Journal of Reliability and Statistical Studies; ISSN (Print): 0974-8024, (Online):2229-5666,
Vol. 5, Issue 1 (2012): 55-64

RELIABILITY TEST PLANS FOR TYPE-II EXPONENTIATED LOG-LOGISTIC DISTRIBUTION

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(Received February 18, 2012)

Abstract

In this paper we consider a generalization of the log- logistic distribution called Type-II exponentiated log- logistic distribution suggested by Kotz and Nadarajah (2000). The operating characteristic for a sampling plan is determined for the case that a lot of products are submitted for inspection with lifetimes specified by a Type-II exponentiated log- logistic distribution (TELLD). The results are illustrated by a numerical example.

Key words: Type-II Exponentiated Log- logistic Distribution, Reliability Test Plans,

Mathematics Subject Classification: 62N05; 62P30.

1. Introduction

Gupta *et. al*. (1998) introduced the exponentiated exponential distribution as a generalization of the standard exponential distribution. Kotz and Nadarajah (2000) introduced a new method of adding a new parameter to an existing distribution. In this paper we introduce a new parameter to the standard log- logistic distribution [in lines of the exponentiated frechet distribution suggested by Nadarajah and Kotz (2003)] and it is called as Type-II exponentiated log- logistic distribution.

We know that the cumulative distribution function (cdf) of the log- logistic distribution is

$$F(x)=\frac{(x/\sigma)^{\beta}}{\left[1+(x/\sigma)^{\beta}\right]};x>0,\sigma>0,\beta>1 \tag{1}$$

We define a new distribution by the cdf as follows:

$$G(x;\alpha,\beta,\sigma)=1-\left[1-\frac{(x/\sigma)^{\beta}}{\{1+(x/\sigma)^{\beta}\}}\right]^{\alpha}=1-\left[1+(x/\sigma)^{\beta}\right]^{-\alpha};x>0,\sigma>0,\alpha>0,\beta>1 \tag{2}$$

We called (2) as the Type II exponentiated log- logistic distribution. The corresponding probability density function (pdf) is given by

$$g(x;\alpha,\beta,\sigma)=\frac{\alpha\beta}{\sigma}\frac{(x/\sigma)^{\beta-1}}{\left[1+(x/\sigma)^{\beta}\right]^{\alpha+1}};\qquad x>0,\sigma>0,\alpha>0,\beta>1 \tag{3}$$

When α =1, the pdf of (3) reduces to the log- logistic distribution. However, extended exponential distribution in reliability test plans, based on life tests, has not paid much attention. Acceptance sampling plans in statistical quality control concern with accepting or rejecting a submitted lot of a large size of products on the basis of the quality of products inspected in a sample taken from the lot. If the quality of the product inspected is the lifetime of the product that is put for testing, after the completion of sampling inspection, then we have a sample of life times of the sampled products. If a decision to accept or reject the lot subject to the risks associated with the two types of errors (rejecting a good lot/ accepting a bad lot) is possible, such a procedure may be termed as 'Acceptance sampling based on life tests' or 'Reliability test plans'. Such a procedure obviously requires the specification of the probability model governing the life of the products.

In this paper, we develop Reliability test plans to decide the acceptance / rejection of a submitted lot of products, whose life time is governed by a Type II exponentiated log- logistic distribution, derive its operating characteristic function and give the corresponding decision rule. Similar plans were developed by Epstein (1954), Sobel and Tischendrof (1959), Goode and Kao (1961), Gupta and Groll (1961), Gupta (1962), Fertig and Mann (1980), Kantam and Rosaiah (1998), Kantam *et al.* (2001), Baklizi (2003), Wu and Tsai (2005), Rosaiah and Kantam (2005), Rosaiah *et al.* (2006), Tsai and Wu (2006), Balakrishnan *et. al.* (2007), Srinivasa Rao *et.al*. (2008) and Srinivasa Rao *et.al*. (2009a & 2009d). The proposed sampling plans, along with the operating characteristics, are given in Section 2. The description of tables is given in Section 3. The results are explained by an example in Section 4.

