Journal of Reliability and Statistical Studies [ISSN: 0974-8024 (Print), 2229-5666 (Online)]
Vol. 3, Issue 2 (2010): 79-91

STOCHASTIC MODELING OF A SYSTEM WITH PRIORITY FOR OPERATION TO NEW UNIT AND PREVENTIVE MAINTENANCE AT PFS SUBJECT TO MOT

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Abstract

A stochastic model for a cold standby system of two identical units is developed in which unit fails completely either directly from normal mode or via partial failure. The unit undergoes preventive maintenance (PM) after a maximum operation time (MOT) at its partial failure stage (PFS). There is a single server who visits the system immediately as and when needed to do preventive maintenance and repair. The priority for operation to new unit is given over the partially failed unit. The failure and maximum operation times of the unit are distributed exponentially while the distributions of PM and repair times are taken as arbitrary. The expressions for some reliability measures of the system are derived using semi-Markov process and regenerative point technique. The numerical results for the particular values of parameters and various costs are obtained to show the behavior of MTSF, availability and profit.

Key Words: Stochastic Model, Priority for Operation, Preventive Maintenance, Partial Failure Stage, MOT and Reliability Measures.

2000 Mathematics Subject Classification: Primary 90 B25 and Secondary 60K10

1. Introduction

Over the last few decades, the stochastic models of maintained systems have been probed by many scholars and practitioner including Gopalan and Naidu (1983), Goel and Sharma (1989) and Singh (1989) due to their applications to variety of areas such as military, industry, health and the environment. In most of these models, it is assumed that

(i) The operating unit enters directly into the complete failed state with constant failure rate.

(ii) The unit works continuously till failure without undergoing preventive maintenance.

(iii) There is no need to give priority for operation to a new unit.

However, in practice, there are many situations where a unit operates on various degraded stages before its total failure and thus it may fail completely either directly from normal mode or via partial failure. Further, the continued operation and ageing of the systems gradually reduce their performance, reliability and safety. Therefore, preventive maintenance of the unit is necessary after a specific period of time at any stage of operation not only to maintain the operational power but may also to improve the reliability and availability of the system. Singh and Agarafiotis [1995] have studied a system with preventive maintenance subject to maximum operation and repair time. Further, the availability of a system can greatly be improved by assigning priority for operation to new unit over the partially failed unit. Chander [2005] has evaluated reliability and economic measures of a system introducing the concept of priority to operation and repair.

The purpose of the present paper is to study the stochastic model of a two-unit cold standby system in which unit fails completely either directly from normal mode or via partial failure. The preventive maintenance of the unit is carried out after a maximum operation time (MOT) at its partial failure stage (PFS). There is a single server who visits the system immediately whenever needed to carry out preventive maintenance and repair. The priority for operation to new unit is given over the partially failed unit. The unit works as good as new after preventive maintenance and repair. The repair of the unit is done only at its complete failure. The switch devices are considered as perfect. The random variables are assumed as independent and uncorrelated to each other. The failure and maximum operation times of the unit are distributed exponentially while the distributions of PM and repair times are taken as arbitrary. The expressions for some reliability measures of the system such as mean sojourn times, mean time to system failure (MTSF), availability, busy period of the server due to PM and repair, expected number of visits by the server and the profit function are derived using semi-Markov process and regenerative point technique. For the particular values of parameters and various costs, the numerical results are obtained to show the behavior of MTSF, availability and profit.

2. Notations

E_0	:	The state of the system at t = 0
E	:	The set of regenerative states
O	:	The unit is operative and in normal mode
Cs	:	The unit in cold standby
$\lambda/\lambda_1/\lambda_2$	:	Constant failure rate of the unit from normal mode to complete failure / normal mode to partial failure / partial failure to complete failure
α_0	:	Maximum constant rate of operation after partial failure
Pm / PM	:	Unit under preventive maintenance / preventive maintenance is continued from previous state
WPm	:	Unit is partially failed and waiting for preventive maintenance
Fwr/ FUr /FUR	:	Unit is completely failed and waiting for repair / under repair / under repair continuously from previous state
PFO/PFS	:	Unit is in partial failure mode and operative/ in cold standby
f(t) / F(t)	:	pdf/cdf of the time for preventive maintenance of the unit after maximum operation time
g(t) / G(t)	:	pdf / cdf of the time for repair of a direct failed unit
$q_{ij}(t), Q_{ij}(t)$	:	p.d.f and c.d.f of first passage time from regenerative state *i* to a regenerative state *j* or to a failed state *j* without visiting any other regenerative state in (0,t].
$q_{ij.kr}(t), Q_{ij.kr}(t)$	:	p.d.f and c.d.f of first passage time from regenerative state *i* to a regenerative state *j* or to a failed state *j* visiting state k, r once in (0,t].
$M_i(t)$	:	Probability that the system up initially in state $S_i \varepsilon E$

		is up at time t without visiting to any other regenerative sate
pdf / cdf	:	Probability density function / cumulative distribution function
$W_i(t)$	:	Probability that the server is busy in the state S_i up to time t without making any transition to any other regenerative state or returning to the same via one or more non-regenerative states
Ⓢ/©	:	Symbols for Stieltjes convolution/Laplace convolution
~|*	:	Symbols for Laplace Stieltjes transform (LST)/ Laplace transform (LT)
'	:	Symbol for derivative of the function
•	:	A time point (called regenerative point) at which the system history prior to it, is irrelevant to the system conditions.

Considering these symbols, the following are possible transition states of the system model:

S_0 = (0, Cs), S_1 = (O, PFS), S_2 = (O, FUr),
S_3 = (O, Pm), S_4 = (PFO, PUr), S_5 = (PFO, PFS),
S_6 = (PFO, Pm), S_7 = (PFO, FUR), S_8 = (FUR, Fwr),
S_9 = (FUR, WPm), S_{10} = (PFO, PM), S_{11} = (Fwr, PM)
S_{12} = (PM, WPm)

The states S_0 to S_6 are regenerative while the others are non-regenerative.

3. Transition Probabilities and Mean Sojourn Times

Simple probabilistic consideration yield the following expressions for the non-zero elements $p_{ij} = Q_{ij}(\infty) = \int q_{ij}(t)\,dt$ as

$$p_{01} = p_{15} = \frac{\lambda_1}{\lambda+\lambda_1}, \quad p_{02} = p_{14} = \frac{\lambda}{\lambda+\lambda_1}, \quad p_{20} = g^*(\lambda+\lambda_1),$$

$$p_{27} = \frac{\lambda_1}{\lambda+\lambda_1}\left\{1 - g^*(\lambda+\lambda_1)\right\}, \quad p_{28} = \frac{\lambda}{\lambda+\lambda_1}\left\{1 - g^*(\lambda+\lambda_1)\right\}, \quad p_{30} = f^*(\lambda+\lambda_1),$$

$$p_{3,10} = \frac{\lambda_1}{\lambda+\lambda_1}\left\{1 - f^*(\lambda+\lambda_1)\right\}, \quad p_{3,11} = \frac{\lambda}{\lambda+\lambda_1}\left\{1 - f^*(\lambda+\lambda_1)\right\}, \quad p_{54} = \frac{\lambda_2}{\lambda_2+\alpha_0},$$

$$p_{41} = p_{71} = g^*(\lambda_2+\alpha_0), \quad p_{48} = p_{78} = \frac{\lambda_2}{\lambda_2+\alpha_0}\left\{1 - g^*(\lambda_2+\alpha_0)\right\},$$

$$p_{49} = p_{79} = \frac{\alpha_0}{\lambda_2+\alpha_0}\left\{1 - g^*(\lambda_2+\alpha_0)\right\}, \quad p_{56} = \frac{\alpha_0}{\lambda_2+\alpha_0}, \quad p_{82} = p_{93} = g^*(0),$$

$$p_{61} = p_{10,1} = f^*(\lambda_2+\alpha_0), \quad p_{6,11} = p_{10,11} = \frac{\lambda_2}{\lambda_2+\alpha_0}\left\{1 - f^*(\lambda_2+\alpha_0)\right\},$$

$$p_{6,12} = p_{10,12} = \frac{\alpha_0}{\lambda_2+\alpha_0}\left\{1 - f^*(\lambda_2+\alpha_0)\right\}, \quad p_{11,2} = p_{12,3} = f^*(0) = 1 \qquad (1)$$

