Journal of Reliability and Statistical Studies; ISSN (Print): 0974-8024, (Online):2229-5666
Vol. 4, Issue 2 (2011): 133-138

AN EXTENSION OF EXTREME-VALUE DISTRIBUTION

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Abstract

In this paper, we propose a model which is an extension of Extreme-value distribution. This model includes some well-known distributions as a special case.

Keywords: Probability density function, Random variable, Statistical model and S-PLUS Software.

Introduction

Statistical Modeling is the Mathematical Formulation of patterns which can in some way represent the data and allow their important characteristics to be described in terms of a limited number of quantities (known as parameters) that the mind can encompass relatively easily. Statistical models describe a phenomenon in the form of mathematical equations. Thus, a large number of observations say 100 or 1000 can be summarized in an equation with say two unknown quantities (called parameters of the model). Such reduction is certainly necessary for human mind. Out of large number of methods and tools developed so far for analyzing data (on the life sciences etc.), the statistical models are the latest innovations. A statistical model expresses a response variable as some function of a set of one or more predictor variables. In the literature (Hogg & Crag (1970), Johnson & Kotz (1970))we come across different types of models e.g., Linear models, Non-linear models, Generalized linear models, Generalized addition models, Analysis of variance models, Stochastic models, Empirical models, Simulation models, Diagnostic models, Operation research models, Catalytic models, Deterministic models, etc.

One of the trends in current research is to collect various results and to put them in a compact and systematic form, in order that further research may be carried out easily. Since the beginning of 1970 attention of the research workers in the discrete distributions appears to be shifted to a new class of discrete distributions known as *Lagrangian Distributions*, the name so given as these distributions are associated with the Lagrange Expansion. The Lagrangian distributions provide generalization of the classical discrete distributions and thus have been found more general in nature and wider in scope. The interesting class of Lagrangian probability distributions (LPD) of the discrete type were studied by Consul (1981), Consul and Shenton (1972, 1973a, 1973b) by the Lagrangian expansion of a probability generating function f(t) under the transformation $u = t/g(t)$, where g(t) is another probability generating function. It was shown that this class consisted of many important families of probability distributions like the generalized Poisson distribution, generalized negative binomial distribution and generalized logarithmic distributions.

Consul and Shenton (1972) gave a method for obtaining generalized discrete distributions using the following Lagrange expansion

$$f\,(\mathrm{f}) = \mathrm{f}(0) + \sum_{s=1}^{\infty} \frac{U^s}{s!}\left[\frac{d^{s-1}}{dt^{s-1}}\,(g\,(t)\,f\,'(t))\right]_{t=0} \tag{1.1.1}$$

where f´(t) is the first derivative of f(t). Let f(t) and g(t) be any two probability generating functions defined on non-negative integers, such that $g(0) \neq 0$. Consider the transformation t = ug(t) which gives u =0 for t = 0 and u = 1 for t = 1. It is found that it has a non-negative derivative at t = 0, so it is reasonable to suppose that there is a power series expansion for t interms of u given by the Lagrangian expansion (1.1.1). It is a well known result that if G_1 (t) and $G_2(t)$ are p.g.f.s then $G_1(G_2(t))$ is also a p.g.f. Therefore, it is clear that the power series (1.1.1) must be another p.g.f. Thus, Lagrangian probability distribution defined on non-negative integers is given by

$$P(X = 0) = f(0)$$

$$P\,(X = x) = \frac{1}{x!}\frac{d^{x-1}}{dt^{x-1}}\left[(g\,(t))^x\ f\,'(t)\right]_{t=0} \quad ; x = 1,2\,, \ldots \tag{1.1.2}$$

As g(t) and f(t) may be replaced by different sets of p.g.f.s, (1.1.2) may provide us with families of generalized discrete type of distributions. Some of the important members of this family for some suitably chosen functions g(t) and f(t) are described as under:

Generalized Negative Binomial Distribution

$$P\,(X = x) = \frac{n\Gamma(n+\beta x)\,\alpha^x\,(1-\alpha)^{n+\beta x - x}}{x!\,\Gamma(n+\beta x - x + 1)}$$

$$= \frac{n}{n+\beta x}\binom{n+\beta x}{x}\alpha^x\,(1-\alpha)^{n+\beta x - x}\ ; \quad n>0,\ 0<\alpha<1,\ \alpha\beta<1,$$

$$x = 0, 1, 2, \ldots$$

By taking

$$g(t) = e^{\lambda_2(t-1)} \quad \text{and} \quad f(t) = e^{\lambda_1(t-1)}$$

Generalized Poisson Distribution

$$P\,(X = x) = \frac{\lambda_1(\lambda_1 + x\lambda_2)^{x-1} e^{-(\lambda_1 + x\lambda_2)}}{x!}\ ;$$

$$\lambda_1 > 0,\ |\lambda_2| < 1,\ x = 0,1,2,\ldots$$

By taking

$$g(t) = (1-\alpha+\alpha t)^{\beta} \quad \text{and} \quad f(t) = (1-\alpha+\alpha t)^{n}$$

Generalized Geometric Series Distribution

$$P\,(\mathrm{X = x}) = \frac{\Gamma(1+\beta\mathrm{x})\,\alpha^{x}\,(1-\alpha)^{1+\beta\mathrm{x}-\mathrm{x}}}{\mathrm{x}!\,\Gamma(\beta\mathrm{x}-\mathrm{x}+2)} \; ; \; \mathrm{x} = 0, 1, 2, \ldots$$

By taking

$$\mathrm{g(t)} = (\mathrm{q}+\mathrm{pt})^{\mathrm{m}}, \qquad \text{and} \qquad \mathrm{f(t)} = \mathrm{t}^{\mathrm{n}}.$$

Similarly for suitable choice of g(t) and f(t) a large number of new and well-known distributions can be obtained from (1.1.2).

