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# PERFORMANCE MODEL OF DATABASE RELATIONSHIP TO DESIGN AND MANUFACTURING

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## Abstract

Recent trends of economization in the development and applications of computer software systems have become an important consideration for the design of modern computer systems. This has increased the efforts towards the study, evaluation and testing of the computer software. The model of information system presented here, is comprised of three subsystems viz,; CAD, Database and CAM namely as A, B and C respectively. Computer-Aided-Design (CAD) is a major element of a computer-integrated manufacturing system. CAD involves any type of design activity that makes use of the computer to develop, analyze, or modify an engineering design. First of all interactive graphics is entered into CAD then it interacts with database and at the last it comes under Computer-Aided-Manufacturing (CAM), which is another major part of manufacturing system. In the performance model of database relationship to design and manufacturing considered in this paper has all failure rates to be following exponential distribution and repair times to be following general time distributions. By the inclusion of supplementary variable and Laplace Transforms technique, all state probabilities, graphs of reliability v/s time, expected profit v/s time and MTTF v/s all types of failures have been sketched in the end so as to forecast the operable behavior of such performance model.

**Key Words:** Stochastic Processes, Reliability, Supplementary Variable Technique, State Probabilities.

## 1. Introduction

Since the development of computer software in today's environment is gaining a primary concern, it is, therefore necessary to make the model of computer software like hardware for using it in the design, evaluation and testing of the software. During the design phase, many software parameters influence the quality and clarity of the program. Prominent amongst these is: executive time, reliability, size etc. In order to measure these parameters, a software measure is considered. A software measure is a measure of quality and clarity of the program. The execution time of a program denotes the time a program takes for its execution. The reliability of a program in general is defined as a measure of a number of errors (bugs) encountered in the program and is often viewed as a qualitative measure influenced by the quality of the software. This measure provides useful data for obtaining suitable testing strategies so as to decide how to perform the test and also, the evaluation of some given aspects of tests.

The model of information system presented here, is comprised of three subsystems CAD, Database and CAM named as A, B and C respectively. Computer-Aided-Design is a major element of a computer-integrated-manufacturing system. CAD involves any type of design activity that makes use of the computer to develop, analyze, or modify an engineering design, First of all interactive graphics is entered into CAD then it interacts with database and at the last it comes under CAM i.e. Computer-Aided-Manufacturing, which is another major part of manufacturing system. An important reason for using a CAD system is that it provides a database for manufacturing the product.

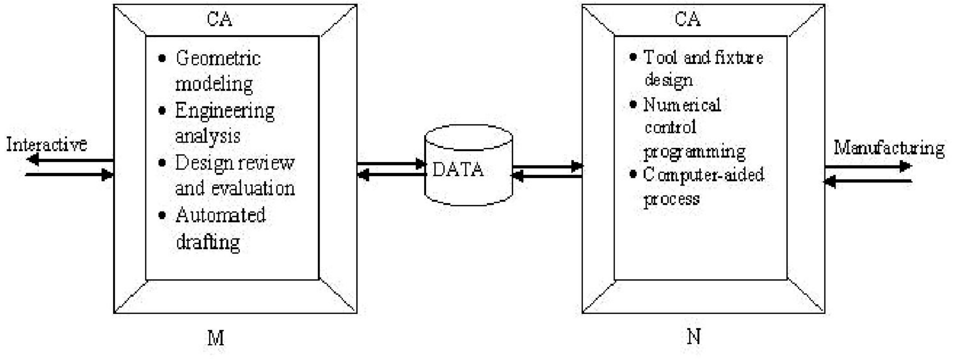


**Figure 1: Relationship to Design and Manufacturing**

**Subsystem A (CAD System)*:*** This subsystem is composed of M non-identical units arranged in series and failure of any one of these causes to break down.

**Subsystem B (Database)*:*** This subsystem consists of two identical units $B_1$ and $B_2$. $B_1$ operates initially while $B_2$ is being put in standby mode. The feature of this subsystem is that the system has switching devices connecting these units in order to utilize surviving units as many as possible.

**Subsystem C (CAM)*:*** This subsystem is comprised of N non-identical units and failure of any one causes the system to reach in degraded state.

As a most simple example, we can design a system, to operate successfully, requires that only one of its several components work successfully. Here failure and repair times follow exponential and general time distributions respectively. By the inclusion of supplementary variable and Laplace Transform (L.T.) techniques, the transition state probabilities of the complex system being in various states have been derived. These L.T.s have been inverted so as to obtain time dependent probabilities. However, graphs of reliability v/s time, expected profit v/s time and MTTF v/s all types of failures have been sketched in the end so as to forecast the operable behavior of such electronic equipment.