It can be easily verified that

$p_{01} + p_{02} = p_{14} + p_{15} = p_{20} + p_{27} + p_{28} = p_{30} + p_{3,11} + p_{3,12} =$

$p_{41} + p_{48} + p_{49} = p_{54} + p_{56} = p_{60} + p_{6,11} + p_{6,12} = p_{71} + p_{78} + p_{79}$
$= p_{82} = p_{93} = p_{10,1} + p_{10,11} + p_{10,12} = p_{11,2} = p_{12,3} = 1$ (2)

The unconditional mean time taken by the system to transit from any regenerative state S_i when time is counted from epoch at entrance into state S_j is stated as:

$m_{ij} = \int_0^\infty t dQij(t) = -q_{ij}^{*\prime}(0)$ and the mean sojourn times μ_i in states S_i are given by

$$\mu_i = \int_0^\infty P(T > t)dt \quad (3)$$

where T denotes the time to system failure
Using these, we have

$\mu_0 = m_{01} + m_{02}$, $\mu_1 = m_{14} + m_{15}$, $\mu_2 = m_{20} + m_{27} + m_{28}$,
$\mu_3 = m_{30} + m_{3,10} + m_{3,11}$, $\mu_4 = m_{41} + m_{48} + m_{49}$, $\mu_5 = m_{54} + m_{56}$,
$\mu_6 = m_{61} + m_{6,11} + m_{6,12}$, $\mu_7 = m_{71} + m_{78} + m_{79}$, $\mu_8 = m_{82}$,
$\mu_9 = m_{93}$, $\mu_{10} = m_{10,1} + m_{10,11} + m_{10,12}$,
$\mu_{11} = m_{11,2}$, $\mu_{12} = m_{12,3}$ (4)

4. Analysis for System Model

(i) Reliability and Mean Time To System Failure (MTSF)

Let $\phi_i(t)$ be the cdf of the first passage time from regenerative state i to a failed state. Regarding the failed state as absorbing state, we have the following recursive relations for $\phi_i(t)$:

$\phi_0(t) = Q_{01}(t) Ⓢ \phi_1(t) + Q_{02}(t) Ⓢ \phi_2(t)$
$\phi_1(t) = Q_{14}(t) Ⓢ \phi_4(t) + Q_{15}(t) Ⓢ \phi_5(t)$
$\phi_2(t) = Q_{20}(t) Ⓢ \phi_0(t) + Q_{27}(t) Ⓢ \phi_7(t) + Q_{28}(t)$
$\phi_3(t) = Q_{30}(t) Ⓢ \phi_0(t) + Q_{3,10}(t) Ⓢ \phi_{10}(t) + Q_{3,11}(t)$
$\phi_4(t) = Q_{41}(t) Ⓢ \phi_1(t) + Q_{48}(t) + Q_{49}(t)$
$\phi_5(t) = Q_{54}(t) Ⓢ \phi_4(t) + Q_{56}(t) Ⓢ \phi_6(t)$
$\phi_6(t) = Q_{61}(t) Ⓢ \phi_1(t) + Q_{6,11}(t) + Q_{6,12}(t)$
$\phi_7(t) = Q_{71}(t) Ⓢ \phi_1(t) + Q_{78}(t) + Q_{79}(t)$
$\phi_{10}(t) = Q_{10,1}(t) Ⓢ \phi_1(t) + Q_{10,11}(t) + Q_{10,12}(t)$ (5)

Taking LST of relations (5), solving for $\tilde{\phi}_0(s)$ and using this, we have

$$R^*(s) = (1 - \tilde{\phi}_0(s))/s \quad (6)$$

The reliability R(t) can be obtained by taking Laplace inverse transform of (6). The mean time to system failure (MTSF)n is given by

$$\text{MTSF}(T_1) = \lim_{s\to 0}(1 - \tilde{\phi}_0(s))/s = \frac{N_{11}}{D_{11}} \quad (7)$$

where

$N_{11} = [1 - p_{12}p_{56} - p_{41}(p_{14} + p_{15}p_{34})][\mu_0 + p_{02}(\mu_2 + p_{27}\mu_7)]$
$+(p_{01} + p_{02}p_{27}p_{51})[\mu_1(p_{14} + p_{15}p_{54})\mu_4 + p_{15}(\mu_5 + p_{56}\mu_6)]$

and

$D_{11} = (1- p_{02}p_{20})[1- p_{15}p_{56}p_{61} – p_{41}(p_{14}+ p_{15}p_{54})]$

(ii) Steady State Availability

Let $A_i(t)$ be the probability that the system is in upstate at instant 't' given that the system entered regenerative state i at t = 0. The recursive relations for $A_i(t)$ are given as

$A_0(t) = M_0(t) + q_{01}(t) © A_1(t) + q_{02}(t) © A_2(t)$
$A_1(t) = M_1(t) + q_{14}(t) © A_4(t) + q_{15}(t) © A_5(t)$
$A_2(t) = M_2(t) + q_{20}(t) © A_0(t) + q_{21.7}(t) © A_1(t)$
$\quad + [q_{22.8}(t)+q_{22.78}(t)]© A_2(t) + q_{23.79}(t) © A_3(t)$
$A_3(t) = M_3(t) + q_{30}(t) © A_0(t) + q_{31.10}(t) © A_1(t)$
$\quad + [q_{32.11} (t) + q_{32.10,11} (t)] © A_2(t) + q_{33.10,12}(t) © A_3(t)$
$A_4(t) = M_4(t) + q_{41}(t) © A_1(t) + q_{42.8}(t) © A_2(t)$
$\quad + q_{43.9}(t) © A_3(t)$
$A_5(t) = M_5(t) + q_{54}(t) © A_4(t) + q_{56}(t) © A_6(t)$
$A_6(t) = M_6(t) + q_{61}(t) © A_1(t) + q_{62.11}(t) © A_2(t) + q_{63.12}(t) © A_3(t)$ (8)

Where

$M_0(t) = M_1(t) = e^{-(\lambda+\lambda_1)t}$, $M_2(t) = e^{-(\lambda+\lambda_1)t}\bar{G}(t)$, $M_3(t) = e^{-(\lambda+\lambda_1)t}\bar{F}(t)$,

$M_4(t) = e^{-(\lambda_2+\alpha_0)t}\bar{G}(t)$, $M_5(t) = e^{-(\lambda_2+\alpha_0)t}$, $M_6(t) = e^{-(\lambda_2+\alpha_0)t}\bar{F}(t)$

