Journal of Reliability and Statistical Studies; ISSN (Print): 0974-8024, (Online):2229-5666
Vol. 4, Issue 1 (2011): 95-106

STOCHASTIC ANALYSIS OF AN AIR CONDITION COOLING SYSTEM MODEL

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Abstract

The present study deals with the stochastic analysis of a real existing industrial system model of a central air-condition (AC) system. The system consists of three different subsystems namely- Air Blower**,** Compressor, water pump. All these subsystems are arranged in series network. Transition probabilities as well as the recurrence relations for various reliability and cost effective measures are developed. Failure time distributions of all the subsystems are taken as exponential whereas repair time distributions are general**.** By using regenerative point technique we have obtained various measures of system effectiveness such as –Reliability, MTSF, Availability, Busy period of repairman and Net expected profit. The results are also drawn in a particular case when repair time distributions are assumed as exponentials.

Key words: Reliability, MTSF, Availability, Profit analysis, regenerative point technique.

1. Introduction

The reliability of hypothetical models have been analyzed widely by various authors including Gupta and Goel (1990), Gupta *et al.* (1994), Gupta and Chaudhary (1994). They obtained cost benefit analysis and various measures of system effectiveness by using different techniques. But sometimes the hypothetical models are not accurately reflected with the real existing systems. Few authors considered the real existing system models like Gupta and Shivakar (2003) analyzed a stochastic model of cloth weaving system, Gupta and Kumar (2007) studied a distillery plant system and obtained profit function and reliability characteristics. Arora e*t al.* (2000), Kumar *et al.* (1996) also worked with real existing industrial systems with different techniques. The present study is devoted to the stochastic analysis of real existing industrial AC system model. The system is of a complex type, reparable engineering system installed in majority of office buildings, scientific laboratories, industrial and commercial complexes etc. The AC system consists of three main subsystems that are: (i) Air Blower, (ii) Compressor and (iii) Water pump. These sub systems are arranged in a series configuration and the system failure occurs if any one of the subsystem fails.

2. System Discription

The AC system consist three main subsystems that are: Air blower, compressor and water pump. Out of three subsystems two subsystems (compress and water-pump) have their cold standby units. Air Blower is working as single (without redundancy) as it is too much expensive. A single repairman is always available with the system to repair the failed unit and operate the cold standby unit with the help of a switching device which is always perfect and instantaneous whenever required. The service discipline of the repair man is FCFS (first come first serve).

2.1 Roll of Subsystems

Air Blower (B): Air blower sucks the hot air from air-conditioning rooms and blows it into the condenser to cool down with the help of FREON gas that present inside the condenser.

Compressor (C): The main work of compressor is to suck high-temperature-low-pressure (HT-LP) super heated vapour form FREON gas from air-blower to compressor and compress it into high-pressure liquid form gas with the help of pistons and pass to water condenser to cool and convert in to the form of low-temperature-high-pressure super cool compressed liquid gas.

Water Pump (Wp): The main function of water pump is to pump cold water from cooling tower to water condenser in order to chill FREON gas to maintain low temperature.

2.2 Notations and States of The System

$\lambda_b, \lambda_c, \lambda_w$	:	Constant failure rates of blower, compressor and water pump.
$G_b(\cdot), G_c(\cdot), G_w(\cdot)$	:	c.d.f. of repair time of blower, compressor and water pump.
q_{ij}	:	p.d.f. of transition time from state S_i to S_j.
p_{ij}	:	†steady state direct transition probability from state S_i to S_j, such that $p_{ij} = \int q_{ij}(u)du$
$Z_i(t)$	:	Probability that system sojourns in state S_i up to time t.
ψ_i	:	Mean sojourn time in state S_i.
*, ~	:	symbols for Laplace and Laplace - Stieltjes transforms.

To write the various states of the system we define the following symbols:

$B_o / B_r / B_{wo}$	:	Blower is operative/under repair/ waiting for operation.
$C_o / C_s / C_r / C_{wr} / C_F / C_{wo}$	:	Compressor is operative/ standby/ under repair/wait for repair/ failed/waiting for operation.
$Wp_o / Wp_s / Wp_r / Wp_{wr} / Wp_F / Wp_{wo}$	:	Water pump is operative/ standby /under repair/waiting

†The limits of integration are 0 to ∞ whenever not mentioned.

for repair/ failed/ waiting for operation.

The possible states of the system are S_0 to S_{14} in which S_0, S_1, S_2, S_4, S_7 are operative states and other are failed. Transition diagram of the system model is shown in figure-1.

