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ESTIMATION OF GEOMETRIC MEAN OF SKEWED POPULATIONS WITH AND WITHOUT USING AUXILIARY INFORMATION

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Abstract

There are certain situations, for example, positively skewed distributions, where geometric mean is more appropriate measure of location than the arithmetic mean as it gives larger weight to smaller values than larger values of variables. It is specifically useful in averaging ratios, percentages and rates of change in one period over the other. In this paper we propose two different types of estimators for estimating population geometric mean of the characteristic under study variable y, one with and the other without using auxiliary information. To investigate the properties of these estimators we obtain their Bias and Mean Square Errors (MSE) along with the upper bounds for their mean square errors under certain realistic assumptions. Empirical example is also given showing the relative efficiencies of the proposed estimators.

Key Words: Geometric Mean, Skewness, Estimators, Auxiliary Information, Bias, Mean Square Error, Relative Efficiency, Upper Bound.

1. Introduction

In most of the survey sampling problems the parameter of interest is either the total or the arithmetic mean of the characteristic under study as a measure of average or central values. The per capita income, the average operational holding size, per capita fuel wood consumption are some of the parameters which might give very misleading conclusions for the populations due to high skewness of the variables in question. In medical studies variables with normal and symmetrical distribution are very rare say for example survival times. In such cases of distribution the arithmetic mean may not be most appropriate measure of the average of the variables and it is preferable to use either the median or the geometric mean. For positively skewed distribution, geometric mean is a more appropriate measure of location than the mean as it gives larger weight to smaller values than larger values of variables. Geometric Mean is specifically useful in averaging ratios, percentages and rates of change in one period over the other. Also use of central limit theorem is valid only when sampling distribution of statistics in use is normal which is not in the case of highly skewed distributions and hence calculation of confidence interval may not be easy task in such situations. Though the properties of geometric mean have been studied empirically through simulations, bootstrapping and other techniques, a lot of work remains to be done for studying theoretical properties of geometric mean .

Considering a finite population of N unit, let (Y_i , X_i) , i = 1, 2, …. ,N be the values of observation for the i^{th} unit of the population according to the study variable y and the auxiliary variable x respectively. Further let a simple random sample of size n from this population is taken without replacement having sample values (y_i, x_i), i = 1,2…,n assuming without any loss of generality that first n units have been selected in the sample from N units of the population. Further we assume that no value is zero and negative.

The population geometric mean, the parameter of interest, is given by,

$$G_m = (Y_1 Y_2 ... Y_N)^{1/N} \tag{1.1}$$

The obvious choice for the estimator should be the sample geometric mean given by,

$$\hat{G}_m = g_m = (y_1 y_2 ... y_n)^{1/n} \tag{1.2}$$

To study properties of g_m is not very easy task and hence to avoid the mathematical complexities assuming deviation about mean to be less as compared to mean we may consider the following estimators for population geometric mean

$$g_0 = \bar{y}\left(1 - \frac{\alpha}{2}\frac{s_y^2}{\bar{y}^2}\right) \tag{1.3}$$

And for positively correlated variables y and x we may take

$$g_1 = \bar{y}\left(1 - \frac{\alpha}{2}\frac{s_y^2}{\bar{y}^2}\right)\left(\frac{\bar{X}}{\bar{x}}\right) \tag{1.4}$$

whereas when y and x are negatively correlated we may take

$$g_1^* = \bar{y}\left[1 - \alpha\frac{s_y^2}{\bar{y}^2}\right]\left(\frac{\bar{x}}{\bar{X}}\right) \tag{1.5}$$

where $\alpha = (n-1)/n$ or a suitably chosen scalar.

A generalized class of estimators of geometric mean may be taken as g_c

$$g_c = \bar{y}\left[1 - \alpha\frac{s_y^2}{\bar{y}^2}\right]\left(\frac{\bar{x}}{\bar{X}}\right)^{\lambda} \tag{1.6}$$

where α and λ are the characterizing scalars to be chosen suitably and determined by minimizing mean square error $MSE(g_c)$.

