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ANALYSIS OF A TWO UNIT STANDBY SYSTEM WITH CORRELATED FAILURE AND REPAIR AND RANDOM APPEARANCE AND DISAPPEARANCE OF REPAIRMAN

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Abstract: The paper deals with the stochastic analysis of a two non-identical unit standby system model. The one unit is considered as priority (p) unit and the other as ordinary (o) unit. The p-unit gets priority in operation. A single repair facility appears in and disappears from the system randomly with constant rates. The repair discipline of units is FCFS. The joint distribution of failure and repair times for each unit is taken to be bivariate exponential. Using regenerative point technique various measures of system effectiveness useful to industrial managers are obtained.

Keywords: Standby redundant system, bivariate exponential distribution, availability, mean time to system failure, mean sojourn time, transition probability.

1. Introduction

Repair maintenance is one of the important measures for increasing the effectiveness of a system. Numerous authors have analysed various system models considering different repair policies [1-3, 6, 9, 10]. Mogha & Gupta [11] and Singh & Srinivasu [13] analysed two-unit warm and cold standby system models in which a failed unit goes for preparation before entering into repair. The preparation time is taken as a r. v. which follows some probability distribution. Murari and Goyal [12] analysed three stochastic models, A, B and C, each consisting of two identical units in a cold standby configuration. In model A it has been assumed that the repairman is always available with the system. In model B, he is available immediately on failure of a unit, while in model C he takes some random time to reach the system.

Gupta and Goel [7] considered a two-dissimilar unit parallel system model assuming that a delay occurs due to administrative action in locating & getting the repairman available to the system. Singh [14] analysed a two-unit cold standby system assuming that the service facility appears and disappears from the system at random. Recently, Tuteja and Malik [15] investigated two single unit system models with a common assumption that a single repair facility appears and disappears randomly. They have also assumed that the failure and repair times are uncorrelated random variables.

The purpose of the present paper is devoted to extend the concept of random appearance and disappearance of repairman in a two dissimilar unit standby system with failure and repair times of each unit as correlated random variables having their joint distribution as bivariate exponential. The concept of correlation between failure and repair times was introduced by Goel, Gupta, Srivastava and others [4, 5, 8] in the literature of reliability. The system description and assumptions are as follows:

(i) The system consists of two non-identical (dissimilar) units—unit-1 and unit-2. The unit-1 is called priority or ordinary unit (p-unit) and unit-2 as the non-priority unit (o-unit). The operation of only one unit is required to run the system. Initially unit-1 is operative and unit-2 is kept into cold standby.

(ii) Each unit of the system has two modes—Normal (N) and total failure (F). When both the units are in N-mode, the p-unit gets preference in operation over the o-unit.

(iii) A switching device is used to put the standby unit into operation whenever the operative p-unit fails. The switching device is found always perfect and instantaneous whenever required.

(iv) There is a single repair facility which appears in and disappears from the system randomly. Once the repairman starts the repair of a failed unit, he does not leave the system till all the units are repaired that failed during his stay in the system on FCFS bases.

(v) The joint distribution of failure and repair times for each unit is taken to be bivariate exponential having the density function

$$f_i(x,y)=\alpha_i\,\beta_i\,(1-r_i)e^{-\alpha_i x-\beta_i y}\,I_0(2\sqrt{\alpha_i\,\beta_i\,r_i\,x\,y});$$

$$x,\ y,\ \alpha_i,\beta_i>0;\ 0\le r_i<1$$

and each repaired unit works as good as new.

(vi) The distributions of time to appearance and disappearance of repairman are taken to be exponential with different parameters.

2. Notations And States of the System

For defining the states of the system we assume the following symbols:

N_{io}, N_{2S} : Unit-i is operative and unit-2 is standby in N-mode. (i = 1 and 2 for p and o-unit)

F_{ir}, F_{iw} : Unit-i is in F-mode and under repair/waiting for repair.

A, NA : Repairman is available/not available in the system.