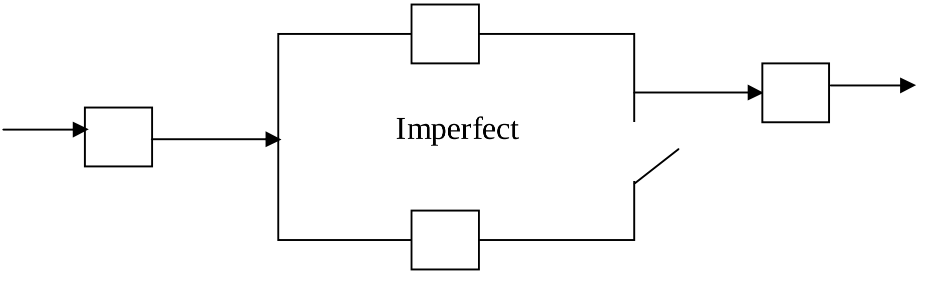


**Figure 2: Logical Block diagram**

In this paper the switching device connecting the standby unit to the main system is considered as imperfect. This feature of the subsystem B gives all the possibilities in order to utilize surviving units as many as possible. The given references are lacking the imperfect switching.

## 2. Assumptions

The following assumptions are being associated with the system:

1. Initially, the system is operable.
2. The system has three types of states: normal, degraded and failed.
3. All transition rates vary from component to component, as all the components are non-identical.
4. Failures and repairs are S-independent.
5. Separates repair facilities are available for each subsystem and switch over device.
6. Nothing can fail when the system is in failed state.
7. A repaired unit is as good as new and is immediately reconnected to the system.
8. Repair facility is at single channel.
9. Upon failure, if all repair facilities are busy, the immediately failed unit is repaired firstly i.e. the repair policy is on last-come-first-served basis.
10. When a unit fails, repair for the failed unit and the installation of the standby unit for operation starts.

## 3. State Description

| **State** | **Subsystem A** | **$B_1$** | **$B_2$** | **Subsystem C** | **Corresponding state prob.** | **State (in nature)** |
|---|---|---|---|---|---|---|
| S(0, 0, 0) | All-op | Op | Ds | All-op | $P_0(t)$ | Op |
| S(0, 1, 0) | All-op | F | Op | All-op | $P_1(t)$ | Op |
| S(0, 0, j) | All-op | Op | Ds | $j^{th}$-F | $P_2(t)$ | Op |
| S(0, 1, j) | All-op | F | Op | $j^{th}$-F | $P_3(t)$ | Op |
| S(0, s, 0) | All-op | F | Bs | All-op | $P_4(t)$ | F |
| S(0, 2, 0) | All-op | F | F | All-op | $P_5(t)$ | F |
| S(0, 2, j) | All-op | F | F | $j^{th}$-F | $P_6(t)$ | F |
| S(0, s, j) | All-op | F | Bs | $j^{th}$-F | $P_7(t)$ | F |
| S(i, 0, 0) | $i^{th}$-F | Op | Ds | All-op | $P_8(t)$ | F |
| S(i, 1, 0) | $i^{th}$-F | F | Op | All-op | $P_9(t)$ | F |
| S(i, 0, j) | $i^{th}$-F | Op | Ds | $j^{th}$-F | $P_{10}(t)$ | F |
| S(I, 1, j) | $i^{th}$-F | F | Op | $j^{th}$-F | $P_{11}(t)$ | F |

All-op: all operable; op: operable; F: failed; Ds: deteriorated standby; Bs: Being switched

**Table 1**

## 4. Notations

$$D\left|D_x\right|D_y\left|D_z\right|D_w\Big| : \frac{d}{dt}\left|\frac{\partial}{\partial t}\right|\frac{\partial}{\partial x}\left|\frac{\partial}{\partial y}\right|\frac{\partial}{\partial z}\left|\frac{\partial}{\partial w}\right.$$

$\alpha_i\left|\lambda_B\right|\beta_j$ : Constant failure rate of ith – A/any of B/jth – C a : unit constant switching in rate from standby mode to on-line mode

$\mu_i(x)|\nu_j(y)|\gamma_B(z)|\phi(w)$: transition repair rate of the ith – A/jth – C/any of B/switching device

$S_i(x) = i(x)\exp\left[\int_0^x i(x)dx\right]$ by Davis' relation, where $i = \mu, \nu, \gamma, \phi$.

$\bar{S}_k(x)$: Laplace transform of $S_k(t)$

$\bar{a}: 1 - a$ and $\int = \int_0^\infty$ otherwise stated

$\alpha = \sum_i \alpha_i$

$\beta = \sum_j \beta_j$

$\bar{f}(s) =$ Laplace transform of f(t)

$P_n$ : steady state prob. of the system in nth state; $0 \le n \le 11$

$P_i(t)$: P (the system is in ith state at time t); i = 0, 1

$P_j(x,t)dx|P_k(y,t)dy|P_l(z,t)dz|P_m(w,t)dw$ : P (the system is in j/k/l/m$^{th}$ state at time t and the elapsed repair time lies between x/y/z/w and x + dx/y + dy/z + dz/w + dw); j = 8, 9, 10, 11; k = 2, 3, 6; 1 = 5; m = 4, 7

