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MULTI-STATE SYSTEM ANALYSIS BASED ON MULTIPLE-VALUED DECISION DIAGRAM

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Abstract

Mathematical description of an examined system like a Multi-State System (MSS) permits the analysis of this system reliability in more detail, because the MSS defines some performance levels (more that only working and failure). A structure function is one of basic definitions of MSS. But the structure function dimension increases critically depending on the number of system components. Therefore the development of methods for examination and quantification of such function is an actual problem in MSS reliability analysis. In this paper a method for the analysis of the MSS structure function with high dimension is proposed. This method is based on initial representation of the MSS structure function by Multiple-Valued Decision Diagram and is developed for calculation of the importance measures of the MSS.

Key words: Reliability, Multi-State System, Multiple-Valued Logic, Logical Differential Calculus, Multi-Valued Decision Diagram

1. Introduction

There are two principal mathematical models in reliability analysis as shows in Zio (2009). The *Binary-State System* (BSS) allows investigating two states as working and failure for initial system. The *Multi-State System* (MSS) is a mathematical model that is used for describing the system with some (more than two) levels of performance (availability, reliability). Lisnianski and Levitin (2003) and Lisnianski et al. (2010) presented a detailed analysis of MSS reliability estimation and quantification methods. They considered a lot of examples of MSS application in reliability analysis of information, manufacturing, production, power generation, transportation and other systems.

The MSS performance level changes from zero to (M-1) and has M possible values. Each of n system components can be in one of m_i (i = 1, …, n) possible states: from the complete failure (it is 0) to the perfect functioning (it is m_i-1). A structure function is one of typical representations of MSS and defines correlation of system performance level depending on MSS components states [Lisnianski and Levitin, 2003; Zaitseva, 2012]:

$$\phi(\boldsymbol{x}): \{0, \ldots, m_1\text{-}1\}\times\ldots\times\{0, \ldots, m_n\text{-}1\}\rightarrow\{0, \ldots, M\text{-}1\}, \qquad (1.1)$$

where x_i is the i-th component state; $\boldsymbol{x} = (x_1, \ldots, x_n)$ is vector of components states.

Every system component states x_i is characterized by the probability of the performance rate:

$$p_{i,s} = \Pr\{x_i = s_i\}, \quad s = 0, \ldots, m-1 \qquad (1.2)$$

The MSS importance analysis is one of the directions for estimation of MSS behavior against the system structure and components states. There are different methods and algorithms for MSS importance analysis. Authors of the paper [Lisnianski and Levitin, 2003; Levitin et al., 2003] have considered basic *Importance measures* (IM) for system with two performance level and multi-state components and their definitions by output performance measure. Ramirez-Marquez and Coit (2005) have generalized this result for MSS and have proposed new type of IM that is named as composite importance measures. Meng (2009) has presented a review of IMs. New methods for importance analysis of MSS have been considered in [Zaitseva, 2012]. These methods based on mathematical tools of *Multiple-Valued Logic* (MVL).

The mathematical tools of MVL as Logical Differential Calculus for MSS reliability analysis have been proposed in [Zaitseva and Levashenko, 2006]. The authors have shown that the MSS structure function is interpreted as MVL function. The Logical Differential Calculus is mathematical tool that permits the analysis of changes in MVL function depending of changes of its variables. Therefore this tool can be used to evaluate influence of every system component state change to MSS performance level. The principal disadvantage of the Logical Differential Calculus application in reliability analysis is the increase of computational complexity depending on the number of system component. Miller and Drechsler (2002) have proposed Multiple-Valued Decision Diagram (MDD) for such function representation and analysis.

MDD is generalization of Binary Decision Diagram (BDD). BDD is widely used in reliability analysis for BSS [Chang, 2004]. MDD is natural extension for MSS analysis [Zaitseva and Levashenko, 2007; Xing and Dai, 2009]. On the other hand, the MSS representation by MDD causes development of new algorithms for system reliability analysis. We proposed new algorithms for IM calculation based on MDD by Logical Differential Calculus.

