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RELIABILITY EQUIVALENCE FACTORS OF A PARALLEL SYSTEM IN TWO-DIMENSIONAL DISTRIBUTION

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Abstract

This paper is devoted to obtain the two-dimensional reliability modeling equivalence factors of n independent and identical components parallel system. Bivariate Weibull model has been used by considering the failure behavior of components. We discuss two-dimensional failure modeling for a system where degradation is due to age and usage. Three different methods are used to improve the given system. The mean times to failures of the original and improved systems are obtained. Numerical studies are presented to compare the different reliability factors obtained.

Keywords: Two-dimensional Reliability Modeling, Equivalence Factors, Parallel System, Hot Duplication, Cold Duplication, Reduction Method.

1. Introduction

In reliability theory, one way to improve the performance of a system is to use the redundancy method. There are two main such methods:
1. Hot duplication method, in this case, it is assumed that some of the system components are duplicated in parallel.
2. Cold duplication method, in this case, it is assumed that some of the system components are duplicated in parallel via a perfect switch.

Sarhan et al. (2008) investigated that the design of the system improved by the reduction method should be equivalent to the design of the system improved by one of the duplication methods. The comparison of the designs produces the so-called reliability equivalence factors.

The concept of the reliability equivalence factors was introduced by Rade (1989-1) and applied to various reliability systems in Rade (1990-3 and 1991). Rade (1993) applied this concept for the two-component parallel and series systems with independent and identical components whose lifetimes follow the exponential distribution. Sarhan (2000, 2002, 2004, 2005 and 2009) derived the reliability equivalence factors of other more general systems. The systems studied by Sarhan are the series system in (2000), a basic series-parallel system in (2002), a bridge network system in (2004), the parallel system in (2005), a parallel-series system in (2008), and a general series-parallel system in (2009). All these systems have independent and identical exponential components.

In this paper, the reliability equivalence factors of a parallel system with n independent and identical components are obtained. Three different methods are used to improve the given system. The failure rates of the system components are reduced by a factor ρ, $0 < \rho < 1$. Now, once the reduction method is adopted, the failure rates of the system components are functions about age and usage, with bivariate Weibull distribution. The numerical results are presented to interpret how one can utilize the obtained models.

2. n-Component parallel system

The system considered here consists of n independent and identical components connected in parallel. Each component has a bivariate Weibull reliability as given by Lu and Bhattacharyya (1990), say 1. That is, the lifetime U and the usage V are random variables with the following reliability function:

$$R(u,v)=\exp\{-[(\frac{u}{\theta_1})^{\beta_1/\delta}+(\frac{v}{\theta_2})^{\beta_2/\delta}]^{\delta}\} \tag{1}$$

with $\theta_1,\theta_2,\beta_1,\beta_2 \succ 0$ and $0 \prec \delta \le 1$. Note that U and V are note independent

The corresponding failure density function is given by Baik et al. (2004):

$$f(u,v)=[(\frac{u}{\theta_1})^{\beta_1/\delta-1}\times(\frac{v}{\theta_2})^{\beta_2/\delta-1}]\times[(\frac{u}{\theta_1})^{\beta_1/\delta}+(\frac{v}{\theta_2})^{\beta_2/\delta}]^{\delta-2}\times(\beta_1\beta_2/\theta_1\theta_2)$$
$$\times\{[(\frac{u}{\theta_1})^{\beta_1/\delta}+(\frac{v}{\theta_2})^{\beta_2/\delta}]^{\delta}+1/\delta-1\}\times\exp\{-[(\frac{u}{\theta_1})^{\beta_1/\delta}+(\frac{v}{\theta_2})^{\beta_2/\delta}]^{\delta}\}] \tag{2}$$

and the hazard function is

$$h(u,v)=[(\frac{u}{\theta_1})^{\beta_1/\delta-1}\times(\frac{v}{\theta_2})^{\beta_2/\delta-1}]\times[(\frac{u}{\theta_1})^{\beta_1/\delta}+(\frac{v}{\theta_2})^{\beta_2/\delta}]^{\delta-2}\times(\beta_1\beta_2/\theta_1\theta_2)$$
$$\times\{[(\frac{u}{\theta_1})^{\beta_1/\delta}+(\frac{v}{\theta_2})^{\beta_2/\delta}]^{\delta}+1/\delta-1\} \tag{3}$$

