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A GENERALIZED CLASS OF PSNR SAMPLING SCHEME

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Abstract

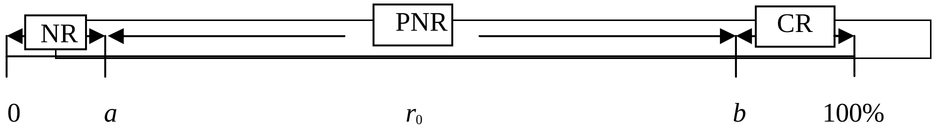
In the present days of advanced electronic technologies in the field of communication like mobile phones, e-mail, internet etc. (and so forth) the incentive based mail surveys are being popular due to their cost effectiveness and rapid approach to individuals. But, a major disadvantage appears as mail surveys are affected by a huge amount of non-response of units in the sample. The post-stratified non-response (PSNR) scheme is used, in the stratified sampling set-up, when (i) frames of stratum are unknown and (ii) strata contain some non-responding units. This paper presents a general class of PSNR type sampling scheme by introducing three groups of earmarked strata based on response pattern along-with two parameters pf the class. The Bayesian approach regarding utilization of prior knowledge (or guess) of response pattern is introduced in the proposed class for estimating the population mean. Several properties of the class are derived and the results are numerically supported.

Keywords: Post-stratified non-response (PSNR), Very nearly complete response (CR), Very nearly complete non-response (NR), Partial non-response (PNR), Respondents (RS), Non-respondents (NRS), Earmarked strata.

1. Introduction

Consider a mail survey where questionnaire are mailed to the respondents and expected to return back before a deadline. It is a common practice in the present days of electronic revolution and fast, cheaper also in terms of time and cost. The response pattern of a mail survey varies over several characteristics like geographical, educational, age, status, etc. An expert survey practitioner can make a guess about the response pattern and could utilize this information for making the estimation efficient. One can divide all the strata into three groups: G_1, G_2, and G_3 based on their response pattern. With two numbers, *a*and *b* (in percentage) k stratum could be classified as:
Group (G_1): $b< r_0 <100\%$, strata providing very nearly complete response (CR);
Group (G_2): $a \le r_0 \le b$, strata partial non response (PNR);
Group (G_3): $0< r_0 <a$, strata providing very nearly complete non-response (NR);

The measuring scale r_0, therefore, categorized *k* stratum as below:



The value *a* and *b* are arbitrary and based on experience and judgment of survey practitioners. For instance *a* may be 10% and *b* as 90%. The stratum could be earmarked in advance in G_1, G_2, and G_3 groups using the past experience or the prior information about the response pattern before the selection of a sample. Some useful contribution to the post-stratifications are by Holt & Smith (1979), Jagers, Oden & Trulsson (1985), Jagers (1986), Smith (1991), Casady and Valliant (1993),Wywil (2001) and in the field of non-response by Hinde and Chambers (1991), Jackway and Boyce (1987), Jones (1983), Khare (1987), Kott (1994), Little (1982, 1986) and Sarandal and Swensson (1987). Lessler and Kalsbeak (1992), Groves and Couper (1998), Shukla and Dubey (2001, 2004, 2006, 2007), Shukla and Trivedi (2006), Trivedi and Shukla (2006) and Shukla, Bankey and Trivedi (2002).

2. Notations

Assume a finite population $U = (U_{11}, U_{12}, U_{13}.......U_{iN_i}.....U_{kN_k})$ of k strata, each of size N_i with $\sum_{i=1}^{k} W_i = \sum_{i=1}^{k} \frac{N_i}{N} = 1$. The i^{th} strata contains a part of respondents (RS) of size $N_i^{'}$ and non respondents (NRS) of size $N_i^{''}$ such that $N_i = N_i^{'} + N_i^{''}$. Let Y_{ijm} be the m^{th} value of the j^{th} RS or NRS in the i^{th} strata {j=1 for RS and 2 for NRS; m = 1, 2, 3… $(N_i^{'}\ or\ N_i^{''})$ based on j}. Symbols $\overline{Y}_i$ and $\overline{Y} = \sum_{i=1}^{k} W_i \overline{Y}_i$ are for means while S_{1i}^2, S_{2i}^2 are population mean squares of RS and NRS in i^{th} strata and S_i^2, S^2 are defined as usual. The stratification of the population is on the basis of information of a characteristic other than Y under study. Like earlier, we denote CR for very nearly complete response ($r_0 > b$), PNR for partial non-response and NR for very nearly complete non- response ($r_0 < a$) throughout what follows under.

2.1 Bayesian Approach

An expert survey practitioner could make these groups G_1, G_2 and G_3 based on prior knowledge or guess or past experience. This denotes earmarking procedure of k strata before drawing a sample. The earmarking could help in appropriate choice of α_1, α_2 and α_3 and making estimation efficient.

3. Generalized Post-Stratified Non- Response (Psnr) Sampling Scheme

Following Shukla and Dubey (2001), the proposed scheme is:

Step I: Draw a sample of size n by SRSWOR from the population N.

Step II: Post stratify sample into n_i units $\left(\sum_{i=1}^{k} n_i = n\right)$ based on information of the criteria of stratification and assume $n_i^{'s}$ are moderate in number.

Step III: Using economical data collection procedure like by post, e-mail, internet etc. send questionnaires to all n units, in order to collect information on the variable Y and wait for return until the deadline of time is over.

Step IV: Suppose we get CR on all the r_1 strata of G_1 with n_{jG_1} units and mean $\bar{y}_{jG_1}$; r_2 strata of G_2 with total n_{lG_2} units having partial response PNR with size $n_{lG_2}^{'}$ and $n_{lG_2}^{''}$ $\left(n_{lG_2} = n_{lG_2}^{'} + n_{lG_2}^{''}\right)$ representing to respondents $(N^{'})$ and non-respondents $(N^{''})$. The mean of n'_{lG_2} units is $\bar{y}'_{lG_2}$. The group G_3 contains (k-r_1-r_2) strata with n_{mG_3} units having NR. (j=1,2,...r_1; l=1,2,3..r_2;m = 1,2, ..k-r_1-r_2).

Note : If incidentally group with CR (i.e. G_3) has a few non-responding units (i.e.$r_0 \neq$ 100% but $b < r_0 <$ 100%) then repeat those by first few responding units in order to make up the group with 100% response. Similarly, if group of NR (i.e.G_1) contains a few responding units (i.e. $r_0 \neq 0$ but $0 < r_0 <$ a) then discard those units from counting in order to make the group complete non-response.