Now taking L.T of relations (9) and solving for $A_0^*(s)$. Using this the steady-state availability of the system is given by

$$A_{10}(\infty) = \lim_{s\to 0} s\ A_0^*(s) = \frac{N_2}{D_2} \quad (9)$$

where

$N_2 = [(1-p_{3,10}p_{10,12})(1-p_{28}-p_{27}p_{78}) - p_{27}p_{79}(p_{3,11}+ p_{3,10}\ p_{10,11}) - p_{27}p_{71}$
$\{(p_{15}p_{56}p_{6,12}+p_{14}p_{49}+p_{15}p_{54}p_{49})(p_{3,11}+p_{3,10}p_{10,11}) + (1-p_{3,10}p_{10,12})$
$(p_{15}p_{56}p_{6,11}+p_{14}p_{48}+p_{15}p_{54}p_{48})\} - p_{3,10}p_{10,1}\{(1-p_{28}-p_{27}p_{78})$
$(p_{15}p_{56}p_{6,12}+p_{14}p_{49}+p_{15}p_{54}p_{49}) + p_{27}p_{79}(p_{15}p_{56}p_{6,11}+p_{14}p_{48}+p_{15}p_{54}p_{48})\}$
$- (p_{14}p_{41}+p_{15}p_{54}p_{41}+p_{15}p_{56}p_{61})\{(1-p_{28}-p_{27}p_{78}) (1-p_{3,10}p_{10,12}) - p_{27}p_{79}$
$(p_{3,11}+p_{3,10}p_{10,11})\}]\mu_0 + [p_{01}(1-p_{28}-p_{27}p_{78}) (1-p_{3,10}p_{10,12}) - p_{27}p_{79}$
$(p_{3,11}+p_{3,10}p_{10,11})\} + p_{02}\{p_{27}p_{71}(1-p_{3,10}p_{10,12}) + p_{27}p_{79}p_{3,10}p_{10,1}\}][\mu_1$
$+ (p_{14}+p_{15}p_{54})\mu_4 + p_{15}\mu_5 + p_{15}p_{56}\mu_6] + [p_{02}(1-p_{3,10}p_{10,12}) +$
$(p_{15}p_{56}p_{6,12}+p_{14}p_{49}+p_{15}p_{54}p_{49})\{p_{01}(p_{3,11}+p_{3,10}p_{10,11}) - p_{02}p_{3,10}p_{10,1}\}$
$+ p_{01}(1-p_{3,10}p_{10,12}) (p_{15}p_{56}p_{6,11}+p_{14}p_{48}+p_{15}p_{54}p_{48}) - p_{02}(1-p_{3,10}p_{10,12})$
$(p_{15}p_{56}p_{61}+p_{14}p_{41}+p_{15}p_{54}p_{41})]\mu_2 + [p_{02}p_{27}p_{79} + (p_{15}p_{56}p_{6,12}+p_{14}p_{49}$
$+p_{15}p_{54}p_{49})\{p_{01}(1-p_{28}-p_{27}p_{78}) + p_{02}p_{27}p_{71}\} + p_{27}p_{79}\{p_{01}(p_{15}p_{56}p_{6,11}$
$+p_{14}p_{48}+p_{15}p_{54}p_{48}) - p_{02}(p_{15}p_{56}p_{61}+p_{14}p_{41}+p_{15}p_{54}p_{41})\}]\mu_3$

$D_2 = [\{p_{15}p_{56}p_{6,12} + (p_{14}+p_{15}p_{54})p_{49}\}\{p_{20}(p_{3,11}+p_{3,10}p_{10,11})+p_{30}(1-p_{28}-p_{27}p_{78})\}$
$+ (p_{15}p_{56}p_{6,11}+p_{14}p_{48}+p_{15}p_{54}p_{48})\{p_{20}(1-p_{3,10}p_{10,12}) + p_{30}p_{27}p_{79}\}]\mu_0$
$+ [p_{02}p_{27}p_{30}(p_{49}p_{71}-p_{41}p_{79}) + (p_{01}p_{20}+p_{27}p_{71})\{p_{49}(p_{3,11}+p_{3,10}p_{10,11})$
$+ p_{48}(1-p_{3,10}p_{10,12})\} + (p_{01}p_{30}+p_{3,10}p_{10,1})\{p_{49}(1-p_{28}-p_{27}p_{78}) + p_{27}p_{79}p_{48}\}$
$+ p_{41}\{(1-p_{28}-p_{27}p_{78}) (1-p_{3,10}p_{10,12}) - p_{27}p_{79}(p_{3,11}+p_{3,10}p_{10,11})\} - p_{02}p_{20}$
$\{p_{3,10}p_{10,1}p_{49} + p_{41} (1-p_{3,10}p_{10,12})\}]\mu_1 + [\{1-p_{14}p_{41}-p_{15}(p_{56}p_{61}+p_{54}p_{41})\}$
$\{(1-p_{3,10}p_{10,12})\ \mu_2^1 + p_{27}p_{79}\mu_3^1\} + (p_{15}p_{56}p_{6,12}+p_{14}p_{49}+p_{15}p_{54}p_{49})$
$\{(p_{01}p_{20}+p_{27}p_{71})\ \mu_3^1 - (p_{01}p_{30}+p_{3,10}p_{10,1})\ \mu_2^1\ \}] + (p_{14}+p_{15}p_{54})[\ (1-p_{28}-p_{27}p_{78}$

$-p_{02}p_{20})$ $(1-p_{3,10}p_{10,12})$ - $p_{27}p_{79}(p_{3,11}+p_{02}p_{30}+p_{3,10}p_{10,11})]$ μ_4^1
$+ p_{15}[(p_{3,11}+p_{3,10}p_{10,11})$ $(p_{27}p_{71}p_{49}+p_{01}p_{20}p_{49}-p_{41}p_{27}p_{79})$ + $(1-p_{3,10}p_{10,12})$
$(p_{48}p_{27}p_{71}+p_{48}p_{01}p_{20}-p_{41}p_{02}p_{20})$ + $(1-p_{28}-p_{27}p_{78})\{p_{15}p_{3,10}p_{10,1}+p_{01}p_{30}p_{49}$
$+p_{41}(1-p_{3,10}p_{10,12})\}$ + $p_{3,10}p_{10,1}(p_{27}p_{79}p_{48}-p_{02}p_{20}p_{49})+p_{30}p_{27}(p_{01}p_{48}p_{79}$
$+p_{02}p_{71}p_{49}-p_{02}p_{41}p_{79})]\mu_5$ + $p_{15}p_{56}[(1-p_{3,10}p_{10,12})(p_{20}p_{01}+p_{27}p_{71})$ + $p_{27}p_{79}$
$(p_{30}p_{01}+p_{3,10}p_{10,1})]$ μ_6^1

(iii) Busy Period Analysis Due to Repair

Let $B_i^1(t)$ be the probability that the server is busy at an instant *t* given that the system entered regenerative state *i* at *t* = 0. The following are the recursive relations for $B_i(t)$:

$$\begin{aligned}
B_0^1(t) &= q_{01}(t) © B_1^1(t) + q_{02}(t) © B_2^1(t)\\
B_1^1(t) &= q_{14}(t) © B_4^1(t) + q_{15}(t) © B_5^1(t)\\
B_2^1(t) &= W_2(t) + q_{20}(t) © B_0^1(t) + q_{21.7}(t) © B_1^1(t)\\
&\quad + [q_{22.8}(t)+q_{22.78}(t)] © B_2^1(t) + q_{23.79}(t) © B_3^1(t)\\
B_3^1(t) &= q_{30}(t) © B_0^1(t) + q_{31.10}(t) © B_1^1(t)\\
&\quad + [q_{32.11}(t) + q_{32.10,11}(t)] © B_2^1(t) + q_{33.10,12}(t) © B_3^1(t)\\
B_4^1(t) &= W_4(t) + q_{41}(t) © B_1^1(t) + q_{42.8}(t) © B_2^1(t) + q_{43.9}(t) © B_3^1(t)\\
B_5^1(t) &= q_{54}(t) © B_4^1(t) + q_{56}(t) © B_6^1(t)\\
B_6^1(t) &= q_{61}(t) © B_1^1 + q_{62.11}(t) © B_2^1(t) + q_{63.12}(t) © B_3^1(t) \qquad (10)
\end{aligned}$$

where

$$W_2(t) = e^{-(\lambda+\lambda_1)t}\bar{G}(t) + [(\lambda_1 e^{-(\lambda+\lambda_1)t} © 1)]\bar{G}(t) + (\lambda e^{-(\lambda+\lambda_1)t} © 1)\bar{G}(t)$$

$$W_4(t) = e^{-(\lambda_2+\alpha_0)t}\bar{G}(t) + \left(\lambda_2 e^{-(\lambda_2+\alpha_0)t} © 1\right)\bar{G}(t) + \left(\alpha_0 e^{-(\lambda_2+\alpha_0)t} © 1\right)\bar{G}(t)$$

