

COMBINED-TYPE FAMILY OF ESTIMATORS OF POPULATION MEAN IN STRATIFIED RANDOM SAMPLING UNDER NON-RESPONSE

*Manoj K. Chaudhary, *V. K. Singh and **R. K. Shukla

*Department of Statistics, Banaras Hindu University, Varanasi-221005, India

**Department of Statistics, Central University of Bihar, B.I.T. Campus, P.O.-
B.V. College Patna-800014, India

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Abstract

The present paper focuses on the study of combined-type family of estimators of population mean in stratified random sampling in the presence of non-response. In this paper, we have suggested a family of factor-type estimators of population mean in stratified random sampling under non-response using an auxiliary variable. The properties of the family have been discussed in detail. The theoretical results are also supported by an empirical study.

Key Words: Combined-type family of estimators, Factor-type estimators, Stratified random sampling, Auxiliary variable, Non-response.

1. Introduction

Auxiliary information recorded from the population elements can be successfully used to design a manageable and efficient sampling design and after sample selection, to further improve the efficiency of estimators. If a continuous auxiliary variable is available i.e. strongly correlated with the study variable, it is possible to improve the efficiency by using ratio method of estimation or regression method of estimation. In these methods, auxiliary information is incorporated into the estimation procedure using statistical models. The use of these techniques can considerably improve the accuracy of estimates, i.e. produce estimates that are close to the corresponding population values and in addition, decrease the design variances of the estimators.

There are several authors who have suggested estimators using some known population parameters of auxiliary variable(s). Upadhyaya and Singh (1999) have suggested the class of estimators in simple random sampling. Kadilar and Cingi (2003) and Shabbir and Gupta (2005) extended these estimators for the stratified random sampling. Singh et al (2012) have suggested the general family of ratio-type estimators in systematic sampling. Tailor et al (2012) have recently proposed dual to ratio-cum-product estimator using known parameters of auxiliary variables. Kadilar and Cingi (2005) and Shabbir and Gupta (2006) have suggested new ratio estimators in stratified sampling to improve the efficiency of the estimators. Koyuncu and Kadilar (2008) have proposed families of estimators for estimating population mean in stratified random sampling by considering the estimators proposed in Searls (1964) and Khoshnevisan et al (2007). Singh and Vishwakarma (2008) have suggested a family of estimators using transformation in the stratified random sampling. Koyuncu and Kadilar (2009) have

proposed a general family of estimators, which uses the information of two auxiliary variables in the stratified random sampling to estimate the population mean of the study variable.

In the present paper, we have suggested a family of combined-type estimators of population mean in stratified random sampling in the presence of non-response using the information available on an auxiliary variable. The optimum property of the family has been discussed. Choice of appropriate estimator in the family in order to get a desired level of accuracy even if non-response is high, is also discussed.

2. Sampling Strategy and Estimation Procedure

Let us consider a population consisting of N units which is divided into k strata. Let the size of i^{th} stratum be N_i , ($i = 1, 2, \dots, k$) and we select a sample of size n from the entire population in such a way that n_i units are selected from the i^{th} stratum. Thus, we have $\sum_{i=1}^k n_i = n$. Let us assume that the situation in which the non-response is observed on study variable and auxiliary variable is free from non-response. It is observed that there are n_{i1} respondent units and n_{i2} non-respondent units in the sample of n_i units for the i^{th} stratum regarding the study variable. Using Hansen and Hurwitz (1946) procedure, we select a sub-sample of size m_i units out of n_{i2} non-respondent units with the help of simple random sampling without replacement (SRSWOR) scheme such that $n_{i2} = L_i m_i$, $L_i \geq 1$ and the information are observed on all the m_i units. Let X_0 and X_1 be the study and auxiliary variables with respective population means \bar{X}_0 and \bar{X}_1 .

The Hansen-Hurwitz estimator of population mean \bar{X}_{0i} of study variable X_0 for the i^{th} stratum, is given by

$$T_{0i}^* = \frac{n_{i1} \bar{x}_{0i1} + n_{i2} \bar{x}_{0mi}}{n_i}, \quad (i = 1, 2, \dots, k) \quad (2.1)$$

where \bar{x}_{0i1} and \bar{x}_{0mi} are the sample means based on n_{i1} respondent units and m_i non-respondent units respectively in the i^{th} stratum for the study variable. Obviously T_{0i}^* is an unbiased estimator of \bar{X}_{0i} . Combining the estimators over all the strata, we get the unbiased estimator of population mean \bar{X}_0 of study variable, given by

$$T_{0st}^* = \sum_{i=1}^k p_i T_{0i}^* \quad (2.2)$$

where $p_i = \frac{N_i}{N}$.