3. Transition Probabilities and Sojourn Times

(a) The direct or one step steady state transition probabilities are as follows -

The steady state transition probabilities can be obtained by using the results,

$$p_{01} = \lim_{t\to\infty} Q_{01}(t)$$

$$= \lambda_c \int e^{-(\lambda_b+\lambda_c+\lambda_w)u}\, du = \frac{\lambda_c}{\lambda_b+\lambda_c+\lambda_w} = \frac{\lambda_c}{\Omega},$$

Where, $\Omega = \lambda_b+\lambda_c+\lambda_w$

Similarly,

$$p_{02} = \frac{\lambda_w}{\Omega}, \quad p_{03} = \frac{\lambda_b}{\Omega}, \quad p_{10} = \bar{G}_c(\Omega), \quad p_{15} = \frac{\lambda_b\left[1-\bar{G}_c(\Omega)\right]}{\Omega},$$

$$p_{16} = \frac{\lambda_c\left[1-\bar{G}_c(\Omega)\right]}{\Omega}, \quad p_{20} = \bar{G}_w(\Omega), \quad p_{27} = \frac{\lambda_c\left[1-\bar{G}_w(\Omega)\right]}{\Omega},$$

$$p_{28} = \frac{\lambda_b\left[1-\bar{G}_w(\Omega)\right]}{\Omega}, \quad p_{29} = \frac{\lambda_w\left[1-\bar{G}_w(\Omega)\right]}{\Omega},$$

$$p_{30} = \int dG_b(u) = 1, \quad p_{42} = \bar{G}_c(\Omega), \quad p_{4,10} = \frac{\lambda_w\left[1-\bar{G}_c(\Omega)\right]}{\Omega},$$

$$p_{4,11} = \frac{\lambda_b\left[1-\bar{G}_c(\Omega)\right]}{\Omega}, \quad p_{4,12} = \frac{\lambda_c\left[1-\bar{G}_c(\Omega)\right]}{\Omega},$$

$$p_{51} = \int dG_b(u) = 1, \quad p_{71} = \bar{G}_w(\Omega),$$

$$p_{7,13} = \frac{\lambda_b\left[1-\bar{G}_w(\Omega)\right]}{\Omega}, \quad p_{7,12} = \frac{\lambda_c\left[1-\bar{G}_w(\Omega)\right]}{\Omega},$$

$$p_{7,14} = \frac{\lambda_w\left[1-\bar{G}_w(\Omega)\right]}{\Omega}, \quad p_{82} = \int dG_b(u) = 1,$$

$$p_{10,4} = \int dG_w(u) = 1$$

Similarly,

$$p_{11,4} = 1, \quad p_{12,7} = 1, \quad p_{13,7} = 1, \quad p_{11}^{(6)} = \frac{\lambda_c\left[1-\bar{G}_c(\Omega)\right]}{\Omega} = p_{16}$$

$$p_{12}^{(4)} = \lambda_w \int ve^{-(\Omega)v} dG_c(v),\ p_{1,10}^{(4)} = \lambda_w^2 \int ve^{-(\Omega)v} \bar{G}_c(v)dv$$

$$p_{1,11}^{(4)} = \lambda_b \lambda_w \int ve^{-(\Omega)v} \bar{G}_c(v)dv\ ,$$

$$p_{1,12}^{(4)} = \lambda_c \lambda_w \int ve^{-(\Omega)v} \bar{G}_c(v)dv$$

$$p_{21}^{(7)} = \lambda_c \int ve^{-(\Omega)v} dG_w(v),\ p_{2,13}^{(7)} = \lambda_b \lambda_c \int ve^{-(\Omega)v} \bar{G}_w(v)dv$$

$$p_{2,12}^{(7)} = \lambda_c^2 \int ve^{-(\Omega)v} \bar{G}_w(v)dv,$$

$$p_{2,14}^{(7)} = \lambda_c \lambda_w \int ve^{-(\Omega)v} \bar{G}_w(v)dv$$

$$p_{47}^{(12)} = \frac{\lambda_c}{\Omega} \int \left(1 - e^{-(\Omega)v}\right) dG_c(v) = p_{4,12},$$

$$p_{74}^{(14)} = \frac{\lambda_w}{\Omega} \int \left(1 - e^{-(\Omega)v}\right) dG_w(v) = p_{7,14}$$

