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STOCHASTIC ANALYSIS OF A COMPLEX SYSTEM WITH CORRELATED WORKING AND REST PERIOD OF REPAIRMAN

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Abstract

This paper deals with the stochastic analysis of a complex system with correlated working and rest time of repairman. The system consists of two subsystems, say A and B connected in series. Subsystem A consists of two identical units whereas subsystem B consists of only one unit. The operation of only one unit of subsystem A with subsystem B is sufficient to do the job. The failure time distributions of the units are taken exponential while the repair time distributions are assumed to be general. Various measures of system effectiveness useful to system managers are obtained by using regerative point technique. Graphical study of the system through MTSF and profit function is also carried out.

Key Words: Series configuration, Markov-renewal process, Regenerative point technique MTSF.

1. Introduction

Various researchers including (3,4,5,6,7) have analysed complex system models under different sets of asumptions such as two types of repair, allowed down time, abnormal weather condtions, random appearance and disappearance of repairman, patience time of repairman etc. and obtained various economic measures of system effectiveness using the theory of Markov-renewal process, regenerative point technique and supplementary variable technique. The common assumption taken in all these systems is that a single repairman repairs the failed unit continuously till it is repaired and the failure and repair times of a unit are uncorrelated random variables. But in practice, it is not possible for a repairman to repair a failed unit continuously for a long time due to his tiredness. Thus, working efficiency of the repairman may reduce and he needs rest for some period of time. Further, as the rest time of repairman depends upon his working time, therefore, some sort of correlation exists between the random variables denoting the working and rest period of the repairman.

Keeping this view, in mind, we, in the present paper investigate a complex system model by considering the rest period of the repairman. The joint distribution of the random variables representing the working and rest period of repairman is assumed to have bivariate exponential distribution with the density function given by

$$f(x, y) = \alpha\beta(1-r)e^{-\alpha x-\beta y}I_0\left(2\sqrt{\alpha\beta rxy}\right), \alpha, \beta, x, y > 0 \; ; \; 0 < r < 1$$

Where,

$$I_0(z) = \sum_{k=0}^{\infty}\frac{(z/2)^{2k}}{(k!)^2}$$ **is the modified Bessel function of type I and order zero.**

The system description and assumptions are as follows:

(i) System consists of two subsystems, say A and B connected in series. Subsystem A consists of two identical units in passive standby whereas subsystem B consists of only one unit which is dissimilar to units of subsystem A. Initially system starts functioning from state S_0 in which one unit of subsystem A and subsystem B is operative and other unit of subsystem A is kept as cold standby,

(ii) Each unit of the system has two modes - Normal (N) and Total Failure (F).
(iii) Switching device which is used to detect the failed unit and to switch the standby unit into operation is assumed to be perfect,

(iv) A single repairman is always available with the system to repair a failed unit. The repairman goes for rest after working a random period of time, and taking rest for a random period of time, he again starts the repair of failed unit which is pre-emptive repeat type,

(v) If during the repair of failed unit of subsystem A, the subsystem B also fails then priority in repair is given to the unit of subsystem B.

(vi) The failure time distributions of units of both the subsystems are taken negative exponential with different parameters while the repair time distributions are assumed to be general.

2. Notations

E	:	set of regenerative states
X	:	r.v. representing the rest time of the repairman.
Y	:	r.v. representing the working time of the repairman.

θ_1 / θ_2 : constant failure rates of units of subsystems A and B respectively.

$G_1(.) / G_2(.)$: c.d.f.'s of time to repair of units of subsystem A and subsystem B respectively.

$f(x, y)$: joint p.d.f. of (x, y)

$= \alpha\beta(1-r)\, e^{-\alpha x-\beta y} I_0\left(2\sqrt{\alpha\beta rxy}\right) \alpha, \beta, r, x, y > 0, 0 \le r < 1$

$g(.)$: marginal p.d.f. of X = x

$= \alpha(1-r)\, e^{-\alpha(1-r)x}\ ;\ x > 0$

$k(y \mid x)$: conditional p.d.f. of Y|X = x

$= \beta\, e^{-\beta y-\alpha rx} I_0\left(2\sqrt{\alpha\beta rxy}\right)$

$q_{ij}(.)$: p.d.f. of direct transition from regenerative state S_i to S_j.

