695379582t)} + 0.07164780178\, e^{(-1.830698558t)} + 0.1693005420\, e^{(-1.390327662t)} \cos(0.06257156474t) + 0.7083715935 e^{(-1.390327662t)} \sin(0.06257156474t) + 0.01064757580\, e^{(-0.6114665356t)} + 0.7590650916 \quad (57)$$

(c) Again taking failure rates of subsystems A and B for partial and catastrophic as $\lambda_A = \lambda_{Pj} = \lambda_C = 0.5$, $x = 1, \theta = 1, \phi_A = \phi_{Pj} = \phi_C = 1$ in (38) and determining inverse Laplace transform, one may get

$$P_{up}(t) = 0.1246243285 e^{(-3.175102361t)} \cos(0.7549458585t) + 0.1703914724\, e^{(-3.175102361t)} \sin(0.7549458585t) + 0.14511904888\, e^{(-1.843043060t)} \cos(0.6150146839t) + 0.4044051920\, e^{(-1.843043060t)} \sin(0.6150146839t) + 0.1273527830\, e^{(-1.681909159t)} + 0.6029038397 \quad (58)$$

In equation (56), (57) and (58), setting t =0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 units of time, one may obtain Table 1. Table 1 demonstrates how the availability of the system changes with respect to time.

Time (t)	Availability $P_{up}(t)$		
	Case (a)	Case (b)	Case (c)
0	1.00000	1.00000	1.00000
1	0.99365	0.82870	0.69097
2	0.99129	0.77988	0.61839
3	0.99039	0.76566	0.60511
4	0.99002	0.76133	0.60314
5	0.98985	0.75993	0.60292
6	0.98974	0.75943	0.60290
7	0.98967	0.75923	0.60290
8	0.98961	0.75915	0.60290
9	0.98956	0.75911	0.60290
10	0.98950	0.75908	0.60290

Table 1: Availability as function of time

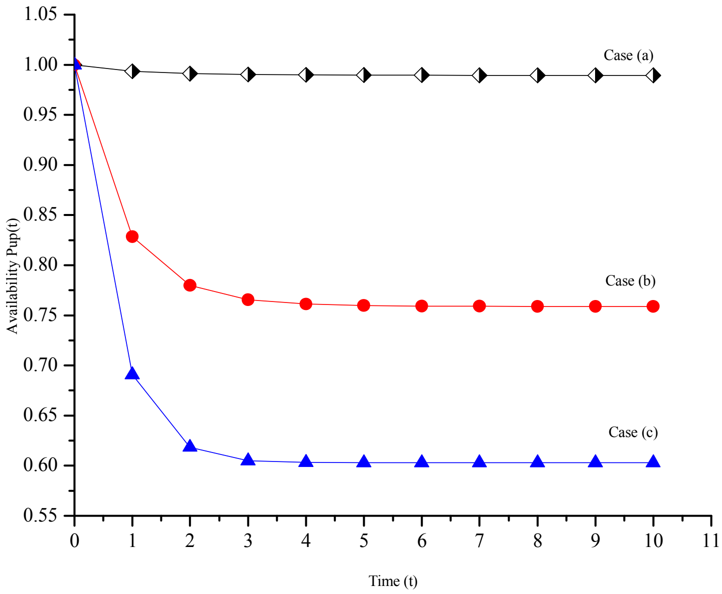

Fig. 2: Availability as function of time

B. Cost Analysis

Let the service facility be always available, then expected profit during the interval (0, *t*] is

$$E_p(t) = K_1 \int_0^t P_{UP}(t)dt - K_2 t \tag{59}$$

Using (56) in (59) for the above mentioned parameters, one can obtain (60) for the values of the various parameters as taken for availability analysis, we have

$$\begin{aligned} E_p(t) = \mathrm{K}_1\ [&-0.00001047685417\ e^{(-2.718123048\ t)} \\ &-0.00449799699e^{(-1.045862357\ t)} \\ &-0.005306360389\ e^{(-1.017069307\ t)} \cos(0.006154998102t) \\ &-0.03248269614\ e^{(-1.017069307\ t)} \sin(0.006154998102t) \\ &-0.04682101838\ e^{(-0.04007598092\ t)} + 0.9882507561t \\ &+0.05661569890] - \mathrm{K}_2 t \end{aligned} \tag{60}$$

Taking K_1 =1; K_2 = 0.05, 0.10, 0.20, 0.30, 0.40, 0.50 and using (60), the computed values of E_p (t) are given in Table 2.