## 5. Formulation

By elementary probability considerations and continuity arguments, the difference-differential equations for the stochastic process, are as follows:

$$(D + \lambda_B + \beta + \alpha)P_0(t) = \sum_i \int P_8(x,t)\mu_i(x)dx + \sum_j \int P_2(y,t)\nu_j(y)dy + \int P_5(z,t)\gamma_B(z)dz \quad (1)$$

$$(D + \lambda_B + \beta + \alpha)P_1(t) = \sum_i \int P_9(x,t)\mu_i(x)dx + \sum_j \int P_3(y,t)\nu_j(y)dy + \int P_4(w,t)\phi(w)dw + a\lambda_B P_0(t) \quad (2)$$

$$\left(D_y + D_t + \lambda_B + \alpha + \nu_j(y)\right)P_2(y,t) = 0 \quad (3)$$

$$\left(D_y + D_t + \lambda_B + \alpha + \nu_j(y)\right)P_3(y,t) = a\lambda_B P_2(y,t) \quad (4)$$

$$\left(D_w + D_t + \phi(w)\right)P_4(w,t) = 0 \quad (5)$$

$$\left(D_z + D_t + \gamma_B(z)\right)P_5(z,t) = 0 \quad (6)$$

$$\left(D_y + D_t + \nu_j(y)\right)P_6(y,t) = \lambda_B P_3(y,t) \quad (7)$$

$$\left(D_w + D_t + \phi(w)\right)P_7(w,t) = 0 \quad (8)$$

$$\left(D_x + D_t + \mu_i(x)\right)P_8(x,t) = 0 \tag{9}$$

$$\left(D_x + D_t + \mu_i(x)\right)P_9(x,t) = 0 \tag{10}$$

$$\left(D_y + D_t + \mu_i(x)\right)P_{10}(x,t) = 0 \tag{11}$$

$$\left(D_y + D_t + \mu_i(x)\right)P_{11}(x,t) = 0 \tag{12}$$

These difference-differential equations are associated with the following boundary and initial conditions:

**Boundary Conditions**

$$P_2(0,t) = \beta_j P_0(t) + \sum_i \int \mu_i(x) P_{10}(x,t)\,dx \tag{13}$$

$$P_3(0,t) = \beta_j P_1(t) + \int \phi(w) P_7(w,t)\,dw + \sum_i \int \mu_i(x) P_{11}(x,t)\,dx \tag{14}$$

$$P_4(0,t) = \bar{a}\lambda_B P_0(t) \tag{15}$$

$$P_5(0,t) = \lambda_B P_1(t) + \sum_j \int \nu_j(y) P_6(y,t)\,dy \tag{16}$$

$$P_6(0,t) = 0 \tag{17}$$

$$P_7(0,t) = \bar{a}\lambda_B P_2(t) \tag{18}$$

$$P_8(0,t) = \alpha_i P_0(t) \tag{19}$$

$$P_9(0,t) = \alpha_i P_1(t) \tag{20}$$

$$P_{10}(0,t) = \alpha_i P_2(t) \tag{21}$$

$$P_{11}(0,t) = \alpha_i P_3(t) \tag{22}$$

**Initial Conditions**

$$P_k(0) = \begin{cases} 1 & \text{as } k = 0 \\ 0 & \text{otherwise} \end{cases} \tag{23}$$