Taking L.T. of relations (10) and solving for $B_0^{1*}(s)$ and using this, we can obtain the fraction of time for which the repairman is busy in steady state

$$B_{10}^1 = \lim_{s \to 0} sB_0^{1*}(s) = \frac{N_3}{D_2} \qquad (11)$$

$$\begin{aligned}
N_3 = {} & [p_{02}(1- p_{3,10}p_{10,12}) + (p_{15}p_{56}p_{6,12} + p_{14}p_{49} + p_{15}p_{54}p_{49})\{p_{01}(p_{3,11}\\
& + p_{3,10}p_{10,11}) - p_{02}p_{3,10}p_{10,1}\} + p_{01}(1- p_{3,10}p_{10,12})(p_{15}p_{56}p_{6,11} + p_{14}p_{48}\\
& + p_{15}p_{54}p_{48}) - p_{02}(1- p_{3,10}p_{10,12})(p_{15}p_{56}p_{61} + p_{14}p_{41} + p_{15}p_{54}p_{41})] W_2^*(0)\\
& + (p_{14} + p_{15}p_{54})[p_{01}\{(1- p_{28} - p_{27}p_{78})(1- p_{3,10}p_{10,12}) - p_{27}p_{79}(p_{3,11}\\
& + p_{3,10}p_{10,11})\} + p_{02}\{p_{27}p_{71}(1- p_{3,10}p_{10,12}) + p_{27}p_{79}p_{3,10}p_{10,1}\}] W_4^*(0)
\end{aligned}$$

and D_2 is already mentioned**.**

(iv) Busy Period Analysis due to Preventive Maintenance

Let $B_i^2(t)$ be the probability that the server is busy for preventive maintenance at an instant 't' given that the system entered regenerative state i at t = 0. The recursive relations for $B_i^2(t)$ are given as

$$\begin{aligned}
B_0^2(t) &= q_{01}(t) © B_1^2(t) + q_{02}(t) © B_2^2(t)\\
B_1^2(t) &= q_{14}(t) © B_4^2(t) + q_{15}(t) © B_5^2(t)\\
B_2^2(t) &= q_{20}(t) © B_0^2(t) + q_{21.7}(t) © B_1^2 + [q_{22.8}(t)+q_{22.78}(t)] © B_2^2(t) + q_{23.79}(t) © B_3^2(t)\\
B_3^2(t) &= W_3(t)+q_{30}(t) © B_0^2(t) + q_{31.10}(t) © B_1^2(t)\\
&\quad + [q_{32.11}(t) + q_{32.10,11}(t)] © B_2^2(t) + q_{33.10,12}(t) © B_3^2(t)\\
B_4^2(t) &= q_{41}(t) © B_1^2(t) + q_{42.8}(t) © B_2^2(t) + q_{43.9}(t) © B_3^2(t)\\
B_5^2(t) &= q_{54}(t) © B_4^2(t) + q_{56}(t) © B_6^2(t)
\end{aligned}$$

$B_6^2(t) = W_4(t) + q_{61}(t) \copyright B_1^2(t)$
$\quad + q_{62.11}(t) \copyright B_2^2(t) + q_{63.12}(t) \copyright B_3^2(t)$ (12)

Where

$$W_3(t) = e^{-(\lambda+\lambda_1)t}\bar{F}(t) + \left(\lambda e^{-(\lambda+\lambda_1)t} \copyright 1\right)\bar{F}(t) + \left(\lambda_1 e^{-(\lambda+\lambda_1)t} \copyright 1\right)\bar{F}(t),$$

$$W_6(t) = e^{-(\lambda_2+\alpha_0)t}\bar{F}(t) + \left(\lambda_2 e^{-(\lambda_2+\alpha_0)t} \copyright 1\right)\bar{F}(t) + \left(\alpha_0 e^{-(\lambda_2+\alpha_0)t} \copyright 1\right)\bar{F}(t)$$

Taking L.T. of relations (12) and solving for $B_0^{2*}(s)$ and using this, we can obtain the fraction of time for which the repairman is busy in steady state

$$B_{10}^2 = \lim_{s \to 0} sB_0^{2*}(s) = \frac{N_4}{D_2} \quad (13)$$

where,

$N_4 = [p_{27}p_{79}\{(p_{15}p_{56}p_{6,11}+p_{14}p_{48}+p_{15}p_{54}p_{48}) + p_{02}(1-p_{14}p_{41}-p_{15}p_{54}p_{41}$
$\quad -p_{15}p_{56}p_{61})\} + (p_{15}p_{56}p_{6,12}+p_{14}p_{49}+p_{15}p_{54}p_{49})\{p_{01}(1-p_{28}-p_{27}p_{78})$
$\quad + p_{02}p_{27}p_{71}\}]W_3^*(0) + p_{15}p_{56}[p_{01}\{(1-p_{28}-p_{27}p_{78})(1-p_{3,10}p_{10,12})$
$\quad -p_{27}p_{79}(p_{3,11}+p_{3,10}p_{10,11})\}+p_{02}\{p_{27}p_{71}(1-p_{3,10}p_{10,12})$
$\quad +p_{27}p_{79}p_{3,10}p_{10,1}\}]W_6^*(0)$

and D_2 is already specified.

(v) Expected Number of Visits

Let $N_i(t)$ be the expected number of visits by the server in (0,t] given that the system entered the regenerative state *i* at *t=0*. We have the following recursive relations for $N_i(t)$:

$N_0(t) = Q_{01}(t) \circledS N_1(t) + Q_{02}(t) \circledS [1+N_2(t)]$

$N_1(t) = Q_{14}(t) \circledS [1+N_4(t)] + Q_{15}(t) \circledS N_5(t)$

$N_2(t) = Q_{20}(t) \circledS N_0(t) + Q_{21.7}(t) \circledS N_1(t)$
$\quad + [Q_{22.8}(t) + Q_{22.78}(t)] \circledS N_2(t) + Q_{23.79}(t) \circledS N_3(t)$

$N_3(t) = Q_{30}(t) \circledS N_0(t)+Q_{31.10}(t) \circledS N_1(t)+[Q_{32.11}(t)+Q_{32.10,11}(t)] \circledS N_2(t)$
$\quad +Q_{33,10,12}(t) \circledS N_3(t)$

$N_4(t) = Q_{41}(t) \circledS N_1(t) + Q_{42.8}(t) \circledS N_2(t) + Q_{43.9}(t) \circledS N_3(t)$

$N_5(t) = Q_{54}(t) \circledS [1+N_4(t)] + Q_{56}(t) \circledS [1+N_6(t)]$

$N_6(t) = Q_{61}(t) \circledS N_1(t) + Q_{62.11}(t) \circledS N_2(t) + Q_{63.12}(t) \circledS N_3(t)$ (14)

Taking LST of relations (16) and solving for $\tilde{N}_0(s)$.

The expected number of visits per unit time can be obtained as

$$N_{10} = \lim_{s \to 0} s\tilde{N}_0(s) = \frac{N_5}{D_2} \quad (15)$$

where

$N_5 = (1-p_{3,10}p_{10,12})(1-p_{28}-p_{27}p_{78}) - p_{27}p_{79}(p_{3,11}+p_{3,10}p_{10,11}) + p_{02}[p_{27}$
$\quad (1-p_{14}p_{48}-p_{15}p_{54}p_{48}-p_{15}p_{56}p_{6,11})\{p_{71}(1-p_{3,10}p_{10,12}) + p_{79}p_{3,10}p_{10,1}\}$
$\quad - (p_{15}p_{56}p_{6,12}+p_{14}p_{49}+p_{15}p_{54}p_{49})\{p_{27}p_{71}(p_{3,11}+p_{3,10}p_{10,11}) + p_{3,10}p_{10,1}$
$\quad (1-p_{28}-p_{27}p_{78})\} - (p_{14}p_{41}+p_{15}p_{54}p_{41}+p_{15}p_{56}p_{61})\{(1-p_{28}-p_{27}p_{78})$
$\quad (1-p_{3,10}p_{10,12}) - p_{27}p_{79}(p_{3,11}+p_{3,10}p_{10,11})\}]$

and D_2 is already specified.