Now, we define the estimator of population mean \bar{X}_1 of auxiliary variable as

$$T_{1st} = \sum_{i=1}^k p_i \bar{x}_{1i} \tag{2.3}$$

where \bar{x}_{1i} is the sample mean based on n_i units in the i^{th} stratum for the auxiliary variable. It can easily be seen that T_{1st} is an unbiased estimator of \bar{X}_1 because \bar{x}_{1i} gives unbiased estimate of the population mean \bar{X}_{1i} of auxiliary variable for the i^{th} stratum.

2.1 Suggested Family of Estimators

Motivated by Singh and Shukla (1987), we suggest a family of factor-type estimators for estimating the population mean \bar{X}_0 in stratified random sampling under non-response as

$$T_{FC}(\alpha) = T_{0st}^* \left[\frac{(A + C)\bar{X}_1 + fBT_{1st}}{(A + fB)\bar{X}_1 + CT_{1st}} \right] \tag{2.1.1}$$

where A , B and C are the functions of α , given by $A = (\alpha - 1)(\alpha - 2)$, $B = (\alpha - 1)(\alpha - 4)$, $C = (\alpha - 2)(\alpha - 3)(\alpha - 4)$; $\alpha > 0$ and $f = \frac{n}{N}$.

2.1.1 Particular Cases of $T_{FC}(\alpha)$

Case-1: For $\alpha = 1$, we have

$$T_{FC}(1) = T_{0st}^* \frac{\bar{X}_1}{T_{1st}} \tag{2.1.2}$$

which is usual combined ratio estimator under non-response.

Case-2: If $\alpha = 2$, then

$$T_{FC}(2) = T_{0st}^* \frac{T_{1st}}{\bar{X}_1} \tag{2.1.3}$$

which is usual combined product estimator under non-response.

Case-3: If $\alpha = 3$, then

$$T_{FC}(3) = T_{0st}^* \frac{\bar{X}_1 - fT_{1st}}{(1 - f)\bar{X}_1} \tag{2.1.4}$$

which is combined dual to ratio-type estimator under non-response. The dual to ratio type estimator was proposed by Srivenkataramana (1980).

Case-4: If $\alpha = 4$,

$$T_{FC}(4) = T_{0st}^* \tag{2.1.5}$$

which is usual mean estimator of stratified population under non-response.

2.1.2 Properties of $T_{FC}(\alpha)$

Using large sample approximation, the bias and mean square error (MSE) of $T_{FC}(\alpha)$ up to the first order of approximation can be obtained by the equations (2.1.6) and (2.1.7) respectively.

$$\begin{aligned} B[T_{FC}(\alpha)] &= E[T_{FC}(\alpha) - \bar{X}_0] \\ &= \phi(\alpha) \bar{X}_0 \sum_{i=1}^k p_i \left(\frac{1}{n_i} - \frac{1}{N_i} \right) \left[\frac{C}{A + fB + C} C_{1i}^2 - \rho_{01i} C_{0i} C_{1i} \right] \end{aligned} \quad (2.1.6)$$

where $\phi(\alpha) = \frac{C - fB}{A + fB + C}$, $C_{0i} = \frac{S_{0i}}{\bar{X}_{0i}}$, $C_{1i} = \frac{S_{1i}}{\bar{X}_{1i}}$, S_{0i}^2 and S_{1i}^2 are the population mean squares of study and auxiliary variables respectively in the i^{th} stratum. ρ_{01i} is the population correlation coefficient between X_0 and X_1 in the i^{th} stratum.

$$\begin{aligned} M[T_{FC}(\alpha)] &= E[T_{FC}(\alpha) - \bar{X}_0]^2 \\ &= \bar{X}_0^2 \left[\frac{V(T_{0st}^*)}{\bar{X}_0^2} + \phi(\alpha)^2 \frac{V(T_{1st})}{\bar{X}_1^2} - 2\phi(\alpha) \frac{Cov(T_{0st}^*, T_{1st})}{\bar{X}_0 \bar{X}_1} \right] \\ &= \left[V(T_{0st}^*) + \phi(\alpha)^2 R_{01}^2 V(T_{1st}) - 2\phi(\alpha) R_{01} Cov(T_{0st}^*, T_{1st}) \right] \end{aligned}$$

where $R_{01} = \frac{\bar{X}_0}{\bar{X}_1}$.

