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PROBABILISTIC ANALYSIS OF A SYSTEM WITH SERVER FAILURE DURING REPAIR

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Abstract

This paper considers two reliability models developed for a single-unit system where the repair facility (server) is subject to failure during repair. There is a single server who attends the system immediately whenever needed. In model I, the server undergoes for treatment upon failure while in model II, the inspection of the server is done at its failure to know the feasibility of treatment. If treatment of the server is not feasible, it is replaced by another server. The system is discussed probabilistically in detail and the expressions for various reliability quantities are derived adopting semi-Markov process and regenerative point technique. The failure time of the unit and server are distributed exponentially while the distributions of repair time of the unit, treatment and inspection of the server are taken as arbitrary. The results for a particular case are drawn to depict the behavior of some measures of system effectiveness graphically.

Key words: Server Failure, Reliability Models, Treatment, Inspection and Probabilistic Analysis.

2000 Mathematics Subject Classification: Primary 90B25 and Secondary 60K10

1. Introduction

In most of the studies including [1-5] pertaining to reliability models of repairable systems, it is usually assumed that the repair facility neither fails nor deteriorates. In practice, the repair facility in a repairable system is subject to failure and can be treated or replaced after it fails. For example, a server may fail when an accident takes place during repair due to some causes such as mishandling of the system, electric shock and carelessness. And, if the accident takes place is major, the server may be replaced by another to continue the remaining repair. But in case of minor accident, the server may resume the job after taking some treatment. Cao and Wu (1989) evaluated reliability of a two-unit cold stand by system with replaceable repair facility.

While considering above practical situations in mind, two reliability models are developed for a single-unit system where the repair facility (server) is subject to failure during repair. A single server is available to the system who attends the system immediately whenever needed. Two reliability models are developed to study the system probabilistically in detail. In model I, the server undergoes for some treatment at failure whereas in model II, the inspection of the server is done at its failure to probe the feasibility of the treatment. If treatment of the server is not feasible, it is replaced by another similar one who resumes the repair. The failure, repair, inspection and treatment times are considered as independent and uncorrelated random variables. The failure time of the unit and server follow negative exponential distributions while that of repair, inspection and treatment times are taken as arbitrary. The failures are self announcing and switching is perfect and instantaneous.

It is assumed that the server neither fails nor deteriorates in the idle periods. After repair, the failed unit and server work as good as new. To analyze the system probabilistically in detail, expressions for some reliability characteristics such as mean sojourn times, mean time to system failure (MTSF), availability, busy period of the server, expected number of inspections for the server, expected number of treatments given to the server and expected number of visits by the server are derived by making use of semi-Markov process and regenerative point technique. The profit function is also derived for each system model to carry out the cost-benefit analysis. The numerical results for a particular case are obtained to depict the behavior of MTSF, Availability and Profit of the system models graphically.

2. Notations

E	:	Set of regenerative states.
O	:	The unit is operative and in normal mode.
SG	:	The server is good.
λ	:	Constant failure rate of the unit.
μ	:	Constant failure rate of the server.
a /b	:	Probability that treatment of server is feasible / not feasible.
FUr / FWr	:	The Unit is failed and under repair / waiting for repair.
SFUt / SFUi	:	The server is failed and under treatment / under inspection.
g(t) / G(t)	:	pdf / cdf of repair time of the unit .
f(t) / F(t)	:	pdf / cdf of treatment time of the server .
h(t) / H(t)	:	pdf / cdf of inspection time of the server.
$q_{ij}(t)$/ $Q_{ij}(t)$	:	pdf and cdf of direct transition time from a regenerative state i to a regenerative state j without visiting any other regenerative state.
$q_{ij.k}(t)$ / $Q_{ij.k}(t)$	:	pdf and cdf of first passage from a regenerative state i to a regenerative state j or to a failed state j visiting state k once in (0,t].
$M_i(t)$	:	Probability that the system is up Initially instate $S_i \in$ Eis up at time t without visiting to any other regenerative state.
$W_i(t)$	:	Probability that the server is in state Si upto time t without making transition to any other regenerative state or returning to the same via one or more non-regenerative states.
m_{ij}	:	Contribution to mean sojourn time in state S_i when system transists directly to state S_j ($S_i, S_j \in E$) so that $\mu_i = \sum m_{ij}$ where $m_{ij} = \int t\, dQ_{ij}(t) = -q_{ij}^{*/}(0)$ and μ_i is the mean sojourn time in state $S_j \in$ E
(s) / ©	:	Symbol for Stieltjes convolution / Laplace convolution.
~ / *	:	Symbol for Laplace Stieltjes Transform (LST) / Laplace Transform (LT).
/ **(**desh**)**	:	Symbol for derivative of the function.