(vi) Cost- Benefit Analysis

Profit incurred to the system model in steady state is given by

$$P_1 = K_1A_0 - K_2\, B_{10}^1 - K_3\, B_{10}^2 - K_4\, N_{10} \qquad (16)$$

where

K_1 = Revenue per unit up-time of the system

K_2 = Cost per unit time for which server is busy in repair

K_3 = Cost per unit time for which server is busy in preventive maintenance

K_4 = Cost per unit visit by the server

Particular Case

Let us take $g(t) = \boldsymbol{q}e^{-\boldsymbol{q}t}$, $f(t) = \boldsymbol{b}e^{-\boldsymbol{b}t}$

By using the non-zero elements p_{ij}, we get the following results:

MTSF $(T_1) = N_{11}/D_{11}$, Availability $(A_{10}) = N_2/D_2$

Busy Period for repair $(B_{10}^1) = N_3/D_2$,

Busy Period preventive maintenance $(B_{10}^2) = N_4/D_2$

Expected no. of visits $(N_{10}) = N_5/D_2$

where

$$\begin{aligned}N_{11} = {} & (\beta+\lambda_2+\alpha_0)(\lambda_2+\alpha_0)(\lambda\lambda_2+\lambda\alpha_0+\lambda_1\lambda_2)\,[(\theta+\lambda+\lambda_1)(\theta+\lambda_2+\alpha_0)\\ & +\lambda(\theta+\lambda_1+\lambda_2+\alpha_0)] + \lambda_1[(\theta+\lambda+\lambda_1)(\theta+\lambda_2+\alpha_0) + \lambda\theta][\lambda_1(\theta+\lambda_2+\alpha_0)\\ & (\beta+\lambda_2+2\alpha_0) + (\beta+\lambda_2+\alpha_0)\{(\lambda_2+\alpha_0)(\theta+\lambda+\lambda_2+\alpha_0) + \lambda_1\lambda_2\}]\end{aligned}$$

$$\begin{aligned}D_{11} = {} & (\theta+\lambda_2+\alpha_0)[(\lambda+\lambda_1)(\theta+\lambda+\lambda_1) - \theta\lambda][(\theta+\lambda_2+\alpha_0)\{(\lambda+\lambda_1)(\lambda_2+\alpha_0)\\ & (\beta+\lambda_2+\alpha_0) - \beta\lambda_1\alpha_0\} - \theta(\beta+\lambda_2+\alpha_0)\{\lambda(\lambda_2+\alpha_0) + \lambda_1\lambda_2\}]\end{aligned}$$

$$\begin{aligned}N_2 = {} & \beta\theta[(\lambda+\lambda_1)(\lambda_2+\alpha_0)(\theta+\lambda_2+\alpha_0)(\beta+\lambda_2+\alpha_0)\{\{(\beta+\lambda+\lambda_1)(\beta+\lambda_2+\alpha_0)\\ & - \lambda_1\alpha_0\}\{(\theta+\lambda_1)(\theta+\lambda_2+\alpha_0) - \lambda_1\lambda_2\} - \lambda_1\alpha_0\{\lambda(\beta+\lambda_2+\alpha_0) + \lambda_1\lambda_2\}\}\\ & - \alpha_0\lambda_1\theta\{\lambda(\beta+\lambda_2+\alpha_0) + \lambda_1\lambda_2\}\{\lambda_1\alpha_0(\theta+\lambda_2+\alpha_0) + (\beta+\lambda_2+\alpha_0)\\ & (\lambda\lambda_2+\lambda\alpha_0+\lambda_1\lambda_2)\} - \theta\lambda_1\lambda_2\{(\beta+\lambda+\lambda_1)(\beta+\lambda_2+\alpha_0) - \lambda_1\alpha_0\}\{\lambda_1\alpha_0\\ & (\theta+\lambda_2+\alpha_0) + (\beta+\lambda_2+\alpha_0)(\lambda\lambda_2+\lambda\alpha_0+\lambda_1\lambda_2)\} - \beta\lambda_1\alpha_0\{(\theta+\lambda_1)(\theta+\lambda_2+\alpha_0)\\ & - \lambda_1\lambda_2\}\{\lambda_1\alpha_0(\theta+\lambda_2+\alpha_0) + (\beta+\lambda_2+\alpha_0)\}(\lambda\lambda_2+\lambda\alpha_0+\lambda_1\lambda_2)\\ & - \beta\lambda^2{}_1\lambda_2\alpha_0\{\lambda_1\alpha_0(\theta+\lambda_2+\alpha_0) + (\beta+\lambda_2+\alpha_0)(\lambda\lambda_2+\lambda\alpha_0+\lambda_1\lambda_2)\}\\ & - \{\beta\lambda_1\alpha_0(\theta+\lambda_2+\alpha_0) + \theta(\beta+\lambda_2+\alpha_0)(\lambda\lambda_2+\lambda\alpha_0+\lambda_1\lambda_2)\}\{\{(\theta+\lambda_1)\\ & (\theta+\lambda_2+\alpha_0) - \lambda_1\lambda_2\}\{(\beta+\lambda+\lambda_1)(\beta+\lambda_2+\alpha_0) - \lambda_1\alpha_0\} - \lambda_1\alpha_0\{\lambda(\beta+\lambda_2+\alpha_0)\\ & + \lambda_1\lambda_2\}\} + \{\lambda_1(\theta+\lambda_1)(\theta+\lambda_2+\alpha_0) - \lambda_1\lambda_2\}\{(\beta+\lambda+\lambda_1)(\beta+\lambda_2+\alpha_0) - \lambda_1\alpha_0\}\\ & - \lambda^2{}_1\alpha_0\{\lambda(\beta+\lambda_2+\alpha_0) + \lambda_1\lambda_2\} + \lambda\lambda_1\theta\{(\beta+\lambda+\lambda_1)(\beta+\lambda_2+\alpha_0) - \lambda_1\alpha_0\}\\ & + \lambda\lambda^2{}_1\beta\alpha_0\}\{\lambda_1\alpha_0(\theta+\lambda_2+\alpha_0) + (\lambda_1+\lambda_2+\alpha_0)(\theta+\lambda_2+\alpha_0)(\beta+\lambda_2+\alpha_0)\\ & + (\beta+\lambda_2+\alpha_0)(\lambda\lambda_2+\lambda\alpha_0+\lambda_1\lambda_2)\} + (\theta+\lambda_2+\alpha_0)\{\lambda(\lambda+\lambda_1)(\lambda_2+\alpha_0)\\ & (\beta+\lambda_2+\alpha_0)(\theta+\lambda_2+\alpha_0)\{(\beta+\lambda+\lambda_1)(\beta+\lambda_2+\alpha_0) - \lambda_1\alpha_0\}\\ & + \lambda_1\alpha_0\{\lambda_1\alpha_0(\theta+\lambda_2+\alpha_0)+(\beta+\lambda_2+\alpha_0)(\lambda\lambda_2+\lambda\alpha_0+\lambda_1\lambda_2)\}\{\lambda(\lambda_2+\alpha_0)+\lambda_1\lambda_2\}\\ & + \lambda_1\lambda_2\{(\beta+\lambda+\lambda_1)(\beta+\lambda_2+\alpha_0) - \lambda_1\alpha_0\}\{\lambda_1\alpha_0(\theta+\lambda_2+\alpha_0)+(\beta+\lambda_2+\alpha_0)\\ & (\lambda\lambda_2+\lambda\alpha_0+\lambda_1\lambda_2)\} - \lambda\{(\beta+\lambda+\lambda_1)(\beta+\lambda_2+\alpha_0) - \lambda_1\alpha_0\}\{\beta\lambda_1\alpha_0(\theta+\lambda_2+\alpha_0)\\ & + \theta(\beta+\lambda_2+\alpha_0)(\lambda\lambda_2+\lambda\alpha_0+\lambda_1\lambda_2)\}\} + \lambda_1\alpha_0(\beta+\lambda_2+\alpha_0)\{\lambda(\lambda+\lambda_1)(\lambda_2+\alpha_0)\\ & (\theta+\lambda_2+\alpha_0)(\beta+\lambda_2+\alpha_0) + \{\lambda_1\alpha_0(\theta+\lambda_2+\alpha_0) + (\beta+\lambda_2+\alpha_0)(\lambda\lambda_2+\lambda\alpha_0\\ & +\lambda_1\lambda_2)\}\{(\theta+\lambda_1)(\theta+\lambda_2+\alpha_0) + \lambda\theta - \lambda_1\lambda_2\} + \lambda_1\lambda_2\{\lambda_1\alpha_0(\theta+\lambda_2+\alpha_0)\\ & + (\beta+\lambda_2+\alpha_0)(\lambda\lambda_2+\lambda\alpha_0+\lambda_1\lambda_2)\} - \lambda\lambda_1\beta\alpha_0(\theta+\lambda_2+\alpha_0) - \lambda\theta(\beta+\lambda_2+\alpha_0)\end{aligned}$$