With the above symbols and the assumptions stated in previous section, the possible states of the system are shown in Fig. 1. In the figure, we observe that the epochs of transitions from state S_2 to S_4 and S_6 to S_7 are non-regenerative whereas all the other entrance epochs into the states are regenerative.

The other notations used are defined as follows:

E : Set of regenerative states $\equiv \{S_0, S_1, S_2, S_3, S_4, S_5, S_6\}$

$\bar{E}$: Set of non-regenerative states $\equiv \{S_4, S_7\}$

X_i (i = 1, 2) : Random variables representing the failure times of p-unit and o-unit respectively for i = 1 and 2.

Y_i (i = 1, 2) : Random variables representing the repair times of p-unit and o-unit respectively for i = 1 and 2.

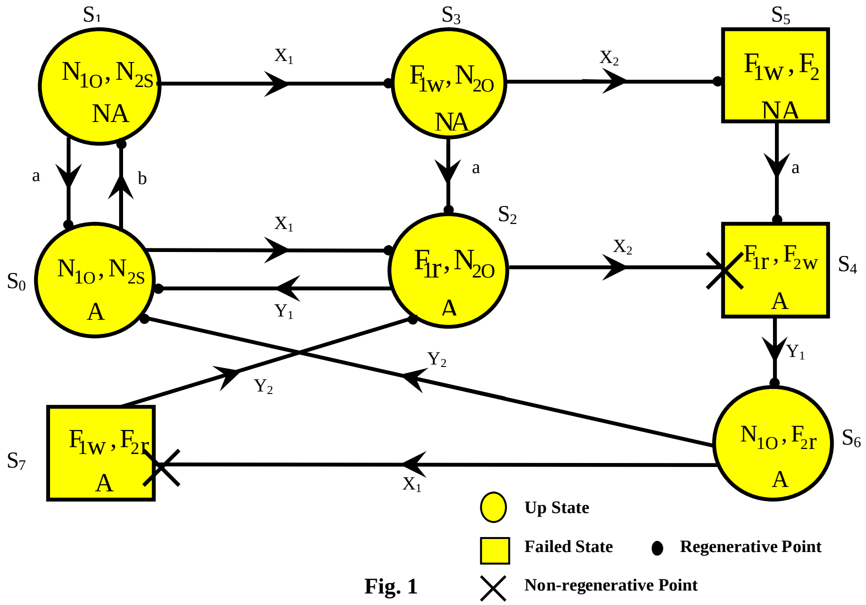

Fig. 1

$f_i(x, y)$: Joint p.d.f. of (X_i, Y_i); i = 1 and 2

$$= \alpha_i \beta_i (1-r_i) e^{-\alpha_i x - \beta_i y} I_0 (2\sqrt{\alpha_i \beta_i r_i x y})$$

$$x, y, \alpha_i, \beta_i > 0; \; 0 \le r_i < 1$$

where,

$$I_0 (2\sqrt{\alpha_i \beta_i r_i x y}) = \sum_{j=0}^{\infty} \frac{(\alpha_i \beta_i r_i x y)^j}{(j!)^2}$$

$g_i(\cdot)$: marginal p.d.f. of X_i $= \alpha_i (1-r_i) e^{-\alpha_i (1-r_i) x}$

$h_i(\cdot)$: marginal p.d.f. of Y_i $= \beta_i (1-r_i) e^{-\beta_i (1-r_i) y}$

$k_i(y \mid x)$: conditional p.d.f. of Y_i given $X_i = x$

$$= \beta_i e^{-\beta_i y - \alpha_i r_i x} I_0 (2\sqrt{\alpha_i \beta_i r_i x y})$$

