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OPTIMAL REPLACEMENT POLICIES BASED ON NUMBER OF DOWN TIMES FOR COLD STANDBY SYSTEM WHEN THE LIFETIME AND THE REPAIR TIME ARE DEPENDENT

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Abstract

The purpose of this article is to present optimal replacement policies for a cold standby system consisting of two components and one repairman. By using the bivariate exponential model of Freund (1961) for the life time of one component and the repair time of another component, we developed methods for obtaining optimal number of down time in such a way that the long run expected reward per unit time is maximized. The results are illustrated with the help of numerical example and simulation study.

Key words: Cold standby system, Freund's bivariate exponential Model, Number of down times, Priority in use, Renewal reward theorem, Repair rate, Replacement policy.

1. Introduction

In this paper we find the optimal replacement policies for a cold standby system based on number of down times of the first component. The cold standby system consisting of two components, one repairman and repair rate of one component may depend on failure rate of another component. This type of dependency can occur in practical circumstances, when there is load or pressure on repairman. To explain such interdependency of two components we have used the bivarite exponential model of Freund. The rest of this paper is organized as follows. We give brief introduction of bivariate exponential model of Freund (1961) in Section 2. We propose replacement policy for two identical components, mathematical formulation for long run expected reward per unit time, a numerical example and simulation study in Section 3. Replacement policy with component-1 has priority in use, mathematical formulation for long run expected reward per unit time, a numerical example and simulation study have discussed in Section 4. Finally some concluding remarks.

In order to decrease the operating cost of a repairable system, different replacement policies have been developed. Barlow and Hunter (1960) used elementary renewal theory to obtain optimum policies. Nakagawa and Osaki (1975) assumed that both the working time and repair time of priority component have general distribution while working time and repair time of the non priority component have an exponential distribution. They obtained some reliability indices of the system using Markov renewal theory. Brown and Proschan (1983), Block, et al. (1985), Kijima (1989) proposed and studied many repair/replacement policies based on working age, number of repairs, repair cost and their combinations. Lam (1988), Zhang and Wang (2006, 2007), Zhang et al. (2007) used geometric process approach to obtain optimal policies. Rattihalli and Hanagal (2009) used bivariate exponential model of Freund (1961) to obtain optimal replacement policy based on length of down times. Hanagal and Kanade (2010a) proposed replacement policy based on number of down times (or shutdown) of the repairable system. Hanagal and Kanade (2010b) also proposed optimal replacement policy based on number of down times with priority in use when the lifetime and repair time are independent. All these policies are widely used to avoid unscheduled failure and larger production losses. The important reason for this attempt is that these policies can be applied to a variety of areas such as military, industry etc. Practically, majority of industrial systems are composed of several units and there are some types of dependency between units. In other words, the failure time of one component may depend on repair time of other component. So there is need to design replacement policy for such situation.

2. Bivariate exponential model of Freund (1961)

Let x_1, x_2 denote lifetimes of the two components which are identically, independently and exponentially distributed with failure rate α and Y_1, Y_2 denote repair times of the components which are identically, independently and exponentially distributed with repair rate β. The interdependence of the components is such that the failure of a component increases the repair rate of the other component β to β' due to load or pressure and also when one component is in cold standby the failure rate of other component remains same α = α'. Here the interdependence is due to the failure of a component when the other component is under repair. Here the repairman will have more load or pressure because he is repairing one component and another component is waiting for repair and his repair rate will increase. The p.d.f. of bivariate exponential model of Freund (1961) with parameter (α , β , β') is given by,

$$\begin{aligned} f(x_i, y_{i'}) &= \alpha\beta' \exp(-\beta' y_{i'} - (\alpha+\beta-\beta')x_i), \quad 0 < x_i < y_{i'} \\ &= \alpha\beta \exp(-\alpha x_i - \beta y_{i'}), \qquad\qquad 0 < y_{i'} < x_i \end{aligned} \tag{2.1}$$