$q_{ij}^{(k)}(.)$: p.d.f. of transition from regenerative state S_i to S_j via non-regenerative state S_k.

p_{ij} : steady state direct probability of transition from state S_i to S_j such that

$$p_{ij} = \int_0^\infty q_{ij}(u)du$$

$Z_i(t)$: probability that the system sojourns in state S_i upto time t.

ϕ_i : mean sojourn time in state S_i.

©,Ⓢ : symbols for ordinary and stieltjes convolution.

$$A(t) © B(t) = \int_0^t A(u)B(t-u)du$$

and

$$A(t) \circledS B(t) = \int_0^t dA(u)B(t-u)$$

Symbols

We define the following symbols for the states of the system.

N_0/ NA_g/ NA_s : Unit of subsystem A is in N-mode and operative/good/standby.

NB_0 / NB_g : Unit of subsystem B is in N-mode and operative/good.

FA_r / FA_{wr} : Unit of subsystem A is in F-mode and under repair/waiting for repair.

FB_r / FB_{wr} : Unit of subsystem B is in F-mode and under repair/waiting for repair.

Using the above symbols, the states of the system and transitions between them alongwith transition times are shown in Fig. 1. The epochs of transition from S_1 to S_4 and S_3 to S_5 are non regenerative.

3. Transition Probabilities and Sojourn Times

First, we obtain the unconditional direct and indirect steady state transition probabilities i.e. p_{ij} and $p_{ij}^{(k)}$ are given by :

$$p_{01} = \int_0^\infty \theta_1 e^{-(\theta_1+\theta_2)u} du$$

$$= \theta_1(\theta_1+\theta_2)^{-1}$$

Similarly,

$$p_{02} = \theta_2(\theta_1+\theta_2)^{-1}$$

$$p_{10} = \tilde{G}_1(\theta_1+\theta_2+\alpha(1-r))$$

TRANSITION DIAGRAM

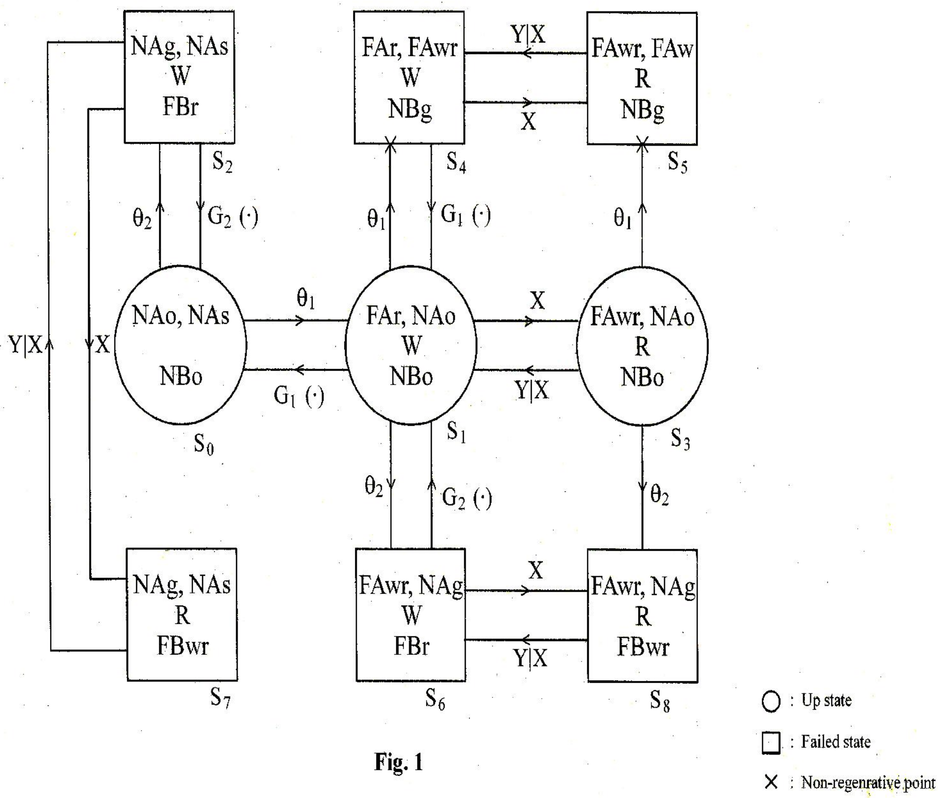

Fig. 1

$$p_{13} = \frac{\alpha(1-r)}{\theta_1+\theta_2+\alpha(1-r)}\left[1-\tilde{G}_1(\theta_1+\theta_2+\alpha(1-r))\right]$$