$K_i(y \mid x)$: conditional c.d.f. of Y_i given $X_i = x$

a : constant rate of appearance of repairman in the system

b : constant rate of disappearance of repairman from the system

3. Transition Probabilities And Sojourn Times

(a) The steady-state unconditional and conditional transition probabilities can be obtained as follows:

$$p_{01} = b/\{b+\alpha_1(1-r_1)\} \quad ,$$
$$p_{02} = \alpha_1(1-r_1)/\{b+\alpha_1(1-r_1)\}$$
$$p_{10} = a/\{a+\alpha_1(1-r_1)\} \quad ,$$
$$p_{13} = \alpha_1(1-r_1)/\{a+\alpha_1(1-r_1)\}$$
$$p_{32} = a/\{a+\alpha_2(1-r_2)\} \quad ,$$
$$p_{35} = \alpha_2(1-r_2)/\{a+\alpha_2(1-r_2)\}$$
$$p_{54} = 1 = p_{46|x}$$
$$p_{20|x} = \beta'_1 e^{-\alpha_1 r_1 x(1-\beta'_1)} \ ; \ \beta'_1 = \frac{\beta_1}{\{\beta_1+\alpha_2(1-r_2)\}}$$
$$p_{24|x} = 1-\beta'_1 e^{-\alpha_1 r_1 x(1-\beta'_1)} = p^{(4)}_{26|x}$$
$$p_{60|x} = \beta'_2 e^{-\alpha_2 r_2 x(1-\beta'_2)} \quad , \quad \beta'_2 = \frac{\beta_2}{\beta_2+\alpha_1(1-r_1)}$$
$$p^{(7)}_{62|x} = 1-\beta'_2 e^{-\alpha_2 r_2 x(1-\beta'_2)}$$

(b) From the conditional steady-state transition probability, the unconditional steady-state transition probability can be obtained as follows:

$$p_{20} = \int p_{20|x}\, g_1(x)\,dx = \beta'_1(1-r_1)/(1-r_1\beta'_1)$$

Similarly,

$$p_{24} = p^{(4)}_{26|x} = 1-\beta'_1(1-r_1)/(1-r_1\beta'_1)$$

$$p_{60} = \beta'_2(1-r_2)/(1-r_2\beta'_2) \quad , \quad p_{62} = 1-p_{60}$$
$$p_{46} = 1$$

Now, we observe that

$$p_{01}+p_{02} = 1 \ , \ p_{10}+p_{13} = 1 \ , \ p_{20}+p_{24}(p^{(4)}_{26}) = 1$$
$$p_{32}+p_{35} = 1 \ , \ p_{54}=p_{46} = 1 \ , \ p_{60}+p^{(7)}_{62} = 1 \qquad (1\text{-}6)$$

(c) The mean sojourn time in various states are as follows:

$$\psi_0 = \int e^{-\{b+\alpha_1(1-r_1)\}t}\,dt = 1/b+\alpha_1(1-r_1)\ , \ \psi_5 = 1/a$$
$$\psi_1 = 1/a+\alpha_1(1-r_1), \qquad \psi_3 = 1/a+\alpha_2(1-r_2)$$
$$\psi_{2|x} = \frac{[1-\beta'_1 e^{-\alpha_1 r_1 x(1-\beta'_1)}]}{\alpha_2(1-r_2)} \quad , \quad \psi_{4|x} = (1+\alpha_1 r_1 x)/\beta_1$$

$$\psi_{6|x} = \frac{[1-\beta_2' e^{-\alpha_2 r_2 x(1-\beta_2')}]}{\alpha_1(1-r_1)}$$

So that

$$\psi_2 = \int \psi_{2|x}\, g_1(x)\,dx = \frac{1}{\alpha_2(1-r_2)}[1-\frac{\beta_1'(1-r_1)}{(1-r_1\beta_1')}]$$

$$\psi_4 = \frac{1}{\beta_1(1-r_1)}, \quad \psi_6 = \frac{1}{\alpha_1(1-r_1)}[1-\frac{\beta_2'(1-r_2)}{(1-r_2\beta_2')}]$$