2. Logic Differential Calculus in MSS reliability analysis

Logical Differential Calculus of MVL function includes different methods and algorithms for estimation of influence of variable/variables value change to the function value modification. *Direct Partial Logic Derivatives* (DPLD) are part of Logic Differential Calculus and can be used for the analysis of dynamic properties of MVL function or MSS structure function. These derivatives reflect the change in the value of the underlying function when the variable value changes [Zaitseva, 2012]. DPLD with respect to variable x_i for MSS structure function (1) permits the analysis of the system performance level change from j to $\tilde{j}$ when the i-th component state changes from s to $\tilde{s}$.

DPLD for the MSS structure function has some specifics, because the structure function (1.1) of coherent MSS has the following assumptions: (*a*) the structure function is monotone; (*b*) all components are independent and relevant to the system. Assumptions (*a*) and (*b*) cause gradual changes of the function value depending on the same variable change and system reliability doesn’t change by leaps and bounds. Therefore the DPLD for the coherent MSS performance level reduction is defined as:

$$\partial\phi(j \to j-1)/\partial x_i(s \to s-1) = \begin{cases} 1, & if\ \phi(s_i, \boldsymbol{x}) = j\ and\ \phi((s-1)_i, \boldsymbol{x}) = j-1 \\ 0, & other \end{cases}, \quad (2.1)$$

where $\phi(s_i, \boldsymbol{x}) = \phi(x_1, \ldots, x_{i-1}, s_i, x_{i+1}, \ldots, x_n)$; $\phi((s\text{-}1)_i, \boldsymbol{x}) = \phi(x_1,\ldots, x_{i-1}, (s\text{-}1)_i, x_{i+1},\ldots, x_n)$; $s_i \in \{1, \ldots, m_i\text{-}1\}$ and $j \in \{1, \ldots, M\text{-}1\}$.

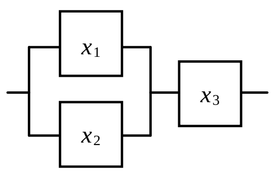
DPLD (2.1) allows discovering the boundary system state for which change of the i-th system component state from s to s-1 causes modification of the MSS performance level from j to j-1.

Example

Consider the MSS of three component (n=3) in Fig.1. The structure function of this MSS is

$$\phi(\boldsymbol{x}) = \text{AND}\big(\text{OR}(x_1, x_2), x_3\big) \quad (2.2)$$

and values of this function are in truth-table in Fig.1, where $m_1 = m_2 = 2$, $m_3 = 4$, $M = 3$.



The MSS structure function

$x_1\ x_2$ \ x_3	0	1	2	3
0 0	0	0	0	0
0 1	0	1	1	1
1 0	0	1	1	1
1 1	0	1	2	2

Figure 1: The MSS example for n = 3, M = 3 and $\boldsymbol{m}$ = (2, 2, 4)

The influence of the third variable failure to the MSS performance can be analysed by DPLDs $\partial\phi(1\to0)/\partial x_3(1\to0)$ (failure) and $\partial\phi(2\to1)/\partial x_3(1\to0)$ (performance level decrease). Consider the first of these derivatives. This derivative has three nonzero values for variables vector $\boldsymbol{x} = (x_1\ x_2\ x_3)$: (1 1 $\underline{1\to0}$), (0 1 $\underline{1\to0}$) and (1 0 $\underline{1\to0}$). Therefore the failure of the third component cause the system breakdown for working state of the first and the second components or working state one of them. The system isn't functioning if the first and the second component are failed and failure of the third component hasn't influence to the system performance level change.