$(\lambda\lambda_2+\lambda\alpha_0+\lambda_1\lambda_2)\}]$

$N_3 = \beta(\theta+\lambda+\lambda_1)(\theta+\lambda_2+\alpha_0)(\lambda\lambda_2+\lambda\alpha_0+\lambda_1\lambda_2)[\lambda_1\lambda_2\{(\beta+\lambda+\lambda_1)(\beta+\lambda_2+\alpha_0) - \lambda_1\alpha_0\}\{\lambda_1\alpha_0(\theta+\lambda_2+\alpha_0) + (\beta+\lambda_2+\alpha_0)(\lambda\lambda_2+\lambda\alpha_0+\lambda_1\lambda_2)\}+ \lambda(\lambda+\lambda_1)(\lambda_2+\alpha_0)(\theta+\lambda_2+\alpha_0)(\beta+\lambda_2+\alpha_0)\{(\beta+\lambda+\lambda_1)(\beta+\lambda_2+\alpha_0) - \lambda_1\alpha_0\}+ \lambda_1\alpha_0(\lambda\lambda_2+\lambda\alpha_0+\lambda_1\lambda_2)\{\lambda_1\alpha_0(\theta+\lambda_2+\alpha_0)+(\beta+\lambda_2+\alpha_0)(\lambda\lambda_2+\lambda\alpha_0+\lambda_1\lambda_2)\} - \lambda\{(\beta+\lambda+\lambda_1)(\beta+\lambda_2+\alpha_0) - \lambda_1\alpha_0\}\{\beta\lambda_1\alpha_0(\theta+\lambda_2+\alpha_0)+\theta(\beta+\lambda+\lambda_1)(\lambda\lambda_2+\lambda\alpha_0+\lambda_1\lambda_2)\}]+\beta(\theta+\lambda_2+\alpha_0)(\beta+\lambda_2+\alpha_0)[\lambda_1\{(\theta+\lambda_1)(\theta+\lambda_2+\alpha_0) - \lambda_1\lambda_2\}\{(\beta+\lambda+\lambda_1)(\beta+\lambda_2+\alpha_0) - \lambda_1\alpha_0\} - \lambda^2{}_1\alpha_0\{\lambda(\beta+\lambda_2+\alpha_0) + \lambda_1\lambda_2\} + \lambda\lambda_1\theta\{(\beta+\lambda+\lambda_1)(\beta+\lambda_2+\alpha_0) - \lambda_1\alpha_0\} + \lambda\lambda^2{}_1\beta\alpha_0]$

$N_4 = \theta\lambda_1\alpha_0(\beta+\lambda+\lambda_1)(\beta+\lambda_2+\alpha_0)[\lambda(\lambda+\lambda_1)(\lambda_2+\alpha_0)(\theta+\lambda_2+\alpha_0)(\beta+\lambda_2+\alpha_0) + \{\lambda_1\alpha_0(\theta+\lambda_2+\alpha_0) + (\beta+\lambda+\alpha_0)(\lambda\lambda_2+\lambda\alpha_0+\lambda_1\lambda_2)\}\{(\theta+\lambda_1)(\theta+\lambda_2+\alpha_0) + \theta\lambda - \lambda_1\lambda_2\} + \lambda_1\lambda_2\{\lambda_1\alpha_0(\theta+\lambda_2+\alpha_0) + (\beta+\lambda_2+\alpha_0)(\lambda\lambda_2+\lambda\alpha_0+\lambda_1\lambda_2)\} - \beta\lambda\lambda_1\alpha_0(\theta+\lambda_2+\alpha_0) - \theta\lambda(\beta+\lambda_2+\alpha_0)(\lambda\lambda_2+\lambda\alpha_0+\lambda_1\lambda_2)] + \theta\lambda_1\alpha_0(\theta+\lambda_2+\alpha_0)(\beta+\lambda_2+\alpha_0)[\beta\lambda\lambda^2{}_1\alpha_0+\lambda_1\{(\theta+\lambda_1)(\theta+\lambda_2+\alpha_0) - \lambda_1\lambda_2\}\{(\beta+\lambda+\lambda_1)(\beta+\lambda_2+\alpha_0) - \lambda_1\alpha_0\} + \lambda\lambda_1\theta\{(\beta+\lambda+\lambda_1)(\beta+\lambda_2+\alpha_0) - \lambda\alpha_0\} - \lambda^2{}_1\alpha_0\{\lambda(\beta+\lambda_2+\alpha_0) + \lambda_1\lambda_2\}]$

$N_5 = \beta\theta\lambda[(\lambda+\lambda_1)(\lambda_2+\alpha_0)(\theta+\lambda_2+\alpha_0)(\beta+\lambda_2+\alpha_0)\{\{(\beta+\lambda+\lambda_1)(\beta+\lambda_2+\alpha_0) - \lambda_1\alpha_0\}\{(\theta+\lambda_1)(\theta+\lambda_2+\alpha_0) - \lambda_1\lambda_2\} - \lambda_1\alpha_0\{\lambda(\beta+\lambda_2+\alpha_0) + \lambda_1\lambda_2\}\} - \theta\lambda_1\alpha_0\{\lambda(\beta+\lambda_2+\alpha_0) + \lambda_1\lambda_2\}\{\lambda_1\alpha_0(\theta+\lambda_2+\alpha_0) + (\beta+\lambda_2+\alpha_0)(\lambda\lambda_2+\lambda\alpha_0+\lambda_1\lambda_2)\} - \theta\lambda_1\lambda_2\{(\beta+\lambda+\lambda_1)(\beta+\lambda_2+\alpha_0) - \lambda_1\alpha_0\}\{\lambda_1\alpha_0(\theta+\lambda_2+\alpha_0) + (\beta+\lambda_2+\alpha_0)(\lambda\lambda_2+\lambda\alpha_0+\lambda_1\lambda_2)\} - \beta\lambda_1\alpha_0\{(\theta+\lambda_1)(\theta+\lambda_2+\alpha_0) - \lambda_1\lambda_2\}\{\lambda_1\alpha_0(\theta+\lambda_2+\alpha_0) + (\beta+\lambda_2+\alpha_0)(\lambda\lambda_2+\lambda\alpha_0+\lambda_1\lambda_2)\} - \beta\lambda_1^2\lambda_2\alpha_0\{\lambda_1\alpha_0(\theta+\lambda_2+\alpha_0) + (\beta+\lambda_2+\alpha_0)(\lambda\lambda_2+\lambda\alpha_0+\lambda_1\lambda_2)\} - \{\beta\lambda_1\alpha_0(\theta+\lambda_2+\alpha_0) + \theta(\beta+\lambda_2+\alpha_0)(\lambda\lambda_2+\lambda\alpha_0+\lambda_1\lambda_2)\}\{\{(\theta+\lambda_1)(\theta+\lambda_2+\alpha_0) - \lambda_1\lambda_2\}\{(\beta+\lambda+\lambda_1)(\beta+\lambda_2+\alpha_0) - \lambda_1\alpha_0\} - \lambda_1\alpha_0\{\lambda(\beta+\lambda_2+\alpha_0) + \lambda_1\lambda_2\}\}] + \beta\theta(\lambda+\lambda_1)(\lambda_2+\alpha_0)(\theta+\lambda_2+\alpha_0)(\beta+\lambda_2+\alpha_0)[\lambda\lambda_1\theta(\beta+\lambda+\lambda_1)(\beta+\lambda_2+\alpha_0) - \lambda_1\alpha_0\} + \lambda_1\{(\theta+\lambda_1)(\theta+\lambda_2+\alpha_0) - \lambda_1\lambda_2\}\{(\beta+\lambda+\lambda_1)(\beta+\lambda_2+\alpha_0) - \lambda_1\alpha_0\} + \lambda\lambda_1^2\beta\alpha_0 - \lambda_1^2\alpha_0\{\lambda(\beta+\lambda_2+\alpha_0) + \lambda_1\lambda_2\}]$