4. Analysis of Characteristics

(a) Reliability and MTSF

To determine $R_i(t)$, the reliability of the system when system initially starts from state regenerative S_i, we assume the failed states S_4, S_5 and S_7 of the system as absorbing. Using simple probabilistic arguments in regenerative point technique, the value of $R_0(t)$ in terms of its Laplace transform is

$$R_0^*(s) = \frac{Z_0^* + q_{01}^* Z_1^* + (q_{01}^* q_{13}^* q_{32}^* + q_{02}^*) Z_2^* + q_{01}^* q_{13}^* Z_3^*}{1 - q_{01}^* q_{10}^* - (q_{01}^* q_{13}^* q_{32}^* + q_{02}^*) q_{20}^*} \tag{7}$$

where Z_0^*, Z_1^*, Z_2^* and Z_3^* are the L.T. of

$$Z_0(t) = e^{-\{b+\alpha_1(1-r_1)\}t}, \quad Z_1(t) = e^{-\{a+\alpha_1(1-r_1)\}t}$$

$$Z_2(t) = e^{-\alpha_2(1-r_2)t} \int \bar{K}_1(t|x) g_1(x)\,dx, \quad Z_3(t) = e^{-\{a+\alpha_2(1-r_2)\}t}$$

The mean time to system failure (MTSF) is given by

$$E(T_0) = \lim_{s\to 0} R_0^*(s)$$

$$= \frac{\psi_0 + p_{01}\psi_1 + (p_{01}p_{13}p_{32} + p_{02})\psi_2 + p_{01}p_{13}\psi_3}{1 - p_{01}p_{10} - p_{01}p_{13}p_{32}p_{20} - p_{02}p_{20}} \tag{8}$$

(b) Availability Analysis

Let $A_i^1(t)$ and $A_i^2(t)$ be the probabilities that the system is up(operative) at epoch t due to the operation of first and second unit respectively when initially system starts from state $S_i \in E$. Using the regenerative point technique and the tools of Laplace transforms, one can obtain the values of $A_0^1(t)$ and $A_0^2(t)$ in terms of their Laplace Transforms i.e. $A_0^{1*}(s)$ and $A_0^{2*}(s)$.

The steady-state availabilities of the system due to the operation of unit-1 and unit-2 are given by

$$A_0^1 = N_1 / D_1 \text{ and } A_0^2 = N_2 / D_1 \tag{9-10}$$

where,

$$N_1 = (1-p_{26}^{(4)}p_{62}^{(7)})(\psi_0 + p_{01}\psi_1) + \{p_{01}p_{13}(p_{35} + p_{32}p_{26}^{(4)}) + p_{02}p_{26}^{(4)}\}\psi_6$$

$$N_2 = [p_{01}p_{13}(p_{32} + p_{35}p_{62}^{(7)}) + p_{02}]\psi_2 + p_{01}p_{13} + (1-p_{26}^{(4)}p_{62}^{(7)})\psi_3$$

$$D_1 = (1-p_{26}^{(4)}p_{62}^{(7)})[\psi_0 + p_{01}\psi_1 + p_{01}p_{13}(\psi_3 + \psi_4 + \psi_5)] + [p_{01}p_{13}(p_{32} + p_{35}p_{62}^{(7)}) + p_{02}]\psi_4 + [p_{02}p_{26}^{(4)} + p_{01}p_{13}(p_{35} + p_{32}p_{26}^{(4)})]\{\beta_2(1-r_2)\}^{-1} \tag{11}$$

The expected up time of the system due to unit-1 and unit-2 during (0, t) are given by

$$\mu_{up}^1(t) = \int_0^t A_0^1(u)\,du\ ,\ \mu_{up}^2(t) = \int_0^t A_0^2(u)\,du$$