3. Multiple-Valued Decision Diagram for the MSS structure function

The structure function representation by MDD has been considered in [Zaitseva, 2012]. Miller and Drechsler (2002) defined MDD as a directed acyclic graph for MVL-function representation. For the structure function (1.1) this graph has M sink nodes, labelled from 0 to (M-1), representing M corresponding constant from 0 to (M-1). Each non-sink node is labelled with a structure function variable x_i and has m_i outgoing edges (Fig.2). The first (left) of edges is labelled by "0" and agrees with component fail. The m_i-th last outgoing edge is labelled "m_i -1" edge and presents the perfect operation state of system component.

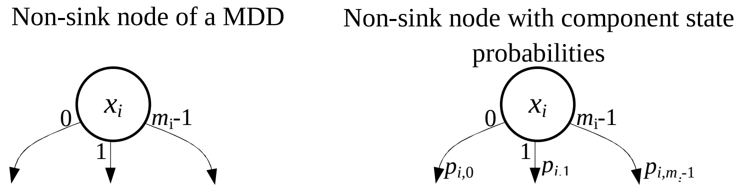


Figure 2: Interpretation of component state in MDD

Sink nodes of the MDD correspond to performance levels of the MSS. In this case the non-sink node outgoing edges are interpreted as component states. The probabilistic interpretation of the MSS assumes that every edge from variable x_i with labelled s_i is marked by the i-th component stat probability p_{i,s_i} (Fig. 2). Rules for MSS measures calculation based on MDD are trivial and are presented in [Zaitseva, 2012]. The probability of the MSS state is such measure.

Lisnianski and Levitin (2003) have been defined the probability of the MSS state as probability that the system reliability is equal to the performance level j:

$$R(j) = \Pr\{\phi(\mathbf{x}) = j\},\ j = 0, \ldots, M\text{-}1 \quad (3.1)$$

Note, the MSS unreliability is defined as $F = R(0)$.

Calculation of measures (3.1) based on the MDD consists of analysis of paths from top non-sink node to the sink node "j". Therefore the MDD is divided into M sub-diagrams that have one sink node and include paths to this node from top non-sink node of the MDD. Every edge from variable x_i with labelled s_i is marked by the i-th component stat probability p_{i,s_i} [Zaitseva, 2012].

Example

The MDD for the MSS structure function (2.2) is in Fig. 3. The MSS state probabilities (3.1) are calculated by analysis of sub-diagrams in Fig.4. The component state probabilities are in Table 1. According to the sub-diagrams and probabilities in Table 1 MSS state probabilities are:

$$R(1) = p_{1,1}\, p_{2,1}\, p_{3,1} + (p_{3,1} + p_{3,2} + p_{3,3})(p_{1,0}\, p_{2,1} + p_{1,1}\, p_{2,0}) = 0.416$$