and

$D_2 = \beta\theta[\alpha_0\{\lambda_1\alpha_0(\theta+\lambda_2+\alpha_0) + (\beta+\lambda_2+\alpha_0)(\lambda\lambda_2+\lambda\alpha_0+\lambda_1\lambda_2)\}\{\theta(\theta+\lambda_2+\alpha_0)\{\lambda(\beta+\lambda_2+\alpha_0) + \lambda_1\lambda_2\} + \beta(\beta+\lambda_2+\alpha_0)\{(\theta+\lambda_1)(\theta+\lambda_2+\alpha_0) - \lambda_1\lambda_2\}\} + \lambda_2\{\lambda_1\alpha_0(\theta+\lambda_2+\alpha_0) + (\beta+\lambda_2+\alpha_0)(\lambda\lambda_2+\lambda\alpha_0+\lambda_1\lambda_2)\}\{\beta\lambda_1\alpha_0(\beta+\lambda_2+\alpha_0) + \theta(\theta+\lambda_2+\alpha_0)\{(\beta+\lambda+\lambda_1)(\beta+\lambda_2+\alpha_0) - \lambda_1\alpha_0\}\}] + \beta\theta(\lambda_2+\alpha_0)(\beta+\lambda_2+\alpha_0)[\beta\lambda_1\alpha_0(\theta+\lambda_1)(\theta+\lambda_2+\alpha_0)(\beta+\lambda+\lambda_1+\lambda_2+\alpha_0) + \theta\lambda_1(\theta+\lambda+\lambda_1+\lambda_2+\alpha_0)(\beta+\lambda_2+\alpha_0)\{\lambda_2(\beta+\lambda+\lambda_1) + \lambda\alpha_0\} + \theta(\lambda+\lambda_1)\{\{(\theta+\lambda_1)(\theta+\lambda_2+\alpha_0) -\lambda_1\lambda_2\}\{(\beta+\lambda+\lambda_1)(\beta+\lambda_2+\alpha_0) - \lambda_1\alpha_0\} - \lambda_1\alpha_0\{\lambda(\beta+\lambda_2+\alpha_0) + \lambda_1\lambda_2\}\} - \theta\lambda(\theta+\lambda_2+\alpha_0)\{\beta\lambda_1\alpha_0 + \theta\{(\beta+\lambda+\lambda_1)(\beta+\lambda_2+\alpha_0) - \lambda_1\alpha_0\}\}] + \beta(\theta+\lambda_2+\alpha_0)(\theta+\lambda+\lambda_1)[(\lambda+\lambda_1)\{(\beta+\lambda+\lambda_1)(\beta+\lambda_2+\alpha_0) - \lambda_1\alpha_0\}\{(\lambda+\lambda_1)(\lambda_2+\alpha_0)(\theta+\lambda_2+\alpha_0)(\beta+\lambda_2+\alpha_0) -\theta(\beta+\lambda_2+\alpha_0)(\lambda\lambda_2+\lambda\alpha_0+\lambda_1\lambda_2) - \beta\lambda_1\alpha_0(\theta+\lambda_2+\alpha_0)\} - \beta\lambda_1\alpha_0(\beta+\lambda+\lambda_1+\lambda_2+\alpha_0)\{\lambda_1\alpha_0(\theta+\lambda_2+\alpha_0)+(\beta+\lambda_2+\alpha_0)(\lambda\lambda_2+\lambda\alpha_0+\lambda_1\lambda_2)\}] + \lambda_1\alpha_0\theta(\beta+\lambda+\lambda_1)(\beta+\lambda_2+\alpha_0)[\theta(\theta+\lambda+\lambda_1+\lambda_2+\alpha_0)\{\lambda_1\alpha_0(\theta+\lambda_2+\alpha_0) + (\beta+\lambda_2+\alpha_0)(\lambda\lambda_2+\lambda\alpha_0+\lambda_1\lambda_2)\} + (\lambda+\lambda_1)\{(\lambda+\lambda_1)(\lambda_2+\alpha_0)(\theta+\lambda_2+\alpha_0)(\beta+\lambda_2+\alpha_0) - \theta(\beta+\lambda+\alpha_0)(\lambda\lambda_2+\lambda\alpha_0+\lambda_1\lambda_2) - \beta\lambda_1\alpha_0(\theta+\lambda_2+\alpha_0)\}] + \beta(\theta+\lambda_2+\alpha_0)(\beta+\lambda_2+\alpha_0)(\lambda\lambda_2+\lambda\alpha_0+\lambda_1\lambda_2)[\{(\theta+\lambda_2+\alpha_0)(\theta\lambda_1+\lambda\lambda_1+\lambda^2{}_1)$

$- \lambda_1\lambda_2 (\lambda+\lambda_1)\}\{(\beta+\lambda+ \lambda_1) (\beta+\lambda_2+\alpha_0) - \lambda_1\alpha_0\} - \lambda_1\alpha_0\{\lambda(\beta+\lambda+ \lambda_1)$
$(\beta+\lambda_2+\alpha_0) + \lambda_1\lambda_2(\lambda+ \lambda_1)\}] + \lambda_1\beta\theta(\beta+\lambda_2+\alpha_0)[\theta\lambda_1\alpha_0(\theta+\lambda_2+\alpha_0)$
$\{\lambda(\beta+\lambda_2+\alpha_0) +\lambda_1\lambda_2\} + \theta\{(\beta+\lambda+ \lambda_1) (\beta+\lambda_2+\alpha_0) - \lambda_1\alpha_0\}\{\lambda_1\lambda_2(\theta+\lambda+\lambda_1$
$+ \lambda_2+\alpha_0) - \lambda\theta(\theta+\lambda_2+\alpha_0)\} + \{(\theta+\lambda_1) (\theta+\lambda_2+\alpha_0) - \lambda_1\lambda_2\}\{\beta\lambda^2{}_1(\theta+\lambda_2+\alpha_0)$
$+ \beta\lambda_1\alpha_0(\beta+\lambda_2+\alpha_0) + \theta(\lambda+ \lambda_1)\{(\beta+\lambda+ \lambda_1) (\beta+\lambda_2+\alpha_0) - \lambda_1\alpha_0\}\}$
$+ \beta\lambda_1\alpha_0\{\lambda_1\lambda_2(\beta+\lambda+\lambda_1+ \lambda_2+\alpha_0) - \lambda\theta(\theta+\lambda_2+\alpha_0)\}] + \theta\lambda_1\alpha_0(\theta+\lambda_2+\alpha_0)$
$(\beta+\lambda_2+\alpha_0)[\beta\lambda^2{}_1\alpha_0(\beta+\lambda+\lambda_1+ \lambda_2+\alpha_0) + \theta\lambda_1(\theta+\lambda+\lambda_1+ \lambda_2+\alpha_0)\{ (\beta+\lambda+ \lambda_1)$
$(\beta+\lambda_2+\alpha_0) - \lambda_1\alpha_0\}]$

5. Conclusion

The numerical results obtained for particular values of various parameters indicate that the mean time to system failure (MTSF), availability and profit incurred to the system model decrease with the increase of maximum operation time (α_0), direct failure rate (λ) and partial failure rate (λ_1) as shown in tables 1, 2 and 3. But their values increase as repair rate (θ) and preventive maintenance rate (β) increase. It is concluded that a system in which preventive maintenance is carried out after a specific period of operating time at partially failure stage can be made profitable giving priority for operation to new unit over partially failed unit. However, such a system has less MTSF as compared to the system where no priority is given for operation to the new unit.