where $\alpha, \beta, \beta' > 0,\ i \neq i' = 1,2.$

3. Proposed Replacement Policy

We study a two identical components system with following assumptions,

1. Initially both the components are new. The first component is in working state while the second component is in cold standby state.
2. The repairman repairs the component-1 as soon as it fails. At the same time standby component-2 begins to work. If one component fails while other component is still under repair it must wait for repair and system breaks down. A possible course of the system is shown in Figure 1.
3. The replacement policy used here is based on the number of the down times *k* of the first component. Renewal occurs when the number of down times of the first component of the system reaches *k*.
4. The time interval between completion of the $(j\text{-}1)^{\text{th}}$ repair and completion of the j^{th} repair of the component *i* is called j^{th} cycle of the component i, $i = 1, 2;\ j = 1, 2, 3; \ldots\ldots$ The time interval between two successive system replacements is called a renewal cycle.

5. $X_j^{(i)}, Y_j^{(i)}$ are respectively working time and repair time of component *i* in the j^{th} cycle $i = 1, 2; j = 1, 2, 3....$and $\left(X_j^{(i)}, Y_j^{(i')}\right)$ have bivariate exponential model of Freund (1961), $i \neq i' = 1,2$.

6. C_0, C_{w0}, C_{p0}, C_d are initial cost, reward cost, repair cost and down time cost of the system respectively. The component in the system does not produce working reward during cold standby.

7. The reward cost of the component decreases geometrically after every failure (repair) of the components. So we replace the reward cost for the component:
8. $C_w(j) = a^{j-1} C_{w0} \quad i = 1, 2;\ j = 1, 2, ...;\ 0 < a < 1$
9. The repair cost of the component increases geometrically after every failure(repair) of the components. So we replace the repair cost for the component:
10. $C_p(j) = b^{j-1} C_{p0} \quad i = 1, 2;\ j = 1, 2, ...; 1 < b.$

In Figure 1, a graphical representation of a renewal cycle is given. It consists of a sequence of working periods, repair periods, cold standby periods and down time of the two components.

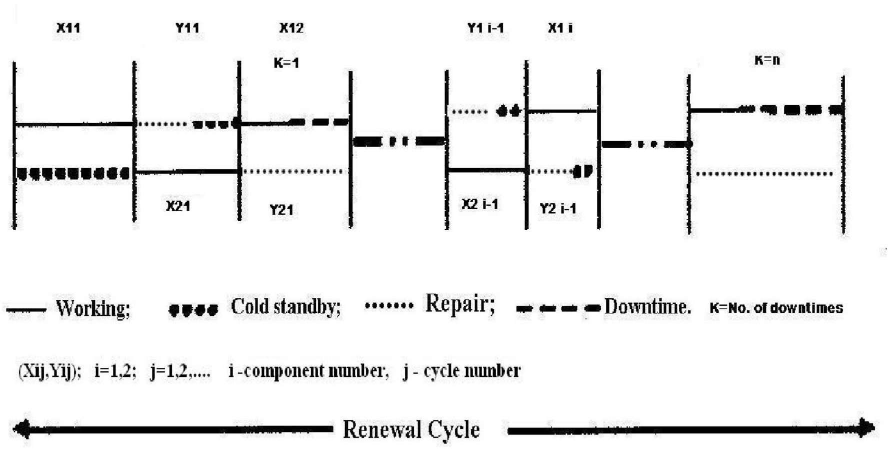

Figure 1: A possible course of the system

Mathematical Formulation

Under this policy it is assumed that, the replacement of the system occurs when the number of down times of the first component reaches a predetermined number *k*. Here we derived mathematical expression for long run reward. According to Renewal reward theorem [Ross, 1996], the long run reward per unit time *C(k)* is obtained as follows,

$$C(k) = \frac{\text{the expected reward incurred in a renewal cycle}}{\text{the expected length of a renewal cycle}}$$

Let the random variable R be the number of cycles in a renewal cycles until the k^{th} down time occur due to failure of component-1. Therefore k-1 down times are included in the length of renewal cycle. $k - 1$ down times occur for the component-1 and K_2 down times occur for the component-2. Here the system starts with component-1 and following are the number of working times for each component.
Component-1: $R + 1$ working times
Component-2: R working times

In a similar manner one can write how many repair times occur in a renewal cycle for each component. The following are the number of repair times for each component.
Component-1: R repair times
Component-2: R repair times