The transition states for model I are regenerative while for model II, the states S_0, S_1, S_2 are regenerative and S_3 is non regenerative.

The possible transition between states along with transition rates for the system models are shown in figures 1 and 2 :

STATE TRANSITION DIAGRAM

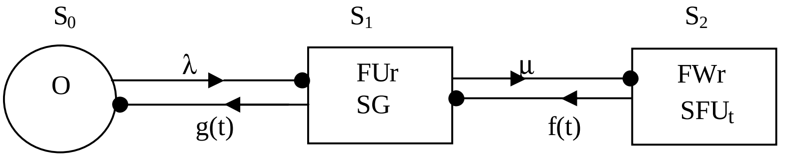

Fig. 1: (For Model I)

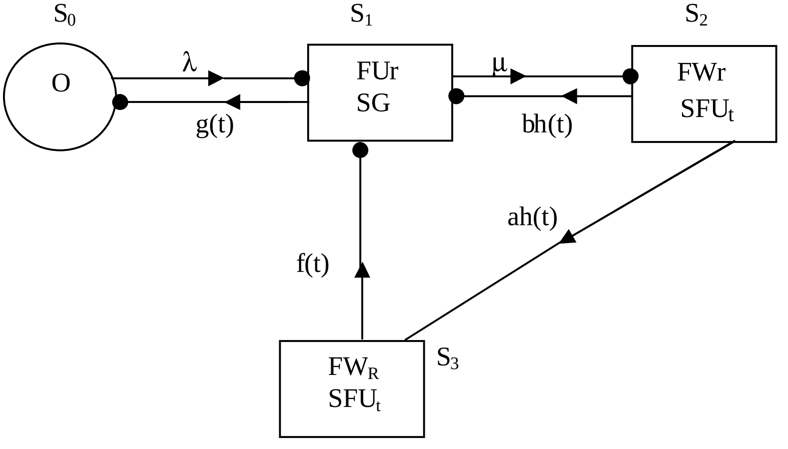

Fig. 2: (For Model II)

● : Transition point
○ : Up-State
□ : Failed State

3. Transition Probabilities and Mean Sojourn Times

Simple probabilistic considerations yield the following expressions for the non-zero elements

$p_{ij} = Q_{ij}(\infty) = \int_0^\infty q_{ij}(t)dt$ as

For both models

$dQ_{01}(t) = \lambda e^{-\lambda t} dt$, $\quad d Q_{10}(t) = e^{-\mu t} g(t)dt$, $\quad dQ_{12}(t) = \mu e^{-\mu t} \bar{G}(t) dt$

For model I

$dQ_{21}(t) = f(t)dt$

For model II

$dQ_{21}(t) = b\, h(t)dt$, $\quad dQ_{23}(t) = a\, h(t)dt$

$dQ_{31}(t) = f(t)dt$, $dQ_{21.3}(t) = dQ_{23}(t) © dQ_{31}(t)$

Letting $t \to \infty$, using $p_{ij} = Q_{ij}(\infty)$, we get

For model I

$p_{01} = 1$, $p_{10} = g^*(\mu)$, $p_{12} = 1 - g^*(\mu)$, $p_{21} = f^*(0)$

For model II

The values of p_{01}, p_{10}, p_{12} are same as defined for model I, while the others are

$p_{21} = bh^*(0)$, $p_{23} = a\, h^*(0)$, $p_{31} = f^*(0)$, $p_{21.3} = a\, h^*(0)\, f^*(0)$

It can be easily verified that

$p_{01} = 1 = p_{10} + p_{12} = p_{21}$ and $p_{21} + p_{21.3} = 1$

Mean sojourn times are

$\mu_0 = m_{01} = \int_0^\infty P(T > t)dt = 1/\lambda$

$\mu_1 = m_{10} + m_{12} = \dfrac{(1 - g^*(m))}{m}$ (for both models)

$\mu_2 = m_{21} = - f^{*/}(0)$ (for model I)

$\mu^{|}_2 = m_{21} + m_{21.3} = [h^{*/}(0) + a\, f^{*/}(0)$ (for model II)