2. Reliability Test Plan

We assume that the lifetime of a product follows a Type II exponentiated log-logistic distribution with scale parameter σ, defined by (3). A common practice in life testing is to terminate the life test by a pre–determined time't' and note the number of failures (assuming that a failure is well defined). One of the objectives of these experiments is to set a lower confidence limit on the average life. It is then to establish a specified average life with a given probability of at least p^*. The decision to accept the specified average life occurs if and only if the number of observed failures at the end of the fixed time *'t'* does not exceed a given number *'c'*– called the acceptance number. The test may get terminated before the time *'t'* is reached when the number of failures exceeds *'c'* in which case the decision is to reject the lot. For such a truncated life test and the associated decision rule; we are interested in obtaining the smallest sample size to arrive at a decision. It is assumed that the distribution parameter α is known, while σ is unknown. In this case the average lifetime of the product depends only on σ and it is easily seen that the average lifetime is monotonously increasing in σ. Let σ_0 represent the required minimum average lifetime, then the following holds:

$$G(t,\sigma) \le G(t,\sigma_0) \Leftrightarrow \sigma \ge \sigma_0 \tag{4}$$

A sampling plan consists of the following quantities:

- The number of units 'n' on test.
- The acceptance number 'c',

- The maximum test duration ‘t’, and
- The ratio t/σ_0 , where σ_0 is the specified average life.

The consumer’s risk i.e., the probability of accepting a bad lot (the one for which the true average life is below the specified life σ_0) not to exceed $1 - p^*$, so that p^* is a minimum confidence level with which a lot of true average life below σ_0 is rejected, by the sampling plan. For a fixed p^* our sampling plan is characterized by (n,c, t/σ_0). Here we consider sufficiently large sized lots so that the binomial distribution can be applied. The problem is to determine for given values of p^* (0 < p^* <1), σ_0 and c, the smallest positive integer ‘n’ such that

$$L(p_0) = \sum_{i=0}^{c} \binom{n}{i} p_0^i \left(1 - p_0\right)^{n-i} \leq 1 - p^* \qquad (5)$$

Where $p_0 = G(t;\alpha,\sigma_0)$ is given by (2) indicates the failure probabilities before time‘t’ which depends only on the ratio t/σ_0 . The function L (p) is the operating characteristic function of the sampling plan, i.e. the acceptance probability of the lot as function of the failure probability $p(\sigma) = G(t;\alpha,\sigma)$. The average lifetime of the products is increasing in σ and, therefore, the failure probability $p(\sigma) = G(t;\alpha,\sigma)$ decreases with increasing σ implying that the operating characteristic function is increasing in σ . For a given value of p^* and t/σ_0 the values of n and c are determined by means of the operating characteristic function. For some sampling plans, the values of the operating characteristic function depending on σ/σ_0 are displayed in Table 3.

The minimum values of n satisfying the inequality (5) are obtained and displayed in Table 1 for p^* = 0.75, 0.90, 0.95, 0.99 and t/σ_0 = 0.315, 0.472, 0.629, 0.786, 1.180, 1.573, 1.966, 2.359 for $\beta = 2, \alpha = 2$.

If $p = G(t;\alpha,\sigma)$ is small and n is large the binomial probability may be approximated by Poisson probability with parameter λ = n. p so that the left side of (5) can be written as

$$L^*(p) = \sum_{i=0}^{c} \frac{\lambda^i}{i\,!} e^{-\lambda} \leq 1 - p^* \qquad (6)$$

Where λ = n. $G(t;\alpha,\sigma_0)$. The minimum values of ‘n’ satisfying (6) are obtained for the same combination of p*, t/σ_0 values as those used for (4). The results are given in Table 2.

The producer's risk is the probability of rejecting a lot although $\sigma \geq \sigma_0$ holds. It is obtained by the operating characteristic function:

$$L\left[p(\sigma)\right] = L\left[G(t, \alpha, \sigma)\right] \tag{7}$$

For a specified value of the producer's risk say 0.05, one may be interested in knowing what value of σ or σ/σ_0 will ensure a producer's risk less than or equal to 0.05 for a given sampling plan. The value of σ and, hence, the value of σ/σ_0 , is the smallest positive number for which the following inequality holds:

$$\sum_{i=0}^{c} \binom{n}{i} \; p(\sigma)^i \; \left[1 - p(\sigma)\right]^{n-i} \geq 0.95 \tag{8}$$

For some sampling plan (n, c, t/σ_0) and values of p*, minimum values of σ/σ_0 satisfying (8) are given in Table 4.