$$p_{22}^{(9)} = \frac{\lambda_w}{\Omega} \int \left(1 - e^{-(\Omega)v}\right) dG_w(v) = p_{29}$$

$$p_{24}^{(7,14)} = \lambda_c \lambda_w \int \left[\frac{(1 - e^{-(\Omega)w})}{(\Omega)^2} - \frac{we^{-(\Omega)w}}{(\Omega)} \right] dG_w(w) = p_{2,14}^{(7)}$$

$$p_{17}^{(4,12)} = \lambda_c \lambda_w \int \left[\frac{(1 - e^{-(\Omega)w})}{(\Omega)^2} - \frac{we^{-(\Omega)w}}{(\Omega)} \right] dG_c(w) = p_{1,12}^{(4)}$$

Hence, we observe that

$$p_{01} + p_{02} + p_{03} = 1$$

$$p_{10} + p_{11}^{(6)} + p_{12}^{(4)} + p_{15} + p_{17}^{(4,12)} + p_{1,10}^{(4)} + p_{1,11}^{(4)} = 1$$

$$p_{20} + p_{21}^{(7)} + p_{22}^{(9)} + p_{24}^{(7,14)} + p_{28} + p_{2,13}^{(7)} + p_{2,12}^{(7)} = 1$$

$$p_{42} + p_{47}^{(12)} + p_{4,10} + p_{4,11} = 1$$

$$p_{71} + p_{74}^{(14)} + p_{7,13} + p_{7,12} = 1$$

$$p_{30} = p_{51} = p_{82} = p_{10,4} = p_{11,4} = p_{13,7} = p_{12,7} = 1 \qquad (1\text{-}6)$$

3.1 Mean Sojourn Times

If T_i is the sojourn time in state S_i, then mean sojourn time in state S_i is given by,

$$\psi_i = \int P(T_i > t)\, dt$$

Therefore, the mean sojourn times for various states are as follows:

$$\psi_0 = \int e^{-(\lambda_b+\lambda_c+\lambda_w)u}\, du = \frac{1}{\lambda_b+\lambda_c+\lambda_w},$$

$$\psi_1 = \int e^{-(\lambda_b+\lambda_c+\lambda_w)u}\, \bar{G}_c(u)du$$

$$\psi_2 = \int e^{-(\lambda_b+\lambda_c+\lambda_w)u}\, \bar{G}_w(u)du \quad ,$$

$$\psi_3 = \int \bar{G}_b(u)du = \psi_5 = \psi_8 = \psi_{11} = \psi_{13}$$

$$\psi_4 = \int e^{-(\lambda_b+\lambda_c+\lambda_w)u}\, \bar{G}_c(u)du, \quad \psi_6 = \int \bar{G}_c(u)du = \psi_{12}$$

$$\psi_7 = \int e^{-(\lambda_b+\lambda_c+\lambda_w)u}\, \bar{G}_w(u)du, \quad \psi_9 = \int \bar{G}_w(u)du = \psi_{10} = \psi_{14} \tag{7-14}$$

4. **Analysis of Results**

4.1 Reliability and MTSF

Let $R_i(t)$ be the probability that the system is operative during (0, t) given that at t=0 it starts from state $S_i \in E$. By simple probabilistic arguments the value of $R_0(t)$ in terms of its Laplace transforms is given by

$$R_0^*(s) = \frac{N_1(s)}{D_1(s)} \tag{15}$$

Where,

$$N_1(s) = Z_0^*\left(1 - q_{21}^{(7)*} q_{12}^{(4)*}\right) + \left(Z_1^* + q_{14}^* Z_4^*\right)\left(q_{01}^* + q_{21}^{(7)*} q_{02}^*\right) + \left(Z_2^* + q_{27}^* Z_7^*\right)\left(q_{01}^* q_{12}^{(4)*} + q_{02}^*\right) \tag{16}$$

$$D_1(s) = 1 - q_{21}^{(7)*} q_{12}^{(4)} - q_{10}^*\left(q_{01}^* + q_{21}^{(7)*} q_{02}^*\right) + q_{20}^*\left(q_{01}^* q_{12}^{(4)*} + q_{02}^*\right) \tag{17}$$

Taking the inverse Laplace Transform of (15), one may get the reliability of the system when initially system starts from state S_0.

In particular case, when repair time distributions are also exponential, by using the matlab software we obtained the values of reliability of the system for different values of mission time and the curves are drawn in figure-2.