4. Reliability and Mean Time to System Failure (MTSF)

Let $\Phi_i(t)$ be the cdf of the first passage time from regenerative state i to a failed state regarding the failed state as absorbing state. We have the following recursive relations for $\Phi_i(t)$:

$\Phi_0(t) = Q_{01}(t)$ (for both models) (4.1)

Taking LST of above relation (4.1), we get

$$\tilde{\Phi}_0(s) = \tilde{Q}_{01}(s)$$

Now $$R^*(s) = \frac{1 - \tilde{\Phi}_0(s)}{s} = \frac{1}{(l + s)} \qquad (4.2)$$

Taking Laplace inverse transform of (4.2), the reliability R(t) of the system models can be given by $R(t) = e^{-\lambda t}$, $t > 0$ (4.3)

And, mean time to system failure (MTSF) is given by

$$\text{MTSF} = \lim_{s \to 0} R^*(s) = 1/\lambda \quad \text{(for both models)} \qquad (4.4)$$

5. Steady State Availability

Let $A_i(t)$ be the probability that the system is in up state at instant t given that the system entered regenerative state i at t = 0.The recursive relation for $A_i(t)$ are as follows:

For model I

$A_0(t) = M_0(t) + q_{01}(t) © A_1(t)$

$A_1(t) = q_{10}(t) \copyright A_0(t) + q_{12}(t) \copyright A_2(t)$

$A_2(t) = q_{21}(t) \copyright A_1(t)$ (5.1)

For model II

The expression for $A_0(t)$ and $A_1(t)$ are same as given in 5.1 while the remaining is

$A_2(t) = [q_{21}(t) + q_{21.3}(t)] \copyright A_1(t)$ (5.2)

where, $M_0(t) = e^{-\lambda t}$

Now taking L.T. of relations (5.1) & (5.2) and obtain the value of $A_0^*(s)$, which is given by

$$A_0^*(s) = \frac{M_0^*(s)\,[1 - q_{12}^*(s)\, q_{21}^*(s)]}{[1 - q_{12}^*(s)\, q_{21}^*(s) - q_{10}^*(s)\, q_{01}^*(s)]} \quad \text{(for model I)} \qquad (5.3)$$

$$A_0^*(s) = \frac{M_0^*(s)\,[1 - q_{12}^*(s)\{q_{21}^*(s) + q_{21.3}^*(s)\}]}{[1 - q_{12}^*(s)\{q_{21}^*(s) + q_{21.3}^*(s)\} - q_{10}^*(s)\, q_{01}^*(s)]} \quad \text{(for model II)} \qquad (5.4)$$

The steady state availability is given by

For model I

$$A_{10} = \lim_{s \to 0} A_0^*(s) = N_{11}/D_{11} \qquad (5.5)$$

where, $N_{11} = \mu_0 p_{10}$ and $D_{11} = \mu_0 p_{10} + \mu_1 + \mu_2 p_{12}$

For model II

$$A_{20} = \lim_{s \to 0} A_0^*(s) = N_{21}/D_{21} \qquad (5.6)$$

where, $N_{21} = \mu_0 p_{10}$ and $D_{21} = \mu_0 p_{10} + \mu_1 + \mu_2' p_{12}$

6. Busy Period Analysis for the Server

Let $B_i(t)$ be the probability that the system is busy at instant t given that the system entered regenerative state i at t = 0.The recursive relation for $B_i(t)$ are as follows:

For model I

$B_0(t) = q_{01}(t) \copyright B_1(t)$

$B_1(t) = W_1(t) + q_{10}(t) \copyright B_0(t) + q_{12}(t) \copyright B_2(t)$

$B_2(t) = q_{21}(t) \copyright B_1(t)$ (6.1)

For model II

$B_0(t) = q_{01}(t) \copyright B_1(t)$

$B_1(t) = W_1(t) + q_{10}(t) \copyright B_0(t) + q_{12}(t) \copyright B_2(t)$

$B_2(t) = [q_{21}(t) + q_{21.3}(t)] \copyright B_1(t)$ (6.2)

where

$W_1(t) = e^{-\mu t}\, \bar{G}(t)$ (for both models)

Now taking L.T. of relations (6.1) & (6.2) and obtain the value of $B_0^*(s)$

$$B_0^*(s) = \frac{W_1^*(s)q_{01}^*(s)}{[1 - q_{12}^*(s)\, q_{21}^*(s) - q_{10}^*(s)\, q_{01}^*(s)]} \quad \text{(for model I)} \qquad (6.3)$$

$$B_0^*(s) = \frac{W_1^*(s)q_{01}^*(s)}{}$$

$$[1 - q_{12}^{*}(s)\{q_{21}^{*}(s) + q_{21.3}^{*}(s)\} - q_{10}^{*}(s)\, q_{01}^{*}(s)]$$

(for model II) (6.4)

$B_{i0} = \lim_{s \to 0} B_0^{*}(s)$ (i = 1,2)

For model I

$B_{10} = N_{12}/D_{11}$, where $N_{12} = \mu_1 p_{01}$ and D_{11} is already defined.

