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CONFIGURATIONAL MODELING AND STOCHASTIC ANALYSIS OF A COMPLEX REPARABLE INDUSTRIAL SYSTEM MODEL

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Abstract

The present paper deals with the configurational modeling and stochastic analysis of a complex reparable system model based on cold-drink making system. The considered system consists of a number of sub-systems of varying nature. The stochastic analysis of the considered system model is carried out by using regenerative point technique under the assumption that all failure rates are constant and repair rates are general. In the present system model, the concept of common cause failure is also incorporated. The expressions for several systems characteristics such as reliability, MTSF, steady state availability, busy period and expected profit have been obtained. MTSF and profit function have also been widely studied through graphs taking repair time distributions as exponential.

Key words: Reliability redundant system, MTSF, availability.

1. Introduction and System Description

In the field of reliability various hypothetical redundant system models have been analyzed under different sets of assumptions. Practical usefulness of configurational modeling and stochastic analysis exists in realistic industrial modeling. The stochastic analysis of realistic industrial systems is very helpful for system managers, system engineers and researchers for present and future strategies. But, a very few work related to realistic modeling [1-6] have been seen in the literature. Considering the importance of realistic modeling, the purpose of the present paper is to develop and analyze an industrial system model based on cold-drink making system situated at Muzaffarnagar in U.P., India.

Cold-drink making system is a complex type reparable engineering system consists of seven subsystems/units. The working of the system plant is as follows: - First of all hardness of water is removed by mixing lime and bleaching powder in hard water. Hence, the hard water from water supply unit (WS) changes in soft water. Now, the soft water is passed through ammonium compressor unit (NH) to make it chill. A fixed amount of sugar and flavor is mixed with the chilled water. After that mixed chilled water comes in carbonator unit (CO) where carbon dioxide gas (CO_2) is mixed with it. This prepared solution is filled by filter unit (F) into bottles, coming from bottling unit (B). Finally, filled bottles are sealed by crimping machine unit (CM). The electricity unit provides the electricity to the plant (E).

2. Assumptions

(i) Failure and repair are stochastically independent.
(ii) A single repair facility is always present to repair a failed subsystem/unit. Priority in repair to the units WS, NH, CO, F, B and CM is given over the unit E.

(iii) Each unit of the system has two modes normal (N) and total failure (F).
(iv) System/unit failure occurs either due to normal failure or due to common cause failure. Common cause failure is defined as any instance multiple unit or component fails due to a single cause.
(v) Each repaired unit is as good as new.
(vi) All the failure time distributions are taken as exponential whereas repair time distributions are taken as general.

In the light of above assumptions and using the regenerative point technique, the following measures of system effectiveness are obtained.
(i) Transition probabilities and sojourn times in different states.
(ii) Reliability and mean time to system failure (MTSF).
(iii) Pointwise and steady state availabilities of the system.
(iv) Expected busy period of the repair facility during (0, t).
(v) Net expected profit incurred in (0, t) and in steady state.

The nature of MTSF and profit function is studied in the light of graph in a particular case taking repair time distributions as exponential.

3. Notations for States of the System

α_i	:	constant failure rates of the units WS/NH/CO/F/B and CM, respectively for i = 1, 2, 3, 4, 5, 6.
β	:	constant failure rate of the unit E.
γ	:	common cause failure rate of the system when it is either in state S_0 or S_1.
$g_i(\cdot), G_i(\cdot)$	:	pdf and cdf of repair time of the units WS/NH/CO/F/B and CM, respectively for i = 1, 2, 3, 4, 5, 6.
$h(\cdot), H(\cdot)$	:	pdf and cdf of repair time of the system in failed state S_{15} due to common cause failure.
$k(\cdot), K(\cdot)$	:	pdf and cdf of repair time of the unit E.

3.1 Symbols for States of the System

$E_o/E_g/E_s/E_{wr}$	:	Unit E is operative/good/stand by/under repair/waiting for repair.
$B_o/B_g/B_r$	:	Unit B is operative/good/under repair.
$F_o/F_g/F_r$	:	Unit F is operative/good/under repair
$WS_o/WS_g/WS_r$	:	Unit WS is operative/good/under repair.
$NH_o/NH_g/NH_r$	:	Unit NH is operative/good/under repair.
$CO_o/CO_g/CO_r$	:	Unit CO is operative/good/under repair.
$CM_o/CM_g/CM_r$	:	Unit CM is operative/good/under repair.