(c) Busy Period Analysis

Let $B_i^1(t)$ and $B_i^2(t)$ be the respective probabilities that the repairman is available with the system at epoch t but respectively he is idle (not busy) and busy in repairing a unit when system initially starts from $S_i \in E$. Using the regenerative point technique and the tools of Laplace transform, one can obtain the values of $B_i^1(t)$ and $B_i^2(t)$ in terms of their Laplace transforms i.e. $B_i^{1*}(s)$ and $B_i^{2*}(s)$. In the long run the probabilities that the repair facility will be idle (not busy) and busy in repairing either unit are given respectively by

$$B_0^1 = N_3 / D_1 \quad \text{and} \quad B_0^2 = N_4 / D_1 \tag{12-13}$$

where,

$$N_3 = (1-p_{26}^{(4)}\,p_{62}^{(7)})\psi_0$$

and

$$N_4 = [p_{01}p_{13}(p_{32} + p_{35}p_{62}^{(7)}) + p_{02}]\psi_4 + p_{01}p_{13}(1-p_{26}^{(4)}p_{62}^{(7)})\psi_4 + [p_{01}p_{13}(p_{35} + p_{32}p_{26}^{(4)}) + p_{02}p_{26}^{(4)}]\{\beta_2(1-r_2)\}^{-1}$$

The value of D_1 is same as given in (b).

The expected durations of the repairman in (0, t) when he is idle and busy are respectively given by

$$\mu_b^1(t) = \int_0^t B_0^1(u)\,du\ ,\ \mu_b^2(t) = \int_0^t B_0^2(u)\,du$$

(d) Profit Function Analysis

The net expected profit earned by the system during (0, t) is given by

P(t) = Expected total revenue in (0, t) – Expected total expenditure in repair during (0, t)

$$= K_0\,\mu_{up}^1(t) + K_1\,\mu_{up}^2(t) - K_2\,\mu_b^1(t) - K_3\,\mu_b^2(t) \tag{14}$$

where, K_0 and K_1 are the revenues per-unit up time due to the operation of first and second unit respectively, K_2 and K_3 are the amounts paid to the repairman per-unit of time during his availability with the system when he is idle and busy in repair respectively.

The expected profit by the system per-unit time in steady state is given by

$$P = K_0 A_0^1 + K_1 A_0^2 - K_2 B_0^1 - K_3 B_0^2 \tag{15}$$

5. Graphical Study of System Behaviour

For a more clear view of the behaviour of system characteristics w.r.t. the various parameters involved, we plot curves for MTSF and profit function in Fig. 2 and Fig. 3 w.r.t. the failure parameter (α_1) of priority unit for three different values of correlation coefficient (r_1 = 0.25, 0.50 and 0.75) between X_1 and Y_1 and two different values of repair parameter (β_1) of priority unit while the other parameters are kept fixed as α_2 = 0.005, β_2 = 0.5, b = 0.05, K_0 = 500, K_1 = 300, K_2 = 50 and K_3 = 150.

From Fig. 2 it is clear that the MTSF decreases uniformly as the failure parameters α_1 of p-unit increases. Further we observe that the expected life of the system increases as we increase the correlation coefficient r_1 between the random variables denoting failure and repair times of p-unit. The same trends in MTSF are observed if we increase the value of repair parameter (β_1).

From Fig. 3 we observe that the profit (P) decreases uniformly as we increase the value α_1. Also, with the increase of the value r_1 or β_1, the net expected profit tend to increase as it should be.

Thus we conclude that the higher correlation between the failure and repair times provides the better system performances.