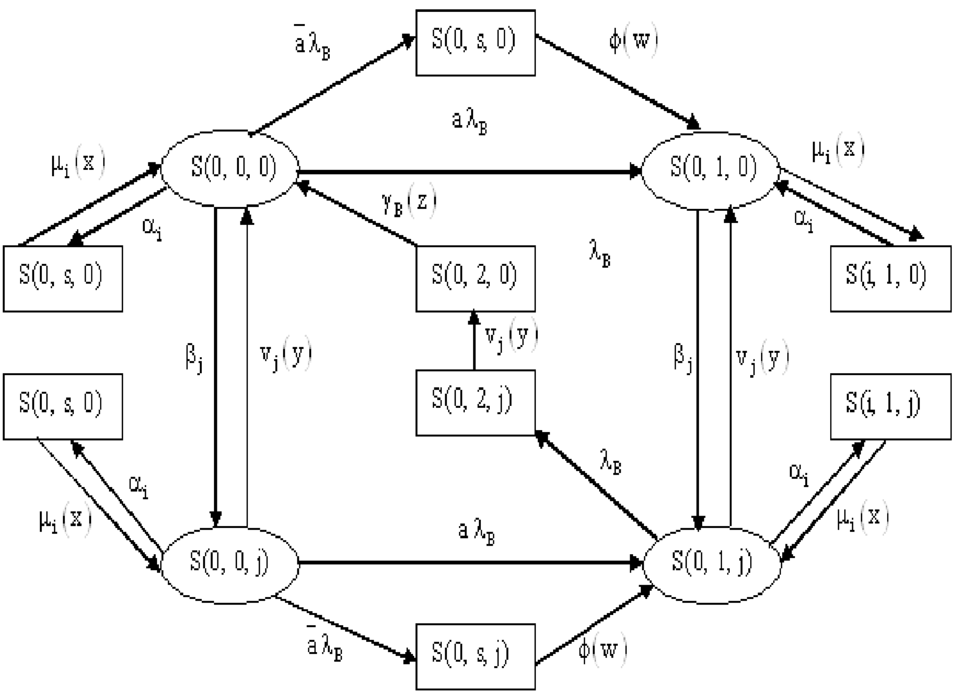


**Figure 3: Transition State Diagram**

## 6. Solution

Taking Laplace Transforms of equations (1) to (12) and using initial and boundary conditions, the solution is as follows:

$$\overline{P_0}(s) = \frac{1}{F(s)} \text{ and } \overline{P_i}(s) = E_i \cdot \overline{P_0}(s), \text{ where } i = 1, 2, 3, \ldots, 11.$$

$$E_1 = \frac{F_1}{F_2}$$

$$E_2 = \frac{\beta_j F_v(s+\alpha+\lambda_B)}{1-\sum_i \alpha_i \bar{S}_\mu(s) F_v(s+\alpha+\lambda_B)}$$

$$E_3 = \left(\beta_j E_1 + \bar{a}\lambda_B E_2 \bar{S}_\phi(s) + \sum_i \alpha_i E_3 \bar{S}_\mu(s)\right) F_v(s+\alpha+\lambda_B)$$

$$-a\lambda_B \frac{d}{ds} F_v(s+\alpha+\lambda_B)$$

$$E_4 = a\lambda_B F_\phi(s)$$

$$
\begin{aligned}
E_5 = F_\gamma(s)\Bigg[ & \lambda_B E_1 + \sum_j \lambda_B \Bigg\{ \left(\beta_j E_1 + \bar{a}\lambda_B E_2 \bar{S}_\phi(s)\right) . \frac{\bar{S}_v(s) - \bar{S}_v(s+\alpha+\lambda_B)}{\alpha+\lambda_B} \\
& + \sum_i \alpha_i E_3 \bar{S}_\mu(s) + a\lambda_B \left(\beta_j + \sum_i \alpha_i E_3 \bar{S}_\mu(s)\right). \\
& \left( \frac{\bar{S}_v(s) - \bar{S}_v(s+\alpha+\lambda_B)}{(\alpha+\lambda_B)^2} + \frac{\bar{S}'_v(s+\alpha+\lambda_B)}{(\alpha+\lambda_B)} \right) \Bigg\} \Bigg]
\end{aligned}
$$

$$
\begin{aligned}
E_6 = \lambda_B \Bigg[ & \frac{\left(\beta_j E_1 + \bar{a}\lambda_B E_2 \bar{S}_\phi(s) + \sum_i \alpha_i E_3 \bar{S}_\mu(s)\right)\left(F_v(s) - F_v(s+\alpha+\lambda_B)\right)}{\alpha+\lambda_B} + \\
& a\lambda_B \left(\beta_j + \sum_i \alpha_i E_2 \bar{S}_\mu(s)\right) \cdot \left( \frac{F_v(s) - F_v(s+\alpha+\lambda_B)}{(\alpha+\lambda_B)^2} + \frac{\frac{d}{ds} F_v(s+\alpha+\lambda_B)}{\alpha+\lambda_B} \right) \Bigg\} \Bigg]
\end{aligned}
$$

$$E_7 = \bar{a}\lambda_B E_2 F_\phi(s)$$

$$E_8 = \alpha_i F_\mu(s)$$

$$E_9 = \alpha_i E_1 F_\mu(s)$$

$$E_{10} = \alpha_i E_2 F_\mu(s)$$

$$E_{11} = \alpha_i E_3 F_\mu(s)$$

$$F_k(s) = \frac{1 - \bar{S}_k(s)}{s}$$

$$
\begin{aligned}
F_1 = {} & a\lambda_B + \bar{a}\lambda_B \bar{S}_\phi(s) + \sum \left\{ -a\lambda_B \bar{S}'_v(s+\alpha+\lambda_B) \right\} \left( \beta_j + \sum_i \alpha_i \bar{S}_\mu(s) E_2 \right) \\
& + \sum_j \bar{S}_v(s+\alpha+\lambda_B) \left( \bar{a}\lambda_B E_2 \bar{S}_\phi(s) + \sum_i \alpha_i \bar{S}_\mu(s) E_3 \right)
\end{aligned}
$$

$$F_2 = s + \alpha + \beta + \lambda_B - \sum_j \beta_j \bar{S}_v(s+\alpha+\lambda_B) - \sum_i \alpha_i \bar{S}_\mu(s)$$