The reliability function of the original system which consists of n independent and identical components, denoted by $R_P(u,v)$, can be obtained as:

$$R_P(u,v)=1-[1-\exp\{-[(\frac{u}{\theta_1})^{\beta_1/\delta}+(\frac{v}{\theta_2})^{\beta_2/\delta}]^{\delta}\}]^n \tag{4}$$

Figure (1) shows $R_P(u,v)$ with

$n=4, \theta_1=2, \theta_2=3, \beta_1=1.5, \beta_2=2, \delta=0.5$ and $0 \le u \le 5,\ 0 \le v \le 10$

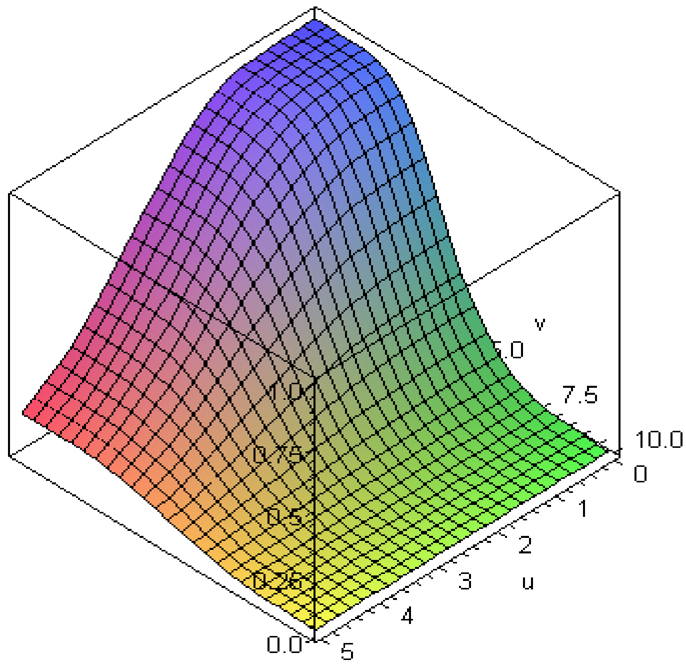

Figure 1.:The reliability function of the orginal system

The reliability of the system can be improved according to one of the following three different methods:
1- Reducing the failure for some lifetimes.
2- Add hot duplication components
3- Add cold duplication components.

2.1. The Reduction Method

Assuming that the system is improved by improving r, 1<r<n, of its components according to the reduction methods i.e., the time and usage varying failure rates of r components are reduced from

$$[\rho(\frac{u}{\theta_1})^{\beta_1/\delta} + \rho(\frac{v}{\theta_2})^{\beta_2/\delta}]^{\delta},\ 0 \prec \rho \prec 1 \quad \text{to} \quad [(\frac{u}{\theta_1})^{\beta_1/\delta} + (\frac{v}{\theta_2})^{\beta_2/\delta}]^{\delta}$$

Let $R_{(r),\rho}(u,v)$ denote the reliability function of the system improved by reducing the varying failure rates of r of its components for some lifetimes, then, one can obtain

$$R_{(r),\rho}(u,v) = 1-[1-\exp\{-[\rho(\frac{u}{\theta_1})^{\beta_1/\delta} + \rho(\frac{v}{\theta_2})^{\beta_2/\delta}]^{\delta}\}]^{r}[1-\exp\{-[(\frac{u}{\theta_1})^{\beta_1/\delta} + (\frac{v}{\theta_2})^{\beta_2/\delta}]^{\delta}\}]^{n-r} \qquad (5)$$

Figure (2) shows $R_{(r),\rho}(u,v)$ with
$n=4, \theta_1=2, \theta_2=3, \beta_1=1.5, \beta_2=2, \delta=0.5$ and
$\rho=0.5, \mathrm{r}=2,\ 0\le u\le 5,\ 0\le v\le 10$