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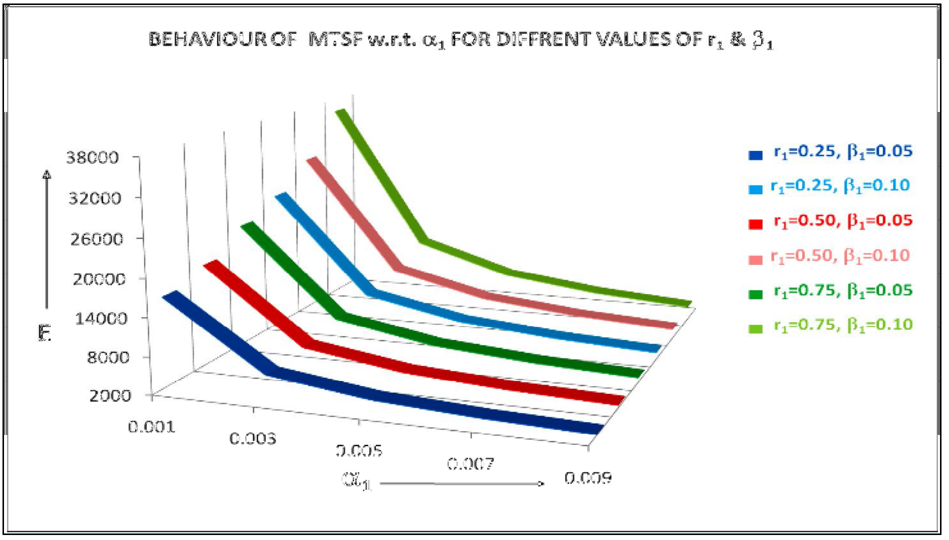
BEHAVIOUR OF MTSF w.r.t. α_1 FOR DIFFRENT VALUES OF r_1 & β_1
38000
32000
26000
20000
14000
8000
2000
E
0.001
0.003
0.005
0.007
0.009
α_1
r_1=0.25, β_1=0.05
r_1=0.25, β_1=0.10
r_1=0.50, β_1=0.05
r_1=0.50, β_1=0.10
r_1=0.75, β_1=0.05
r_1=0.75, β_1=0.10

Fig.-2

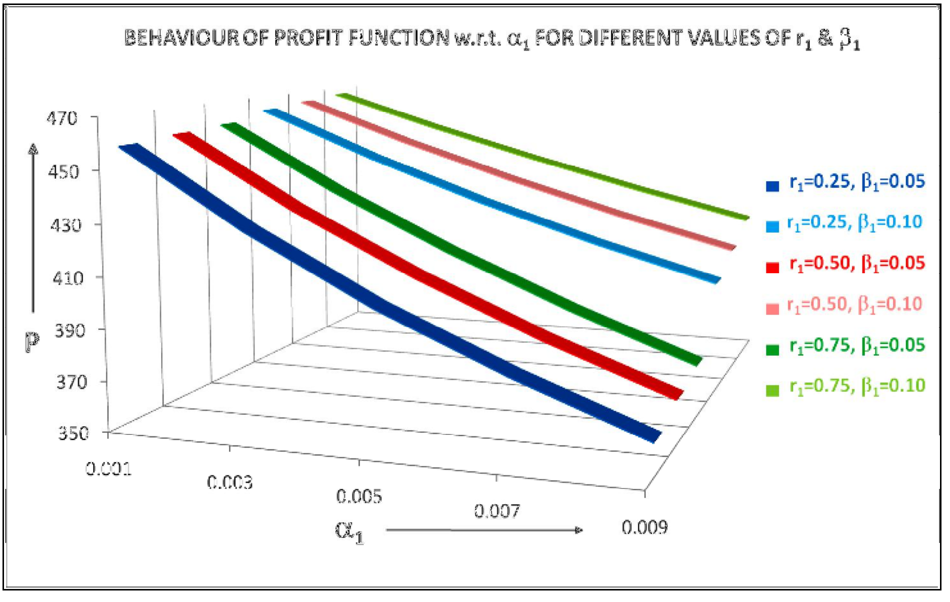
BEHAVIOUR OF PROFIT FUNCTION w.r.t. α_1 FOR DIFFERENT VALUES OF r_1 & β_1
470
450
430
410
390
370
350
P
0.001
0.003
0.005
0.007
0.009
α_1
r_1=0.25, β_1=0.05
r_1=0.25, β_1=0.10
r_1=0.50, β_1=0.05
r_1=0.50, β_1=0.10
r_1=0.75, β_1=0.05
r_1=0.75, β_1=0.10

Fig.-3