So far we have seen that various authors proposed various methods for the generalization of various type of discrete type of distributions. Here, our aim is to develop a generalized statistical model on the pattern of Consul (1973), Gupta (1974) etc in case of continuous type of distributions. Bhat et. al (2008), Bhat et al. (2006), Bhat et al. (2004), Bedi et. al (1980) have proposed new models and used in practical fields. The proposed model includes extreme-value distribution as a special case. The Gumbel distribution is also referred to as Extreme-value distribution. The probability density function of Extreme-value distribution is given by

$$f\,(x) = \alpha\lambda e^{\lambda x}\exp\left(-\alpha e^{\lambda x}\right);\, \alpha > 0,\, \lambda > 0 \text{ and } \mathrm{x} > 0$$

Alternatively, if random variable X has Weibull distribution, then log X has an Exteme-value distribution (Lawless 1982, pp: 141). In this paper, we proposed a model which is given below:

PROPOSED STATISTICAL MODEL

Let g(x) be a continuous monotonic increasing function in (k, ∞) such that g(∞) $=\infty$ and g(k) =0 where k is any positive real number, then the function

$$f\,(x; g, \alpha, \lambda) = \alpha\lambda e^{\lambda g(x)} g'(x)\exp\left\{-\alpha e^{\lambda g(x)}\right\};\, k < x < \infty \qquad (1.1.3)$$

= 0 elsewhere

is the pdf of the random variable X of continuous type.

DERIVATION OF EXTREME-VALUE DISTRIBUTION

In fact, a number of new and some well-known distributions follow from the model (1.1.3) for a suitable choice of the function g(x) and the parameters of the proposed model. Here we mention extreme-value model

Remark 1.1.1. Taking g(x)=x so that $g'(x) = 1$ in (1.1.3), we have

$$f(x) = \alpha\lambda e^{\lambda x} \exp\left(-\alpha e^{\lambda x}\right); \alpha > 0, \lambda > 0 \text{ and } x > 0$$
$$= 0 \text{ elsewhere}$$

which is the pdf of type I extreme-value distribution or log-gamma distribution or log-Weibull distribution with parameters $\alpha > 0$ and $\lambda > 0$.

Similarly as above, many more distributions can be shown as a particular case of the proposed statistical model (1.1.3) for a suitable choice of the generating function g(x) and the parameters.

Numerical Illustrations

Numerical illustrations are implemented in S-PLUS Software for Extreme-value distribution. These illustrations are meant for the purpose of showing strength of Bayesian methods in various practical situations. Statistical Software's like SPSS and Minitab do not cater the need of Bayesians and hence they gets attracted towards the modern Softwares like S-PLUS and R. Details of these Softwares can be seen in Venables and Ripley (2000), which is an excellent book on applied statistics using S-PLUS and R.

Table 1: Posterior mode and Posterior standard error of Extreme-value distribution with different priors

Prior	Posterior mode		Posterior Standard error	
	alpha	lambda	Alpha	lambda
1	5.3385243	0.5859691	0.01657597	0.01106693
1/lambda	5.3386316	0.5857601	0.01657073	0.01105912
1/alpha*lambda	5.3385774	0.5857717	0.01657108	0.01105945

 # Bayesian Analysis of Extreme-value Distribution with uniform Prior

```
library(Mass,first=T)
nextr1<-deriv3(~log((alpha*lambda)*exp(lambda*y)*exp(lambda*exp(lambda*y))),
c("alpha","lambda"),function(y,alpha,lambda)NULL)
y<-dbmdata$Potassium
y<-as.vector(y)
t<-log(y)
fitextr1<-ms(~nextr1(t,alpha,lambda),start=c(alpha=2.39,lambda=1.21),data=dbmdata)
post.std<-sqrt(diag(summary(fitextr1)$Information))
summary(fitextr1)
post.std
> summary(fitextr1)
Final value: 1358.453

Solution:
     Par.      Grad. Hessian.alpha Hessian.lambda
```

alpha 5.338527 -5.632678e-011 4097.744 2048.937

lambda 0.585969 -4.578609e-010 2048.937 9193.523

Information:

alpha lambda

alpha 0.00027464206 -0.00006120877

lambda -0.00006120877 0.00012241367

Convergence: RELATIVE FUNCTION CONVERGENCE.

Computations done:

Iterations Function Gradient
7 9 8
> post.std

[1] 0.01657233 0.01106407

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