The mean time to system failure (MTSF) can be obtained by using the well known formula –

$$E(T_0) = \int R_0(t)\, dt = \lim_{s\to 0} R_0^*(s) = \frac{N_1(0)}{D_1(0)}$$

Now using the results $q_{ij}^*(0) = p_{ij}$ and $Z_i^*(0) = \psi_i$ we get

$$N_1(0) = \psi_0\left(1 - p_{21}^{(7)} p_{12}^{(4)}\right) + \left(\psi_1 + p_{14}\psi_4\right)\left(p_{01} + p_{21}^{(7)} p_{02}\right) + \left(\psi_2 + p_{27}\psi_7\right)\left(p_{01} p_{12}^{(4)} + p_{02}\right) \quad (18)$$

$$D_1(0) = 1 - p_{21}^{(7)} p_{12}^{(4)} - p_{10}\left(p_{01} + p_{21}^{(7)} p_{02}\right) + p_{20}\left(p_{01} p_{12}^{(4)} + p_{02}\right) \quad (19)$$

4.2 Availability Analysis

Let $A_i(t)$ be the probability that the system is up (operative) at epoch't', when initially the system starts from state $S_i \in E$. Using the technique of the Laplace transform the value of $A_0(t)$ in terms of its L.T., *i.e.* $A_0^*(s)$ (20)
Now, the steady state probability that the system will be operative is given by,

$$A_0 = \lim_{s\to 0} s\,A_0^*(s) = N_2/D_2 \quad (21)$$

Where,

$$N_2 = U_0\psi_0 + U_1\psi_1 + U_2\psi_2 + U_3\psi_3 + U_4\psi_4 \quad (22)$$

D_2 =

$$U_0\psi_0 + U_1 n_1 + U_2 n_2 + p_{03} U_0 n_3 + U_4 n_4 + p_{15} U_2 n_5 + \left(p_{1,10}^{(4)} U_1 + p_{4,10} U_3\right) n_8 + \left(p_{1,11}^{(4)} U_1 + p_{4,11} U_3\right) n_9 + \left(p_{2,13}^{(7)} U_2 + p_{7,13} U_4\right) n_{10} + \left(p_{2,12}^{(7)} U_2 + p_{7,12} U_4\right) n_{11} \quad (23)$$

Where

$$U_0 = \left(1 - p_{11}^{(6)} - p_{15}\right)\Big[\left(1 - p_{22}^{(9)} - p_{28}\right)\Big\{\left(p_{42} + p_{47}^{(12)}\right)\left(p_{71} + p_{74}^{(14)}\right) - p_{74}^{(14)} p_{47}^{(12)}\Big\}$$

$$- p_{42}\Big\{p_{24}^{(7,14)}\left(p_{71} + p_{74}^{(14)}\right) + p_{74}^{(14)}\left(p_{2,13}^{(7)} + p_{2,12}^{(7)}\right)\Big\}\Big]$$

$$- p_{21}^{(7)}\Big[\Big\{p_{12}^{(4)}\left(p_{42} + p_{47}^{(12)}\right)\left(p_{71} + p_{74}^{(14)}\right) - p_{74}^{(14)} p_{47}^{(12)}\Big\}$$

$$- p_{42}\Big\{\left(p_{1,10}^{(4)} + p_{1,11}^{(4)}\right)\left(p_{71} + p_{74}^{(14)}\right) + p_{74}^{(14)} p_{17}^{(4,12)}\Big\}\Big]$$

$$- p_{71}\Big[p_{12}^{(4)}\Big\{p_{24}^{(7,14)} p_{47}^{(12)} + \left(p_{42} + p_{47}^{(12)}\right)\left(p_{2,13}^{(7)} + p_{2,12}^{(7)}\right)\Big\}$$

$$+ \left(1 - p_{22}^{(9)} - p_{28}\right)\Big\{\left(p_{1,10}^{(4)} + p_{1,11}^{(4)}\right) p_{47}^{(12)} + p_{17}^{(4,12)}\left(p_{42} + p_{47}^{(12)}\right)\Big\}$$

$$+ p_{42}\Big\{\left(p_{1,10}^{(4)} + p_{1,11}^{(4)}\right)\left(p_{2,13}^{(7)} + p_{2,12}^{(7)}\right) - p_{24}^{(7,14)} p_{17}^{(4,12)}\Big\}\Big]$$

$$U_1 = \left[p_{42} p_{71} + p_{42} p_{74}^{(14)} + p_{47}^{(12)} p_{71}\right]\left[p_{01} + p_{02} p_{21}^{(7)} - p_{01} p_{22}^{(9)} - p_{01} p_{28}\right]$$