and $F(s) = s + \alpha + \beta + \lambda_B - \sum_i \alpha_i \bar{S}_\mu(s) - \sum_j \left( \beta_j + \sum_i \alpha_i E_2 \bar{S}_\mu(s) \right) \bar{S}_v(s+\alpha+\lambda_B)$

$$-\bar{S}_{\gamma}(s)\lambda_B\left[E_1+\sum_j\left\{\frac{\beta_j E_1+\bar{a}\lambda_B E_2\bar{S}_{\phi}(s)+\sum_i\alpha_i E_3\bar{S}_{\mu}(s)}{\alpha+\lambda_B}\left(\bar{S}_v(s)-\bar{S}_v(s+\alpha+\lambda_B)\right)\right.\right.$$

$$\left.\left.+a\lambda_B\left(\beta_j+\sum_i\alpha_i E_2\bar{S}_{\mu}(s)\right)\left(\frac{\bar{S}_v(s)-\bar{S}_v(s+\alpha+\lambda_B)}{(\alpha+\lambda_B)^2}+\frac{\bar{S}'_v(s)-\bar{S}_v(s+\alpha+\lambda_B)}{\alpha+\lambda_B}\right)\right\}\right]$$

**Evaluation of L.T. of UP and DOWN state probabilities**

The Laplace Transforms of the probabilities that the system is in up i.e. good and down i.e. failed state at time t are as follows:

$$\bar{P}_{up}(s)=\sum_{i=0}^{3}\bar{P}_i(s)=G(s)\bar{P}_0(s) \tag{24}$$

$$\bar{P}_{down}(s)=H(s)\cdot\bar{P}_0(s) \tag{25}$$

Where $G(s)=1+E_1+\sum_j\left(E_2+E_3\right)$ and $H(s)=\frac{1}{s}-G(s)$

## 7. Steady State Behavior

Employing Abel's lemma in L.T. of prob., we get

$$P_{up}=\frac{G(0)}{F'(0)} \tag{26}$$

$$P_{down}=\frac{G(0)}{F'(0)} \tag{27}$$

Where $G(0)=\left[G(s)\right]_{s=0}$, $H(0)=\left[H(s)\right]_{s=0}$, and $F'(0)=\left[\frac{d}{ds}F(s)\right]_{s=0}$

## 8. Special Cases

**(i) Constant Repair rates:** When all repair rates follow exponential time distribution, Setting; $\bar{S}_k(s)=\frac{k}{s+k}$; up and down state prob. are as follows:

$$\bar{P}_{up}(s)=\frac{I(s)}{J(s)} \tag{28}$$

$$\bar{P}_{down}(s)=\frac{1}{s}-\frac{I(s)}{J(s)} \tag{29}$$

Where $I(s)=\left[G(s)\right]_{\text{at}\,\bar{S}_k(s)=\frac{k}{s+k}}$ and $J(s)=\left[F(s)\right]_{\text{at}\,\bar{S}_k(s)=\frac{k}{s+k}}$ (30)

The system reliability function R(t) is given by

$$R(t) = \sum_{i=0}^{3} \sum_{j} P_i(t) \tag{31}$$

The mean time to system failure (MTSF) may be evaluated as

$$MTSF = \int_0^\infty R(t)\,dt \tag{32}$$

**(ii) Perfect switch over device:** When the switching device is perfect, the result are obtained substituting a = 1 in the foregoing analysis.

**(iii) Cold Standby:** When the failure rates of a standby unit is zero, the result for a cold standby may be evaluated.

**(iv) Non-repairable system:** If the system is non-repairable, then taking prob. Independent of x

$$R(t) = \left(1 - \frac{\beta}{\lambda_B}\right).e^{-(\alpha+\beta+\lambda_B).t} + \left(\frac{\beta}{\lambda_B} + a\lambda_B t\right).e^{-(\alpha+\lambda_B).t} \tag{33}$$

$$MTSF = \frac{1}{(\alpha+\lambda_B+\beta)} + \frac{\beta^2}{\lambda_B(\alpha+\lambda_B)(\alpha+\lambda_B+\beta)} + \frac{a\lambda_B}{(\alpha+\lambda_B)^2} \tag{34}$$

and

$$H(t) = C_1\left[\left(1-\frac{\beta}{\lambda_B}\right)\frac{1-e^{-(\alpha+\beta+\lambda_B).t}}{(\alpha+\beta+\lambda_B)} - \left(\frac{\beta}{\lambda_B}+a\lambda_B t\right)\cdot\frac{e^{-(\alpha+\lambda_B).t}}{(\alpha+\lambda_B)} + \frac{\beta}{\lambda_B(\alpha+\lambda_B)} + a\lambda_B\left(\frac{1-e^{-(\alpha+\lambda_B).t}}{(\alpha+\lambda_B)^2}\right)\right] - C_2\cdot t - C_3$$

where $C_1$ : revenue cost per unit time

$C_2$ : service cost per unit time

$C_3$ : system establishment cost.