$$p_{11}^{(4)} = \frac{\theta_1}{\theta_1+\theta_2}\left[\tilde{G}_1(\alpha(1-r))-\tilde{G}_1(\theta_1+\theta_2+\alpha(1-r))\right]$$

$$p_{15}^{(4)} = \frac{\theta_1}{\theta_1+\theta_2}\left[1-\tilde{G}_1(\alpha(1-r))-\frac{\alpha(1-r)}{\theta_1+\theta_2+\alpha(1-r)}\right.$$
$$\left.\times\left(1-\tilde{G}_1(\theta_1+\theta_2+\alpha(1-r))\right)\right]$$

$$p_{16} = \frac{\theta_2}{\theta_1+\theta_2+\alpha(1-r)}\left[1-\tilde{G}_1(\theta_1+\theta_2+\alpha(1-r))\right]$$

$$p_{20} = \tilde{G}_2(\alpha(1-r))$$

$$p_{27} = 1-\tilde{G}_2(\alpha(1-r))$$

$$p_{13} = \frac{\alpha(1-r)}{\theta_1+\theta_2+\alpha(1-r)}\left[1-\tilde{G}_1(\theta_1+\theta_2+\alpha(1-r))\right]$$

$$p_{11}^{(4)} = \frac{\theta_1}{\theta_1+\theta_2}\left[\tilde{G}_1(\alpha(1-r))-\tilde{G}_1(\theta_1+\theta_2+\alpha(1-r))\right]$$

$$p_{15}^{(4)} = \frac{\theta_1}{\theta_1+\theta_2}\left[1-\tilde{G}_1(\alpha(1-r))-\frac{\alpha(1-r)}{\theta_1+\theta_2+\alpha(1-r)}\right.$$
$$\left.\times\left(1-\tilde{G}_1(\theta_1+\theta_2+\alpha(1-r))\right)\right]$$

$$p_{16} = \frac{\theta_2}{\theta_1+\theta_2+\alpha(1-r)}\left[1-\tilde{G}_1(\theta_1+\theta_2+\alpha(1-r))\right]$$

$$p_{20} = \tilde{G}_2(\alpha(1-r))$$

$$p_{27} = 1-\tilde{G}_2(\alpha(1-r))$$

$$p_{34|x}^{(5)} = \frac{\theta_1}{\theta_1+\theta_2}\left[1-k^*((\theta_1+\theta_2)|x)\right]$$

So that,

$$p_{34}^{(5)} = \frac{\theta_1}{\theta_1+\theta_2+(1-r)\beta} = p_{35}$$

$$p_{35|x} = \frac{\theta_1}{\theta_1+\theta_2}\left[1-\frac{\beta}{\theta_1+\theta_2+\beta}e^{-\frac{\alpha r(\theta_1+\theta_2)x}{\theta_1+\theta_2+\beta}}\right]$$

So that,

$$p_{35} = \frac{\theta_1}{\theta_1+\theta_2+\beta(1-r)}$$

$$p_{38|x} = \frac{\theta_2}{\theta_1+\theta_2}\left[1-k^*((\theta_1+\theta_2)|x)\right]$$

So that,

$$p_{38} = \frac{\theta_2}{\theta_1+\theta_2+(1-r)\beta} \qquad (22-25)$$

It can be easily verified that

$$p_{31}+p_{34}^{(5)}+p_{38} = 1 \qquad (26)$$

The mean sojourn times in various states are as follows :

$$\phi_0 = \int_0^\infty e^{-(\theta_1+\theta_2)t}dt$$
$$= (\theta_1+\theta_2)^{-1}$$

Similarly,

$$\phi_1 = 1-\tilde{G}_1(\theta_1+\theta_2+\alpha(1-r))$$
$$\phi_2 = 1-\tilde{G}_2(\alpha(1-r))$$

$$\phi_{3|x} = \frac{1}{\theta_1+\theta_2}\left[1-k^*\{(\theta_1+\theta_2)|x\}\right]$$

$$= (\theta_1+\theta_2)^{-1}\left[1-\frac{\beta}{\theta_1+\theta_2+\beta}e^{-\frac{\alpha r(\theta_1+\theta_2)x}{\theta_1+\theta_2+\beta}}\right]$$