Step V: To cope with non-response, select a sub sample of size n'''_{lG_2} by SRSWOR from n''_{lG_2} and sub-samples of n'''_{mG_3} from n_{mG_3} of G_3 to conduct the personal interviews. Let the respective sample means are $\bar{y}'''_{lG_2}$ over n'''_{lG_2} with prefixed ratio $f_{lG_2} = \left(\dfrac{n''_{lG_2}}{n'''_{lG_2}}\right)$ and $\bar{y}'''_{mG_3}$ on n'''_{mG_3} with prefixed ratio $f^*_{mG_3} = \left(\dfrac{n_{mG_3}}{n'''_{mG_3}}\right)$.

4. Estimation Strategy

With $p_i = \left(\dfrac{n_i}{n}\right)$, Agrawal and Panda (1993) suggested weights structure $W_{i\alpha} = [\alpha p_i + (1-\alpha)W_i]$; $E(W_{i\alpha}) = W_i$. Based on response pattern of earmarked strata in groups, we choose three constants α_1, α_2 and α_3 for G_1, G_2 and G_3 respectively and define three separate weight structure as

$W_{j\alpha_1} = [\alpha_1 p_j + (1-\alpha_1)W_j]$ for G_1; $W_{l\alpha_2} = [\alpha_2 p_l + (1-\alpha_2)W_l]$ for G_2,

and $W_{m\alpha_3} = [\alpha_3 p_m + (1-\alpha_3)W_m]$ for G_3.

At $\alpha_1 = 0$ $W_{j\alpha_1} = W_j = \left(\dfrac{N_j}{N}\right)$; $\alpha_2 = 0, W_{l\alpha_2} = W_l = \left(\dfrac{N_l}{N}\right)$; and at $\alpha_3 = 0, W_{m\alpha_3} = W_m = \left(\dfrac{N_m}{N}\right)$.

$$E(p_j) = E\left(\frac{n_j}{n}\right) = \left(\frac{N_j}{N}\right) = W_j,\ E(p_l) = E\left(\frac{n_l}{n}\right) = \left(\frac{N_l}{N}\right) = W_l;\ E(p_m) = E\left(\frac{n_m}{n}\right) = \left(\frac{N_m}{N}\right) = W_m.$$

Also, $E[W_{j\alpha_1}] = W_j$, $E[W_{l\alpha_2}] = W_l$, $E[W_{m\alpha_3}] = W_m$

The proposed estimator for $\bar{Y}$ is

$$\bar{y}(r_1, r_2, \alpha_1, \alpha_2, \alpha_3) = \sum_{j=1}^{r_1} W_{j\alpha_1}\bar{y}_{jG_1} + \sum_{l=1}^{r_2} W_{l\alpha_2}\left\{\frac{n'_{lG_2}\bar{y}'_{lG_2} + n''_{lG_2}\bar{y}'''_{lG_2}}{n'_{lG_2} + n''_{lG_2}}\right\} + \sum_{m=1}^{k-r_1-r_2} W_{m\alpha_3}\bar{y}'''_{mG_3} \tag{4.1}$$

Remark 4.1: The proposed class of scheme and estimators contain r_1 and r_2 as scheme parameters and α_1, α_2, α_3 as parameters of estimators based on the response pattern of earmark strata in groups. When r_1=0 or otherwise $r_2 = k$ the proposed class of family converts into one-parameter family of PSNR scheme. When both r_1=0, $r_2 = k$ the two-parameter family reduces into simple PSNR scheme proposed by Shukla and Dubey (2001). The feeling provides that two-parameter family is just like a general class of PSNR scheme with scheme parameters r_1 and r_2.

Note: The conditional expectation is;

$$E[\bar{y}(r_1, r_2, \alpha_1, \alpha_2, \alpha_3)] = E\left[E\left[E\{\bar{y}(r_1, r_2, \alpha_1, \alpha_2, \alpha_3)\}/(n_{jG_1}, n_{lG_2}, n_{mG_3}, n''_{lG_2}, n'''_{lG_2}, n'''_{mG_3})\right]\right]$$

Theorem 4.1: The estimator $\bar{y}\,(r_1, r_2, \alpha_1, \alpha_2, \alpha_3)$ is unbiased for $\bar{Y}$.

Proof:
$$E[\bar{y}(r_1,r_2,\alpha_1,\alpha_2,\alpha_3)] = E\left[E\left[E\left\{\sum_{j=1}^{r_1} W_{j\alpha_1}\bar{y}_{jG_1} + \sum_{l=1}^{r_2} W_{l\alpha_2}\left(\frac{n'_{lG_2}\bar{y}'_{lG_2} + n''_{lG_2}\bar{y}'''_{lG_2}}{n'_{lG_2}+n''_{lG_2}}\right)\right.\right.\right.$$
$$\left.\left.\left. + \sum_{m=1}^{k-r_1-r_2} W_{m\alpha_3}\bar{y}'''_{mG_3}\right\} \Big/ \left(n_{jG_1}, n_{lG_2}, n_{mG_3}, n''_{lG_2}, n'''_{lG_2}, n''_{mG_3}\right)\right]\right]$$
$$= E\left[E\left\{\sum_{j=1}^{r_1} W_{j\alpha_1}\bar{y}_{jG_1} + \sum_{l=1}^{r_2} W_{l\alpha_2}\left(\frac{n'_{lG_2}\bar{y}'_{lG_2} + n''_{lG_2}E\left(\bar{y}'''_{lG_2}\right)}{n'_{lG_2}+n''_{lG_2}}\right)\right\}\right.$$
$$\left. + \left\{\sum_{m=1}^{k-r_1-r_2} W_{m\alpha_3}E\left(\bar{y}'''_{mG_3}\right)\right\} \Big/ \left(\left(n_{jG_1}, n_{lG_2}, n_{mG_3}, n''_{lG_2}, n'''_{lG_2}, n'''_{mG_3}\right)\right)\right]$$
$$= E\left[E\left\{\sum_{j=1}^{r_1} W_{j\alpha_1}\bar{y}_{jG_1} + \sum_{l=1}^{r_2} W_{l\alpha_2}\left(\frac{n'_{lG_2}\bar{y}'_{lG_2} + n''_{lG_2}\bar{y}''_{lG_2}}{n'_{lG_2}+n''_{lG_2}}\right)\right\} \Big/ \left(n_{jG_1}, n_{lG_2}, n''_{lG_2}\right)\right.$$
$$\left. + E\left[\left\{\sum_{m=1}^{k-r_1-r_2} W_{m\alpha_3}E\left(\bar{y}'''_{mG_3}\right)\right\} \Big/ \left(n_{mG_3}, n'''_{mG_3}\right)\right]\right]$$
$$= E\left[E\left\{\sum_{j=1}^{r_1} W_{j\alpha_1}\bar{y}_{jG_1} + \sum_{l=1}^{r_2} W_{l\alpha_2}\bar{y}_{lG_2}\right\} \Big/ \left(n_{jG_1}, n_{lG_2}\right)\right\} + \left[\left\{\sum_{m=1}^{k-r_1-r_2} W_{m\alpha_3}E\left(E\left(\bar{Y}'''_{mG_3}\right)\right) / \left(n_{mG_3}, n'''_{mG_3}\right)\right]$$
$$= \left[\sum_{j=1}^{r_1} E\left(W_{j\alpha_1}\right)\bar{Y}_{jG_1} + \sum_{l=1}^{r_2} E\left(W_{l\alpha_2}\right)\bar{Y}_{lG_2} + \sum_{m=1}^{k-r_1-r_2} E\left(W_{m\alpha_3}\right)\bar{Y}_{mG_3}\right]$$
$$= \left[\sum_{j=1}^{r_1} W_j\bar{Y}_{jG_1} + \sum_{l=1}^{r_2} W_l\bar{Y}_{lG_2} + \sum_{m=1}^{k-r_1-r_2} W_m\bar{Y}_{mG_3}\right] = \sum_{i=1}^{k} W_i\bar{Y}_i = \bar{Y}.$$
Hence the theorem.