$$-p_{01}\,p_{42}\left[p_{24}^{(7,14)}\left(p_{71}+p_{74}^{(14)}\right)+p_{74}^{(14)}\left(p_{2,13}^{(7)}+p_{2,12}^{(7)}\right)\right]$$

$$+p_{02}\,p_{71}\left[p_{24}^{(7,14)}\,p_{47}^{(12)}+\left(p_{42}+p_{47}^{(12)}\right)\left(p_{2,13}^{(7)}+p_{2,12}^{(7)}\right)\right]$$

$$U_2=\left[p_{42}\,p_{71}+p_{42}\,p_{74}^{(14)}+p_{47}^{(12)}\,p_{71}\right]\left[p_{01}\,p_{12}^{(4)}+p_{02}\left(1-p_{15}-p_{11}^{(6)}\right)\right]$$

$$+p_{01}\,p_{42}\left[\left(p_{1,10}^{(4)}+p_{1,11}^{(4)}\right)\left(p_{71}+p_{74}^{(14)}\right)+p_{74}^{(14)}\,p_{17}^{(4,12)}\right]$$

$$-p_{02}\,p_{71}\left[\left(p_{1,10}^{(4)}+p_{1,11}^{(4)}\right)p_{47}^{(12)}-p_{17}^{(4,12)}\left(p_{42}+p_{47}^{(12)}\right)\right]$$

$$U_3=\left[p_{24}^{(7,14)}\left(p_{71}+p_{74}^{(14)}\right)+p_{74}^{(14)}\left(p_{2,13}^{(7)}-p_{2,12}^{(7)}\right)\right]\left[p_{02}\left(1-p_{15}-p_{11}^{(6)}\right)+p_{01}\,p_{12}^{(4)}\right]$$

$$-\left[\left(p_{1,10}^{(4)}+p_{1,11}^{(4)}\right)\left(p_{71}+p_{74}^{(14)}\right)+p_{74}^{(14)}\,p_{17}^{(4,12)}\right]\left[p_{01}\,p_{12}^{(4)}-p_{02}\,p_{21}^{(7)}\right]$$

$$+p_{02}\,p_{71}\left[\left(p_{1,10}^{(4)}+p_{1,11}^{(4)}\right)\left(p_{2,13}^{(7)}+p_{2,12}^{(7)}\right)-p_{24}^{(7,14)}\,p_{17}^{(4,12)}\right]$$

$$U_4=\left[p_{24}^{(7,14)}\,p_{47}^{(12)}+\left(p_{42}+p_{47}^{(12)}\right)\left(p_{2,13}^{(7)}-p_{2,12}^{(7)}\right)\right]\left[p_{01}\,p_{12}^{(4)}+p_{02}\left(1-p_{15}-p_{11}^{(6)}\right)\right]$$

$$+\left[p_{47}^{(12)}\left(p_{1,10}^{(4)}+p_{1,11}^{(4)}\right)+p_{17}^{(4,12)}\left(p_{42}+p_{47}^{(12)}\right)\right]\left[p_{02}\,p_{21}^{(7)}+p_{01}\left(1-p_{22}^{(9)}-p_{28}\right)\right]$$

$$+p_{01}\,p_{42}\left[\left(p_{1,10}^{(4)}+p_{1,11}^{(4)}\right)\left(p_{2,13}^{(7)}+p_{2,12}^{(7)}\right)+p_{24}^{(7,14)}\,p_{17}^{(4,12)}\right]$$

The expected up time of the system during (0, t) is given by,

$$\mu_{up}(t)=\int_0^t A_0(u)\,du \qquad (24)$$

4.3 Busy Period Analysis

Let $B_i^c(t)$, $B_i^B(t)$ and $B_i^{Wp}(t)$ be the probabilities that the system is under repair at epoch t, when the system initially starts from regenerative state $S_i \in E$. Using elementary probabilistic arguments the value of $B_0^c(t)$, $B_0^B(t)$ and $B_0^{Wp}(t)$ can be obtained in terms of their Laplace transforms *i.e.* $B_0^{c*}(s)$, $B_0^{B*}(s)$ and $B_0^{Wp*}(s)$.