In each cycle there are one working time, one repair time and either one cold standby or down time or both. There is no down time or cold standby in the first cycle for the component-1. If there are R cycles, then the number of repairs will be R. Now we classify the number of repair as 1) Repairs followed by standby 2) Repairs preceded by downtime. Now the total number of repairs R is the sum of number of repairs followed by standby (S_i) and the number of repairs preceded by down times (K_i) for the component-i, $i = 1, 2$ and $K_1 = k$(fixed). We can express R for each component in terms of random variable S_1, S_2 and K_2 in the following way as,
Component-1: $R = S_1 + k - 1$
Component-2: $R = S_2 + K_2$

Additional Assumptions

Here S_1 takes the values $1, 2, 3,\ldots.$ and and so on. Look at the Figure 1. S_1 cannot take value zero. When the system starts with component-1, the system will not break down after the failure of component-1 because of standby component-2. There will be at least one repair without down time. Therefore $S_1 \geq 1$. But S_2 can take values 0, 1, 2, …. and so on. S_2 can take value zero when the component-2 is failed while the component-1 still under repair. The random variables S_1 and S_2 are discrete random variables. The possible choice of these random variables, S_1 and S_2 are truncated negative binomial, $TNB(k, p)$ truncated at zero and negative binomial, $NB(K_2, p)$ respectively where p is the probability of the occurrence of down time and $K_2 \geq 1$. The random variable K_2 is number of down times in the component-2 which takes values 1,2, ... and so on. The possible choice for K_2 is truncated Poisson, $TP(\lambda)$ truncated at zero. The expression for p is

$$p = P(Y_j^{(2)} > X_{j+1}^{(1)}) = P(Y_j^{(1)} > X_j^{(2)}) = \frac{\alpha}{\alpha + \beta} \tag{3.1}$$

Let L be the length of the renewal cycle,

$$L=\sum_{j=1}^{S_1+k} X_j^{(1)}+\sum_{j=1}^{S_2+K_2} X_j^{(2)}+\sum_{j=1}^{k-1}(Y_j^{(2)}-X_{j+1}^{(1)})\,|\,I_{(Y_j^{(2)}>X_{j+1}^{(1)})}$$
$$+\sum_{j=1}^{K_2}(Y_j^{(1)}-X_j^{(2)})\,|\,I_{(Y_j^{(1)}>X_j^{(2)})} \qquad (3.2)$$

where *I(.)* is the indicator function and the third term and fourth terms in Eqn (3.2) are the length of waiting for repair (down time) of the first and second components respectively.

The total reward incurred in the renewal cycle is given by,

$$C=\sum_{j=1}^{S_1+k} C_w(j)X_j^{(1)}+\sum_{j=1}^{S_2+K_2} C_w(j)X_j^{(2)}-\sum_{j=1}^{S_1+k-1} C_p(j)Y_j^{(1)}-\sum_{j=1}^{S_2+K_2} C_p(j)Y_j^{(2)}-$$
$$-\sum_{j=1}^{k-1} C_d(Y_j^{(2)}-X_{j+1}^{(1)})\,|\,I_{(Y_j^{(2)}>X_{j+1}^{(1)})}-\sum_{j=1}^{K_2} C_d(Y_j^{(1)}-X_j^{(2)})\,|\,I_{(Y_j^{(1)}>X_j^{(2)})}-C_o \qquad (3.3)$$

The following expected values are used for evaluating *E(L)* and *E(C)*

$$E(X_j^{(1)})=E(X_j^{(2)})=\frac{1}{\alpha}$$
$$E(Y_j^{(1)})=E(Y_j^{(2)})=\frac{(\alpha+\beta')}{\beta'(\alpha+\beta)}$$
$$E(Y_j^{(2)}-X_{j+1}^{(1)})\,|\,I_{(Y_j^{(2)}-X_{j+1}^{(1)})>0}=E(Y_j^{(1)}-X_j^{(2)})\,|\,I_{(Y_j^{(1)}-X_j^{(2)})>0}=\frac{1}{\beta'} \qquad (3.4)$$