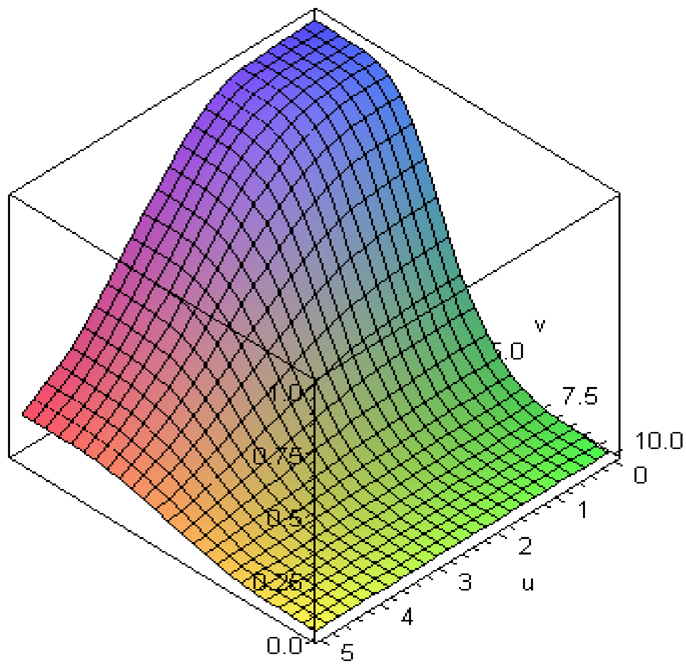

Figure 2: The reliability function of the system modified by reduction method

2.2. The Hot Duplication Method

Assume that the system is improved by improving m , 1<m<n, of its components according to the hot duplication method. Let $R^{H}_{(m)}$ (u,v) represent the reliability function of the system improved by improving m of its components by hot duplication, then, we can get $R^{H}_{(m)}$ (u,v) as following:

$$R^{H}_{(m)}(u,v)=1-[1-\exp\{-[(\frac{u}{\theta_1})^{\beta_1/\delta}+(\frac{v}{\theta_2})^{\beta_2/\delta}]^{\delta}\}]^{n+m} \tag{6}$$

Figure (3) shows $R^{H}_{(m)}(u,v)$ with $n=4, \theta_1=2, \theta_2=3, \beta_1=1.5, \beta_2=2, \delta=0.5$ and
$m=2,\ 0\le u\le 5,\ 0\le v\le 10$

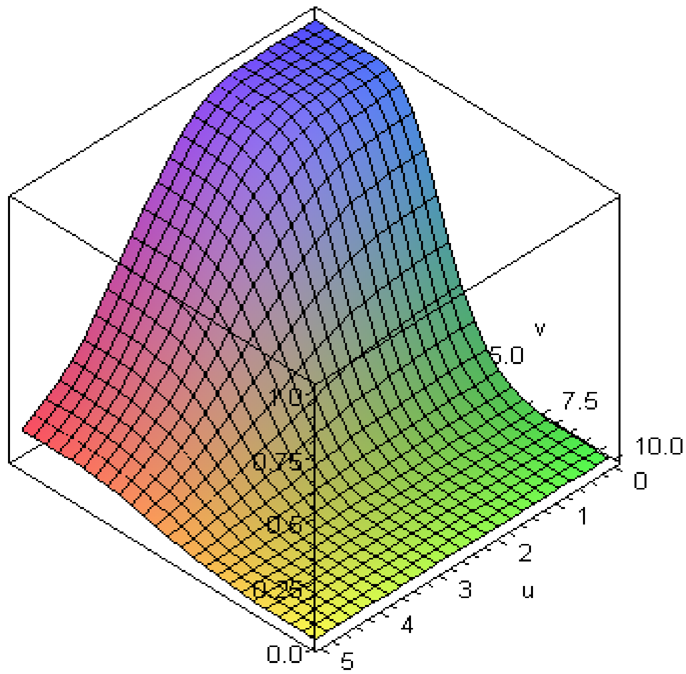

Figure 3: The reliability function of the system modified by hot duplication method