Now, the steady state probabilities B_0^c, B_0^B and B_0^{wp} that in the long run, the probability that the repair facility will be busy in the repairing of failed compressor, blower room and water pump respectively are given by,

So that,

$$B_0^c=N_3/D_2,\quad B_0^B=N_4/D_2 \quad \text{and} \quad B_0^{Wp}=N_5/D_2 \qquad (25\text{-}27)$$

Where,

$$N_3 = \left(\psi_1 + p_{14}\, p_{4,12}\, \psi_{12} + p_{14}\, \psi_4 + p_{16}\, \psi_6\right) U_1 + p_{2,12}^{(7)}\, \psi_{12}\, U_2$$
$$+ \left(\psi_4 + p_{4,12}\, \psi_{12}\right) U_3 + p_{7,12}\, \psi_{12}\, U_4$$

$$N_4 = p_{03}\, \psi_3\, U_0 + \left(p_{15}\, \psi_5 + p_{1,11}^{(4)}\, \psi_{11}\right) U_1 + \left(p_{28}\, \psi_8 + p_{2,13}^{(7)}\, \psi_{13}\right) U_2$$
$$+ p_{4,11}\, \psi_{11}\, U_3 + p_{7,13}\, \psi_{13}\, U_4$$

$$N_5 = p_{1,10}^{(4)}\, \psi_{10}\, U_1 + \left(\psi_2 + p_{27}\, \psi_7 + p_{27}\, p_{7,14}\, \psi_{14} + p_{29}\, \psi_9\right) U_2$$
$$+ p_{4,10}\, \psi_{10}\, U_3 + \left(\psi_7 + p_{7,14}\, \psi_{14}\right) U_4$$

and D_2 is the same as in last section.

Now, the expected busy period of the repair facility in the repair of compressor, blower room and water pump during (0, t) is given by

$$\mu_b^c(t) = \int_0^t B_0^c(u)\, du, \quad \mu_b^B(t) = \int_0^t B_0^B(u)\, du \quad \text{and} \quad \mu_b^{Wp}(t) = \int_0^t B_0^{Wp}(u)\, du \qquad (28\text{-}30)$$

4.4 Cost Benefit Analysis

We are now in a position to obtain the profit function by considering mean up time of the system during (0, t) and expected busy period of the repair facility during (0, t). Let us suppose,

K_0	=	revenue per unit up time
K_1	=	payment to repair facility per unit time when repair facility is busy in the repair of compressor.
K_2	=	payment to repair facility per unit time when repair facility is busy in the repair of blower room.
K_3	=	payment to repair facility per unit time when repair facility is busy in the repair of water pump.

The expected profit incurred by the system during (0, t) is given by,

P(t) = Expected total revenue in (0, t) – Expected total repair cost in (0, t)

$$= K_0\, \mu_{up}(t) - K_1\, \mu_b^c(t) - K_2\, \mu_b^B(t) - K_3\, \mu_b^{wp}(t) \qquad (31)$$

The expected profit per unit time in a steady state is given by,

$$P = K_0\, A_0 - K_1\, B_0^c - K_2\, B_0^B - K_3\, B_0^{wp} \qquad (32)$$

Where,

A_0, B_0^c, B_0^B, and B_0^{wp} has been already defined.

5. Particular Case

In this section, if we consider the case when all repair time distribution are also exponential i.e.

$$G_c(t) = 1 - e^{-\mu_c t}, \qquad G_b(t) = 1 - e^{-\mu_b t}, \qquad G_w(t) = 1 - e^{-\mu_w t}$$

Then the steady state transition probabilities will be,

$$p_{10} = \frac{\mu_c}{\lambda_b + \lambda_c + \lambda_w + \mu_c} = p_{42}, \qquad p_{15} = \frac{\lambda_b}{\lambda_b + \lambda_c + \lambda_w + \mu_c} = p_{4,11}$$

Similarly we can find others.

6. Graphical Representation

For more concrete study of the behavior of the characteristics obtained under study in above particular case, we plot the curves for reliability and MTSF, for different values of λ_b and λ_c while the other parameters are kept fixed as:

$\lambda_w = 0.001$, $\mu_b = 0.04$, $\mu_c = 0.05$, $\mu_w = 0.08$,

$K_0 = 4000$, $K_1 = 100$, $K_2 = 150$, $K_3 = 50$

In Fig.-2, curves represents the graph of reliability with respect to time for different values of λ_b taken as λ_b=.04, .05 and .06. when λ_c=.002. It is observed that the reliability decreases as λ_b increases.