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Table-1

α_0 ↓	Mean Time to System Failure(MTSF)				
	λ=.13,λ1=.17, λ2=.21,θ=2.1, β=2.7	λ=.16,λ1=.17, λ2=.21,θ=2.1, β=2.7	λ=.13,λ1=.20, λ2=.21,θ=2.1, β=2.7	λ=.13,λ1=.17, λ2=.21,θ=2.6, β=2.7	λ=.13,λ1=.17, λ2=.21,θ=2.1, β=3.7
5	7.588071	7.22887	6.670627	7.731869	8.051282
10	6.56007	6.32531	5.693827	6.651233	6.823007
15	6.209871	6.016065	5.362984	6.288034	6.394234
20	6.033109	5.85966	5.196402	6.105867	6.175197
25	5.926476	5.765199	5.096049	5.996381	6.042101
30	5.855135	5.701955	5.028971	5.923313	5.952622
35	5.804049	5.656644	4.980968	5.871083	5.888324
40	5.765664	5.622584	4.944915	5.83189	5.839884
45	5.735765	5.596046	4.916844	5.801393	5.802076
50	5.711819	5.574787	4.894368	5.776988	5.771744

Table-2

α_0 ↓	Availability				
	λ=.13,λ1=.17, λ2=.21,θ=2.1, β=2.7	λ=.16,λ1=.17, λ2=.21,θ=2.1, β=2.7	λ=.13,λ1=.20, λ2=.21,θ=2.1, β=2.7	λ=.13,λ1=.17, λ2=.21,θ=2.6, β=2.7	λ=.13,λ1=.17, λ2=.21,θ=2.1, β=3.7
5	0.98745	0.986323	0.983299	0.990065	0.994247
10	0.977455	0.976042	0.972621	0.979486	0.985052
15	0.974213	0.972716	0.969089	0.976068	0.981973
20	0.972653	0.971117	0.967371	0.974434	0.980471
25	0.971745	0.970187	0.966363	0.973488	0.97959
30	0.971154	0.969581	0.965704	0.972875	0.979015
35	0.970739	0.969156	0.96524	0.972446	0.97861
40	0.970433	0.968842	0.964896	0.97213	0.97831
45	0.970198	0.968601	0.964631	0.971889	0.978079
50	0.970011	0.96841	0.964421	0.971697	0.977896

Table-3

α_0	Profit				
	λ=.13,λ_1=.17, λ_2=.21,θ=2.1, β=2.7,K_1=50 K_2=150,K_3= K_4=50	λ=.16,λ_1=.17, λ_2=.21,θ=2.1, β=2.7,K_1=50 K_2=150,K_3= K_4=50	λ=.13,λ_1=.20, λ_2=.21,θ=2.1, β=2.7,K_1=50 K_2=150,K_3= K_4=50	λ=.13,λ_1=.17, λ_2=.21,θ=2.6, β=2.7,K_1=50 K_2=150,K_3= K_4=50	λ=.13,λ_1=.17, λ_2=.21,θ=2.1, β=3.7,K_1=50 K_2=150,K_3= K_4=50
5	4912.171	4902.85	4890.107	4927.102	4946.603
10	4863.481	4852.985	4838.012	4875.399	4902.102
15	4847.761	4836.938	4820.849	4858.767	4887.289
20	4840.217	4829.246	4812.52	4850.833	4880.092
25	4835.835	4824.779	4807.646	4846.249	4875.885
30	4832.985	4821.876	4804.46	4843.282	4873.139
35	4830.99	4819.843	4802.22	4841.211	4871.211
40	4829.516	4818.342	4800.56	4839.687	4869.785
45	4828.385	4817.19	4799.283	4838.52	4868.689
50	4827.49	4816.278	4798.27	4837.598	4867.82

State Transition Diagram

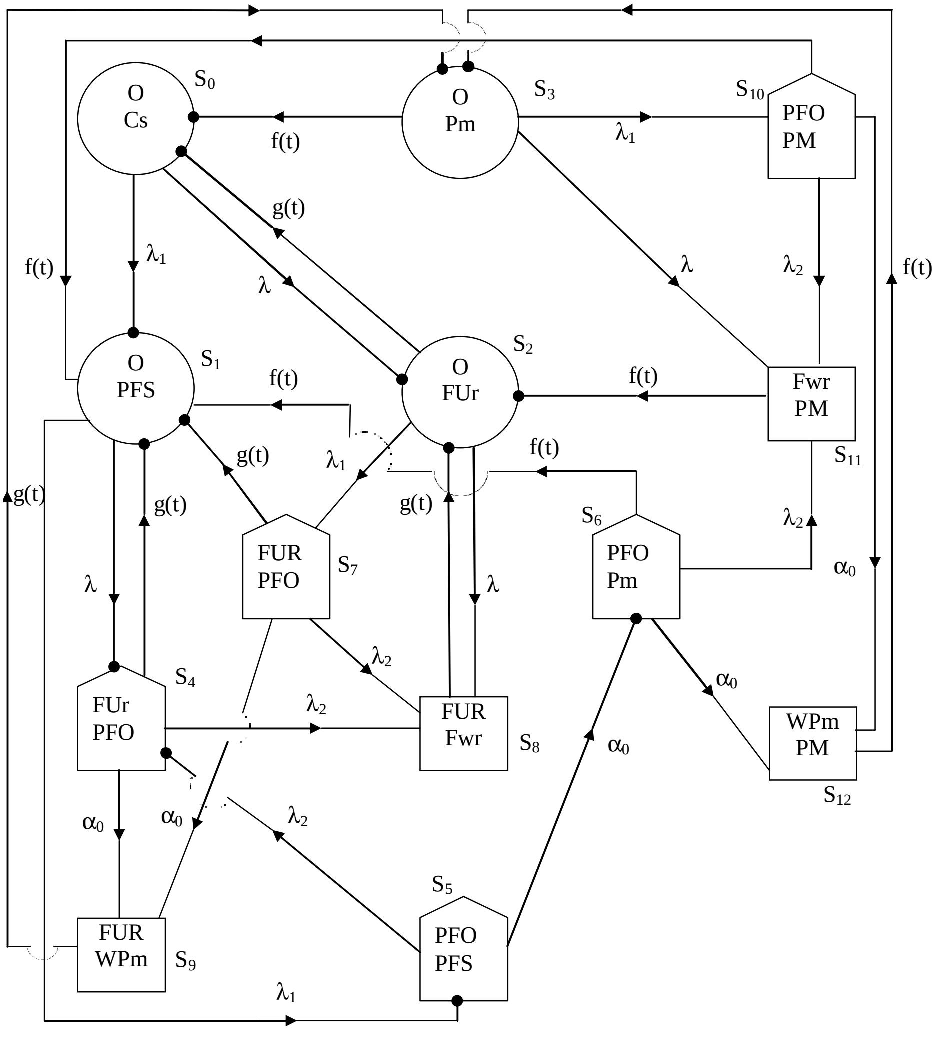

Fig. 1

• Regenerative point ☐ Failed state

○ Up-state ☖ Partial failure up-state