Theorem 4.2: The variance of estimator $\bar{y}\,(r_1, r_2, \alpha_1, \alpha_2, \alpha_3)$ is

$$V[\bar{y}(r_1,r_2,\alpha_1,\alpha_2,\alpha_3)] = \sum_{l=1}^{r_2}\left\{\alpha_2^2\left(\frac{W_{lG_2}}{n}\right) + (1-\alpha_2)^2 W_{lG_2}^2 E\left(\frac{1}{n_{lG_2}}\right) + 2\alpha_2(1-\alpha_2)\frac{W_{lG_2}}{n}\right\}A_l$$
$$+ \sum_{m=1}^{k-r_1-r_2} B_m\left\{\alpha_3^2\left(\frac{W_{mG_3}}{n}\right) + (1-\alpha_3)^2 W_{mG_3}^2 E\left(\frac{1}{n_{mG_3}}\right) + 2\alpha_3(1-\alpha_3)\frac{W_{mG_3}}{n}\right\}$$
$$+ \sum_{j=1}^{r_1} S_{jG_1}^2\left\{\alpha_1^2\left(\frac{W_{jG_1}}{n} - \frac{E\left(p_{jG_1}^2\right)}{N_{jG_1}}\right) + (1-\alpha_1)^2\left(W_{jG_1}^2 E\left(\frac{1}{n_{jG_1}}\right) - \frac{W_{jG_1}}{N}\right) + 2\alpha_1(1-\alpha_1)\left(\frac{W_{jG_1}}{n} - \frac{W_{jG_1}}{N}\right)\right\}$$
$$+ \sum_{l=1}^{r_2} S_{lG_2}^2\left\{\alpha_2^2\left(\frac{W_{lG_2}}{n} - \frac{E\left(p_{lG_2}^2\right)}{N_{lG_2}}\right) + (1-\alpha_2)^2\left(W_{lG_2}^2 E\left(\frac{1}{n_{lG_2}}\right) - \frac{W_{lG_2}}{N}\right) + 2\alpha_2(1-\alpha_2)\left(\frac{W_{lG_2}}{n} - \frac{W_{lG_2}}{N}\right)\right\}$$

$$+\sum_{m=1}^{k-r_1-r_2} S_{mG_3}^2\left\{\alpha_3^2\left\{\frac{W_{mG_3}}{n}-\frac{E\left(p_{mG_3}^2\right)}{N_{mG_3}}\right\}+(1-\alpha_3)^2\left\{W_{mG_3}^2 E\left(\frac{1}{n_{mG_3}}\right)-\frac{W_{mG_3}}{N}\right\}+2\alpha_3(1-\alpha_3)\left\{\frac{W_{mG_3}}{n}-\frac{W_{mG_3}}{N}\right\}\right\}$$

$$+\frac{(N-n)}{n(N-1)}\left[\alpha_1^2\left\{\sum_{j=1}^{r_1} W_{jG_1}\left(1-W_{jG_1}\right)\bar{Y}_{jG_1}^2-\sum_{j\neq j'}^{r_1}\sum_{=1}^{r_1} W_{jG_1}W_{j'G_1}\bar{Y}_{jG_1}\bar{Y}_{j'G_1}\right\}\right.$$

$$+\alpha_2^2\left\{\sum_{l=1}^{r_2} W_{lG_2}\left(1-W_{lG_2}\right)\bar{Y}_{lG_2}^2-\sum_{l\neq l'}^{r_2}\sum_{=1}^{r_2} W_{lG_2}W_{l'G_2}\bar{Y}_{lG_2}\bar{Y}_{l'G_2}\right\}$$

$$+\alpha_3^2\left\{\sum_{m=1}^{k-r_1-r_2} W_{mG_3}\left(1-W_{mG_3}\right)\bar{Y}_{mG_3}^2-\sum_{m\neq m'}^{k-r_1-r_2}\sum_{=1} W_{mG_3}W_{m'G_3}\bar{Y}_{mG_3}\bar{Y}_{m'G_3}\right\}-2\alpha_1\alpha_2\left\{\sum_{j=1}^{r_1}\sum_{l=1}^{r_2} W_{jG_1}W_{lG_2}\bar{Y}_{jG_1}\bar{Y}_{lG_2}\right\}$$

$$\left.-2\alpha_1\alpha_3\left\{\sum_{j=1}^{r_1}\sum_{m=1}^{k-r_1-r_2} W_{jG_1}W_{mG_3}\bar{Y}_{jG_1}\bar{Y}_{mG_3}\right\}-2\alpha_2\alpha_3\left\{\sum_{l=1}^{r_2}\sum_{m=1}^{k-r_1-r_2} W_{lG_2}W_{mG_3}\bar{Y}_{lG_2}\bar{Y}_{mG_3}\right\}\right] \quad (4.2)$$

where $A_l=\left(\frac{N_{lG_2}^{''}}{N_{lG_2}}\right)\left(f_{lG_2}-1\right)S_{2lG_2}^2$ *and* $B_{mG_3}=\left(f_{mG_3}^{*}-1\right)S_{2mG_3}^2$ are constants.

$$E\left(\frac{1}{n_{jG_1}}\right)=\left[\left(\frac{1}{nW_{jG_1}}\right)+\frac{(N-n)\left(1-W_{jG_1}\right)}{n^2(N-1)W_{jG_1}^2}\right];E\left(p_{jG_1}^2\right)=\left[\frac{(N-n)W_{jG_1}\left(1-W_{jG_1}\right)}{n(N-1)}+W_{jG_1}^2\right]$$

$$E\left(\frac{1}{n_{lG_2}}\right)=\left[\left(\frac{1}{nW_{lG_2}}\right)+\frac{(N-n)\left(1-W_{lG_2}\right)}{n^2(N-1)W_{lG_2}^2}\right];E\left(p_{lG_2}^2\right)=\left[\frac{(N-n)W_{lG_2}\left(1-W_{lG_2}\right)}{n(N-1)}+W_{lG_2}^2\right]$$

$$E\left(\frac{1}{n_{mG_3}}\right)=\left[\left(\frac{1}{nW_{mG_3}}\right)+\frac{(N-n)\left(1-W_{mG_3}\right)}{n^2(N-1)W_{mG_3}^2}\right];E\left(p_{mG_3}^2\right)=\left[\frac{(N-n)W_{mG_3}\left(1-W_{mG_3}\right)}{n(N-1)}+W_{mG_3}^2\right]$$