In Fig. 3 and Fig. 4, curves represent the graphs of MTSF and profit function with respect to λ_b for different values of λ_c = 0.002, 0.005 and 0.008. It is observed that the MTSF decreases as λ_b increases. Also, with an increase in λ_c, the MTSF decreases. Further, we observe that the rate of decrement in MTSF is rapid initially and tends to vanish as λ_b becomes large. From Figure-4 it is obvious that the profit decreases with the increase in with λ_b.Further with an increase in λ_c there is a decrease in profit with a constant rate.

Acknowledgment

We sincerely thanks to the Department of Science and Technology (DST), Government of India for financial support under Woman Scientist Scheme-A (SR/WOS-A/MS-09/2009) to Punam Phartyal.

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TRANSITION DIAGRAM

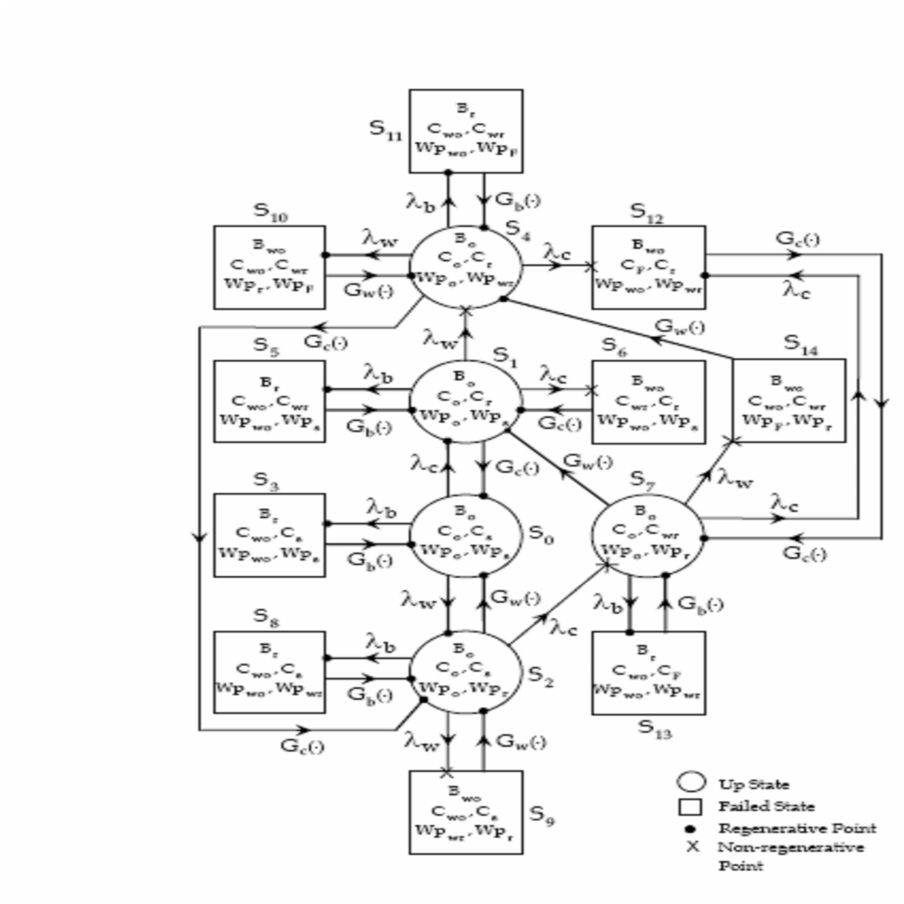

Fig.1

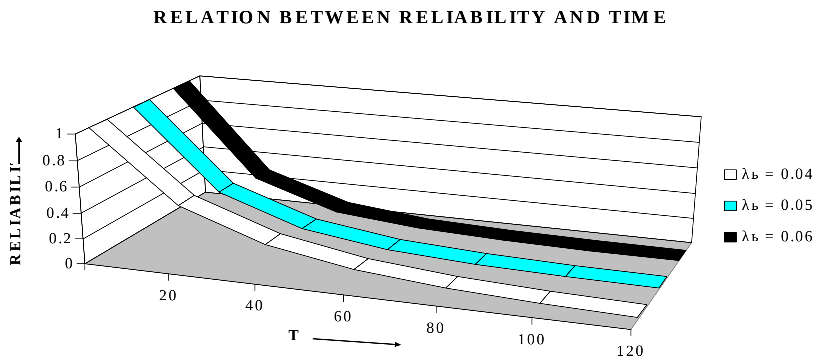
RELATION BETWEEN RELIABILITY AND TIME
RELIABILITY
1
0.8
0.6
0.4
0.2
0
20
40
60
80
100
120
T
λb = 0.04
λb = 0.05
λb = 0.06