## 9. Numerical Computations

Inserting, $\alpha = 0.1, \beta = 0.1, a = 1$, for a non-repairable system;

- **Reliability Analysis:** Here $\lambda_B = 0.2$,
  $R(t)=0.5e^{-0.4t}+(0.5+0.2t)e^{-0.3t}$ (35)
- **Profit function Analysis:** Here $\lambda_B = 0.2$
  $H(t) = C_1\,[1.25(1- e^{-0.4t})-3.33(0.5+0.2t)\,e^{-0.3t} +1.67+1.11\,(1- e^{-0.3t})]-C_2t-C_3$

- **MTSF Analysis**

$$MTSF = \frac{1}{(\alpha+\lambda_B+\beta)} + \frac{\beta^2}{\lambda_B(\alpha+\lambda_B)(\alpha+\lambda_B+\beta)} + \frac{a\lambda_B}{(\alpha+\lambda_B)^2} \quad (36)$$

$$MTSF(\alpha=0.1) = \frac{1}{(0.1+\lambda_B+\beta)} + \frac{\beta^2}{\lambda_B(0.1+\lambda_B)(0.1+\lambda_B+\beta)} + \frac{a\lambda_B}{(0.1+\lambda_B)^2} \quad (37)$$

$$MTSF(\lambda_B=0.2) = \frac{1}{(0.2+\alpha+\beta)} + \frac{\beta^2}{0.2(0.2+\alpha)(0.2+\alpha+\beta)} + \frac{0.2}{(0.2+\alpha)^2} \quad (38)$$

## 10. Discussion

Figure 4 shows the reliability of the system w.r.t time. A critical examination of the graph reliability v/s time reveals that the reliability of the system decreases with time and ultimately, after a sufficient long interval of time, if becomes steady to the value zero.

Figure 5 determines the expected profit during the interval (0, t). A critical view of the graph expected profit v/s time reveals that the expected profit, which decreases apparently as the service cost $C_2$ approaches towards 1.

Figure 6, Figure 7 and Figure 8 depict the plots of MTSF with different types of failure rates.