Proof:

$$V\left[\bar{y}(r_1,r_2,\alpha_1,\alpha_2,\alpha_3)\right]=E\left[E\left[V\left\{\bar{y}(r_1,r_2,\alpha_1,\alpha_2,\alpha_3)\Big/\left(n_{jG_1},n_{lG_2},n_{mG_3},n_{lG_2}^{''},n_{lG_2}^{'''},n_{mG_3}^{''''}\right)\right\}\right]\right]$$

$$+E\left[V\left[E\left\{\bar{y}(r_1,r_2,\alpha_1,\alpha_2,\alpha_3)\Big/\left(n_{jG_1},n_{lG_2},n_{mG_3},n_{lG_2}^{''},n_{lG_2}^{'''},n_{mG_3}^{''''}\right)\right\}\right]\right]$$

$$+V\left[E\left[E\left\{\bar{y}(r_1,r_2,\alpha_1,\alpha_2,\alpha_3)\Big/\left(n_{jG_1},n_{lG_2},n_{mG_3},n_{lG_2}^{''},n_{lG_2}^{'''},n_{mG_3}^{''''}\right)\right\}\right]\right] \quad (4.3)$$

The first term on the right hand side of (4.3) will be

$$E\left[E\left[V\left\{\sum_{j=1}^{r_1} W_{j\alpha_1}\bar{y}_{jG_1}+\sum_{l=1}^{r_2} W_{l\alpha_2}\left(\frac{n_{lG_2}^{'}\bar{y}_{lG_2}^{'}+n_{lG_2}^{''}\bar{y}_{lG_2}^{'''}}{n_{lG_2}^{'}+n_{lG_2}^{''}}\right)\right.\right.\right.$$

$$\left.\left.\left.+\sum_{m=1}^{k-r_1-r_2} W_{m\alpha_3}\bar{y}_{mG_3}^{'''}\right\}\Big/\left(n_{jG_1},n_{lG_2},n_{mG_3},n_{lG_2}^{''},n_{lG_2}^{'''},n_{mG_3}^{''''}\right)\right]\right]$$

$$=\sum_{l=1}^{r_2} A_l\left\{\alpha_2^2\left(\frac{W_{lG_2}}{n}\right)+(1-\alpha_2)^2 W_{lG_2}^2 E\left(\frac{1}{n_{lG_2}}\right)+2\alpha_2(1-\alpha_2)\frac{W_{lG_2}}{n}\right\}$$

$$+\sum_{m=1}^{k-r_1-r_2} B_m\left\{\alpha_3^2\left(\frac{W_{mG_3}}{n}\right)+(1-\alpha_3)^2 W_{mG_3}^2 E\left(\frac{1}{n_{mG_3}}\right)+2a_3(1-\alpha_3)\frac{W_{mG_3}}{n}\right\} \quad (4.4)$$

The second term of (4.3) will be

$$E\left[V\left[E\left\{\sum_{j=1}^{r_1} W_{j\alpha_1}\bar{y}_{jG_1}+\sum_{l=1}^{r_2} W_{l\alpha_2}\left(\frac{n'_{lG_2}\bar{y}'_{lG_2}+n''_{lG_2}\bar{y}'''_{lG_2}}{n'_{lG_2}+n''_{lG_2}}\right)\right.\right.\right.$$
$$\left.\left.\left.+\sum_{m=1}^{k-r_1-r_2} W_{m\alpha_3}\bar{y}'''_{mG_3}\right\}\Big/\left(n_{jG_1},n_{lG_2},n_{mG_3},n''_{lG_2},n'''_{lG_2},n'''_{mG_3}\right)\right]\right]$$
$$=\sum_{j=1}^{r_1} S^2_{jG_1}\left[\alpha_1^2\left\{\frac{W_{jG_1}}{n}-\frac{E\left(p^2_{jG_1}\right)}{N_{jG_1}}\right\}+(1-\alpha_1)^2\left\{W^2_{jG_1}E\left(\frac{1}{n_{jG_1}}\right)-\left(\frac{W_{jG_1}}{N}\right)\right\}+2\alpha_1(1-\alpha_1)\left\{\left(\frac{W_{jG_1}}{n}-\frac{W_{jG_1}}{N}\right)\right\}\right]$$
$$+\sum_{l=1}^{r_2} S^2_{lG_2}\left[\alpha_2^2\left\{\frac{W_{lG_2}}{n}-\frac{E\left(p^2_{lG_2}\right)}{N_{lG_2}}\right\}+(1-\alpha_2)^2\left\{W^2_{lG_2}E\left(\frac{1}{n_{lG_2}}\right)-\left(\frac{W_{lG_2}}{N}\right)\right\}+2\alpha_2(1-\alpha_2)\left\{\left(\frac{W_{lG_2}}{n}-\frac{W_{lG_2}}{N}\right)\right\}\right]$$
$$+\sum_{m=1}^{k-r_1-r_2} S^2_{mG_3}\left[\alpha_3^2\left\{\frac{W_{mG_3}}{n}-\frac{E\left(p^2_{mG_3}\right)}{N_{mG_3}}\right\}+(1-\alpha_3)^2\left\{W^2_{mG_3}E\left(\frac{1}{n_{mG_3}}\right)-\left(\frac{W_{mG_3}}{N}\right)\right\}+2\alpha_3(1-\alpha_3)\left\{\left(\frac{W_{mG_3}}{n}-\frac{W_{mG_3}}{N}\right)\right\}\right] \quad (4.5)$$