Further let,

$$\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i, \qquad \bar{Y} = \frac{1}{N}\sum_{i=1}^{N} Y_i,$$

$$\sigma_Y^2 = \frac{1}{N}\sum_{i=1}^{N}\left(Y_i - \bar{Y}\right)^2, \qquad S_Y^2 = \frac{1}{N-1}\sum_{i=1}^{N}\left(Y_i - \bar{Y}\right)^2,$$

$$s_y^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left(y_i - \bar{y}\right)^2, \qquad C_Y^2 = \frac{S_Y^2}{\bar{Y}^2},$$

$$\mu_{rs} = \frac{1}{N}\sum_{i=1}^{N}\left(Y_i - \bar{Y}\right)^r\left(X_i - \bar{X}\right)^s, \qquad \beta_2 = \frac{\mu_{40}}{\mu_{20}^2}$$

2. Bias of the Estimator g_0

Let, $$e_0 = \frac{\bar{y} - \bar{Y}}{\bar{Y}} \Rightarrow \bar{y} = \bar{Y}(1 + e_0)$$

$$e_1 = \frac{s_y^2 - S_y^2}{S_y^2} \Rightarrow s_y^2 = S_y^2(1+e_1)$$

so that $$E(e_0) = E(e_1) = 0 \tag{2.1}$$

Assuming population size N to large be enough in comparison to sample size n, we may ignore finite population correction factor (f.p.c) and get,

$$E\left(e_0^2\right) = \frac{C_y^2}{n}$$

$$E\left(e_1^2\right) = \frac{2}{n-1}\left\{1 + \frac{n-1}{2n}(\beta_2 - 3)\right\}$$

$$E\left(e_0 e_1\right) = \frac{\mu_3}{n\bar{Y}S_y^2} \tag{2.2}$$

Since for large sample if deviations are small as compared with mean, we may use, for mathematical simplicity, approximation formula for geometric mean as

$$G = \bar{Y}\left(1 - \frac{\sigma_Y^2}{2\bar{Y}^2}\right)$$

$$= \bar{Y}\left[1 - \frac{N-1}{2N}\frac{S_y^2}{\bar{Y}^2}\right] \tag{2.3}$$

Then, $$G = \bar{Y}\left[1 - \frac{n-1}{2n}C_y^2\right]$$

$$G = \bar{Y} - \bar{Y}\frac{\alpha}{2}C_y^2$$

$$G = \bar{Y} - k \qquad \text{where} \qquad k = \bar{Y}\frac{\alpha}{2}C_y^2 \tag{2.4}$$

Now from equations (1.3) and (2.1), we have

$$g_0 = \bar{Y}\left(1+e_0\right)\left(1-\frac{\alpha}{2}\frac{S_y^2}{\bar{Y}^2}\frac{\left(1+e_1\right)}{\left(1+e_0\right)^2}\right)$$

$$= \bar{Y}\left[\left(1+e_0\right)-\frac{\alpha}{2}C_y^2\left(1+e_1\right)\left(1+e_0\right)^{-1}\right] \tag{2.5}$$

Now put $k=\bar{Y}\frac{\alpha}{2}C_y^2$ and expanding $\left(1+e_0\right)^{-1}$ in (2.5) we get,

$$g_0 - G = \bar{Y}e_0 - k\left(-e_0+e_0^2+e_1-e_0e_1+\ldots\right) \tag{2.6}$$

Taking expectation both the side we get, $Bias\left(g_0\right)$ up to $O\left(\frac{1}{n}\right)$

$$Bias\left(g_0\right) = E\left(g_0 - G\right)$$

$$= k\left[\frac{\mu_3}{n\bar{Y}S_y^2}-\frac{C_y^2}{n}\right] \tag{2.7}$$

3. The Mean Square Error (MSE) of Estimator (g_0).

The MSE of the proposed estimator is given by

$$MSE(g_0) = E\left(g_0 - G\right)^2 \tag{3.1}$$

Now from equation (2.6) we have up to $O\left(\frac{1}{n}\right)$

$$MSE\left(g_0\right) = E\left[\bar{Y}e_0 + ke_0 - ke_1\right]^2$$

$$= E\left[k_0e_0 - ke_1\right]^2 \qquad \text{Where } k_0 = \bar{Y}+k$$

$$= k_0^2E\left(e_0^2\right)+k^2E\left(e_1^2\right)-2kk_0E\left(e_0e_1\right)$$

Using equation (2.2)

$$= \left[k_0^2\frac{C_y^2}{n}+k^2\frac{2}{n-1}\left\{1+\frac{n-1}{n}\left(\beta_2-3\right)\right\}-2kk_0\frac{\mu_3}{n\bar{Y}S_y^2}\right] \tag{3.2}$$