For model II

$B_{20} = N_{22}/D_{21}$, where $N_{22} = \mu_1 p_{01}$ and D_{21} is already defined.

7. Expected Number of Inspections for the Server

Let $I_i(t)$ be expected no. of inspection of the server in (0,t] given that the system entered regenerative state i at t = 0.The recursive relation for $I_i(t)$ are as follows:

For Model II

$$I_0(t) = Q_{01}(t) \circledS I_1(t)$$
$$I_1(t) = Q_{10}(t) \circledS I_0(t) + Q_{12}(t) \circledS [1 + I_2(t)]$$
$$I_2(t) = [Q_{21}(t) + Q_{21.3}(t)] \circledS I_1(t) \qquad (7.1)$$

Now taking L.T. of relation (7.1) and solving for $I_0^{**}(s)$, we obtain the fraction of time for which server is under inspection as

$$I_0^{**}(s) = \frac{Q_{01}^{**}(s)\, Q_{12}^{**}(s)}{[1 - Q_{12}^{**}(s)\{Q_{21}^{**}(s) + q_{21.3}^{**}(s)\} - Q_{10}^{**}(s)\, Q_{01}^{**}(s)]} \qquad (7.2)$$

The expected number of inspections for the server are given by

$I_{20} = \lim_{s \to 0} s\, I_0^{**}(s)$

$I_{20} = N_{23}/D_{21}$, where $N_{23} = p_{12}$ and D_{21} is already defined

8. Expected Number of Treatments Given to the Server

Let $T_i(t)$ be expected number of Treatments of the server in (0,t] given that the system entered regenerative state i at t = 0.The recursive relations for $T_i(t)$ are as follows:

For Model I

$$T_0(t) = Q_{01}(t) \circledS T_1(t)$$
$$T_1(t) = Q_{10}(t) \circledS T_0(t) + Q_{12}(t) \circledS [1 + T_2(t)]$$
$$T_2(t) = Q_{21}(t) \circledS T_1(t) \qquad (8.1)$$

For Model II

$$T_0(t) = Q_{01}(t) \circledS T_1(t)$$
$$T_1(t) = Q_{10}(t) \circledS T_0(t) + Q_{12}(t) \circledS T_2(t)$$
$$T_2(t) = Q_{21.3}(t) + [Q_{21}(t) + Q_{21.3}(t)] \circledS [1+ T_1(t)] \qquad (8.2)$$

Now taking L.T. of relations (8.1) & (8.2) and solving for $T_0^{**}(s)$, we obtain the fraction of time for which server is under Treatment as

$$T_0^{**}(s) = \frac{Q_{01}^{**}(s)\, Q_{12}^{**}(s)}{[1-Q_{12}^{**}(s)Q_{21}^{**}(s)-Q_{10}^{**}(s)Q_{01}^{**}(s)]} \quad \text{(for model I)} \qquad (8.3)$$

$$T_0^{**}(s) = \frac{Q_{01}^{**}(s)\, Q_{12}^{**}(s)\;\; Q_{21.3}^{**}(s)}{[1-Q_{12}^{**}(s)\{Q_{21}^{**}(s)+Q_{21.3}^{**}(s)\}-Q_{10}^{**}(s)\, Q_{01}^{**}(s)]} \quad \text{(for model II)} \qquad (8.4)$$

The expected number of treatments given to the server are given by

For Model I

$T_{10} = \lim_{s \to 0} s\, T_0^{**}(s) = N_{13}/D_{11}$

where, $N_{13} = p_{01}p_{12}$ and D_{11} is already defined.

For Model II

$T_{20} = \lim_{s \to 0} s\, T_0^{**}(s) = N_{24}/D_{21}$

where, $N_{24} = p_{12}\, p_{21.3}$ and D_{21} is already defined.