Since $V(T_{0st}^*) = \sum_{i=1}^k \left(\frac{1}{n_i} - \frac{1}{N_i} \right) p_i^2 S_{0i}^2 + \sum_{i=1}^k \frac{L_i - 1}{n_i} W_{i2} p_i^2 S_{0i2}^2$,

$$V(T_{1st}) = \sum_{i=1}^k \left(\frac{1}{n_i} - \frac{1}{N_i} \right) p_i^2 S_{1i}^2$$

and

$$Cov(T_{0st}^*, T_{1st}) = \sum_{i=1}^k p_i^2 Cov(T_{0i}^*, \bar{x}_{1i}) = \sum_{i=1}^k \left(\frac{1}{n_i} - \frac{1}{N_i} \right) p_i^2 \rho_{01i} S_{0i} S_{1i}$$

where S_{0i2}^2 is the population mean square of the non-response group in the i^{th} stratum and W_{i2} is the non-response rate of the i^{th} stratum in the population. Therefore, we have

$$M[T_{FC}(\alpha)] = \sum_{i=1}^k \left(\frac{1}{n_i} - \frac{1}{N_i} \right) p_i^2 \left[S_{0i}^2 + \phi(\alpha)^2 R_{01}^2 S_{1i}^2 - 2\phi(\alpha) R_{01} \rho_{01i} S_{0i} S_{1i} \right]$$

$$+ \sum_{i=1}^k \frac{L_i - 1}{n_i} W_{i2} p_i^2 S_{0i2}^2. \tag{2.1.7}$$

2.1.3 Optimum Choice of α

Differentiating MSE of $T_{FC}(\alpha)$ with respect to α and equating the derivative to zero, we get

$$\sum_{i=1}^k \left(\frac{1}{n_i} - \frac{1}{N_i} \right) p_i^2 \left[2\phi'(\alpha)\phi(\alpha)R_{01}^2S_{1i}^2 - 2\phi'(\alpha)R_{01}\rho_{01i}S_{0i}S_{1i} \right] = 0 \tag{2.1.8}$$

which yields

$$\phi(\alpha) = \frac{\sum_{i=1}^k \left(\frac{1}{n_i} - \frac{1}{N_i} \right) p_i^2 R_{01} \rho_{01i} S_{0i} S_{1i}}{\sum_{i=1}^k \left(\frac{1}{n_i} - \frac{1}{N_i} \right) p_i^2 R_{01}^2 S_{1i}^2} = V \text{ (say)}. \tag{2.1.9}$$

Since $\phi(\alpha)$ is a cubic equation in α . So that for given value of V , equation (2.1.9) may be solved so as to obtain at the most three real and positive values of α for which $M[T_{FC}(\alpha)]$ would be minimum.

2.1.4 Reducing MSE Through Appropriate Choice of α

By utilizing factor-type estimators (FTE) suggested by Singh and Shukla (1987), in order to propose combined- type estimators in the present work, we are able to control the precision of the estimator to a desired level only by making an appropriate choice of α . Let the non-response rate and mean-square of the non-response group in the i^{th} stratum at a time be $W_{i2} = \frac{N_{i2}}{N_i}$ and S_{0i2}^2 respectively.

Then, for a choice of $\alpha = \alpha_0$, the MSE of the estimator would be

$$M[T_{FC}(\alpha)/W_{i2}] = \sum_{i=1}^k \left(\frac{1}{n_i} - \frac{1}{N_i} \right) p_i^2 \left[S_{0i}^2 + \phi(\alpha_0)^2 R_{01}^2 S_{1i}^2 - 2\phi(\alpha_0)R_{01}\rho_{01i}S_{0i}S_{1i} \right] + \sum_{i=1}^k \frac{L_i - 1}{n_i} W_{i2} p_i^2 S_{0i2}^2 \tag{2.1.10}$$

Let us now suppose that the non-response rate increased over time and it is $W'_{i2} = \frac{N'_{i2}}{N_i}$ such that $N'_{i2} > N_{i2}$. Obviously, with change in non-response rate, only the parameter S_{0i2}^2 will change. Let it becomes S'_{0i2} . Then we have

$$M[T_{FC}(\alpha)/W'_{i2}] = \sum_{i=1}^k \left(\frac{1}{n_i} - \frac{1}{N_i} \right) p_i^2 [S_{0i}^2 + \phi(\alpha_1)^2 R_{01}^2 S_{1i}^2 - 2\phi(\alpha_1) R_{01} \rho_{01i} S_{0i} S_{1i}] + \sum_{i=1}^k \frac{L_i - 1}{n_i} W'_{i1} p_i^2 S_{0i2}^2 \quad (2.1.11)$$