Using these symbols the various states of the system model are shown in Fig. 1, where the states S_0 and S_1 are up states and rest of the states are failed.

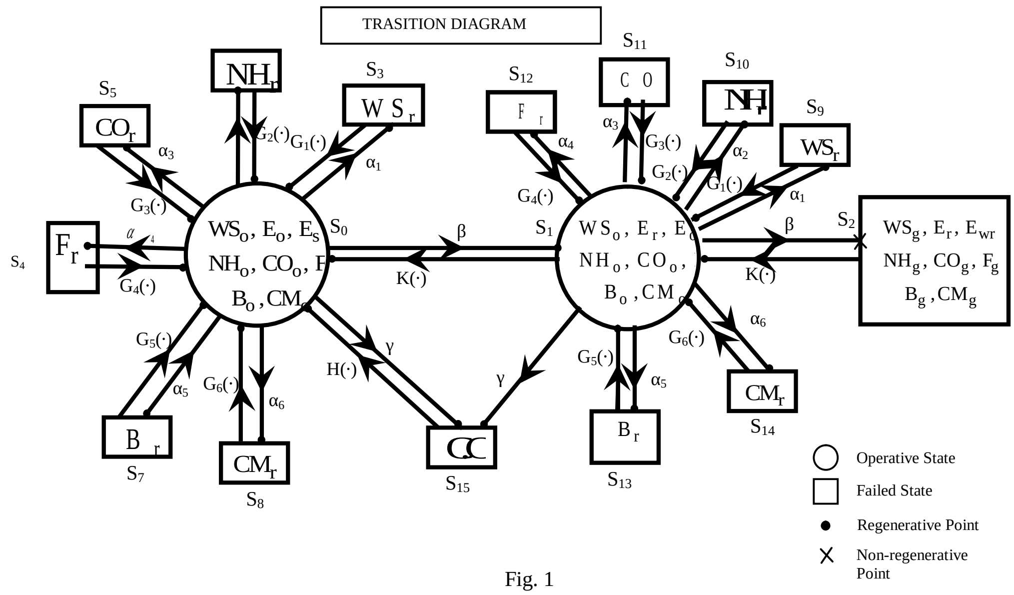

Fig. 1

4. Transition Probabilities and Sojourn Times

All the entrance epochs except at S_2 are regenerative. So, E = (S_0, S_1, S_3,… ,S_{15}). Let T_0 (≡ 0), T_1, T_2, ... denote the instants at which the system enters into any state $S_K \in$ E and let X_n be the state visited at instant T_{n+}, i.e. just after the transition at T_n. Then $\{X_n, T_n\}$ is a Markov renewal process with state E.

The steady state transition probabilities of the system model are as follows:

$$p_{01} = \frac{\beta}{\left(\beta+\gamma+\sum_{i}^{\Delta}\alpha_i\right)}, \qquad p_{0,15} = \frac{\gamma}{\left(\beta+\gamma+\sum\alpha_i\right)},$$

$$p_{0,i+2} = \frac{\alpha_i}{\left(\beta+\gamma+\sum\alpha_i\right)}$$

$$p_{11}^{(2)} = \frac{\beta\left[1-\tilde{K}\left(\beta+\gamma+\sum\alpha_i\right)\right]}{\left(\beta+\gamma+\sum\alpha_i\right)}, \qquad p_{10} = \tilde{K}\left(\beta+\gamma+\sum\alpha_i\right),$$

$$p_{1,15} = \frac{\gamma\left[1-\tilde{K}\left(\beta+\gamma+\sum\alpha_i\right)\right]}{\left(\beta+\gamma+\sum\alpha_i\right)},$$

$$p_{1,i+8} = \frac{\alpha_i\left[1-\tilde{K}\left(\beta+\gamma+\sum\alpha_i\right)\right]}{\left(\beta+\gamma+\sum\alpha_i\right)},$$