9. Expected Number of Visits of the Server

Let $N_i(t)$ be expected number of visits of the server in (0,t] given that the system entered regenerative state i at t = 0.The recursive relation for $N_i(t)$ are as follows:

For Model I

$N_0(t) = Q_{01}(t)\, s\, [1+N_1(t)]$

$N_1(t) = Q_{10}(t) © N_0(t) + Q_{12}(t) © N_2(t)$

$N_2(t) = Q_{21}(t) © [1+N_1(t)]$ (9.1)

For Model II

$N_0(t) = Q_{01}(t)\, (s)\, [1+N_1(t)]$

$N_1(t) = Q_{10}(t)\, (s)\, N_0(t) + Q_{12}(t)\, (s)\, N_2(t)$

$N_2(t) = (Q_{21}(t) + Q_{21.3})\, (s)\, [1+N_1(t)]$ (9.2)

Now taking L.T. of relations (9.1) & (9.2) and solving for $N_0^{**}(s)$, we obtain the fraction of time for which server visits system as

$$N_0^{**}(s) = \frac{Q_{01}^{**}(s)}{[1- Q_{12}^{**}(s)Q_{21}^{**}(s)-Q_{10}^{**}(s)\, Q_{01}^{**}(s)]} \quad \text{(for model I)} \qquad (9.3)$$

$$N_0^{**}(s) = \frac{Q_{01}^{**}(s)}{[1- Q_{12}^{**}(s)\{Q_{21}^{**}(s) + Q_{21.3})\}-Q_{10}^{**}(s)\, Q_{01}^{**}(s)]} \quad \text{(for model II)} \qquad (9.4)$$

The expected number of visits by the server are given by

$N_{10} = \lim_{s \to 0} s\, N_0^{**}(s) = N_{14}/D_{11}$

where

$N_{14} = p_{01}$ and D_{11} is already defined (for model I)

$N_{20} = \lim_{s \to 0} s\, N_0^{**}(s) = N_{25}/D_{21}$

where

$N_{25} = p_{01}$ and D_{21} is already defined. (for model II)

10. Profit Analysis

Profit incurred to the system models in steady state are given by

$P_1 = K_0 A_{10} - K_1 B_{10} - K_4 N_{10}$ (for model I) (10.1)

$P_2 = K_0 A_{20} - K_1 B_{20} - K_2 I_{20} - K_3 T_{20} - K_4 N_{20}$ (for model II) (10.2)

where

K_0 = Revenue per unit up time of the system;

K_1 = Cost per unit time for which server is busy;

K_2 =Cost per unit time for which server is under inspection;

K_3 =Cost per unit time for which server is under treatment;

K_4 = Cost per visit by the server ;

11. Particular Case

Let us take

$g(t) = \alpha e^{-\alpha t}$, $f(t) = \beta e^{-\beta t}$, $h(t) = \gamma e^{-\gamma t}$ (11.1)

we have

$p_{01} = 1$, $p_{10} = \alpha / (\alpha + \mu)$, $p_{12} = \mu / (\alpha + \mu)$ (for both models)

$p_{21} = 1$ (For model I) and $p_{21} = b$, $p_{21.3} = a$ (for model II)

By using these results, we get the following results:

MTSF = μ_0 (for both models)

Availability

$A_{10} = N_{11}/D_{11}$, $A_{20} = N_{21}/D_{21}$

Busy Period

$B_{10} = N_{12}/D_{11}$, $B_{20} = N_{22}/D_{21}$

Expected No. of Inspections

$I_{20} = N_{23}/D_{21}$

Expected No. of Treatments

$T_{10} = N_{13}/D_{11}$, $T_{20} = N_{24}/D_{21}$

Expected No. of Visits

$N_{10} = N_{14}/D_{11}$, $N_{20} = N_{25}/D_{21}$

where

$N_{11} = N_{21} = \alpha/\lambda(\alpha + \mu)$, $D_{11} = [\alpha\beta + \beta\lambda + \lambda\mu] / \beta\lambda(\alpha + \mu)$

$D_{21} = [\alpha\beta\gamma + \beta\lambda\gamma + \lambda\mu(\beta + a\gamma)] / \beta\lambda\gamma(\alpha + \mu)$

$N_{12} = N_{22} = 1/(\alpha + \mu)$, $N_{13} = N_{23} = \mu/(\alpha + \mu)$, $N_{14} = 1$

$N_{24} = a\mu/(\alpha + \mu)$, $N_{25} = 1$

12. Conclusion

The mean time to system failure (MTSF) of the system models is same which remains constant with the change of treatment rate (β) of the server as shown in figure 3. From this, it can be seen that MTSF decreases with the increase of failure rate (λ) of the unit. Figures 4 and 5 indicate that availability and profit of the models keep on increasing with the increase of treatment rate (β) for fixed values of other parameters. And, there is a decrease in their values when failure rates λ and μ of the unit and server respectively increase separately. However, availability and profit increase with the increase of inspection rate (γ). It is interesting to note that model II has more values of availability and profit. On the basis of results obtained for a particular case, it is assessed that the concept of inspection to examine the possibility of treatment or replacement of the server at its failure is reliable and economical as compared to give treatment to the server without knowing its feasibility.