2.3.The Cold Duplication Method

Consider that the system is improved by improving m, 1<m<n of its components according to the cold duplication methods. Let $R^{C}_{(m)}(t)$ denote the reliability function of the system improved by improving m of its components according to cold duplication methods. The function is obtained as follows:

$$R^{C}_{(m)}(u,v) = 1-[1-(1+[(\frac{u}{\theta_1})^{\beta_1/\delta}+(\frac{v}{\theta_2})^{\beta_2/\delta}]^{\delta})]\times[\exp\{-[(\frac{u}{\theta_1})^{\beta_1/\delta}+(\frac{v}{\theta_2})^{\beta_2/\delta}]^{\delta}\}]^{m}$$
$$\times[1-\exp\{-[(\frac{u}{\theta_1})^{\beta_1/\delta}+(\frac{v}{\theta_2})^{\beta_2/\delta}]^{\delta}\}]^{n-m} \quad (7)$$

Figure (4) shows $R^{C}_{(m)}(u,v)$ with

$n = 4, \theta_1 = 2, \theta_2 = 3, \beta_1 = 1.5, \beta_2 = 2, \delta = 0.5$

and

$m = 2,\ 0 \le u \le 5,\ 0 \le v \le 10$

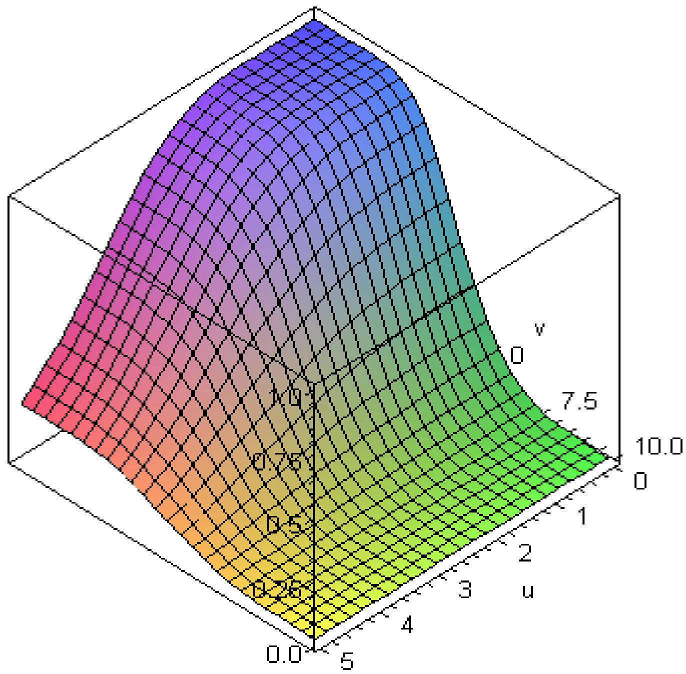

Figure 4: The reliability function of the system modified by cold duplication method

3. The MTTF's

Let U and V be independent random variables as $\delta = 1$, then

$$R(u,v) = R(u)R(v) = \exp[-(\frac{u}{\theta_1})^{\beta_1}] \times \exp[-(\frac{v}{\theta_2})^{\beta_2}] \tag{8}$$

The system MTTF for U or V is defined by

$$MTTF = \int_0^\infty R_P(t)dt = \int_0^\infty \{1-[1-\exp[-(\frac{t}{\theta_i})^{\beta_i}]]^n\}dt, \quad t = u, \mathrm{v}; \; i = 1, 2$$

Using binomial expansion write MTTF as:

$$M\mathrm{TTF} = \theta_{\mathrm{i}} \, \Gamma(1+\frac{1}{\beta_{\mathrm{i}}})\sum_{\mathrm{j}=1}^{n}\binom{n}{j}(\text{-}1)^{\mathrm{j}+1} \, \mathrm{j}^{-\frac{1}{\beta_{\mathrm{i}}}}; \quad i = 1, 2 \tag{9}$$