Time	$E_p(t)$					
	K_2= 0.05	K_2= 0.10	K_2= 0.20	K_2= 0.30	K_2= 0.40	K_2= 0.50
0	0	0	0	0	0	0
1	0.946312	0.896312	0.796312	0.696312	0.596312	0.496312
2	1.888600	1.788600	1.588600	1.388600	1.188600	0.988600
3	2.829375	2.679375	2.379375	2.079375	1.779375	1.479375
4	3.769558	3.569558	3.169558	2.769558	2.369558	1.969558
5	4.709486	4.459486	3.959486	3.459486	2.959486	2.459486
6	5.649282	5.349282	4.749282	4.149282	3.549282	2.949282
7	6.588994	6.238994	5.538994	4.838994	4.138994	3.438994
8	7.528639	7.128639	6.328639	5.528639	4.728639	3.928639
9	8.468227	8.018227	7.118227	6.218227	5.318227	4.418227

Table 2: Expected profit as function of time

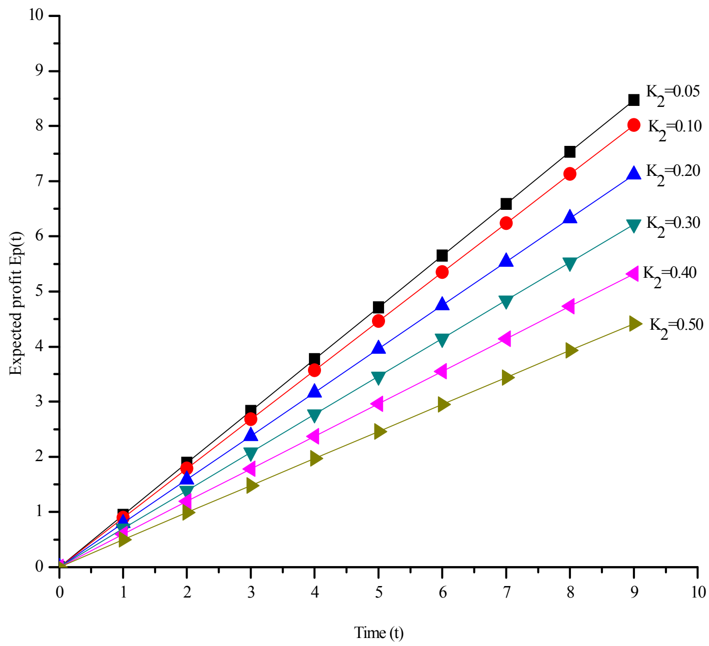

Fig. 3: Expected profit as function of time

6. Conclusions

Table 1 provideS information how availability of the complex engineering repairable system changes with respect to the time when failure rates are fixed at different values. When failure rates are fixed at lower values at 0.01 the availability of the system decreases with respect to time but stabilize at value 0.989 in the long run. When failure rates are putting at the values λ_A=0.10, λ_{Pj}=0.20, λ_C=0.30, the availability of the system decreases smoothly during initial stage but later on become stable at

0.759 in the long run. In a similar approach, when failure rates are fixed at 0.5, the availability of the system decreases sharply during initial stage but later on stabilizing at 0.602 in the long run. Tables 1 and corresponding Figures 2 divulge that when the failure rate increases availability of the system decreases.

When revenue cost per unit time K_1 fixed at 1, service cost K_2 varies and failure rates are kept at lower and somewhat higher values one can obtain Table 2 for repairable system which are depicted in Figures 3. One can conclude by observing this graph that as service cost increases, expected profit decreases. For lower failure rates expected profit is higher in comparison to higher failure rates. Hence the present study clearly proves the importance of head-of line repair policy in comparison of [17-18] which seem to be possible in many engineering systems when it is analyzed with the help of the copula. The further research area is widely open, where one may think of the application of other members of copula family, MTTF and sensitivity analysis.

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