$$R(2) = p_{1,1}\, p_{2,1}\, (p_{3,2} + p_{3,3}) = 0.448$$

$$F = p_{1,0}\, p_{2,0} + p_{3,0}\, (p_{1,0}\, p_{2,1} + p_{1,1}) = 0.136$$

Table 1: MSS Component State Probability

Component	States			
	0	1	2	3
x_1	0.2	0.8		
x_2	0.2	0.8		
x_3	0.1	0.2	0.2	0.5

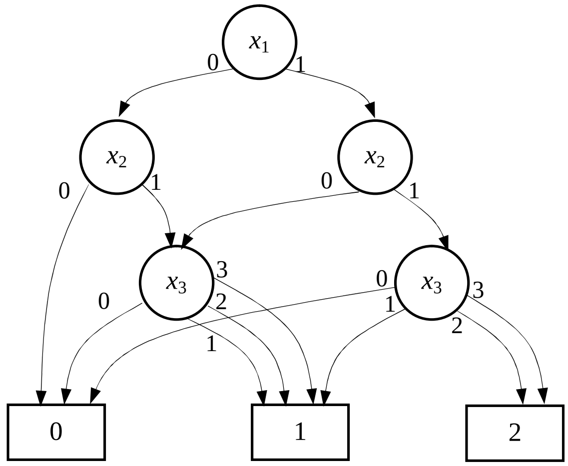
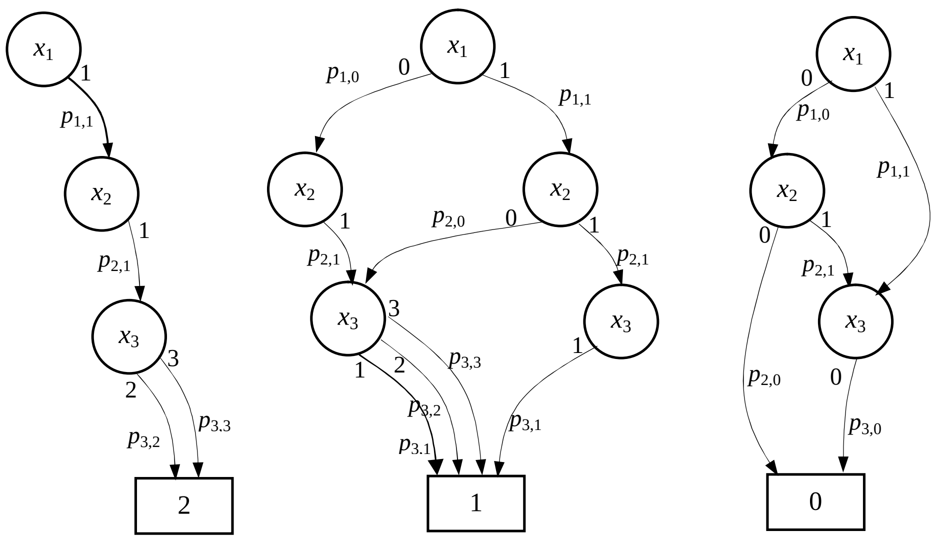


Figure 3: MDD for the MSS structure function (2.2)



Sub-diagram for $R(2)$ Sub-diagram for $R(1)$ Sub-diagram for F

Figure 4: Calculation of the MSS state probabilities R(*j*) by MDD

4. Importance Measures for MSS

There are some IM for MSS estimation. We consider in this paper IM that can be calculated by DPLD [Zaitseva, 2012]: Structural Importance (SI), Criticality Importance (CI), Birnbaum importance (BI), Fussell-Vesely importance (FVI), Component Dynamic Reliability Indices (CDRI) and Dynamic Integrated Reliability Indices (DIRI).

The SI takes into account the topological specifics of the system. It is used for analyzing such systems, which are in designs or we don't know the entire structure of the system. SI of the MSS for the *i*-th component state *s* is the probability of this system performance level *j* decrement if the component state changes from s_i to s_i-1 depending on topological properties of system:

$$I_S(s_i \mid j) = \frac{\rho_i^{s,j}}{m_1 \dots m_{i-1} m_{i+1} \dots m_n}, \tag{4.1}$$

where $\rho_i^{s,j}$ is number of system states when the change component state from *s* to *s*-1 results the system performance level decrement and this number is calculated as numbers of nonzero values of DPLDs (2.1).

The modified SI represent of the *i*-th system component state change influence to MSS performance level decrement for boundary system state. In terms of DPLD (2.1) modified SI is determined as [Zaitseva, 2012]:

$$I_{MS}(s_i \mid j) = \frac{\rho_i^{s,j}}{\rho_i^{(s,j)}}, \tag{4.2}$$

where $\rho_i^{(s_i,j)}$ is number of system states when $\phi(s_i, \boldsymbol{x}) = j$ (it is computed by the structure function of the MSS (1.1)).

Modified SI I_{MS} is the probability of MSS performance decrement depending on the *i*-th component state change and boundary system states. A system component with maximal value of the SI measure (I_S and I_{MS}) has most influence to MSS and this component failure causes high possibility of MSS failure.