$$p_{i+2,0} = p_{i+8,1} = p_{15,0} = p_{21} = 1, \quad i= 1, 2, ..., 6.$$

It is clear that

$$p_{01} + p_{0,15} + \sum p_{0,i+2} = 1,$$

$$p_{10} + p_{11}^{(2)} + p_{1,15} + \sum p_{1,i+8} = 1,$$

4.1 Mean Sojourn Time

Mean sojourn time ψ_K is state S_K is defined as the expected time for which the system stays in state S_K, before transiting to any other state. Let X_K denotes the sojourn time in state S_K, then the mean sojourn time in state S_K is given by

$$\psi_K = \int^{*} P\left[X_K > t\right] dt$$

$$\psi_0 = \frac{\sum \alpha_i}{\left(\beta+\gamma+\sum \alpha_i\right)}, \qquad \psi_1 = 1 - \tilde{K}\left(\beta + \gamma + \sum \alpha_i\right),$$

$$\psi_2 = \int \bar{K}(t)\, dt, \qquad \psi_{i+2} = \frac{1}{\alpha_i} = \psi_{i+8}$$

5. Reliability and MTSF

Let the random variable T_K be the time to system failure when the system initially starts from state $S_K \in E$, then the reliability of the system is given by

$$R_K(t) = P\,[T_K > t]$$

By probabilistic arguments we have the following relations:

$R_0(t) = Z_0(t) + q_{01}(t) © R_1(t)$

$R_1(t) = Z_1(t) + q_{10}(t) © R_0(t)$ (1-2)

where $Z_0(t) = e^{-(\beta+\gamma+\sum\alpha_i)t}$, $Z_1(t) = [1 - K(t)]\, e^{-(\beta+\gamma+\sum\alpha_i)t}$

Taking Laplace Transform (L.T) of relations (1-2) and simplifying for $R_0^*(s)$, we obtain,

$$R_0^*(s) = \frac{Z_0^* + q_{01}^* Z_1^*}{1 - q_{01}^* q_{10}^*} \tag{3}$$

Using the usual formula, the MTSF is given by,

$$E(T_0) = \lim_{s \to 0} R_0^*(s) = \frac{\psi_0 + p_{01}\psi_1}{1 - p_{01}p_{10}} \tag{4}$$

The limit of integration is 0 to ∞ whenever it is not mentioned.

6. Availability Analysis

From the theory of regenerative process, the pointwise availabilities of the system are seen to satisfy the following recursion relations:

$A_0(t) = Z_0(t) + q_{01}(t) © A_1(t) + q_{0,15}(t) © A_{15}(t) + \Sigma q_{0,i+2}(t) © A_{i+2}(t)$

$A_1(t) = Z_1(t) + q_{10}(t) © A_0(t) + q_{11}^{(2)}(t) © A_1(t) + q_{1,15}(t) © A_{15}(t)$
$+ \Sigma q_{1,i+8}(t) © A_{i+8}(t)$
$A_{i+2}(t) = q_{i+2,0} © A_0(t)$
$A_{i+7}(t) = q_{i+8,1}(t) © A_1(t)$
$A_{13}(t) = q_{15,0}(t) © A_0(t)$ (5-9)

Taking L.T. of equations (5-9) and solving for $A_0^*(s)$, we have,

$$A_0^*(s) = N_2(s)/D_2(s) \quad (10)$$

where,

$$N_2(s) = \left(1 - q_{11}^{*(2)} - \Sigma q_{i+8,1}^*\right) Z_0^* + q_{01}^* Z_1^* \quad (11)$$

$$D_2(s) = \left(1 - q_{11}^{*(2)} - \Sigma q_{1,i+8,1}^* q_{i+8,1}^*\right)\left(1 - q_{0,15}^* q_{15,0}^* - \Sigma q_{0,i+2}^* q_{i+2,0}^*\right)$$
$$- q_{01}^*\left(q_{10}^* + q_{1,15}^* q_{15,0}^*\right) \quad (12)$$

For brevity, the argument 's' is omitted from $q_{ij}^*(s)$ and $Z_i^*(s)$. Now the steady state availability is given by,

$$A_0 = N_2/D_2 \quad (13)$$

where,

$$N_2 = \left(p_{10} + p_{1,15}\right)\psi_0 + p_{01}\psi_1 \quad (14)$$

$$D_2 = \left(p_{10} + p_{1,15)}\right)\psi_0 + p_{01}\left(\psi_1 + p_{12}\psi_{12}\right) + \Sigma C_{i+2}\psi_{i+2} + \Sigma C_{i+8}\psi_{i+8} \quad (15)$$

7. Busy Period Analysis

Let $B_K(t)$ be the probability that the repair facility is busy in repair of the failed unit at time t when system initially starts from state $S_K \in E$.