So that,

$$\phi_3 = [\theta_1+\theta_2+\beta(1-r)]^{-1}$$

$$\phi_4 = 1-\tilde{G}_1(\alpha(1-r))$$

$$\phi_{5|x} = \frac{1+\alpha rx}{\beta} = \phi_{7|x} = \phi_{8|x}$$

So that,

$$\phi_5 = (\beta(1-r))^{-1} = \phi_7 = \phi_8$$

$$\phi_6 = 1-\tilde{G}_2(\alpha(1-r)) \qquad (27-35)$$

4. Analysis of Characteristics

(a) Reliability and MTSF

Using the technique of regenerative point, expression of reliability, in terms of its Laplace transform (L.T.), is given by

$$R_o^*(s) = \frac{\left(1-q_{13}^*(s)q_{31}^*(s)\right)Z_0^* + q_{01}^*(s)Z_1^*(s) + q_{01}^*(s)q_{13}^*(s)Z_3^*(s)}{1-q_{01}^*(s)q_{10}^*(s)-q_{13}^*(s)q_{31}^*(s)} \qquad (36)$$

Where, $Z_0^*(s), Z_1^*(s)$ and $Z_2^*(s)$ are the Laplace transforms of

$$Z_0(t) = e^{-[\theta_1+\theta_2]t}$$

$$Z_1(t) = e^{-[\theta_1+\theta_2+\alpha(1-r)]t}\tilde{G}_1(t)$$

and

$$Z_3(t) = e^{-(\theta_1+\theta_2)t}\overline{K}(t|x)$$

Taking inverse Laplace transform of relation (36), we can get the reliability of the system when it initially starts from state S_0. Now the expression of mean time to system failure (MTSF) is given by

$$E(T) = \lim_{s\to 0} R_0^*(s) = \frac{(1-p_{13}p_{31})\phi_0 + p_{01}\phi_1 + p_{01}p_{13}\phi_3}{1-p_{01}p_{10}-p_{13}p_{31}} \qquad (37)$$

(b) Availability Analysis

Let us define A_i (t) as the probability that the system is up (operative) at epoch 't' when it initially starts from the state S_i ε E. Using the regenerative point technique and the tools of Laplace transform one can obtain the value of $A_0(t)$ in terms of its Laplace transform i.e. $A_0^*(s)$.

The steady state availability (probability in the long run that the system is operative) of the system when its initially starts from state S_0, is given by

$$A_0 = \lim_{t\to\infty} A_0(t) = \lim_{s\to 0} sA_0^*(s) = N_1 / D_1 \quad (38)$$

Where,

$$N_1 = p_{20}p_{41}p_{61}(p_{10}\phi_0 + p_{01}\phi_1 + p_{01}p_{13}\phi_3) \quad (39)$$

and

$$\begin{aligned} D_1 = {} & p_{20}p_{41}p_{61}[p_{10}\phi_0 + p_{01}\phi_1 + p_{14}\phi_4 + p_{01}p_{13}(\phi_3 + p_{35}\phi_5)] \\ & + p_{01}p_{20}p_{61}\left(p_{13}p_{34}^{(5)} + p_{15}^{(4)}\right)(\phi_4 + \phi_5) \\ & + p_{01}p_{20}p_{41}(p_{13}p_{38} + p_{16})\phi_6 \\ & + p_{41}[p_{02}p_{10}p_{27}p_{61}\phi_7 + p_{01}p_{20}p_{68}(p_{16} + 2p_{38})\phi_8] \end{aligned} \quad (40)$$

The expected uptime of the system during (0, t) is given by

$$\mu_{up}(t) = \int_0^t A_0(u)du, \text{ so that } \mu_{up}^*(s) = A_0^*(s) / s$$

(c) Busy Period Analysis

Let B_i (t) be the probability that the repairman is busy at time 't' in the repair of a failed unit when system initially starts from state S_i ε E. Using the regenerative point technique and the tools of Laplace transform one can obtain the value of $B_0(t)$ in terms of its Laplace transform *i.e.* $B_0^*(s)$.