3.1. MTTF According to Reduction Method

It is given as $MTTF_{(r),\rho}$. Using binomial expansion, we write

$$MTTF_{(r),\rho} = \theta_i\ \Gamma(1+\frac{1}{\beta_{\mathrm{i}}})\,[\sum_{j=0}^{n-r}\sum_{k=1}^{r}\binom{n-r}{j}\binom{n}{k}(-1)^{j+k+1}\frac{1}{(j+k\rho)^{1/\beta_i}} + \sum_{\mathrm{j=1}}^{\mathrm{n\text{-}r}}\binom{n-r}{j}(-1)^{j+1}\frac{1}{j^{1/\beta_i}}], \quad ,\mathrm{i}=1,2 \tag{10}$$

3.2. MTTF According to Hot Duplication Method

It is given as $MTTF^{H}_{(m)}$. Using binomial expansion., we write

$$M\mathrm{TTF}^{\mathrm{H}}_{\mathrm{(m)}} = \theta_{\mathrm{i}}\ \Gamma(1+\frac{1}{\beta_{\mathrm{i}}})\sum_{\mathrm{j=1}}^{n+m}\binom{n+m}{j}(\text{-}1)^{\mathrm{j+1}}\ \mathrm{j}^{-\frac{1}{\beta_{\mathrm{i}}}}; \quad i=1,2 \tag{11}$$

3.3. MTTF According to Cold Duplication Method

It is given as $MTTF^{C}_{(m)}$. Using binomial expansion, we write

$$M\mathrm{TTF}^{\mathrm{C}}_{\mathrm{(m)}} = (\theta_{\mathrm{i}}/\beta_{\mathrm{i}})\sum_{j=0}^{n-m}\sum_{k=1}^{m}\sum_{\ell=0}^{k}\binom{n-m}{j}\binom{m}{k}\binom{k}{\ell}(-1)^{j+k+1}\frac{\Gamma(\ell+\frac{1}{\beta_{\mathrm{i}}})}{(j+k)^{\ell+1/\beta_i}} + \theta_{\mathrm{i}}\Gamma(1+\frac{1}{\beta_{\mathrm{i}}})\sum_{\mathrm{j=1}}^{\mathrm{n\text{-}m}}\binom{n-m}{j}(-1)^{j+1}j^{-\frac{1}{\beta_{\mathrm{i}}}}, \quad ,\mathrm{i}=1,2 \tag{12}$$

Table (1) gives the MTTF(original), $MTTF^{C}_{(m)}$ and $MTTF^{H}_{(m)}$, $MTTF_{(r),\rho}$ when $n=4,\ \theta_1=2,\ \theta_2=3,\ \beta_1=1.5,\ \beta_2=2,\ \mathrm{m}=\mathrm{r}=2, \rho=0.9$

Table 1: Value of system MTTF for various additions of spare

Variable	MTTF	$MTTF_{(r),\rho}$	$MTTF^{H}_{(m)}$	$MTTF^{C}_{(m)}$
lifetime	3.1531	4.2596	5.1329	.0727
usage	4.1655	4.455	4.5608	6.0837

4. Reliability Equivalence Factors

Generally, the reliability equivalence factor is defined as that factor by which the failure rates of some of the system's components should be reduced in order to reach equality of the reliability of another better system . In what follows, we present some of reliability equivalence factors of the parallel system.

4.1. reliability equivalence factor $\rho^{H}_{(m),(r),(\delta)}(\alpha)$

The hot reliability equivalence factor, say $\rho = \rho^{H}_{(m),(r),(\delta)}(\alpha)$ is defined as that factor ρ by which the failure rates of r of he system's components should be reduced in order to reach the reliability of that system which improved by improving m of the original system's components according to hot duplication method That is, $\rho = \rho^{H}_{(m),(r),(\delta)}(\alpha)$ is the solution of the following system of two equations.

$$R_{(r),\rho}(u,v) = \alpha, \qquad R^{H}_{(m)}(u,v) = \alpha \tag{13}$$

Substituting from (6) into the second equation in (13),we have:

$$\alpha = 1 - [1 - \exp\{-[(\frac{u}{\theta_1})^{\beta_1/\delta} + (\frac{v}{\theta_2})^{\beta_2/\delta}]^{\delta}\}]^{n+m} \tag{14}$$