3. Description of the Tables

Assume that the lifetime distribution is type-II exponentiated log-logistic distribution with $\alpha = 2, \beta = 2$ and that the experimenter is interested in establishing that the true unknown average life is at least 1000 hours with confidence p^* = 0.75. It is desired to stop the experiment at t = 315 hours. Then, for an acceptance number c= 2, the required *n* in Table 1 is 22. If, during 315 hours, no more than 2 failures out of 22 are observed, then the experimenter can assert, with a confidence level of 0.75 that the average life is at least 1000 hours. If the Poisson approximation to binomial probability is used, the value of *n*= 22 is obtained from Table 2 for the same situation.

If the life distribution is assumed to be a gamma distribution with shape parameter 2 (an IFR model), the value of n from Table IB of Gupta and Groll (1961) is 63 using binomial probabilities and it is 64 using Poisson approximation. In general, all the values of n tabulated by us are found to be less than the corresponding values of n tabulated in Kantam and Rosaiah (1998) for a half logistic distribution, Rosaiah *et. al*. (2006) for exponentated log- logistic distribution, which in turn are less than those tabulated by Gupta and Groll (1961) with a gamma model as the lifetime distribution.

For the sampling plan (*n* = 22, *c* = 2, t/σ_0 = 0.*315*) and confidence level p^* = 0.75 under type-II exponentiated log-logistic distribution with $\alpha = 2, \beta = 2$ the values of the operating characteristic function from Table 3 are as follows:

σ / σ_0	2	4	6	8	10	12
L(p)	0.9144	0.9976	0.9998	1.0000	1.0000	1.0000

The above values show, that if the true mean lifetime is twice the required mean lifetime (σ / σ_0 = 2) the producer's risk is approximately 0.0856. The producer's risk is about zero when the true mean life is 8 times or more the specified mean life ($\sigma / \sigma_0 \geq 8$).

From Table 4, we can get the values of the ratio σ / σ_0 for various choices of (c, t/σ_0) in order that the producer's risk may not exceed 0.05. For example if p^* = 0.75, t/σ_0 = 0.315, c=2, Table 4 gives a reading of 2.31. This means the product can have an average life of 2.31 times the required average lifetime in order that under the above acceptance sampling plan the product is accepted with probability of at least 0.95. The actual average lifetime necessary to accept 95 percent of the lots is provided in Table 4.

4. Numerical Example

Consider the following ordered failure times of the release of a software given in terms of hours from starting of the execution of the software up to the time at which a failure of the software occurs (Wood, 1996). This data can be regarded as an ordered sample of size n = 9 with observations

$$\{x_i : i = 1, 2, ...9\} = \{254,788,\ 1054,\ 1393,\ 2216,\ 2880,\ 3593,\ 4281,\ 5180\}$$

Let the required average lifetime be 1000 hours and the testing time be t = 786 hours, this leads to ratio of t/σ_0 = 0.786 with a corresponding sample size n = 9 and an acceptance number c = 4, which are obtained from Table 1 for p^* = 0.*75*. Therefore, the sampling plan for the above sample data is (n = 9, c =42, t/σ_0 = 0.786). Based on the observations, we have to decide whether to accept the product or reject it. We accept the product only, if the number of failures before 786 hours is less than or equal to 4.

However, the confidence level is assured by the sampling plan only if the given life times follow type- II exponentiated log-logistic distribution. In order to confirm that the given sample is generated by life times following at least approximately the type- II exponentiated log-logistic distribution, we have compared the sample quantiles and the corresponding population quantiles and found a satisfactory agreement. Thus, the adoption of the decision rule of the sampling plan seems to be justified. In the sample of 9 units, there is a 1 failure at 254 hours before t = 786 hours. Therefore we accept the product.