In the long run, the probability that the repairman will be busy in repair of failed unit is given by,

$$B_0 = N_2 / D_1 \quad (41)$$

Where,

$$\begin{aligned} N_2 = {} & p_{02}p_{10}p_{41}p_{61}\phi_2 + p_{01}p_{20}\left[p_{61}\left(p_{13}p_{34}^{(5)} + p_{15}^{(4)}\right)\phi_4 \right. \\ & \left. + p_{41}(p_{38} + p_{16})\phi_6 + p_{41}p_{61}\phi_1\right] \end{aligned} \quad (42)$$

The value of D_1 is same as in (40)

The expected busy period of repairman when he is busy in repair of the failed unit during (0, t) is given by,

$$\mu_b(t) = \int_0^t B_0(u)\, du \text{ so that } \mu_b^*(s) = B_0^*(s) / s$$

(d) Profit function Analysis

The expected profit incurred by the system during (0,t) is given by, P(t) = Expected total revenue in (o,t) – Expected total expenditure in (o,t)

$$= C_0\mu_{up}(t) - C_1\mu_b(t) \tag{43}$$

Where C_0 is the revenue per unit time by the system due to operation while C_1 is the amount paid to the repairman per unit time when he is busy in repair of failed unit.

The expected profit per unit time in steady state is given by

$$P = C_o A_o - C_1 B_0 \tag{44}$$

Where, A_o, B_0 have been already defined.

5. Particular Case

If we assume that the repair time distributions of the units of subsystem A and subsystem B are negative exponential with parameters β_1 and β_2 respectively i.e.

$$G_1(t) = 1 - e^{-\beta_1 t},\ t > 0$$

$$G_2(t) = 1 - e^{-\beta_2 t},\ t > 0$$

Then in results (37), (38), (41) and (44) we have the following changes:

$$p_{10} = \beta_1 / (\theta_1 + \theta_2 + \alpha(1-r) + \beta_1)$$

$$p_{13} = \frac{\alpha(1-r)}{(\theta_1 + \theta_2 + \alpha(1-r) + \beta_1)}$$

$$p_{14} = \frac{\theta_1}{(\theta_1 + \theta_2 + \alpha(1-r) + \beta_1)}$$

$$p_{16} = \frac{\theta_2}{(\theta_1 + \theta_2 + \alpha(1-r) + \beta_1)}$$

$$p_{20} = \frac{\beta_2}{(\alpha(1-r) + \beta_2)}$$

$$p_{27} = \frac{\alpha(1-r)}{\alpha(1-r) + \beta_2}$$

$$p_{41} = \frac{\beta_1}{\alpha(1-r) + \beta_1}$$

$$p_{45} = \frac{\alpha(1-r)}{\alpha(1-r) + \beta_1}$$

$$p_{61} = \frac{\beta_2}{\alpha(1-r)+\beta_2}$$

$$p_{62} = \frac{\alpha(1-r)}{\alpha(1-r)+\beta_2}$$

$$\phi_1 = \frac{\theta_1+\theta_2+\alpha(1-r)}{\theta_1+\theta_2+\alpha(1-r)+\beta_1}$$

$$\phi_2 = \frac{\alpha(1-r)}{\alpha(1-r)+\beta_2} = \phi_6$$

$$\phi_4 = \frac{\alpha(1-r)}{\alpha(1-r)+\beta_1}$$

6. Graphical Study of the System Behaviour

For a more concrete study of the system we plot the graphs for MTSF and profit function w.r.t α (failure rate) for different values of r keeping the other parameters fixed. Fig. 2 shows the curves for MTSF w.r.t. α for different values of r (= 0.25, 0.50, 0.75) keeping other parameters fixed as θ_1, = .002, θ_2 = .004, β_1 = 0.10, β_2 = 0.25 and β = .005. From figure it is observed that MTSF decreases linearly as α increases irrespective of other parameters. Further, the curves also indicate that for the same value of α, MTSF is higher for higher values of r. So we conclude that the high correlation (r) between working and rest period of repairman tends to increase the expected life time of the system.

Figure 3 represents the variation in profit w.r.t. α for different values of r (= 0.25, 0.50, 0.75) whereas in addition to the above parameters we fix C_0 = 500 and C_1 = 100. From the figure it is clear that profit decreases linearly as α increases. Also for the fixed value of α, the profit is higher for high correlation (r). Thus, finally we conclude that the high correlation between working and rest periods of the repairman yields the better system performance.