Birnbaum Importance (BI) of a given component is defined as the probability that such component is critical to MSS functioning. The mathematical and logical generalization of this measure for MSS has some interpretations. Definition of BI for MSS in terms of Logical Differential Calculus can be interpreted as rate at which the MSS fails as the *i*-th system component state decreases:

$$I_B(s_i \mid j) = \Pr\left(\partial\phi(j \to j-1) / \partial x_i(s \to s-1)\right), \tag{4.3}$$

BI measures (4.3) depend on the structure of the system and states of the other components, but is independent of the actual state of the *i*-th component.

Consider the definition of *Criticality Importance* (CI) that is the probability that the *i*-th system component is relevant to MSS performance decrement if it has failed or has diminished state. For MSS this measure can be defined as probability of the MSS performance reduces if the state of the *i*-th system component has changed from *s* to *s*-1:

$$I_C(s_i \mid j) = I_B(s_i \mid j) \cdot \frac{p_{i,s-1}}{R(j)}, \tag{4.4}$$

where $p_{i,s\text{-}1}$ is probability of the i-th system component state s-1 (1.2) and $R(j)$ is MSS probability state (3.1).

The CI measure (4.4) correct BI for unreliability or lower state of the i-th component relative. This measure is useful, if the component has high BI and low probability of investigated state with respect of MSS performance decrement. In this case the i-th component CI is low.

CDRI indicates the influence of the i-th component state change to MSS performance level change [Zaitseva and Levashenko, 2006]. This definition of CDRI is similar to definition of modified SI, but CDRIs for MSS failure take into consideration two probabilities: (a) the probability of MSS failure provided that the i-th component state is reduced and (b) the probability of inoperative component state:

$$I_{CDRI}(s_i \mid j) = I_{MS}(s_i \mid j) \cdot p_{i,s-1}\,, \tag{4.5}$$

where $I_{MS}(s_i \mid j)$ is the modified SI (4.2); $p_{i,s}$ is probability of component (1.2).

DIRI is the probability of MSS performance level decrement that caused by the one of system components state deterioration. DIRI allows estimate probability of MSS failure caused by some system component (one of n):

$$I_{DIRI}(s \mid j) = \sum_{i=1}^{n} I_{CDRI}(s_i \mid j) \prod_{\substack{q=1 \\ q \neq i}}^{n} (1 - I_{CDRI}(s_q \mid j)) \tag{4.6}$$

The IM (4.1) – (4.6) are defined based on the DPLD (2.1). Therefore the algorithms for calculation of the DPLD by MDD are principal part for quantification of the MSS that is represented by MDD. There aren't established algorithms for calculation DPLD by the MDD. Changqian and Chenghua (2009) proposed an algorithm for the Partial Derivative calculation by the BDD. But this algorithm doesn't permit the DPLD calculation based on MDD. Therefore we propose new algorithms below for THE calculation for DPLD based on MDD for the following MSS importance analysis.

5. MSS Importance Analysis

We proposed two algorithms for DPLD calculation based on the MSS structure function representation by MDD. One of them analyses the MDD from the top non-sink node to sink node. This algorithm is named "from top to down" and the other algorithm "from down to top" which is based on the inverse analysis – from sink node to the top non-sink node. These algorithms permits to determine nonzero values of DPLD (2.1) and the variables vectors $\boldsymbol{x} = (x_1 \ldots s_i \rightarrow (s\text{-}1)_i \ldots x_n)$ for these nonzero values. The vector corresponds to path between the top non-sink node and sink node. Therefore the two types of paths are considered that satisfy the conditions as:

$$\phi(s_i, \boldsymbol{x}) = j \quad \text{and} \quad \phi((s\text{-}1)_i, \boldsymbol{x}) = j\text{-}1 \tag{5.1}$$

The algorithm "from top to down" is started from analysis of the top non-sink node of the MSS and this analysis "comes down" to the sink node "*j*" or "*j*-1" and edge of variable x_i is defined as "*s*" or "*s*-1" accordingly. The algorithm proceeding recursively for fixed value of the variable x_i and different values of other variables. The algorithm takes into account the next rules:

(1) The sub-diagram from the top non-sink node to the sink node "*j*" or "*j*-1" is formed for following analysis. All paths from the top non-sink node to the sink node are considered and respective variables vectors are determined and they are added to the set for condition (5.1).
(2) The left emanating edge for the current non-sink node is included in the current path if the current node doesn't agree with analyzable variable x_i for DPLD (2.1). The next emanating edge from left to right is considered if the left edge has been analysed. If the current non-sink node has the examined edges only, analysis of the current path comes back to previous non-sink node.
(3) The current non-sink node supposes inclusion of the emanating edge with label "*s*" or "*s*-1" if the current node corresponds with analyzable variable for DPLD (2.1). If this non-sink node hasn't the emanating edge with label "*s*" or "*s*-1", analysis of the current path comes back to previous non-sink node.
(4) The variable x_q (q = 1, …, n) is labelled as irrelevant (for example by "-1") if it is not in the current path. Value of this variable in the variables vector can be defined from set {0, …, m_q-1}.
(5) The algorithm stops if the all possible paths are examined and analyse is come back to the top non-sink node and all of emanating edges of this node is examined.

Example

Consider the calculation of the DPLD $\partial\phi(1\to0)/\partial x_1(1\to0)$ by this algorithm for the MSS structure function (2.2). The MDD of this is in Fig.3. The algorithm starts form the definition of sub-diagram (rule (1)) for conditions: $\phi(\boldsymbol{x}) = 1$ and $\phi(\boldsymbol{x}) = 0$ that are in Fig.4. The first examined non-sink node is node of the variable x_1 that is agree with variable of analysed DPLD (Fig.5). According to the algorithms rule (3) the value of the variable x_1 is defined as "1" and analysis goes to non-sink node of the variable x_2 (it is step 1 in Fig.5). The variable x_2 is analysed according to the algorithm rule (2). The left edge with label "0" is considered and the x_2 value is defined as $x_2 = 0$ (the step 2). The last non-sink node for variable analysed and according to the left outgoing edge we have $x_3 = 1$. Therefore the first of variables vector is $\boldsymbol{x} = (1\ 0\ 1)$ (the step 3). The analysis comes back to the non-sink node of variable x_3 on the step 4 because the sink node "1" is reached. In this case we have $x_1 = 1$ and $x_2 = 0$. The value of the third variable is $x_3 = 2$ because the left emanating edge was considered for previous variables vector and next of edges has label "2" (the step 5). The second variables vector is $\boldsymbol{x} = (1\ 0\ 2)$. The third vector $\boldsymbol{x} = (1\ 0\ 3)$ is calculated similarly on step 7. The calculation of the fourth variables vector causes the analysis to return to the non-sink node x_2 (step 8 and 9). This node has two edges and edge with label "1" wasn't examined. Therefore values of the variables x_1 and x_2 are $x_1 = 1$ and $x_2 = 1$ (step 10). The non-sink node x_3 in current path has only one edge therefore $x_3 = 1$ and $\boldsymbol{x} = (1\ 1\ 1)$ (step 11). The analysis of the condition $\phi(1_1, \boldsymbol{x}) = 1$ is finished because all possible paths are examined according to the algorithm rule 5 (steps 12, 13) and we have the set of variables vectors:

$\{\boldsymbol{x} = (*\ 0\ 1), \boldsymbol{x} = (*\ 0\ 2), \boldsymbol{x} = (*\ 0\ 3), \boldsymbol{x} = (*\ 1\ 1)\}.$