4. Upper Bound for MSE of g_0

It is well known that G.M $\leq$ A.M which gives

$$\frac{G}{\overline{Y}} \leq 1 \tag{4.1}$$

$$\Rightarrow \quad 1 - \frac{G}{\overline{Y}} \geq 0 \tag{4.2}$$

Also , $G = \overline{Y} - k$; where $k = \frac{\alpha}{2}\overline{Y}C_y^2$ from (2.4)

$$\Rightarrow \quad \frac{G}{\overline{Y}} = 1 - \frac{k}{\overline{Y}} \Rightarrow 1 - \frac{G}{\overline{Y}} = \frac{k}{\overline{Y}} \qquad \text{using (4.1)}$$

From (4.2) $\frac{k}{\overline{Y}} \geq 0 \Rightarrow k \geq 0$ since $1 - \frac{G}{\overline{Y}} \geq 0$ (4.3)

Again considering $1 - \frac{G}{\overline{Y}} = \frac{k}{\overline{Y}}$

Since the left hand side is less than unity, as $\frac{G}{\overline{Y}} \leq 1 \,\&\, G > 0$

$$\frac{k}{\overline{Y}} \leq 1 \qquad \Rightarrow k \leq \overline{Y}$$

Thus $k \in \left[0, \overline{Y}\right]$ (4.4)

Further, $\overline{Y} + k = k_0$

$$\Rightarrow \quad k = k_0 - \overline{Y}$$

$$\Rightarrow \quad k_0 - \overline{Y} \geq 0 \qquad \textbf{n}\; k \geq 0 \text{ from (4.3)}$$

$$\Rightarrow \quad k_0 \geq \overline{Y} \tag{4.5}$$

Also $k \leq \overline{Y} \;\; \Rightarrow k_0 - \overline{Y} \leq \overline{Y}$ from (4.4)

$$\Rightarrow \quad k_0 \leq 2\overline{Y} \tag{4.6}$$

Combining (4.5) and (4.6) we get

$$k_0 \in \left[\overline{Y}, 2\overline{Y}\right] \tag{4.7}$$

Substituting these limit of k and k_0 we get from equation (3.2)

$$MSE\left(g_0\right) \le \left[4\bar{Y}^2 \frac{C_y^2}{n} + \frac{2\bar{Y}^2}{n-1}\left\{1+\frac{n-1}{2n}\left(\beta_2 - 3\right)\right\} - 4\bar{Y}^2 \frac{\mu_3}{n\bar{Y}S_y^2} \right]$$

Since the definition is positively skewed therefore we can write $\mu_3 > 0$

$$MSE\left(g_0\right) \le \left[4\bar{Y}^2 \frac{C_y^2}{n} + \frac{2\bar{Y}^2}{n-1}\left\{1+\frac{n-1}{2n}\left(\beta_2 - 3\right)\right\} \right] \tag{4.8}$$

Further we may obtain from equation,

$$k \le \bar{Y}$$

$$\Rightarrow \quad \bar{Y}\frac{\alpha}{2}C_y^2 \le \bar{Y}$$

$$\Rightarrow \quad C_y^2 \le \frac{2}{\alpha} = \frac{2n}{n-1}$$

$$\therefore \quad MSE(g_0) \le \bar{Y}^2\left[\frac{8}{n-1} + \frac{2}{n-1}\left\{1+\frac{n-1}{2n}\left(\beta_2 - 3\right)\right\}\right]$$

$$MSE(g_0) \le \frac{2\bar{Y}^2}{n-1}\left[5+\frac{n-1}{2n}\left(\beta_2 - 3\right)\right] \tag{4.9}$$

If distribution is mesokurtic, $\beta_2 = 3$

then,

$$MSE(g_0) \le \bar{Y}^2\left[\frac{10}{n-1}\right] \tag{4.10}$$

5. Bias of the Estimator g_1

Let, $e_2 = \dfrac{\bar{x} - \bar{X}}{\bar{X}} \Rightarrow \bar{x} = \bar{X}\left(1+e_2\right)$ such that $E\left(e_2\right) = 0$ (5.1)