Clearly, if $\alpha_0 = \alpha_1$ and $S'_{0i2} > S_{0i2}^2$ then $M[T_{FS}(\alpha)W'_{i2}] > M[T_{FS}(\alpha)W_{i2}]$. Therefore, we have to select a suitable value α_1 , such that even if $W'_{i2} > W_{i2}$ and $S'_{0i2} > S_{0i2}^2$, expression (2.1.11) becomes equal to equation (2.1.10) that is, the MSE of $T_{FC}(\alpha)$ is reduced to a desired level given by (2.1.10). Equating (2.1.11) to (2.1.10) and solving for $\phi(\alpha_1)$, the appropriate choice of the parameter α , so as to reduce the mean square error of the combined-type estimator to a desired level, may be obtained from the equation

$$\phi(\alpha_1) = \frac{\sum_{i=1}^k \left(\frac{1}{n_i} - \frac{1}{N_i} \right) p_i^2 R_{01} \rho_{01i} S_{0i} S_{1i}}{\sum_{i=1}^k \left(\frac{1}{n_i} - \frac{1}{N_i} \right) p_i^2 R_{01}^2 S_{1i}^2} \pm \left[\frac{\left[\sum_{i=1}^k \left(\frac{1}{n_i} - \frac{1}{N_i} \right) p_i^2 R_{01} \rho_{01i} S_{0i} S_{1i} \right]^2}{\sum_{i=1}^k \left(\frac{1}{n_i} - \frac{1}{N_i} \right) p_i^2 R_{01}^2 S_{1i}^2} \right]^{\frac{1}{2}} + \frac{\sum_{i=1}^k \left(\frac{1}{n_i} - \frac{1}{N_i} \right) p_i^2 \left\{ \phi(\alpha_0)^2 R_{01}^2 S_{1i}^2 - 2\phi(\alpha_0) R_{01} \rho_{01i} S_{0i} S_{1i} \right\} - \sum_{i=1}^k \frac{L_i - 1}{n_i} p_i^2 (W'_{i2} S_{0i2}^2 - W_{i2} S_{0i2}^2)}{\sum_{i=1}^k \left(\frac{1}{n_i} - \frac{1}{N_i} \right) p_i^2 R_{01}^2 S_{1i}^2} \quad (2.1.12)$$

In order that the roots are real, the conditions on the value of α_0 are given by

$$\phi(\alpha_0) > \frac{\sum_{i=1}^k \left(\frac{1}{n_i} - \frac{1}{N_i} \right) p_i^2 R_{01i} \rho_{01i} S_{0i} S_{1i}}{\sum_{i=1}^k \left(\frac{1}{n_i} - \frac{1}{N_i} \right) p_i^2 R_{01i}^2 S_{1i}^2} + \left[\frac{\sum_{i=1}^k \frac{L_i - 1}{n_i} p_i^2 (W_{i2}' S_{0i2}'^2 - W_{i2} S_{0i2}^2)}{\sum_{i=1}^k \left(\frac{1}{n_i} - \frac{1}{N_i} \right) p_i^2 R_{01i}^2 S_{1i}^2} \right]^{\frac{1}{2}} \tag{2.1.13}$$

or

$$\phi(\alpha_0) < \frac{\sum_{i=1}^k \left(\frac{1}{n_i} - \frac{1}{N_i} \right) p_i^2 R_{01i} \rho_{01i} S_{0i} S_{1i}}{\sum_{i=1}^k \left(\frac{1}{n_i} - \frac{1}{N_i} \right) p_i^2 R_{01i}^2 S_{1i}^2} - \left[\frac{\sum_{i=1}^k \frac{L_i - 1}{n_i} p_i^2 (W_{i2}' S_{0i2}'^2 - W_{i2} S_{0i2}^2)}{\sum_{i=1}^k \left(\frac{1}{n_i} - \frac{1}{N_i} \right) p_i^2 R_{01i}^2 S_{1i}^2} \right]^{\frac{1}{2}} \tag{2.1.14}$$

3. Empirical Study

We have considered MU284 population available in Sarndal et al (1992, page 652, Appendix B). It is assumed that the population in the year 1985 is study variable and that in the year 1975 is auxiliary variable. There are 284 municipalities which have been divided randomly into four strata having sizes 73, 70, 97 and 44.

Table 1 shows the values of the parameters of the population under consideration for the four strata which are needed in computational procedure.