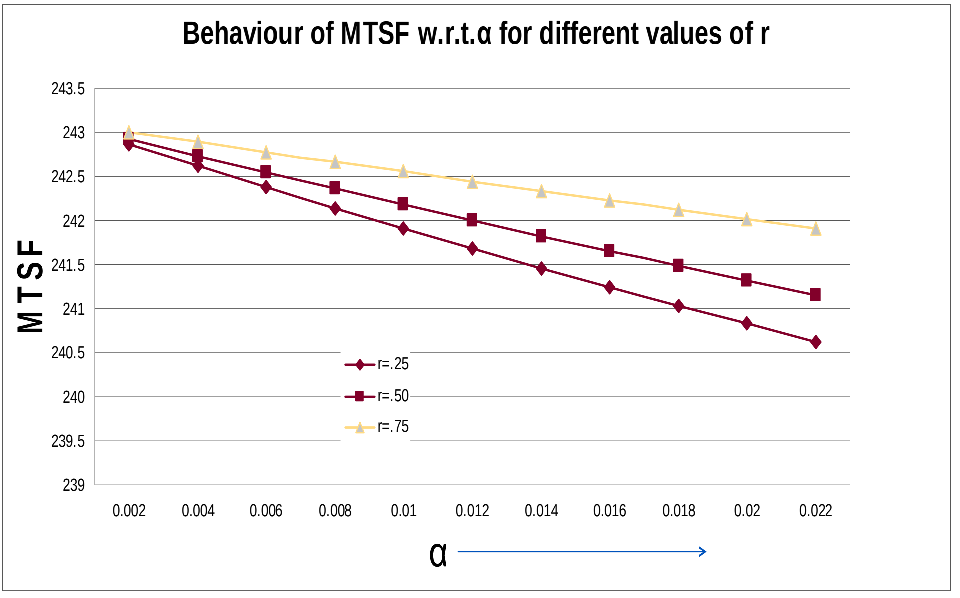
Behaviour of MTSF w.r.t.α for different values of r
243.5
243
242.5
242
241.5
241
240.5
240
239.5
239
M T S F
r=.25
r=.50
r=.75
0.002
0.004
0.006
0.008
0.01
0.012
0.014
0.016
0.018
0.02
0.022
α

Fig. 2

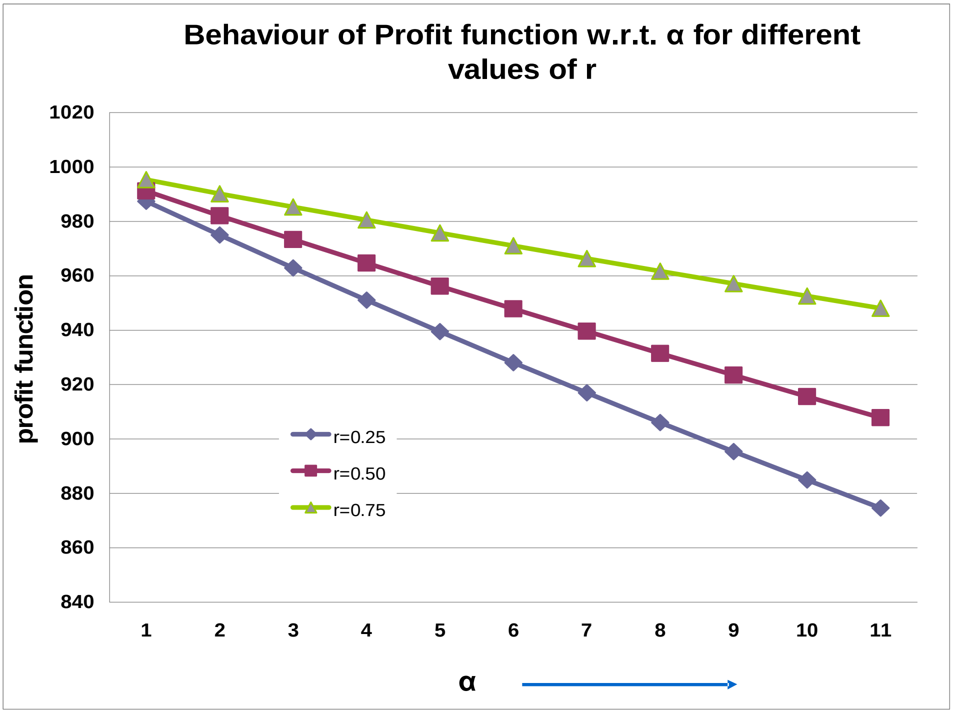
Behaviour of Profit function w.r.t. α for different values of r
1020
1000
980
960
940
920
900
880
860
840
profit function
r=0.25
r=0.50
r=0.75
1
2
3
4
5
6
7
8
9
10
11
α

Fig. 3

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