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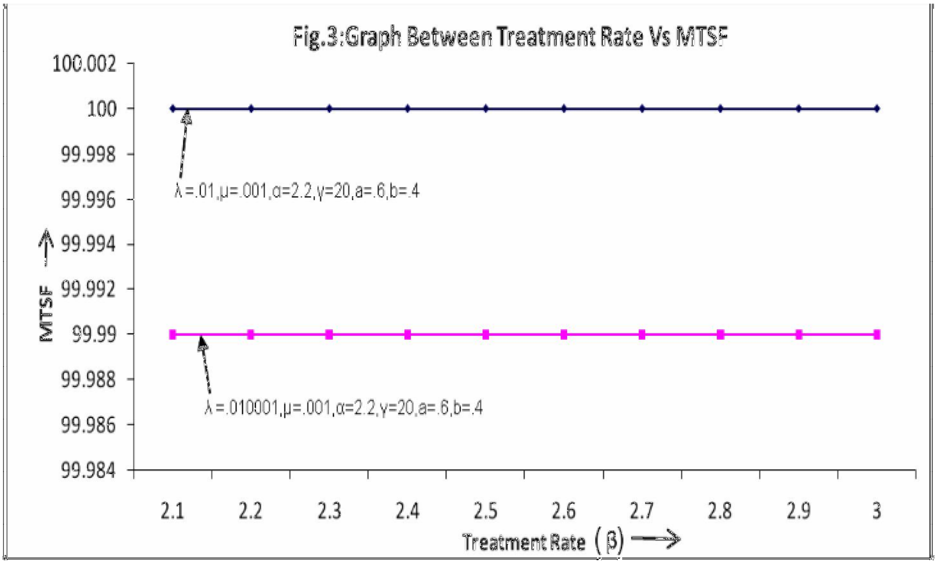

Fig.4:Graph Between Treatment Rate Vs Availability

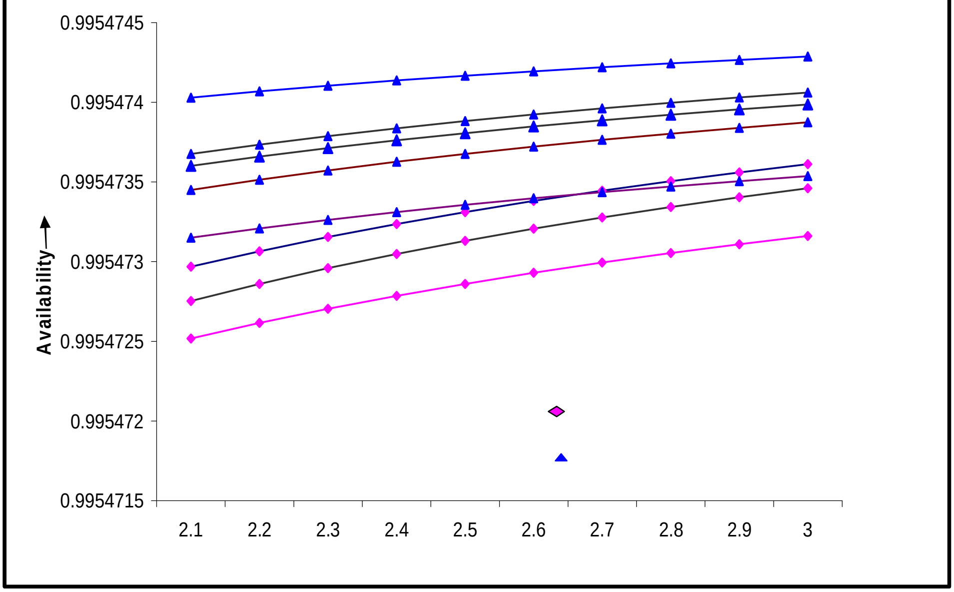

Fig.5:Graph Between Treatment Rate Vs Profit