Using the above expected values in (3.4), expected length *E(L)* of the renewal cycle is given by,

$$E(L)=\frac{1}{\alpha}\left[\frac{kq}{p(1-p^k)}+k+\frac{\lambda^*}{p(1-e^{-\lambda^*})}\right]+\frac{k-1}{\beta'}+\frac{\lambda^*}{\beta'(1-e^{-\lambda^*})} \qquad (3.5)$$

By using the assumptions (7) and (8), we obtain the expected reward incurred in a renewal cycle as,

$$E(C) = \frac{C_{w0}}{(1-a)\alpha}\left[2 - \frac{(ap)^k}{(1-p^k)}[\frac{1}{(1-aq)^k} - 1] - \frac{e^{(-\lambda^*(1-\frac{ap}{1-aq}))} - e^{-\lambda^*}}{(1-e^{-\lambda^*})}\right]$$
$$- \frac{C_{p0}(\alpha+\beta')}{\beta'(b-1)(\alpha+\beta)}\left[[\frac{(bp)^k}{b(1-p^k)}[\frac{1}{(1-bq)^k} - 1]\right.$$
$$\left. + \frac{e^{(-\lambda^*(1-\frac{bp}{1-bq})}) - e^{-\lambda^*}}{(1-e^{-\lambda^*})} - 2\right] - C_d\left[\frac{k-1}{\beta'} + \frac{\lambda^*}{\beta'(1-e^{-\lambda^*})}\right] - C_0 \qquad (3.6)$$

where $q = 1 - p$.

The long run expected reward per unit time, $C_T(k)$ incurred in a renewal cycle is the ratio of two expectations $E(C)$ and $E(L)$,

$$C(k) = \frac{E(C)}{E(L)} \qquad (3.7)$$

To obtain optimal number of down times plot a graph of $C(k)$ Vs k and choose k for which $C(k)$ is maximum.

Examples

Here we consider following parameters to illustrate our theoretical result and for simulation study. In a simulation study, working times ($X_j^{(i)}$) and repair times ($Y_j^{(i)}$) are generated from bivariate exponential distributions with failure rates for given set of parameter values as follows. The number of down times *(k = 1)* is obtained when $Y_j^{(2)} > X_{j+1}^{(1)}$, $j = 1, 2....$ Multiply (0.99) to the reward cost for every cycle (geometrically decreasing) and multiply (1.01) to the repair cost for every cycle (geometrically increasing). Obtain the total reward cost incurred in a renewal cycle and length of renewal cycle. Repeat this procedure 1000 times and take the mean of 1000 total reward costs and mean of 1000 lengths of renewal cycle and take the ratio which gives $C_S(k)$ based on simulation study. Repeat this procedure for $k = 2, 3, ...20$.

A	b	α	β	β'	λ^*	C_0	C_{w0}	C_{p0}	C_d
.99	1.01	.5	1	2	10	400	50	5	1

Table 1: Results obtained from theory and simulation

k	$C_T(k)$	$Cs(k)$	k	$C_T(k)$	$Cs(k)$
1	32.4975	14.9215	11	34.0432	33.9856
2	33.081	27.4098	12	33.8496	33.6197
3	33.5669	31.6898	13	33.6275	33.2141
4	33.935	33.5795	14	33.3815	32.825
5	34.1872	34.5283	15	33.1155	32.4151
6	34.3368	34.8395	16	32.8327	31.9973
7	34.3996	34.8742	17	32.5359	31.5705
8	34.3902	34.7777	18	32.2273	31.1398
9	34.3211	34.5539	19	31.9089	30.69
10	34.2027	34.2782	20	31.5824	30.2486

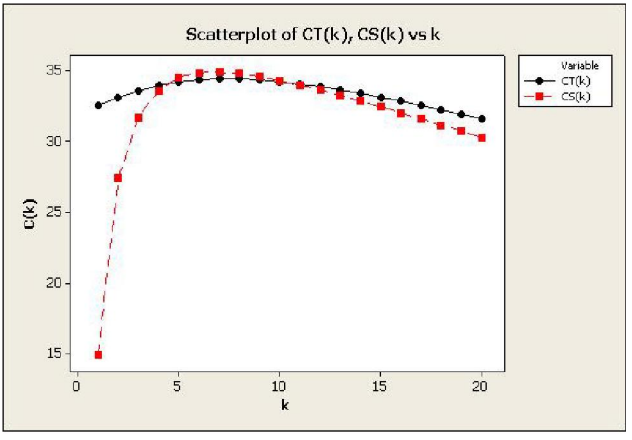

Figure 2: A plot of *C(k)* against *k*

From the Table 1 and Figure 2 it is clear that optimal reward occurs at k = 7 for both theoretical and simulation study.