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p^*	c	t/σ_0							
		0.315	0.472	0.629	0.786	1.180	1.573	1.966	2.359
0.75	0	8	4	3	2	1	1	1	1
0.75	1	15	8	5	4	3	2	2	2
0.75	2	22	11	7	6	4	3	3	3
0.75	3	29	15	10	8	5	5	4	4
0.75	4	36	18	12	9	7	6	5	5
0.75	5	42	22	14	11	8	7	6	6
0.75	6	49	25	17	13	9	8	8	7
0.75	7	55	28	19	15	10	9	9	8
0.75	8	62	32	21	16	12	10	10	9
0.75	9	68	35	23	18	13	11	11	10
0.75	10	75	38	26	20	14	13	12	11
0.90	0	13	6	4	3	2	1	1	1
0.90	1	22	11	7	5	3	3	2	2
0.90	2	30	15	10	7	5	4	4	3
0.90	3	37	19	12	9	6	5	5	4
0.90	4	45	23	15	11	8	6	6	6
0.90	5	52	26	17	13	9	8	7	7
0.90	6	59	30	20	15	10	9	8	8
0.90	7	66	33	22	17	12	10	9	9
0.90	8	73	37	24	19	13	11	10	10
0.90	9	80	41	27	20	14	12	11	11
0.90	10	87	44	29	22	16	13	12	12
0.95	0	16	8	5	4	2	2	1	1
0.95	1	26	13	8	6	4	3	3	2
0.95	2	35	17	11	8	5	4	4	4
0.95	3	43	21	14	10	7	6	5	5
0.95	4	51	25	16	12	8	7	6	6
0.95	5	59	29	19	14	10	8	7	7
0.95	6	66	33	22	16	11	9	8	8
0.95	7	74	37	24	18	12	10	10	9
0.95	8	81	41	27	20	14	12	11	10
0.95	9	88	44	29	22	15	13	12	11
0.95	10	96	48	31	24	16	14	13	12
0.99	0	25	12	7	5	3	2	2	2
0.99	1	36	18	11	8	5	4	3	3
0.99	2	46	22	14	10	7	5	5	4
0.99	3	55	27	17	13	8	6	6	5
0.99	4	64	31	20	15	10	8	7	6
0.99	5	73	36	23	17	11	9	8	8
0.99	6	81	40	26	19	13	10	9	9
0.99	7	89	44	28	21	14	11	10	10
0.99	8	97	48	31	23	15	13	11	11
0.99	9	105	52	34	25	17	14	13	12
0.99	10	112	56	36	27	18	15	14	13

Table 1: Minimum sample size for the specified ratio t/σ_0, confidence level p^*, acceptance number c, $\alpha = 2, \beta = 2$ using the binomial approximation

p^*	c	t/σ_0							
		0.315	0.472	0.629	0.786	1.180	1.573	1.966	2.359
0.75	0	9	5	3	3	2	2	2	2
0.75	1	14	7	5	4	3	3	3	3
0.75	2	22	12	8	7	5	5	4	4
0.75	3	30	16	11	9	7	6	6	6
0.75	4	37	19	13	11	8	7	7	7
0.75	5	44	23	16	12	9	9	8	8
0.75	6	50	26	18	14	11	10	9	9
0.75	7	57	30	20	16	12	11	11	10
0.75	8	63	33	23	18	14	12	12	12
0.75	9	70	36	25	20	15	13	13	13
0.75	10	76	40	27	22	16	15	14	14
0.90	0	14	7	5	4	3	3	3	3
0.90	1	21	11	8	6	5	4	4	4
0.90	2	31	16	11	9	7	6	6	6
0.90	3	39	21	14	11	9	8	7	7
0.90	4	47	25	17	13	10	9	9	9
0.90	5	54	29	20	15	12	11	10	10
0.90	6	62	32	22	18	13	12	11	11
0.90	7	69	36	25	20	15	13	13	13
0.90	8	76	40	27	22	16	15	14	14
0.90	9	83	43	30	23	18	16	15	15
0.90	10	90	47	32	25	19	17	17	16
0.95	0	18	10	7	5	4	4	4	4
0.95	1	27	14	10	8	6	5	5	5
0.95	2	37	19	13	11	8	7	7	7
0.95	3	46	24	16	13	10	9	9	8
0.95	4	54	28	19	15	12	10	10	10
0.95	5	62	32	22	18	13	12	11	11
0.95	6	69	36	25	20	15	13	13	13
0.95	7	77	40	28	22	16	15	14	14
0.95	8	84	44	30	24	18	16	16	15
0.95	9	92	48	33	26	20	18	17	17
0.95	10	99	52	35	28	21	19	18	18
0.99	0	27	14	10	8	6	6	5	5
0.99	1	38	20	14	11	8	8	7	7
0.99	2	49	26	18	14	11	10	9	9
0.99	3	59	31	21	17	13	11	11	11
0.99	4	68	36	24	19	15	13	13	12
0.99	5	77	40	27	22	16	15	14	14
0.99	6	85	45	30	24	18	16	16	15
0.99	7	94	49	33	26	20	18	17	17
0.99	8	102	53	36	29	22	19	19	18
0.99	9	110	57	39	31	23	21	20	20
0.99	10	118	61	42	33	25	22	22	21