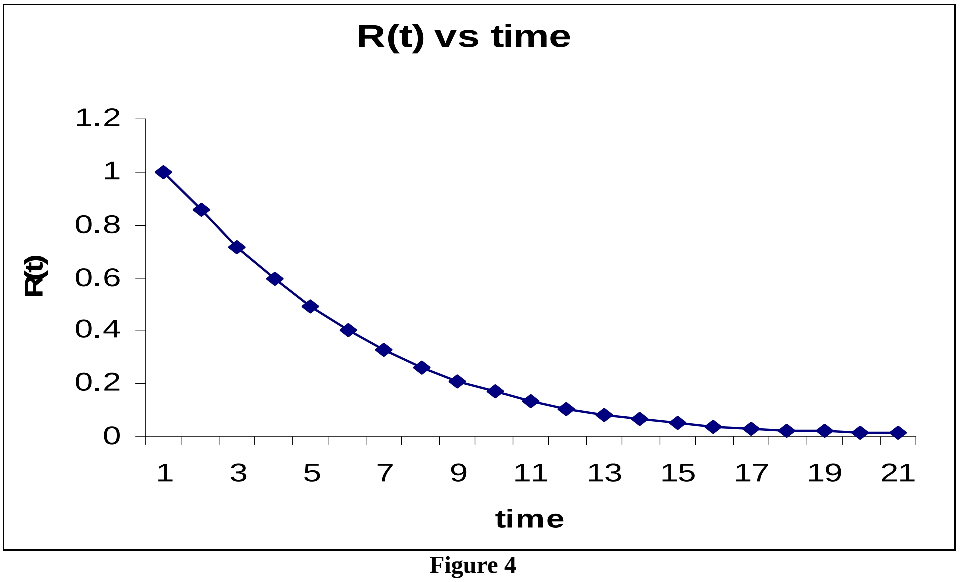


**Figure 4**

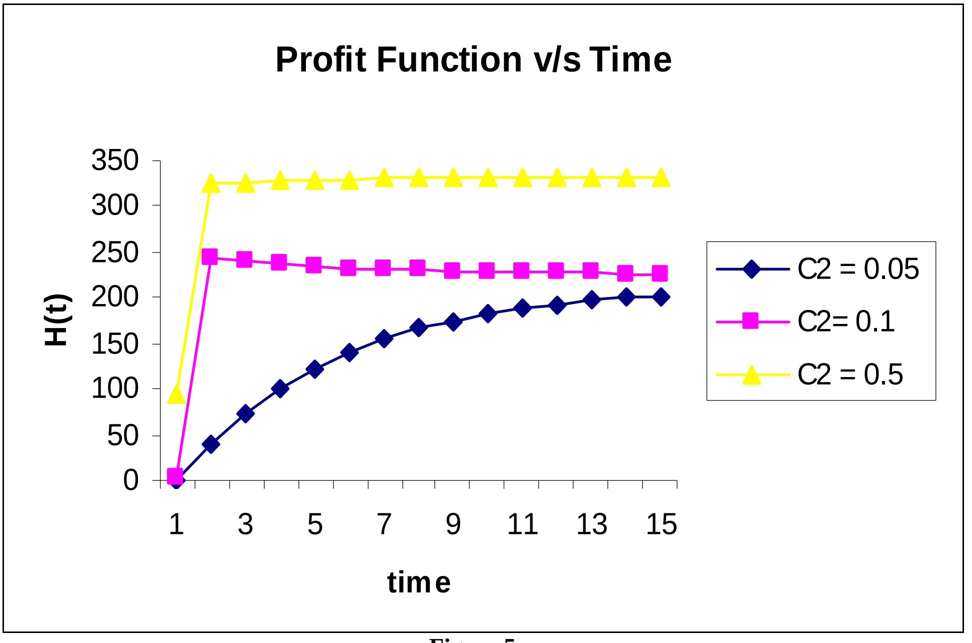


**Figure 5**

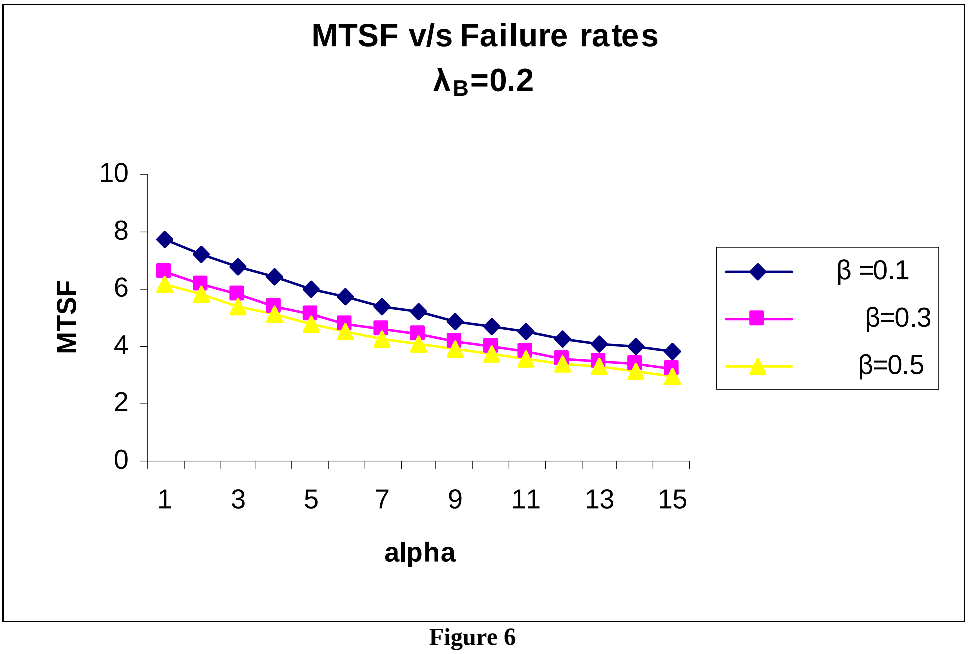


**Figure 6**

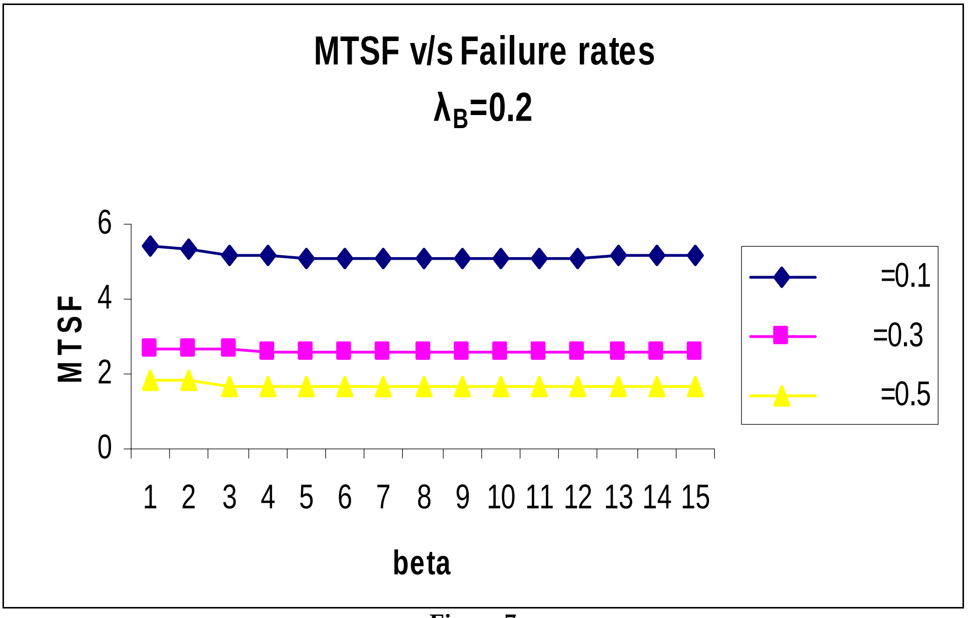


**Figure 7**

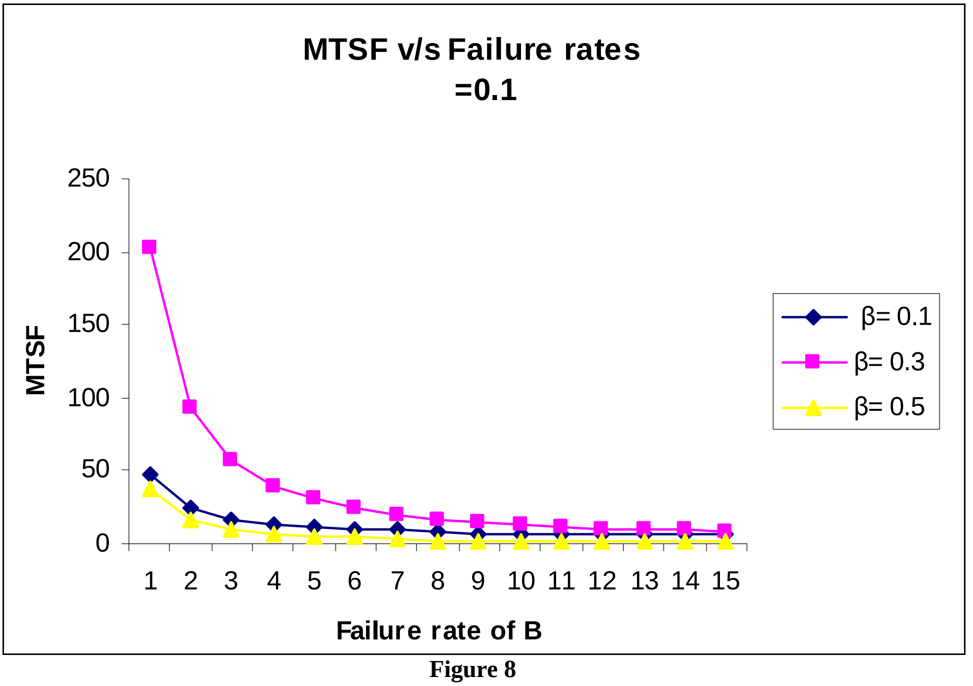


**Figure 8**

## References

1. Cui, L. and Li, H. (2007). Analytical method for reliability and MTTF assessment of coherent systems with dependent components. Reliability Engineering & System Safety, Vol. 92 No.3, p.300-7.
2. Dinesh, K (1991). Analysis and optimization of system availability in sugar, paper and fertilizer industries. Ph.D. Thesis, university of Roorkee, India.
3. Gupta, P., Lal, A.K., Sharma, R.K. and Singh, J. (2005). Numerical analysis of reliability and availability of the serial processes in butter-oil processing plant. International Journal of Quality & Reliability Management. Vol. 22 No.3, p.303-16.
4. Gupta, P. P. and Agarwal, S. C. (1984). A parallel redundant complex system with two types of failure under preemptive-repeat repair discipline. Microelectronics Reliability, Vol. 24 No.3, p. 395-9.
5. Kansal, M. L., Kumar, A. and Sharma, P. B. (1995). Reliability analysis of water distribution systems under uncertainty. Reliability Engineering and System Safety, Vol. 50 p. 51-59.