Assuming population size large enough in comparison to sample size we may ignore finite population correction factor (f.p.c) we obtain,

$$E\left(e_0 e_2\right) = \frac{C_{yx}}{n}$$

$$E\left(e_1 e_2\right) = \frac{\mu_{21}}{n\bar{X}S_y^2} \tag{5.2}$$

from equation (1.4) we have,

$$g_1 = \bar{y}\left(1-\frac{\alpha}{2}\frac{s_y^2}{\bar{y}^2}\right)\left(\frac{\bar{X}}{\bar{x}}\right)$$

Substituting values of $\bar{y}$, s_y^2 *and* $\bar{x}$ from (1.5) and (5.1)

$$g_1 = \bar{Y}\left(1+e_0\right)\left(1-\frac{\alpha}{2}\frac{S_y^2}{\bar{Y}^2}\frac{\left(1+e_1\right)}{\left(1+e_0\right)^2}\right)\left(\frac{\bar{X}}{\bar{X}\left(1+e_2\right)}\right)$$

$$= \left[\bar{Y}\left(1+e_0\right)-\frac{\alpha}{2}\bar{Y}C_y^2\left(1+e_1\right)\left(1+e_0\right)^{-1}\right]\left(1+e_2\right)^{-1} \tag{5.3}$$

Expanding $\left(1+e_0\right)^{-1}$ and $\left(1+e_2\right)^{-1}$ and put $\frac{\alpha}{2}\bar{Y}C_y^2 = k$ in (5.3) we get,

$$g_1 = \bar{Y}\left(1+e_0\right)\left(1-e_2+e_2^2\right)-k\left(1+e_1\right)\left(1-e_0+e_0^2+...\right)\left(1-e_2+e_2^2+...\right)$$

$$g_1 - G = -\ \left[\bar{Y}\left(e_2-e_2^2-e_0+e_0e_2\right)+k\left(-e_0+e_0^2+e_1-e_0e_1-e_2\right.\right.$$

$$\left.\left.+e_0e_2-e_1e_2+e_2^2\right)\right]\ +\ldots \tag{5.4}$$

Taking expectation both the side we get $Bias\left(g_1\right)$ up to $O\left(\frac{1}{n}\right)$,

$$Bias(g_1) = E(g_1 - G)$$

$$= \left(\bar{Y}-k\right)E\left(e_2^2\right)-\left(\bar{Y}+k\right)E\left(e_0e_2\right)-kE\left(e_0^2\right)+kE\left(e_0e_1\right)+kE\left(e_1e_2\right) \tag{5.5}$$

Now putting $\bar{Y}-k = G$ and $\bar{Y}+k = k_0$ in the equation (5.5) we get.

$$Bias\left(g_1\right) = GE\left(e_2^2\right)-k_0E\left(e_0e_2\right)-kE\left(e_0^2\right)+kE\left(e_0e_1\right)+kE\left(e_1e_2\right)$$

$$= \frac{1}{n}\left[\left(GC_x^2-k_0C_{yx}-kC_y^2\right)+\frac{k}{\bar{Y}^2C_y^2}\left(\frac{\mu_{30}}{\bar{Y}}+\frac{\mu_{21}}{\bar{X}}\right)\right] \tag{5.6}$$

6. The Mean Square Error (MSE) of Estimator g_1

The mean squaring error of the proposed estimator g_1 is given by squaring equation (5.4) and taking expectation up to $O\left(\frac{1}{n}\right)$ we get

$$MSE(g_1) = E\left(g_1 - G\right)^2 \tag{6.1}$$

$$= E\left[k_0 e_0 - k e_1 - G e_2\right]^2 \qquad ;\text{where } k_0 = \bar{Y} + k \;\&\; \bar{Y} - k = G$$

$$= \frac{1}{n}\left[k_0^2 C_y^2 + G^2 C_x^2 - 2k_0 G C_{yx}\right] + \frac{2k^2}{n-1}\left\{1 + \frac{n-1}{2n}\left(\beta_2 - 3\right)\right\}$$
$$- \frac{2k}{nS_y^2}\left[\frac{k_0 \mu_{30}}{\bar{Y}} - \frac{G\mu_{21}}{\bar{X}}\right] \tag{6.2}$$