Table 2: Minimum sample size for the specified ratio t/σ_0, confidence level p^*, acceptance number c, $\alpha = 2, \beta = 2$ using the Poisson approximation.

p^*	n	c	t/σ_0	σ/σ_0					
				2	4	6	8	10	12
0.75	22	2	0.315	0.9144	0.9976	0.9998	1.0000	1.0000	1.0000
0.75	11	2	0.472	0.9045	0.9972	0.9997	0.9999	1.0000	1.0000
0.75	7	2	0.629	0.8967	0.9967	0.9997	0.9999	1.0000	1.0000
0.75	6	2	0.786	0.8311	0.9934	0.9993	0.9999	1.0000	1.0000
0.75	4	2	1.180	0.7588	0.9871	0.9985	0.9997	0.9999	1.0000
0.75	3	2	1.573	0.7636	0.9844	0.9981	0.9996	0.9999	1.0000
0.75	3	2	1.966	0.5925	0.9567	0.9937	0.9986	0.9996	0.9999
0.75	3	2	2.359	0.4383	0.9092	0.9844	0.9964	0.9989	0.9996
0.90	30	2	0.315	0.8285	0.9941	0.9994	0.9999	1.0000	1.0000
0.90	15	2	0.472	0.8053	0.9928	0.9992	0.9999	1.0000	1.0000
0.90	10	2	0.629	0.7604	0.9899	0.9989	0.9998	0.9999	1.0000
0.90	7	2	0.786	0.7571	0.9891	0.9988	0.9998	0.9999	1.0000
0.90	5	2	1.180	0.5936	0.9715	0.9965	0.9993	0.9998	0.9999
0.90	4	2	1.573	0.4930	0.9493	0.9930	0.9985	0.9996	0.9999
0.90	4	2	1.966	0.2764	0.8723	0.9784	0.9950	0.9985	0.9995
0.90	3	2	2.359	0.4383	0.9092	0.9844	0.9964	0.9989	0.9996
0.95	35	2	0.315	0.7663	0.9909	0.9991	0.9998	1.0000	1.0000
0.95	17	2	0.472	0.7490	0.9896	0.9989	0.9998	0.9999	1.0000
0.95	11	2	0.629	0.7099	0.9866	0.9985	0.9997	0.9999	1.0000
0.95	8	2	0.786	0.6794	0.9835	0.9982	0.9996	0.9999	1.0000
0.95	5	2	1.180	0.5936	0.9715	0.9965	0.9993	0.9998	0.9999
0.95	4	2	1.573	0.4930	0.9493	0.9930	0.9985	0.9996	0.9999
0.95	4	2	1.966	0.2764	0.8723	0.9784	0.9950	0.9985	0.9995
0.95	4	2	2.359	0.1435	0.7592	0.9493	0.9872	0.9960	0.9985
0.99	46	2	0.315	0.6214	0.9810	0.9979	0.9996	0.9999	1.0000
0.99	22	2	0.472	0.6030	0.9788	0.9976	0.9995	0.9999	1.0000
0.99	14	2	0.629	0.5581	0.9734	0.9969	0.9994	0.9998	0.9999
0.99	10	2	0.786	0.5266	0.9684	0.9962	0.9993	0.9998	0.9999
0.99	7	2	1.180	0.3169	0.9216	0.9891	0.9977	0.9993	0.9998
0.99	5	2	1.573	0.2863	0.8966	0.9841	0.9965	0.9990	0.9996
0.99	5	2	1.966	0.1128	0.7628	0.9533	0.9886	0.9965	0.9987
0.99	4	2	2.359	0.1435	0.7592	0.9493	0.9872	0.9960	0.9985

Table 3: Values of the operating characteristic function of the sampling plan $(n, c, t/\sigma_0)$ for given p^* with $\alpha = 2, \beta = 2$.