Using elementary probabilistic arguments in respect to the above definition of $B_K(t)$, we have the following relations -

$B_0(t) = q_{01}(t) © B_1(t) + q_{0,15}(t) © B_{15}(t) + \Sigma q_{0,i+2}(t) © B_{i+2}(t)$
$B_1(t) = \delta_1 Z_1(t) + q_{01}(t) © B_0(t) + q_{11}^{(2)}(t) © B_1(t) + q_{1,15}(t) © B_{15}(t)$
$+ \Sigma q_{1,i+8}(t) © B_{i+8}(t)$
$B_{i+2}(t) = \delta_{i+2} Z_{i+2}(t) + q_{i+2,0}(t) © B_0(t)$
$B_{i+7}(t) = \delta_{i+8} Z_{i+8}(t) + q_{i+8,1}(t) © B_1(t)$
$B_{13}(t) = \delta_{15} Z_{15}(t) + q_{15,0}(t) © B_0(t)$ (16-20)

where,

$Z_{i+2}(t) = \tilde{1}G_i(t) = Z_{i+8}(t)$
$Z_{15}(t) = 1 \square (t)$

Taking L.T. of the relations (16-20) and then after substituting, we get,

$$B_0^*(s) = N(s)/D_2(s) \quad (21)$$

where,

$$N(s) = \Sigma q_{0,i+2}\left(1-q_{11}^{(2)}\right)\delta_{i+2}\, Z_{i+2} \,\square\, q_{0,i+2}\,\Sigma q_{1+i+8}\,\delta_{i+2}\, Z_{i+2}$$
$$\tilde{.}\; q_{0,13}\,\Sigma q_{1,i+8} q_{i+8,1}\delta_{15} Z_{15} + q_{01}\Sigma q_{1,i+8}\delta_{i+8} Z_{i+8} + q_{01} Z_1 \delta_1$$
$$+\left[q_{0,15}\left(1-q_{11}^{(2)}\right)+q_{01} q_{1,13}\right] Z_{15}\delta_{15} \qquad (22)$$

$D_2(s)$ is the same as in availability analysis.

Now, if $B_0^E(t),\ B_0^{WS}(t),\ B_0^{NH}(t),\ B_0^{CO}(t),\ B_0^F(t),\ B_0^B(t)\text{and}B_0^{CM}(t),$ be the probabilities that the system is under repair due to the failure of the unit E, WS, NH, CO, F, B and CM, respectively, when system initially starts from state S_0. Also, let $B_0^C(t)$ be the probability that system is under repair at epoch t, due to common cause failure, when system initially starts from state S_0. The separate values of these probabilities in terms of their L.T. can be obtained from (21) by substituting ($\delta_1 = 1$, $\delta_{i+8} = \delta_{i+2} = \delta_{15} = 0$) for $B_0^E(t)$, ($\delta_3 = \delta_9 = 1$, rest δ's are zero) for $B_0^{ws}(t)$, ($\delta_4 = \delta_{10} = 1$, rest δ's are zero) for $B_0^{NH}(t)$, ($\delta_5 = \delta_{11} = 1$, rest δ's are zero) for $B_0^{CO}(t)$, ($\delta_6 = \delta_{12} = 1$, rest δ's are zero) for $B_0^F(t)$, ($\delta_7 = \delta_{13} = 1$, rest δ's are zero) for $B_0^{CM}(t)$, ($\delta_8 = \delta_{14} = 1$, rest δ's are zero) for $B_0^B(t)$ and ($\delta_{15} = 1$, rest δ's are zero) for $B_0^C(t)$. In a long run, the probability that the repair facility will be busy in repair of failed unit E, is given by

$$B_0^E = \lim_{t\to\infty} B_0^E(t) = N_4/D_2 \qquad (23)$$

where,

$$N_4 = p_{01}\psi_1 + p_{01}\Sigma p_{1,i+7}\psi_{i+7} \qquad (24)$$

Similarly, other steady state probabilities can be obtained as follows:

$$B_0^C = N_3/D_2,\quad B_0^{WS} = N_5/D_2,\quad B_0^{NH} = N_6/D_2,$$
$$B_0^{CO} = N_7/D_2,\quad B_0^F = N_8/D_2,\quad B_0^B = N_9/D_2,\quad \text{and}$$
$$B_0^{CM} = N_{10}/D_2 \qquad (25\text{-}31)$$

where,

$$N3 = \left[p_{0,15}\left(1-p_{11}^{(2)}\right)+p_{01}\ p_{1,15}\right]\psi_{15} - p_{0,15}\Sigma p_{1,i+8}\psi_{15} \qquad (32)$$

$$Nj = \left[\Sigma p_{0,i+2}\left(1-p_{11}^{(2)}\right)-\Sigma p_{0,i+2}\ \Sigma p_{0,i+8}\right]\psi_{j-2} + p_{01}\Sigma p_{1,i+8}\psi_{j+4} \quad \forall\ j = 5\tilde{}1 \qquad (33)$$

8. Profit Function Analysis

The net expected profit incurred by the system during (0, t) is given by

P (t) = Expected total revenue during (0, t) ˜ Expected total expenditure during (0, t)

$$= C_0\ \mu_{up}(t) - C_1\ \mu_b^E(t) - C_2\ \mu_b^{WS}(t) - C_3\ \mu_b^{NH}(t) - C_4\ \mu_b^{CO}(t)$$

$$-C_5\ \mu_{up}^{F}(t) - C_6\ \mu_b^{B}(t) - C_7\ \mu_b^{CM}(t) - C_8\ \mu_b^{C}(t) \quad (34)$$

where C_0 is the revenue per unit up time by the system and C_1, C_2, C_3, C_4, C_5, C_6 and C_7 are the cost per unit down time when the system is under repair due to the failure of units E, WS, NH, CO, F, B and CM respectively. Also C_8 be the cost per unit down time when the system is under repair due to common cause.
Also,

$$\mu_{up}(t) = \int_0^t A_0(u)du \text{ s.t. } \mu_{up}^{*}(s) = A_0^{*}(s)/s \quad (35)$$

In similar way
$\mu_b^{E}(t),\ \mu_b^{WS}(t),\ \mu_b^{NH}(t),\ \mu_b^{CO}(t),\ \mu_b^{F}(t),\ \mu_b^{B}(t),\ \mu_b^{CM}(t)$ and $\mu_b^{C}(t)$ Can be defined.
Now, the expected profit per unit time in steady state is given by

$$P = \lim_{t\to\infty} P(t)/t = \lim_{s\to 0} s^2 P^{*}(s)$$
$$= C_0A_0 - C_1B_0^{E} - C_2B_0^{WS} - C_3B_0^{NH} - C_4B_0^{CO} - C_5B_0^{F}$$
$$-C_6B_0^{B} - C_7B_0^{CM} - C_8B_0^{C} \quad (36)$$

9. Particular Case

When all the repair time distributions are taken as exponential as

$$g_i(t) = \lambda_i^{-\lambda_i t};$$
$$0 < t < \infty;\ \lambda_i > 0$$
$$h(t) = \eta\, e^{-\eta t}; \qquad 0 < t < \infty;\ \eta > 0$$
$$k(t) = \theta\, e^{-\theta t}; \qquad 0 < t < \infty;\ \theta > 0$$

Now, the changes are as follows:

$$p_{11}^{(2)} = \frac{\beta}{\beta+\gamma+\theta+\Sigma\lambda_i}, \qquad p_{1,15} = \frac{\gamma}{\beta+\gamma+\theta+\Sigma\lambda_i},$$
$$p_{10} = \frac{\theta}{\beta+\gamma+\theta+\Sigma\lambda_i},$$
$$p_{1,i+8} = \frac{\lambda_i}{\beta+\gamma+\theta+\Sigma\lambda_i}, \qquad \psi_1 = \frac{1}{\beta+\gamma+\theta+\Sigma\lambda_i}, \qquad \psi_2 = \frac{1}{\theta},$$
$$Z_1(t) = e^{-(\beta+\gamma+\theta+\Sigma\lambda_i)t}, \qquad Z_{i+2}(t) = e^{-\lambda_i t} = Z_{i+8}(t), \qquad Z_{15}(t) = e^{-\eta t}$$