7. Upper Bound for MSE of g_1

From (4.4) we have

$$k \in \left[0, \bar{Y}\right] \tag{7.1}$$

And from equation (4.7) we have

$$k_0 \in \left[\bar{Y}, 2\bar{Y}\right] \tag{7.2}$$

Also since it is well known that $0 \le G.M \le A.M$,

therefore $\quad G \in \left[0, \bar{Y}\right]$ (7.3)

Substituting these limits of k, k_0 and G, we get from equation (6.2);

$$MSE\left(g_1\right) \le \frac{1}{n}\left[4\bar{Y}^2 C_y^2 + \bar{Y}^2 C_x^2 - 4\bar{Y}^2 C_{yx}\right] + \frac{2\bar{Y}^2}{n-1}\left\{1 + \frac{n-1}{2n}\left(\beta_2 - 3\right)\right\}$$
$$- \frac{2\bar{Y}}{nS_y^2}\left[\frac{2\bar{Y}\mu_{30}}{\bar{Y}} - \frac{\bar{Y}\mu_{21}}{\bar{X}}\right]$$

For positively skewed distribution $\mu_{30} > 0$, $\rho > 0$ therefore,

$$MSE\left(g_1\right) \le \frac{1}{n}\left[4\bar{Y}^2 C_y^2 + \bar{Y}^2 C_x^2\right] + \frac{2\bar{Y}^2}{n-1}\left\{1 + \frac{n-1}{2n}\left(\beta_2 - 3\right)\right\} + \frac{2\bar{Y}^2 \mu_{21}}{n\bar{X}S_y^2}$$

$$= \frac{\bar{Y}^2}{n}\left[4C_y^2 + C_x^2 + \frac{2}{n-1}\left\{1 + \frac{n-1}{2n}\left(\beta_2 - 3\right)\right\} + \frac{2\mu_{21}}{S_y^2 \bar{X}}\right] \tag{7.4}$$

If distribution is mesokurtic, $\beta_2 = 3$ then,

$$= \frac{\bar{Y}^2}{n}\left[4C_y^2 + C_x^2 + \frac{2}{n-1} + \frac{2\mu_{21}}{\bar{X}\bar{Y}C_y^2}\right] \tag{7.5}$$

Further we may obtain from equation,

$$k \le \bar{Y}$$

$$\Rightarrow \quad \bar{Y}\frac{\alpha}{2}C_y^2 \le \bar{Y}$$

$$\Rightarrow \quad C_y^2 \le \frac{2}{\alpha} = \frac{2n}{n-1}$$

$$\therefore MSE\left(g_1\right) \le \frac{\bar{Y}^2}{n}\left[\frac{8n}{n-1} + C_x^2 + \frac{2}{n-1} + \frac{2\mu_{21}}{\bar{X}\bar{Y}^2 C_y^2}\right]$$

$$= \frac{\bar{Y}^2}{n}\left[\frac{2\left(4n+1\right)}{n-1} + \frac{2\mu_{21}}{\bar{X}\bar{Y}^2 C_y^2} + C_x^2\right]$$

$$\le \frac{\bar{Y}^2}{n}\left[\frac{2\left(4n+1\right)}{n-1} + C_x^2\right] \quad \text{If } \mu_{21} < 0 \tag{7.6}$$

C_x^2, being a stable quantity, may be known from previous experience, pilot survey or literature and hence may be replaced by that valve say C_0 then,

$$MSE\left(g_1\right) \le \frac{\bar{Y}^2}{n}\left[\frac{2\left(4n+1\right)}{n-1} + C_0\right] \tag{7.7}$$

which is the upper bound of MSE (g_1).

8. Empirical Example

From the real primary data dealing with weight (Y) in kg and height (X) in c.m. in a study of N=277 children between age group of 3 to 36 months, the required value of population parameters are calculated. Further to study the property of proposed estimator, random sample of size 30 was taken and required sample values calculated.