6. Ke, J. B., Lee, W. C. and Wang, K. H. (2007). Reliability and sensitivity analysis of a system with multiple unreliable service stations and standby switching failures. Physica A: Statistical Mechanics and its Applications, Vol. 380, p.455-69.
7. Mangey, R. and Singh, S.B. (2008). Availability and cost analysis of a parallel redundant complex system with two types of failure under preemptive-resume repair discipline using Gumbel-Hougaard family copula in repair. International Journal of Reliability, Quality and Safety Engineering, Vol. 15 No.4, p. 341-65.
8. Sarkar, J. and Sarkar, S. (2001). Availability of a periodically inspected system supported by a spare unit, under perfect repair and perfect upgrade. Statistics and Probability Letters, Vol. 53, p. 207-17.
9. Shao, J. and Lamberson, L.R. (1991). Modeling a shared load K-out-of-N: G system. IEEE Trans. on Reliability, Vol. 40 No.2, p. 205-209.
10. Singh, J. (1989). Behaviour analysis of refining system in sugar industry. Proceedings of international symposium on stochastic Modelling.
11. Singh J. and Geol, P. (1996). Availability analysis of heating system with stand by and imperfect switch in sugar industry. Proceedings of National Conference on Operations Research in Modern Technology, p. B-39 to 45.