The last term of (4.3) results into

$$V\left[E\left[E\left\{\sum_{j=1}^{r_1} W_{j\alpha_1}\bar{y}_{jG_1}+\sum_{l=1}^{r_2} W_{l\alpha_2}\left(\frac{n'_{lG_2}\bar{y}'_{lG_2}+n''_{lG_2}\bar{y}'''_{lG_2}}{n'_{lG_2}+n''_{lG_2}}\right)\right.\right.\right.$$
$$\left.\left.\left.+\sum_{m=1}^{k-r_1-r_2} W_{m\alpha_3}\bar{y}'''_{mG_3}\right\}\Big/\left(n_{jG_1},n_{lG_2},n_{mG_3},n''_{lG_2},n'''_{lG_2},n'''_{mG_3}\right)\right]\right]$$
$$=V\left[E\left\{\sum_{j=1}^{r_1} W_{j\alpha_1}\bar{y}_{jG_1}+\sum_{l=1}^{r_2} W_{l\alpha_2}\left(\frac{n'_{lG_2}\bar{y}'_{lG_2}+n''_{lG_2}E\left(\bar{y}'''_{lG_2}\right)}{n'_{lG_2}+n''_{lG_2}}\right)+\sum_{m=1}^{k-r_1-r_2} W_{m\alpha_3}E\left(\bar{y}'''_{mG_3}\right)\right\}\Big/\left(n_{jG_1},n_{lG_2},n_{mG_3},n''_{lG_2},n'''_{lG_2},n'''_{mG_3}\right)\right]$$
$$=\frac{(N-n)}{n(N-1)}\left[\alpha_1^2\left\{\sum_{j=1}^{r_1} W_{jG_1}\left(1-W_{jG_1}\right)\bar{Y}^2_{jG_1}-\sum_{j\neq j'}^{r_1}\sum_{=1}^{r_1} W_{jG_1}W_{j'G_1}\bar{Y}_{jG_1}\bar{Y}_{j'G_1}\right\}\right.$$
$$+\alpha_2^2\left\{\sum_{l=1}^{r_2} W_{lG_2}\left(1-W_{lG_2}\right)\bar{Y}^2_{lG_2}-\sum_{l\neq l'}^{r_2}\sum_{=1}^{r_2} W_{lG_2}W_{l'G_2}\bar{Y}_{lG_2}\bar{Y}_{l'G_2}\right\}+\alpha_3^2\left\{\sum_{m=1}^{k-r_1-r_2} W_{mG_3}\left(1-W_{mG_3}\right)\bar{Y}^2_{mG_3}-\sum_{m\neq m'}^{k-r_1-}\sum_{=1}^{r_2} W_{mG_3}W_{m'G_3}\bar{Y}_{mG_3}\bar{Y}_{m'G_3}\right\}$$
$$\left.-2\alpha_1\alpha_2\left\{\sum_{j=1}^{r_1}\sum_{l=1}^{r_2} W_{jG_1}W_{lG_2}\bar{Y}_{jG_1}\bar{Y}_{lG_2}\right\}-2\alpha_1\alpha_3\left\{\sum_{j=1}^{r_1}\sum_{m=1}^{k-r_1-r_2} W_{jG_1}W_{mG_3}\bar{Y}_{jG_1}\bar{Y}_{mG_3}\right\}-2\alpha_2\alpha_3\left\{\sum_{l=1}^{r_2}\sum_{m=1}^{k-r_1-r_2} W_{lG_2}W_{mG_3}\bar{Y}_{lG_2}\bar{Y}_{mG_3}\right\}\right] \quad (4.6)$$

On adding (4.4), (4.5) and (4.6) we get the required expression of variance.

5. Optimum Choices

On differentiating (4.2) with respect to parameters α_1, α_2 and α_3 the resulting equations are

Equation 1: $\alpha_1 b_1-\alpha_2 b_2-\alpha_3 b_3-b_4=0$ (5.1)

where

$$b_1=\sum_{j=1}^{r_1} S^2_{jG_1}\left\{-\frac{E\left(p^2_{jG_1}\right)}{N_{jG_1}}-\left(\frac{W_{jG_1}}{n}-\frac{W_{jG_1}}{N}\right)\right\}+\frac{(N-n)}{n(N-1)}\left\{\sum_{j=1}^{r} W_{jG_1}\left(1-W_{jG_1}\right)\bar{Y}^2_{jG_1}-\sum_{j\neq j'}^{r_1}\sum_{1}^{r_1} W_{jG_1}W_{j'G_1}\bar{Y}_{jG_1}\bar{Y}_{j'G_1}\right\}$$

$$b_2=\frac{N-n}{n(N-1)}\left\{\sum_{j=1}^{r_1}\sum_{l=1}^{r_2} W_{jG_1}W_{lG_2}\bar{Y}_{jG_1}\bar{Y}_{lG_2}\right\},\ \mathrm{b}_3=\frac{N-n}{n(N-1)}\left\{\sum_{j=1}^{r_1}\sum_{m=1}^{k-r_1-r_2} W_{jG_1}W_{mG_3}\bar{Y}_{jG_1}\bar{Y}_{mG_3}\right\}$$

$$b_4=\sum_{j=1}^{r_1}S^2_{jG_1}\left\{W^2_{jG_1}E\left(\frac{1}{n_{jG_1}}\right)-\frac{2W_{jG_1}}{n}\right\}$$

Equation 2: $$\alpha_2 d_2-\alpha_1 d_1-\alpha_3 d_3+d_4=0 \qquad (5.2)$$

where $d_1=\frac{(N-n)}{n(N-1)}\left\{\sum_{j=1}^{r_1}\sum_{l=1}^{r_2}W_{jG_1}W_{lG_2}\bar{Y}_{jG_1}\bar{Y}_{lG_2}\right\}$

$$d_2=\left\{\sum_{l=1}^{r_2}W^2_{lG_2}E\left(\frac{1}{n_{lG_2}}\right)-\frac{W_{lG_2}}{n}\right\}A_1-\sum_{l=1}^{r_2}S^2_{lG_2}\left\{\frac{E\left(p^2_{lG_2}\right)}{N_{lG_2}}+\left(\frac{W_{lG_2}}{n}-\frac{W_{lG_2}}{N}\right)\right\}$$
$$+\frac{(N-n)}{n(N-1)}\left\{\sum_{l=1}^{r_2}W_{lG_2}\left(1-W_{lG_2}\right)\bar{Y}^2_{lG_2}-\sum_{l\neq l'}^{r_2}\sum_{=1}^{r_2}W_{lG_2}W_{l'G_2}\bar{Y}_{lG_2}\bar{Y}_{l'G_2}\right\}$$

$$d_3=\frac{N-n}{n(N-1)}\left\{\sum_{l=1}^{r_2}\sum_{m=1}^{k-r_1-r_2}W_{lG_2}W_{mG_3}\bar{Y}_{lG_2}\bar{Y}_{mG_3}\right\}$$

$$d_4=\sum_{l=1}^{r_2}\left\{-W^2_{lG_2}E\left(\frac{1}{n_{lG_2}}\right)+\frac{W_{lG_2}}{n}\right\}A_1-\sum_{l=1}^{r_2}S^2_{lG_2}\left\{W^2_{lG_2}E\left(\frac{1}{n_{lG_2}}\right)-\frac{2W_{lG_2}}{n}\right\}$$