10. Graphical Analysis

For more concrete study of the system behaviour, we plot curves for MTSF and profit function w.r.t. failure rate of ammonium compressor unit (α_2). Fig. 2 shows the variation in MTSF w.r.t. α_2 for different values of β = 0.001, 0.003 and 0.005 when other parameters are kept fixed as $\alpha_1 = \alpha_3 = \alpha_4 = \alpha_5 = \alpha_6 = 0.002$, $\gamma = 0.02$ and $\theta = 0.01$. From graph it is observed that the MTSF decreases as α_2 increases. The rate of decrement is rapid initially and uniformly decreases for large values of α_2. Also, when we increase the value of β then the MTSF decreases.

Fig. 3. Shows the changes in profit function w.r.t. α_2 for different values of η and θ while the other parameters are kept fixed as $\alpha_1 = \alpha_3 = \alpha_4 = \alpha_5 = \alpha_6 = 0.002$, $\beta = 0.001$, $\gamma = 0.02$, $\lambda_1 = \lambda_2 = \lambda_3 = \lambda_4 = \lambda_5 = \lambda_6 = 0.025$, $C_0 = 1200$, $C_1 = 100$, C_2 120, $C_3 = 125$, $C_4 = 110$, $C_5 = 80$, $C_6 = 110$, $C_7 = 75$, $C_8 = 200$. From graph we observe that the profit decreases as α_2 increases and it is also observed that the values of profit curves tend to increase as we increase the values of repair rates η and θ.

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Behavior of MTSF w.r.t. failure rate of ammonium compressor unit (α_2) for different values of β

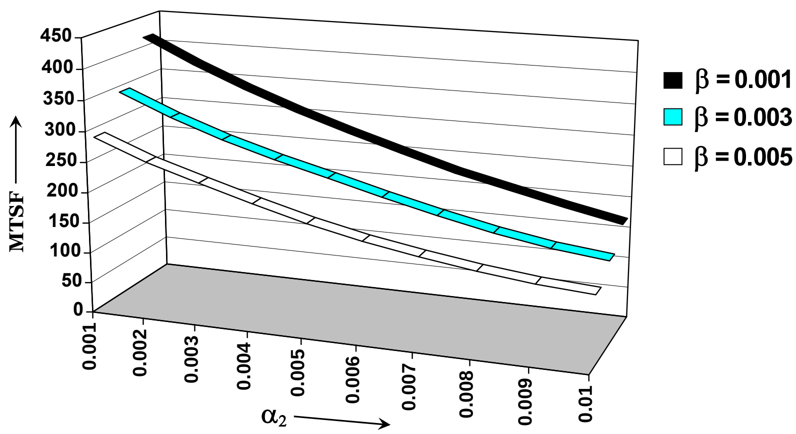

Fig. – 2

Behavior of Profit function w.r.t. failure rate of ammonium compressor unit (α_2) for different values of η and θ

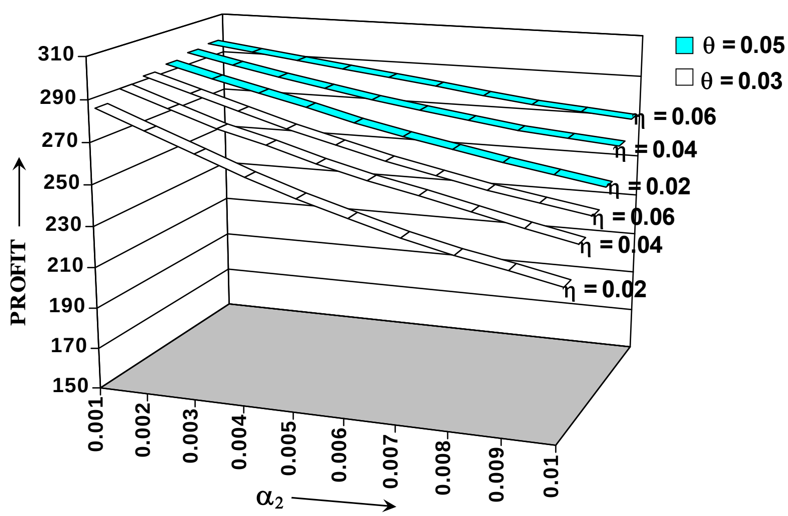

Fig. – 3