$\bar{Y} = 6.587726$, $\bar{X} = 68.23105$, $S_y^2 = 6.691442$

$S_x^2 = 156.989902$, $S_{yx} = 26.029657$, $C_y^2 = 0.154187$

$C_x^2 = 0.033721$, $C_{yx} = 0.0579096$, $\mu_{30} = 8.285403577$

$\mu_{21} = 26.7871327$, $\mu_{40} = 124.976079$, $\mu_{20} = 6.6672861$

$\beta_2 = 2.811439303$, $G_m = 6.069568621$, $G = 6.09678417414$

$\bar{x} = 64.86667$, $\bar{y} = 6.09667$, $s_y^2 = 6.374126$

$s_x^2 = 145.291954$, $s_{yx} = 23.782298$, $c_y^2 = 0.17148877$

$c_x^2 = 0.03453014$, $\hat{\mu}_{30} = 15.0315771$, $\hat{\mu}_{21} = 47.58578339$

$\hat{\mu}_{40} = 129.847287$, $\hat{\mu}_{20} = 6.161655556$, $\hat{\beta}_2 = 3.420094025$

$g_0 = 5.5913371$, $g_1 = 5.88133794278$

$MSE(g_0) = 0.229092405$, $MSE(g_1) = 0.115990168$

$M\hat{S}E(g_0) = 0.20504882494$, $M\hat{S}E(g_1) = 0.08856441218$

Relative efficiency of g_1 over g_0 is

$$\text{Efficiency} = \frac{\text{MSE}(g_0)}{\text{MSE}(g_1)} \times 100 = 197.5101933\%$$

Relative efficiency of g_1 over g_0 based on estimated MSE of g_0 and g_1

$$\text{Efficiency} = \frac{\text{M}\hat{\text{S}}\text{E}(g_0)}{\text{M}\hat{\text{S}}\text{E}(g_1)} \times 100 = 231.525078632\%$$

9. Conclusions

(i) we can easily see that $MSE(g_1) < MSE(g_0)$ if $\rho \geq \frac{GC_x}{2k_0 C_y} \geq 0$, which shows that g_1 will be better than g_0 in the sense of having lesser mean square error when variables y and x are positively correlated .

(ii) A generalized class of estimators of geometric mean may be taken as g_c

$$g_c = \bar{y}\left[1-\alpha\frac{s_y^2}{\bar{y}^2}\right]\left(\frac{\bar{x}}{\bar{\bar{X}}}\right)^{\lambda} \tag{9.1}$$

where α and λ are the characterizing scalars to be chosen suitably and determined by minimizing mean square error $MSE(g_c)$.

(iii) When y and x are positively correlated we may take λ = -1,in (9.1) to get $MSE\left(g_1\right) < MSE\left(g_0\right)$ and obtain following estimator ,

$$g_1 = \bar{y}\left[1-\alpha\frac{s_y^2}{\bar{y}^2}\right]\left(\frac{\bar{X}}{\bar{\bar{x}}}\right) \tag{9.2}$$

(iv) When y and x are negatively correlated we may take λ = 1 in (9.1) to get $MSE\left(g_1^*\right) < MSE\left(g_0\right)$ and obtain following estimator,

$$g_1^* = \bar{y}\left[1-\alpha\frac{s_y^2}{\bar{y}^2}\right]\left(\frac{\bar{x}}{\bar{\bar{X}}}\right) \tag{9.3}$$

(v). It is clear that values of the proposed estimators g_0 =5.5913371 and g_1 =5.881338 are very close to exact and approximated population geometric means $G_m = 6.069568621$ and $G = 6.09678417414$ respectively having very small MSE's and estimated MSE's are given by $MSE\left(g_0\right) = 0.229092405$, $MSE\left(g_1\right) = 0.115990168$. $M\hat{S}E\left(g_0\right) = 0.20504882494$, $M\hat{S}E\left(g_1\right) = 0.08856441218$.

(vi) Relative and estimated relative efficiency of g_1 which utilizes auxiliary information is 197.5% and 231.5% respectively over estimator g_0 which does not utilizes auxiliary information in the example under consideration.

(vii) Upper bounds of MSEs of g_0 and g_1 are given as 14.6921009961 and 1.20139259602 showing that the upper bound of MSE is sharpened a lot by using auxiliary information and becomes quite closer to the actual value of MSE.

(viii) It has been shown that under suitable conditions $MSE\left(g_1\right) < MSE\left(g_0\right)$ showing that efficiency of the estimator g_0 of population geometric mean of variable under study y has been improved by utilizing information on auxiliary variable x in g_1 which is positively correlated with y variable.

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