Equation 3: $$\alpha_3E_3-\alpha_2E_2-\alpha_1E_1-E_4=0 \qquad (5.3)$$

where $E_1=\frac{N-n}{n(N-1)}\left\{\sum_{j=1}^{r_1}\sum_{m=1}^{k-r_1-r_2}W_{jG_1}W_{mG_3}\bar{Y}_{jG_1}\bar{Y}_{mG_3}\right\}$ $E_2=\frac{N-n}{n(N-1)}\left\{\sum_{l=1}^{r_2}\sum_{m=1}^{k-r_1-r_2}W_{lG_2}W_{mG_3}\bar{Y}_{lG_2}\bar{Y}_{mG_3}\right\}$

$$E_3=\left\{\sum_{m=1}^{k-r_1-r_2}W^2_{mG_3}E\left(\frac{1}{n_{mG_3}}\right)-\frac{W_{mG_3}}{n}\right\}B_m-\sum_{m=1}^{k-r_1-r_2}S^2_{mG_3}\left\{\frac{E\left(p^2_{mG_3}\right)}{N_{mG_3}}+\left(\frac{W_{mG_3}}{n}-\frac{W_{mG_3}}{N}\right)\right\}$$
$$+\frac{(N-n)}{n(N-1)}\left\{\sum_{m=1}^{k-r_1-r_2}W_{mG_3}\left(1-W_{mG_3}\right)\bar{Y}^2_{mG_3}-\sum\sum W_{mG_3}W_{m'G_3}\bar{Y}_{mG_3}\bar{Y}_{m'G_3}\right\}$$

$$E_4=\sum_{m=1}^{k-r_1-r_2}\left\{W^2_{mG_3}E\left(\frac{1}{n_{mG_3}}\right)+\frac{W_{mG_3}}{n}\right\}B_m-\sum S^2_{mG_3}\left\{W^2_{mG_3}E\left(\frac{1}{n_{mG_3}}\right)-\frac{2W_{mG_3}}{n}\right\}$$

Solving (5.1), (5.2) and (5.3) which are a system of three equations with only three unknowns α_1, α_2 and α_3. The solution of system provides the optimum choice of α_1, α_2 and α_3 as

$$(\alpha_1)_{opt}=(b_1)^{-1}\left[b_4+\{b_2H_1\gamma'-b_2F_1\}(H_3)^{-1}-b_3\gamma'\right];\ (\alpha_2)_{opt}=(H_3)^{-1}\{H_1\gamma'-F_1\};\ (\alpha_3)_{opt}=-\gamma'$$

where

$$F_1=(b_4d_1+b_1d_4);\ \ F_2=(d_1E_2+d_2E_1)\ \ F_3=(d_4E_1-d_1E_4);\ \ F_4=(b_1d_2+b_2d_1)$$
$$H_1=(b_1d_3+b_3d_1);\ \ H_2=(d_1E_2+d_2E_1);\ H_3=(b_1d_2+b_2d_1);\ \ H_4=(d_3E_1-d_1E_3)$$
$$\gamma'=\left[\frac{F_1F_2+F_3F_4}{H_1H_2+H_3H_4}\right]$$

Note 5.1: The optimum estimator depends upon several population parameters W_{iG_j} , N, n and $E\left(1/n_{iG_j}\right)$. But N and n are prefixed parameters, well known in advance before the start of survey. $E\left(1/n_{iG_j}\right)$ depends on N, n and W_{iG_j} only and therefore could be obtained. To note that in repeated surveys on many occasions at the start duration,

W_{iG_j} are almost stable quantity. Reddy (1978) has proved the stability of parameter $V=\rho\, C_Y/C_X$ over multiple occasions. So, one can assume that guess values of W_{iG_j} are well-known to survey practitioners. Based on this, the optimum estimation could be done.

6. Cost Aspect

We define cost function

$$\phi(j,l,m)= \text{Expected cost} + \lambda\left[V_T\left\{\bar{y}(r_1,r_2,\alpha_1,\alpha_2,\alpha_3)-V_{0A}\right\}\right] \qquad (6.1)$$

where $T=j\,,\,l\,,\,m$.

The variance could be rewritten in other form as

$$Var\{\bar{y}(r_1,r_2,\alpha_1,\alpha_2,\alpha_3)\}=\left[\alpha_1^2\left(D_7^*+D_{16}^*\right)+(1-\alpha_1)^2D_8^*+2\alpha_1(1-\alpha_1)D_9^*+\alpha_2^2\left(D_1^*+D_{10}^*+D_{17}^*\right)\right.$$
$$+(1-\alpha_2)^2\left(D_2^*+D_{11}^*\right)+2\alpha_2(1-\alpha_2)\left(D_3^*+D_{12}^*\right)$$

$$Var\left\{\bar{y}(r_1,r_2,\alpha_1,\alpha_2,\alpha_3)\right\}=\left[\alpha_1^2\left(D_7^*+D_{16}^*\right)+(1-\alpha_1)^2D_8^*+2\alpha_1(1-\alpha_1)D_9^*+\alpha_2^2\left(D_1^*+D_{10}^*+D_{17}^*\right)\right.$$
$$+(1-\alpha_2)^2\left(D_2^*+D_{11}^*\right)+2\alpha_2(1-\alpha_2)\left(D_3^*+D_{12}^*\right)+\alpha_3^2\left(D_4^*+D_{13}^*+D_{18}^*\right)+(1-\alpha_3)^2\left(D_5^*+D_{14}^*\right)$$
$$\left.+2\alpha_3(1-\alpha_3)\left(D_6^*+D_{15}^*\right)-2\alpha_1\alpha_2D_{19}^*-2\alpha_1\alpha_3D_{20}^*-2\alpha_2\alpha_3D_{21}^*\right] \qquad (6.2)$$

where $D_1^*=\sum_{l=1}^{r_2}A_l^*\left(\frac{W_{lG_2}}{n}\right)=D_3^*$; $D_2^*=\sum_{l=1}^{r_2}A_l^*W_{lG_2}^2E\left(\frac{1}{n_{lG_2}}\right)$; $D_4^*=\sum_{m=1}^{k-r_1-r_2}B_m\left(\frac{W_{mG_3}}{n}\right)=D_6^*$;

$D_5^*=\sum_{m=1}^{k-r_1-r_2}B_mW_{mG_3}^2E\left(\frac{1}{n_{mG_3}}\right)$; $D_7^*=\sum_{j=1}^{r_1}S_{jG_1}^2\left\{\frac{W_{jG_1}}{n}-\frac{E\left(p_{jG_1}^2\right)}{N_{jG_1}}\right\}$ $D_8^*=\sum_{j=1}^{r_1}S_{jG_1}^2\left\{W_{jG_1}^2E\left(\frac{1}{n_{jG_1}}\right)-\frac{W_{jG_1}}{N}\right\}$

$D_9^*=\sum_{j=1}^{r_1}S_{jG_1}^2\left\{\frac{W_{jG_1}}{n}-\frac{W_{jG_1}}{N}\right\}$; $D_{10}^*=\sum_{l=1}^{r_2}S_{lG_2}^2\left\{\frac{W_{lG_2}}{n}-\frac{E\left(p_{lG_2}^2\right)}{N_{lG_2}}\right\}$; $D_{11}^*=\sum_{l=1}^{r_2}S_{lG_2}^2\left\{W_{lG_2}^2E\left(\frac{1}{n_{lG_2}}\right)-\frac{W_{lG_2}}{N}\right\}$