4. Priority in Use

In this section we studied two dissimilar components and one repairman. Assume that component-1 has priority in use. We consider a replacement policy based on the number of down times of component-1 by using bivariate exponential model of Freund. The p.d.f. of bivariate exponential model of Freund(1961) with parameter $(\alpha_1, \beta_1, \alpha_2, \beta_2, \beta_1', \beta_2')$ is given by,

$$f(x_i, y_{i'}) = \alpha_i \beta'_{i'} \exp(-\beta'_{i'} y_{i'} - (\alpha_i + \beta_{i'} - \beta'_{i'}) x_i), \quad 0 < x_i < y_{i'}$$
$$= \alpha_i \beta_{i'} \exp(-\alpha_i x_i - \beta_{i'} y_{i'}), \quad 0 < y_{i'} < x_i \tag{4.1}$$

where $\alpha_i, \beta_{i'}, \beta'_{i'} > 0, i \neq i' = 1, 2$.

Along with the original model assumptions in Section 3, we need some extra assumptions as follows which are similar to the assumptions made by Hanagal and Kanade (2010b).

1. The component-1 has priority in use.

2. The repairman repairs the component-1 as soon as it fails. At the same time standby component-2 begins to work. As soon as the repair of the component-1 is over the component-2 will be replaced by component-1 and the system starts working even though component-2 has not failed because component-1 has high reliability as compared to component-2. This we call `shift' of component. If one component fails while other component is still under repair it must wait for repair and the system breaks down. A possible course of the system is shown in Figure 3.

3. Whenever there is no down time due to replacement of component-1 and non failure of the component-2, the lifetime of component-2 is right censored at $Y_j^{(1)}$. In this case reward cost of component-2 is not geometrically decreasing and the repair time is zero for the component-2 and standby time is zero for the component-1. Here for the component-2, no repair takes place unless there is a down time. Similarly for the component-1, cold standby never occurs because it has priority in use.

4. Here $X_j^{(1)}$ and $Y_j^{(2)}$ ($X_j^{(2)}$ and $Y_j^{(1)}$) have bivariate exponential model of Freund.

Additional Assumptions

The possible choice of these random variables, S_1 and S_2 are truncated negative binomial, $TNB(k, p_1)$ truncated at zero and negative binomial, $NB(K_2, p_2)$ respectively where p_i is the probability of the occurrence of down time due to the failure of component-i, $i = 1, 2$. The distribution of K_2 is same as stated in the first model (identical components case). The expressions for p_i are,

$$p_1 = P[X_{j+1}^{(1)} < Y_j^{(2)}] = \frac{\alpha_1}{\alpha_1 + \beta_2}, \qquad p_2 = P[X_j^{(2)} < Y_j^{(1)}] = \frac{\alpha_2}{\alpha_2 + \beta_1}$$

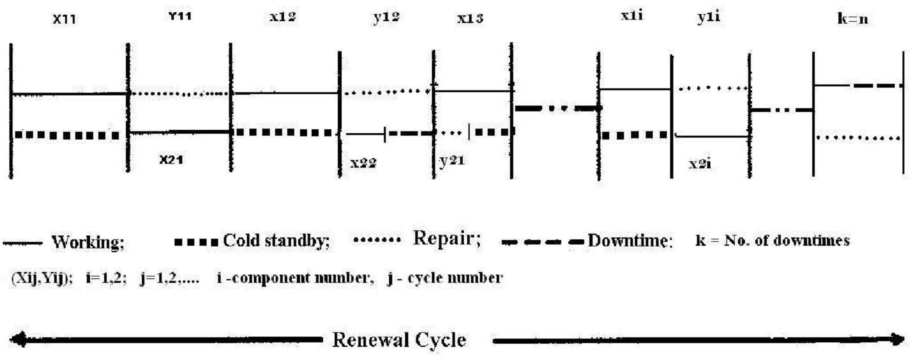

Figure 3: A possible course of the system

Mathematical Formulation

L and C denote the length and total reward of the renewal cycle.