Substituting from (5)into the first equation in (13), we have:

$$\alpha = 1 - [1 - \exp\{-[\rho(\frac{u}{\theta_1})^{\beta_1/\delta} + \rho(\frac{v}{\theta_2})^{\beta_2/\delta}]^{\delta}\}]^{r}[1 - \exp\{-[(\frac{u}{\theta_1})^{\beta_1/\delta} + (\frac{v}{\theta_2})^{\beta_2/\delta}]^{\delta}\}]^{n-r} \tag{15}$$

Using (14) and (15), we find

$$\rho^{H}_{(m),(r),(\delta)}(\alpha) = \{\ln[1-(1-\alpha)^{\frac{m+r}{r(n+m)}}]/\ln[1-(1-\alpha)^{\frac{1}{(n+m)}}]\}^{\frac{1}{\delta}} \tag{16}$$

4.2. α-Fractiles

The α-Fractiles of original and modified system are presented in this section. Let $U(\alpha)$ be the α-Fractile of the original system. Also, let $U_n^H(\alpha)$ denote to the α - Fractile of the modified system obtained by improving m of the system's components according to hot duplication methods. The fractile $U(\alpha)$ can be found by solving the following equation with respect to (U)

$$R\ (u,v) = \alpha \tag{17}$$

Using (4) and (17), we can find that

$$u_{(m)}(\alpha) = \theta_1\{[\ln[\ 1/[1-(1-\alpha)^{\frac{1}{n}}] - (v/\theta_2)^{\beta_2}\}^{\frac{1}{\delta}} \tag{18}$$

The fractile $U_n^H(\alpha)$ can be deduced by solving the second equation in (13) with respect to (U) as follows:

$$u_{(m)}^{H}(\alpha) = \theta_1\{[\ln[1/[1-(1-\alpha)^{\frac{1}{n+m}}]-(v/\theta_2)^{\beta_2}\}^{\frac{1}{\delta}} \tag{19}$$

Table (2) gives the α- fractile and the reliability equivalence factors of the system studied here. It represents the α- fractile and $\rho_{(m),(r),(\delta)}^{H}(\alpha)$ when $\theta_1 = 2, \theta_2 = 3, \beta_1 = 1.5, \beta_2 = 2, \delta = 0.5, \mathrm{v} = 6$ $n = 3, m = 2$ *and* $r = 1, 2, 3$ respectively. In these calculations, the level is chosen to be α = 0.1, 0.2, ……, 0.9..

Table 2: The α -fractile and $\rho_{(m),(r),(\delta)}^{H}(\alpha)$

α	$u_{(2)}(\alpha)$	$u_{(2)}^{H}(\alpha)$	$\rho_{(2),(1),(0.5)}^{H}(\alpha)$	$\rho_{(2),(2),(0.5)}^{H}(\alpha)$	$\rho_{(2),(3),(0.5)}^{H}(\alpha)$
0.1	0.8026	0.0336	0.5206	0.6783	0.7566
0.2	3.7236	1.5082	0.4399	0.6173	0.7083
0.3	6.5637	3.5068	0.3788	0.5686	0.6689
0.4	9.2083	5.5656	0.3262	0.5243	0.6324
0.5	11.728	7.6482	0.2781	0.4812	0.5961
0.6	14.205	9.7939	0.2319	0.4370	0.5579
0.7	16.739	12.085	0.1859	0.3892	0.5155
0.8	19.481	14.686	0.1380	0.3335	0.4642
0.9	22.795	18.037	0.0842	0.2594	0.3917

5. Conclusion

In this paper, we have discussed the reliability equivalence of a parallel system with identical and independent components. It is assumed that the components of the system had a two-dimensional failure modeling for a system where degradation is due to age and usage . Three ways namely the reduction, hold duplication and cold duplication methods are used to improve the system reliability. A reliability equivalence factor is derived. A numerical example is used to illustrate how the results obtained can be applied.

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