Stratum		Mean (\bar{X}_{0i})	Mean (\bar{X}_{1i})	(S_{0i}^2)	(S_{1i}^2)	S_{0i}	S_{1i}	i	(S_{0i2}^2)
i	N_i								
1	73	40.85	39.56	6369.0999	6624.4398	79.8066	81.3907	0.999	618.8844
2	70	27.83	27.57	1051.0725	1147.0111	32.4202	33.8676	0.998	240.9050
3	97	25.79	25.44	2014.9651	2205.4021	44.8884	46.9617	0.999	265.5220
4	44	20.64	20.36	538.4749	485.2655	23.2051	22.0287	0.997	83.6944

Table 1: Parameters of the Population

The value of $R_{01} = \bar{X}_0 / \bar{X}_1 = 1.0192$.

We fix the sample size to be 60. Then the allocation of samples to different strata under proportional and Neyman allocations are shown in the following table

Stratum (i)	Size of Samples under	
	Proportional Allocation	Neyman Allocation
1	15	26
2	15	10
3	21	19
4	9	5

Table 2: Allocation of Sample

The equation (2.1.9) yields optimum values of $\phi(\alpha)$ and α under proportional allocation as

$$\phi(\alpha) = 0.9573, \alpha_{opt} = (37.3743, 2.6097, 1.1025)$$

and under Neyman allocation as

$$\phi(\alpha) = 0.9466, \alpha_{opt} = (30.6986, 2.6137, 1.1252)$$

The following table illustrates the percentage relative efficiencies (PRE) of the estimators $T_{FC}(\alpha)$ for corresponding α_{opt} , $\alpha = 1$ and 4 respectively with respect to $\alpha = 4$ i.e. T_{0st}^* under proportional and Neyman allocations. A comparison of PRE of $T_{FC}(\alpha)$ with α_{opt} and $\alpha = 1$ with that at $\alpha = 4$ reveals the fact that the utilization of auxiliary information at the estimation stage certainly improves the efficiency of the estimator as compared to the usual mean estimator T_{0st}^* .

MSE	Allocation	
	Proportional	Neyman
$M[T_{FC}(\alpha)]_{opt}$	5653.68	4735.64
$M[T_{FC}(1)]$	5012.93	4266.03
$M[T_{FC}(4)] = V[T_{0st}^*]$	100.00	100.00

Table 3: Percentage Relative Efficiency (PRE) Comparison

($L_i = 2$, $W_{i2} = 10\%$ for all i)

Reducing MSE through Appropriate Choice of α

We shall now demonstrate the appropriate choice of α for that the MSE of the estimators $T_{FC}(\alpha)$ can be reduced to a desired level even if the non-response rate is increased.

Let us take $L_i = 2$, $W_{i2} = 0.1$, $W'_{i2} = 0.3$ and $S'^2_{0i2} = \frac{4}{3}(S^2_{0i2})$ for all i .

Conditions for real roots of $\phi(\alpha_1)$ under proportional allocation are

$$\phi(\alpha_0) > 1.1588 \text{ and } \phi(\alpha_0) < 0.7494.$$

If $\phi(\alpha_0) = 1.20$ then $M[T_{FC}(\alpha)W_{i2}] = 2.9527$ and

$$M[T_{FC}(\alpha)W'_{i2}] = 4.5633 \text{ for } \phi(\alpha) = \phi(\alpha_0) = 1.20$$

From (2.1.12), we have $\phi(\alpha_1) = 1.0902$ and 0.8179 and

$$M[T_{FC}(\alpha)W'_{i2}] = 2.9527 \text{ for } \phi(\alpha_1) = 1.0902.$$

Now conditions for real roots of $\phi(\alpha_1)$ under Neyman allocation are

$$\phi(\alpha_0) > 1.1751 \text{ and } \phi(\alpha_0) < 0.7312$$

If $\phi(\alpha_0) = 1.20$ then $M[T_{FC}(\alpha)W_{i2}] = 2.4820$ and

$$M[T_{FC}(\alpha)W'_{i2}] = 4.0007 \text{ for } \phi(\alpha) = 1.20.$$

We get $\phi(\alpha_1) = 1.0612$ and 0.8450 and

$$M[T_{FC}(\alpha)W'_{i2}] = 2.4820 \text{ for } \phi(\alpha_1) = 1.0612.$$

4. Conclusion

In the present paper, we have suggested a general family of combined-type estimators of population mean in stratified random sampling in the presence of non-response. In this context, we have made an attempt to use an auxiliary variable in order to increase the efficiency of the estimator. The optimum property of the family has been discussed. We have also illustrated the reduction of MSE of the family $T_{FC}(\alpha)$ to a desired extent by an appropriate choice of the parameter α even if the non-response rate is high in the population. The theoretical results have been illustrated in the empirical study. The table 3 reveals that the optimum estimator and combined ratio estimator certainly provide the better estimates than the usual mean estimator T_{0st}^* .

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