; $D_{12}^*=\sum_{l=1}^{r_2}S_{lG_2}^2\left\{\frac{W_{lG_2}}{n}-\frac{W_{lG_2}}{N}\right\}$; $D_{13}^*=\sum_{m=1}^{k-r_1-r_2}S_{mG_3}^2\left\{\frac{W_{mG_3}}{n}-\frac{E\left(p_{mG_3}^2\right)}{N_{mG_3}}\right\}$;

$D_{14}^*=\sum_{m=1}^{k-r_1-r_2}S_{mG_3}^2\left\{W_{mG_3}^2E\left(\frac{1}{n_{mG_3}}\right)-\frac{W_{mG_3}}{N}\right\}$; $D_{15}^*=\sum_{m=1}^{k-r_1-r_2}S_{mG_3}^2\left\{\frac{W_{mG_3}}{n}-\frac{W_{mG_3}}{N}\right\}$;

$$D_{16}^*=\frac{(N-n)}{n(N-1)}\left[\sum_{j=1}^{r_1}W_{jG_1}\left(1-W_{jG_1}\right)\bar{Y}_{jG_1}^2-\sum_{j\neq j'}^{r_1}\sum_{=1}^{r_1}W_{jG_1}W_{j'G_1}\bar{Y}_{jG_1}\bar{Y}_{j'G_1}\right];$$

$$D_{17}^*=\frac{(N-n)}{n(N-1)}\left[\sum_{l=1}^{r_2}W_{lG_2}\left(1-W_{lG_2}\right)\bar{Y}_{lG_2}^2-\sum_{l\neq l'}^{r_2}\sum_{=1}^{r_2}W_{lG_{21}}W_{l'G_2}\bar{Y}_{lG_2}\bar{Y}_{l'G_2}\right];$$

$$D_{18}^*=\frac{(N-n)}{n(N-1)}\left[\sum_{m=1}^{k-r_1-r_2}W_{mG_3}\left(1-W_{mG_3}\right)\bar{Y}_{mG_3}^2-\sum_{m\neq m'}^{k-r_1}\sum_{=1}^{-r_2}W_{mG_3}W_{m'G_3}\bar{Y}_{mG_3}\bar{Y}_{m'G_3}\right];$$

$$D_{19}^*=\frac{(N-n)}{n(N-1)}\left[\sum_{j=1}^{r_1}\sum_{l=1}^{r_2}W_{jG_1}W_{lG_2}\bar{Y}_{jG_1}\bar{Y}_{lG_2}\right];\ D_{20}^*=\frac{(N-n)}{n(N-1)}\left[\sum_{j=1}^{r_1}\sum_{m=1}^{k-r_1-r_2}W_{jG_1}W_{mG_3}\bar{Y}_{jG_1}\bar{Y}_{mG_3}\right];$$

$$D_{21}^*=\frac{(N-n)}{n(N-1)}\left[\sum_{l=1}^{r_2}\sum_{m=1}^{k-r_1-r_2}W_{lG_2}W_{mG_3}\bar{Y}_{lG_2}\bar{Y}_{mG_3}\right]$$

The cost for j^{th} strata of G_1, l^{th} strata of G_2 and m^{th} strata of G_3 are

$$C^*_{jG_1} = \left\{\left(C_{0jG_1} + C_{1jG_1}\right)n_{jG_1} + C_{2jG_1}n'_{jG_1}\right\}\ ;\ C^*_{lG_2} = \left\{\left(C_{0lG_2} + C_{1lG_2}\right)n_{lG_2} + C_{2lG_2}n'_{lG_2} + C_{3lG_2}n'''_{lG_2}\right\}$$

$$C^*_{mG_3} = \left\{\left(C_{0mG_3} + C_{1mG_3}\right)n_{mG_3} + C_{3mG_3}n'''_{mG_3}\right\}$$

The total cost for j^{th}, l^{th} and m^{th} strata is $(T_c)^* = C^*_{jG_1} + C^*_{lG_2} + C^*_{mG_3}$ (6.3)

where the terms of cost are defined below:

C_{ojG_1} : Cost of including n_j units of j^{th} stratum in sample n from group G_1.
C_{1jG_1} : Cost of pilot enquiry for post-stratification of n units into n_j .
C_{2jG_1} : Cost of collecting, editing per n_j units of j^{th} strata of the response class.
C_{olG_2} : Cost of including n_l units of l^{th} stratum in sample n from group G_2.
C_{1lG_2} : Cost of pilot enquiry for post-stratification of n units into n_l .
C_{2lG_2} : Cost of collecting, editing per n_l units of l^{th} strata of the response class.
C_{3lG_2} : Cost of personal interview per n_l units from l^{th} non-response class.
C_{0mG_3} : Cost of including n_m units of m^{th} strata in sample n from group G_3.
C_{1mG_3} : Cost of pilot enquiry for post- stratification of n units into n_m.
C_{3mG_3} : Cost of personal interview per n_m units from m^{th} non-response class.

Total cost $T_c = \sum_{j=1}^{r_1} C^*_{jG_1} + \sum_{l=1}^{r_2} C^*_{lG_2} + \sum_{m=1}^{k-r_1-r_2} C^*_{mG_3}$

and expected cost $E(T_c) = E\left(\sum_{j=1}^{r_1} C^*_{jG_1} + \sum_{l=1}^{r_2} C^*_{lG_2} + \sum_{m=1}^{k-r_1-r_2} C^*_{mG_3}\right)$

$$= \frac{n}{N}\left[\sum_{j=1}^{r_1}\left\{\left(C_{0jG_1} + C_{1jG_1}\right)N_{jG_1} + C_{2jG_1}N'_{jG_1}\right\} + \sum_{l=1}^{r_2}\left\{\left(C_{0lG_2} + C_{1lG_2}\right)N_{lG_2} + C_{2lG_2}N'_{lG_2} + C_{3lG_2}\frac{N''_{lG_2}}{f_{lG_2}}\right\}\right.$$
$$\left. + \sum_{m=1}^{k-r_1-r_2}\left\{\left(C_{0mG_3} + C_{1mG_3}\right)N_{mG_3} + C_{3mG_3}\frac{N''_{mG_3}}{f^*_{mG_3}}\right\}\right] \quad (6.4)$$

Let V_{0A} be the prefixed level of variance and with $\lambda \neq 0$ for cost-optimal choice define a function $\delta_A = E(T_c) + \lambda\left[V\{\bar{y}(r_1, r_2, \alpha_1, \alpha_2, \alpha_3)\} - V_{0A}\right]$ (6.5)