Fig.2

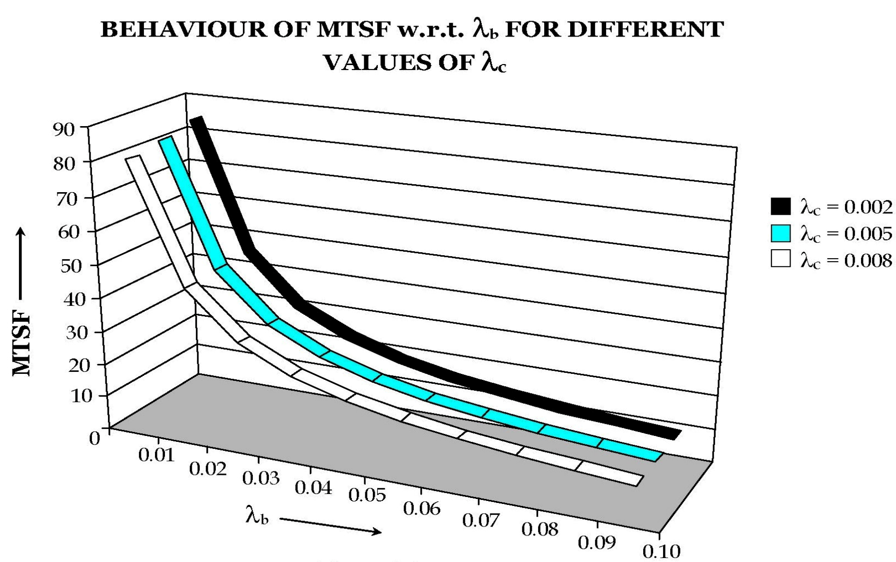
BEHAVIOUR OF MTSF w.r.t. λb FOR DIFFERENT VALUES OF λc
MTSF
90
80
70
60
50
40
30
20
10
0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.09
0.10
λb
λc = 0.002
λc = 0.005
λc = 0.008

Fig. 3

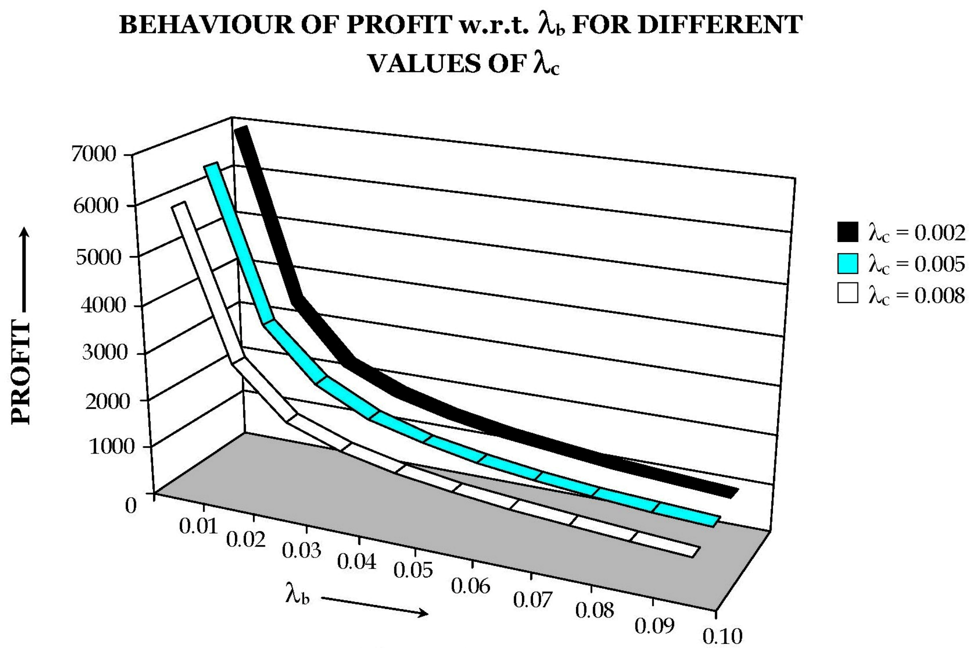
BEHAVIOUR OF PROFIT w.r.t. λb FOR DIFFERENT VALUES OF λc
PROFIT
7000
6000
5000
4000
3000
2000
1000
0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.09
0.10
λb
λc = 0.002
λc = 0.005
λc = 0.008

Fig. 4