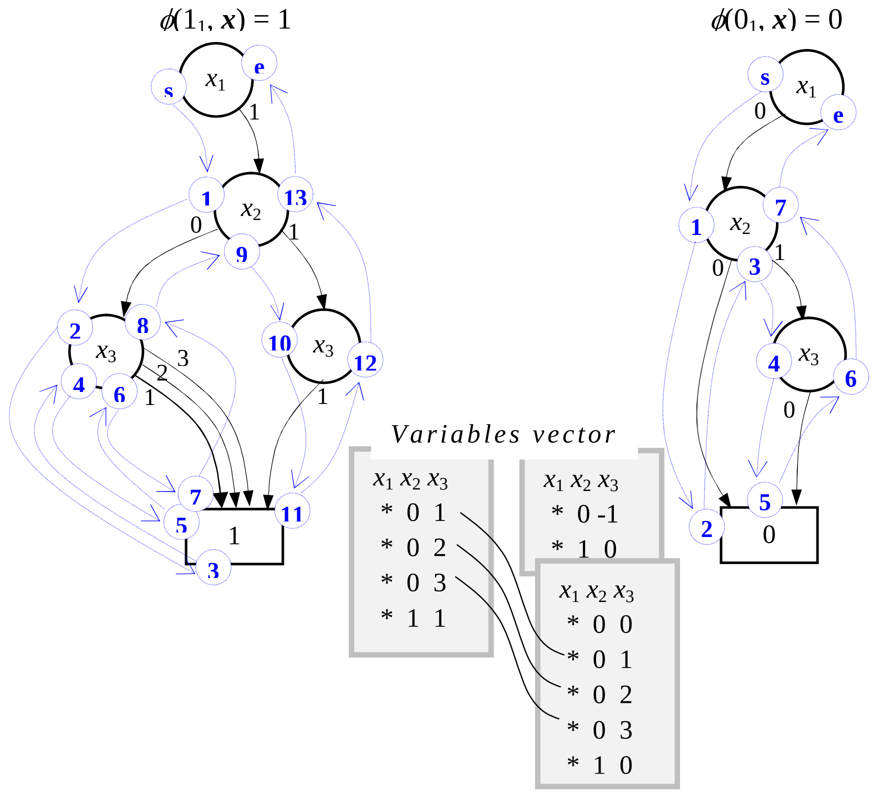


Figure 5: Calculation of the DPLD by MDD

The set of the variables vector for condition $\phi(0_1, \boldsymbol{x}) = 0$ is calculated similarly (Fig.5) and is:

$\{\boldsymbol{x} = (*\ 0\ 0), \boldsymbol{x} = (*\ 0\ 1), \boldsymbol{x} = (*\ 0\ 2), \boldsymbol{x} = (*\ 0\ 3), \boldsymbol{x} = (*\ 1\ 0)\}.$

Three vectors in these sets are equal (Fig.5) therefore the set of the variables vectors of the DPLD $\partial\phi(1\to0)/\partial x_1(1\to0)$ is:

$$\{\boldsymbol{x} = (1\to0\ 0\ 1), \boldsymbol{x} = (1\to0\ 0\ 2), \boldsymbol{x} = (1\to0\ 0\ 3)\}. \quad (5.2)$$

According to (5.2) we have to calculate IM (4.1) – (4.6) for the MSS in Fig.1 with the structure function (2.2) for the first variable. The component state probabilities for the calculation of this IM are in Table 1. The IMs for all components of this MSS are in Table 2.

Table 2: MSS Importance Measures

Component	Importance Measures					
	$I_S(1_i\|1)$	$I_{MS}(1_i\|1)$	$I_B(1_i\|1)$	$I_C(1_i\|1)$	$I_{CDRI}(1_i\|1)$	$I_{DIRI}(1\|1)$
x_1	0.375	0.750	0.180	0.265	0.551	0.333
x_2	0.375	0.750	0.180	0.265	0.551	
x_3	0.750	1	0.960	0.706	0.551	

The algorithm "from down to top" begins the analysis from sink node "*j*" or "*j*-1" to the top non-sink node of the MSS provided that edge of variable x_i is defined as "*s*" or "*s*-1" accordingly. The algorithm is recursive for fixed value of the variable x_i and different values of the other variables according to following rules:

(1) The sub-diagram from the top non-sink node to the sink node "*j*" or "*j*-1" is formed for following analysis. All paths from the top non-sink node to the sink node are considered and respective variables vectors are determined and they are added to the set for condition (5.1).