$$L = \sum_{j=1}^{S_1+k} X_j^{(1)} + \sum_{j=1}^{S_2} X_j^{(2)} I_{(Y_j^{(1)} > X_j^{(2)})} + \sum_{j=1}^{K_2} X_j^{(2)} + \sum_{j=1}^{k-1} (Y_j^{(2)} - X_{j+1}^{(1)}) \mid I_{(Y_j^{(2)} > X_{j+1}^{(1)})}$$

$$+ \sum_{j=1}^{K_2} (Y_j^{(1)} - X_j^{(2)}) \mid I_{(Y_j^{(1)} > X_j^{(2)})} \tag{4.2}$$

The total reward incurred in the renewal cycle is given by.

$$C = \sum_{j=1}^{S_1+k} C_w(j) X_j^{(1)} + \sum_{j=1}^{S_2} C_{w0} X_j^{(2)} I_{(Y_j^{(1)} > X_j^{(2)})} + \sum_{j=1}^{K_2} C_w(j) X_j^{(2)} - \sum_{j=1}^{S_1+k-1} C_p(j) Y_j^{(1)}$$

$$- \sum_{j=1}^{K_2} C_p(j) Y_j^{(2)} - \sum_{j=1}^{k-1} C_d (Y_j^{(2)} - X_{j+1}^{(1)}) \mid I_{(Y_j^{(2)} > X_{j+1}^{(1)})}$$

$$- \sum_{j=1}^{K_2} C_d (Y_j^{(1)} - X_j^{(2)}) \mid I_{(Y_j^{(1)} > X_j^{(2)})} - C_o \tag{4.3}$$

The following expected values are used for evaluating E(L) and E(C).

$$E(X_j^{(1)}) = \frac{1}{\alpha_1}$$

$$E(X_j^{(2)}) = \frac{1}{\alpha_2}$$

$$E(X_j^{(2)} I_{(Y_j^{(1)} - X_{j+1}^{(2)})>0}) = \frac{\alpha_2}{(\alpha_2 + \beta_1)^2}$$

$$E(Y_j^{(1)}) = \frac{(\alpha_2 + \beta_1^{'})}{\beta_1^{'}(\alpha_2 + \beta_1)}$$

$$E(Y_j^{(2)}) = \frac{(\alpha_1 + \beta_2^{'})}{\beta_2^{'}(\alpha_1 + \beta_2)}$$

$$E[(Y_j^{(2)} - X_{j+1}^{(1)}) \mid I_{(Y_j^{(2)} - X_{j+1}^{(1)})>0}] = \frac{1}{\beta_2^{'}}$$

$$E[(Y_j^{(1)} - X_j^{(2)}) \mid I_{(Y_j^{(1)} - X_{j+1}^{(2)})>0}] = \frac{1}{\beta_1^{'}} \tag{4.4}$$

Using the above expected values given in Eqns (4.4) *E(L)* and *E(C)* of the renewal cycle are given by,

$$\begin{aligned} E(L) &= \frac{1}{\alpha_1}[\frac{kq_1}{p_1(1-p_1^{\,k})} + k] + \frac{\alpha_2\lambda^* q_2}{(\alpha_2+\beta_1)^2 p_2(1-e^{-\lambda^*})} \\ &\quad + \frac{\lambda^*}{(1-e^{-\lambda^*})}[\frac{1}{\alpha_2} + \frac{1}{\beta_1^{'}}] + \frac{k-1}{\beta_2^{'}} \end{aligned} \tag{4.5}$$