p^*	c	t/σ_0							
		0.315	0.472	0.629	0.786	1.180	1.573	1.966	2.359
0.75	0	5.89	6.79	6.92	7.33	9.77	12.21	14.64	19.56
0.75	1	3.05	3.13	3.43	4.31	4.29	5.35	6.46	8.59
0.75	2	2.31	2.36	2.68	3.01	3.10	3.87	4.64	6.23
0.75	3	2.07	2.17	2.34	2.44	3.26	3.20	3.86	5.14
0.75	4	1.87	1.94	1.99	2.45	2.82	2.82	3.38	4.53
0.75	5	1.79	1.80	1.90	2.19	2.53	2.55	3.06	4.10
0.75	6	1.69	1.77	1.83	2.00	2.33	2.92	2.83	3.79
0.75	7	1.62	1.69	1.78	1.86	2.17	2.72	2.66	3.55
0.75	8	1.59	1.62	1.66	1.92	2.04	2.55	2.52	3.37
0.75	9	1.54	1.57	1.64	1.82	1.94	2.43	2.41	3.21
0.75	10	1.50	1.57	1.62	1.73	2.10	2.32	2.31	3.08
0.90	0	7.22	7.84	8.49	10.39	9.77	12.21	14.64	19.56
0.90	1	3.62	3.78	3.90	4.31	5.76	5.35	6.46	8.59
0.90	2	2.74	2.92	2.94	3.55	4.03	5.03	4.64	6.23
0.90	3	2.37	2.42	2.53	2.85	3.26	4.08	3.86	5.14
0.90	4	2.16	2.23	2.29	2.74	2.82	3.53	4.23	5.65
0.90	5	1.97	2.04	2.14	2.43	2.92	3.17	3.81	5.08
0.90	6	1.88	1.97	2.03	2.22	2.67	2.92	3.50	4.67
0.90	7	1.78	1.86	1.95	2.23	2.48	2.72	3.26	4.35
0.90	8	1.74	1.77	1.89	2.08	2.33	2.55	3.07	4.10
0.90	9	1.70	1.75	1.77	1.97	2.20	2.43	2.92	3.90
0.90	10	1.64	1.69	1.74	1.99	2.10	2.32	2.79	3.73
0.95	0	8.33	8.78	9.81	10.39	13.86	12.21	14.64	19.56
0.95	1	3.96	4.06	4.33	5.14	5.76	7.17	6.46	8.59
0.95	2	2.93	3.07	3.19	3.55	4.03	5.03	6.03	8.06
0.95	3	2.50	2.65	2.71	3.20	3.81	4.08	4.89	6.55
0.95	4	2.26	2.33	2.43	2.74	3.27	3.53	4.23	5.65
0.95	5	2.10	2.19	2.25	2.65	2.92	3.17	3.81	5.08
0.95	6	1.99	2.09	2.12	2.41	2.67	2.92	3.50	4.67
0.95	7	1.91	1.96	2.03	2.23	2.48	3.09	3.26	4.35
0.95	8	1.84	1.91	1.96	2.23	2.56	2.91	3.07	4.10
0.95	9	1.77	1.83	1.90	2.10	2.43	2.76	2.92	3.90
0.95	10	1.73	1.76	1.85	1.99	2.31	2.63	2.79	3.73
0.99	0	10.21	10.39	10.97	12.74	13.86	17.32	20.81	27.82
0.99	1	4.69	4.81	5.08	5.87	6.90	7.17	8.59	11.53
0.99	2	3.37	3.53	3.64	4.44	4.74	5.95	6.03	8.06
0.99	3	2.87	2.96	3.17	3.51	3.81	4.76	4.89	6.55
0.99	4	2.54	2.65	2.79	3.23	3.65	4.10	4.23	5.65
0.99	5	2.37	2.45	2.55	2.85	3.25	3.65	4.37	5.87
0.99	6	2.22	2.31	2.38	2.76	2.95	3.33	4.01	5.35
0.99	7	2.11	2.16	2.25	2.53	2.74	3.09	3.71	4.97
0.99	8	2.02	2.08	2.16	2.36	2.78	2.91	3.49	4.67
0.99	9	1.95	2.02	2.08	2.34	2.62	3.03	3.31	4.42
0.99	10	1.89	1.94	2.01	2.22	2.49	2.89	3.16	4.21

Table 4: Minimum ratio of true σ and required σ_0 for the acceptability of a lot with producer's risk of 0.05 for $\alpha = 2, \beta = 2$.