(2) The analysis is started from left inflowing edge of sink node. The next inflowing edge from left to right is considered if the left edge has been examined. If the sink node has the examined edges only for some recursion, analysis is ended.

(3) The left inflowing edge for the current non-sink node is included in the current path, if the current node doesn't agree with analyzable variable x_i for DPLD (2.1). The next inflowing edge from left to right is considered if the left edge has been analysed. If the current non-sink node has the examined edges only, analysis of the current path comes back to previous non-sink node or sink node.

(4) The current non-sink node supposes inclusion of the inflowing edge with label "*s*" or "*s*-1" if the current node corresponds with analyzable variable for DPLD (4). If this non-sink node hasn't the emanating edge with label "*s*" or "*s*-1", analysis of the current path comes back to previous node.

(5) The variable x_q (q = 1, …, n) is labelled as irrelevant (for example by "-1") if it is not in the current path. Value of this variable in the variables vector can be defined from set {0, …, m_q-1}.

The calculation of the DPLD nonzero values according to this algorithm is similar to the algorithms "from top to down" and haven't any specifics. The comparison of these algorithms is considered below.

Testing the presented algorithms is performed on benchmarks, which are available at the Benchmark Archives at CBL online: http://www.cbl.ncsu.edu:16080/benchmarks/LGSynth91/twolexamples/. This benchmark has in the PLA – EXPRESSO format that is used for Boolean functions. Therefore the tests carried out on BDD constructed on the basic of the benchmarks. Tested result is shown in Table 3 with the number of nodes and variables. These benchmarks used for two algorithms comparison, control and examination. Experiments permit to estimate the computation complexity of the proposed algorithms (by the time calculation, number of algorithms steps etc.) depending on number of function variables. Computational complexity of the algorithm "from top to down" (Algorithm 1) and algorithm "from down and top" (Algorithm 2) is shown in Fig.6 depending of the number of the function variables (diagram nodes).

Table 3: Benchmark characteristics

Benchmark	Count of Nodes	Count of Variables
5xp1	108	11
9sym	73	18
apex2	265	23
Bw	221	10
Clip	171	13
con1	27	7
misex1	88	9
misex2	173	13
o64	260	10
rd53	35	10
rd73	97	15
rd84	167	15
sao2	169	16
vg2	178	13
xor5	18	7

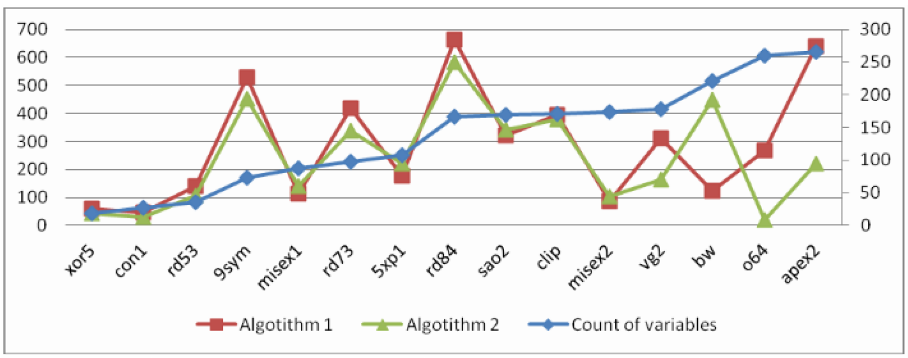


Figure 6: Comparison of algorithms computational complexity

Analysis of the data in Fig.6 shows that the two algorithms have similar parameters. These algorithms can be used for IM (4.1) – (4.6) calculation equally. Therefore principal algorithms for IM quantification based on DPLD have been proposed in this paper.

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