$$\begin{aligned} E(C) = {} & \frac{C_{w0}}{\alpha_1(1-a)}\left[1 - \frac{(a p_1)^k}{(1-p_1^{\,k})}[(\frac{1}{1-aq_1})^k - 1]\right] + \frac{C_{w0}\alpha_2}{(\alpha_2+\beta_1)^2}\frac{\lambda^* q_2}{p_2(1-e^{-\lambda^*})} \\ & + \frac{C_{w0}}{\alpha_2(1-a)}(1 - \frac{e^{-\lambda^*(1-a)} - e^{-\lambda^*}}{1-e^{-\lambda^*}}) \\ & - \frac{C_{p0}(\alpha_2+\beta_1^{'})}{\beta_1^{'}(b-1)(\alpha_2+\beta_1)}\left[\frac{(bp_1)^k}{b(1-p_1^{\,k})}\{(\frac{1}{1-bq_1})^k - 1\} - 1\right] \\ & - \frac{C_{p0}(\alpha_1+\beta_2^{'})}{(b-1)\beta_2^{'}(\alpha_1+\beta_2)}(\frac{e^{-\lambda^*(1-b)} - e^{-\lambda^*}}{1-e^{-\lambda^*}} - 1) - C_d\,[\frac{k-1}{\beta_2^{'}} + \frac{\lambda^*}{\beta_1^{'}(1-e^{-\lambda^*})}] - C_0 \end{aligned} \tag{4.6}$$

The long run expected reward per unit time $C_T\,(k)$ obtained by using *E(C)* and *E(L)*.

Examples

Here we consider following parameters to illustrate our results and simulation study is also based on 1000 *L*'s and *C*'s as explained in earlier section.

a	b	α	α	β_1	β_1'	β_2	β_2'	λ*	C_0	C_{w0}	C_{p0}	C_d
.99	1.01	.4	.8	1	1.5	1	1.5	10	400	50	5	1

Table 2: Results obtained from theory and simulation

k	$C_T(k)$	$Cs(k)$	k	$C_T(k)$	$Cs(k)$
1	24.6879	10.8912	11	32.4583	26.4153
2	27.8623	20.9666	12	32.1806	26.1899
3	29.9853	24.3246	13	31.8647	25.9354
4	31.3404	25.9203	14	31.5194	25.6609
5	32.1679	26.5305	15	31.1519	25.3982
6	32.6379	26.7927	16	30.7674	25.1151
7	32.8627	26.8917	17	30.37	24.831
8	32.9158	26.8819	18	29.963	24.5392
9	32.8457	26.737	19	29.549	24.2555
10	32.6855	26.6037	20	29.1302	24.0005

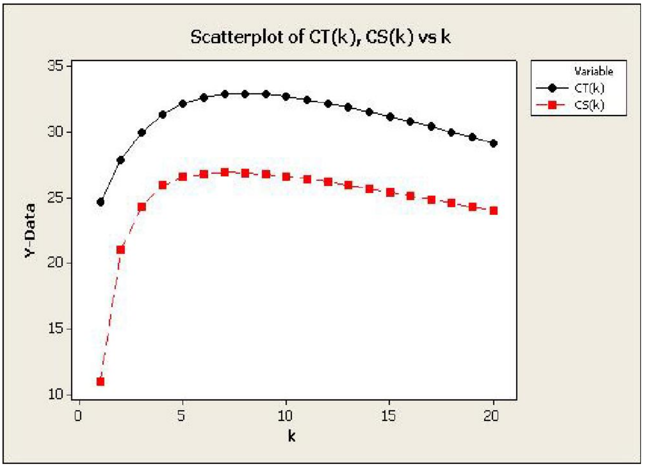

Figure 4: A plot of *C(k)* against *k*

From the Table 2 and Figure 4 it is clear that optimal reward occurs at k = 8 and k = 7 for both theoretical and simulation study respectively.

Remarks

- The difference in the values of $C(k)$ of these two methods is due to the additional assumptions for the distributions of S_1, S_2 and K_2 when calculating $E(L)$ and $E(C)$ and these additional assumptions are not needed to obtain optimal k based on simulated study.

- One can have more examples for different set of parameters values in the proposed models. The procedure is same and will not differ if we take different set of parameters values. The expected reward cost per unit time $C(k)$

depends on *a*, *b*, the statistical parameter values, economic characteristics (initial cost, reward cost, repair cost, down time cost). Any change in these values will change *C(k)*. We have carried out this exercise by taking different parameter set of values but as an illustration we present results only for one set of parameter values.

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