To find optimum f_{lG_2} and $f^*_{mG_3}$, differentiate (6.5) with respect to λ, f_{lG_2} and $f^*_{mG_3}$ respectively and equate to zero. The three equations are:

$$V_{0A} = \alpha_1^2\left(D^*_7 + D^*_{16}\right) + \left(1-\alpha_1\right)^2 D^*_8 + 2\alpha_1\left(1-\alpha_1\right)D^*_9 + \alpha_2^2\left(D^*_1 + D^*_{10} + D^*_{17}\right) + \left(1-\alpha_2\right)^2\left(D^*_2 + D^*_{11}\right)$$
$$+ 2\alpha_2\left(1-\alpha_2\right)\left(D^*_3 + D^*_{12}\right) + \alpha_3^2\left(D^*_4 + D^*_{13} + D^*_{18}\right) + \left(1-\alpha_3\right)^2\left(D^*_5 + D^*_{14}\right)$$
$$+ 2\alpha_3\left(1-\alpha_3\right)\left(D^*_6 + D^*_{15}\right) - 2\alpha_1\alpha_2 D^*_{19} - 2\alpha_1\alpha_3 D^*_{20} - 2\alpha_2\alpha_3 D^*_{21} \quad (6.6)$$

$$\frac{n}{N}\left\{\frac{C_{3lG_2}N''_{lG_2}}{f^2_{lG_2}}\right\} = \lambda\left[\left(\frac{N''_{lG_2}}{N_{lG_2}}\right)W_{lG_2}S^2_{2lG_2}\left\{\left(2\alpha_2 - \alpha_2^2\right)\left(\frac{1}{n}\right) + \left(1-\alpha_2\right)^2 W^2_{lG_2}E\left(\frac{1}{n_{lG_2}}\right)\right\}\right] \quad (6.7)$$

$$\frac{n}{N}\left\{\frac{C_{3mG_3}N''_{mG_3}}{f^{*2}_{mG_3}}\right\} = \lambda\left[W_{mG_3}S^2_{2mG_3}\left\{\left(2\alpha_3 - \alpha_3^2\right)\left(\frac{1}{n}\right) + \left(1-\alpha_3\right)^2 W^2_{mG_3}E\left(\frac{1}{n_{mG_3}}\right)\right\}\right] \quad (6.8)$$

In (6.6), (6.7) and (6.8) there are r_2 equations for G_2, k-r_1-r_2 for G_3 and one with containing V_{oA} to solve for $\left(f_{1G_2}, f_{2G_2}, f_{3G_2} ....f_{r_2G_2}, f_{1G_3}, f_{2G_3} .....f_{(k-r_1-r_2)G_3}, n, \lambda\right)$ parameters. The solution may not possible every time for every data set, because numbers of unknowns are more than the number of equations. One can look for other solving technique given below.

6.1 Conditional Solution

Consider $\sum_{l=1}^{r_2} f_{lG_2} = M_1$ for group G_2 and $\sum_{m=1}^{k-r_1-r_2} f^*_{mG_3} = M_2$ for group G_3 as pre-fixed quantities, in order to get cost optimal solution, where M_1 and M_2 are the prefixed constant related to the total size of sub-sample required for revisit. From equation (6.7)

$$f_{lG_2} = \sqrt{\frac{C_{3lG_2} N''_{lG_2}}{\lambda\left[\left\{\left(\frac{N''_{lG_2}}{N_{lG_2}}\right) W_{lG_2} S^2_{2lG_2}\right\}\left\{\left(2\alpha_2 - \alpha_2^2\right)\left(\frac{1}{n}\right) + (1-\alpha_2)^2 W^2_{lG_2} E\left(\frac{1}{n_{lG_2}}\right)\right\}\right]}}$$

using assumed condition $\sum_{l=1}^{r_2} f_{lG_2} = M_1$ for group G_2 we have

$$\left[f_{lG_2}(n)\right]_{opt} = \frac{M_1 \sqrt{\frac{C_{3lG_2} N''_{lG_2}}{\left[\left\{\left(\frac{N''_{lG_2}}{N_{lG_2}}\right) W_{lG_2} S^2_{2lG_2}\right\}\left\{\left(2\alpha_2 - \alpha_2^2\right)\left(\frac{1}{n}\right) + (1-\alpha_2)^2 W^2_{lG_2} E\left(\frac{1}{n_{lG_2}}\right)\right\}\right]}}}{\sum_{l=1}^{r_2} \sqrt{\frac{C_{3lG_2} N''_{lG_2}}{\left[\left\{\left(\frac{N''_{lG_2}}{N_{lG_2}}\right) W_{lG_2} S^2_{2lG_2}\right\}\left\{\left(2\alpha_2 - \alpha_2^2\right)\left(\frac{1}{n}\right) + (1-\alpha_2)^2 W^2_{lG_2} E\left(\frac{1}{n_{lG_2}}\right)\right\}\right]}}} \tag{6.9}$$

From equation (6.8)

$$\left[f^*_{mG_3}(n)\right]_{opt} = \frac{M_2 \sqrt{\frac{C_{3mG_3} N''_{mG_3}}{\left[\left\{W_{mG_3} S^2_{2mG_3}\right\}\left\{\left(2\alpha_3 - \alpha_3^2\right)\left(\frac{1}{n}\right) + (1-\alpha_3)^2 W^2_{mG_3} E\left(\frac{1}{n_{mG_3}}\right)\right\}\right]}}}{\sum_{m=1}^{k-r_1-r_2} \sqrt{\frac{C_{3mG_3} N''_{mG_3}}{\left[\left\{W_{mG_3} S^2_{2mG_3}\right\}\left\{\left(2\alpha_3 - \alpha_3^2\right)\left(\frac{1}{n}\right) + (1-\alpha_3)^2 W^2_{mG_3} E\left(\frac{1}{n_{mG_3}}\right)\right\}\right]}}} \tag{6.10}$$

The conditional solution, under prefixed M_1 and M_2, provides cost optimal values $\left[f_{lG_2}(n)\right]_{opt}$ and $\left[f^*_{mG_1}(n)\right]_{opt}$ as functions of n. The solution of (6.7) and (6.8) may provide cost optimal n, using (6.9) & (6.10), but does not ensure the solution every time for all datasets.

7. Alternative Cost Optimal Strategy

To overcome the difficulty of uncertainty of cost-optimal solutions for f_{lG_2}, $f^*_{mG_3}$ and n, an alternative cost strategy is proposed as under:

Step I : Be limited to j^{th}, l^{th} and m^{th} strata only in groups G_1, G_2 and G_3 respectively and assume only these strata have non-zero values of Y. The strata j' ($j' \neq j$) in

G_1, strata l' ($l' \neq l$) in G_2 and strata m' ($m' \neq m$) in G_3, have all zero values. Then the total expected cost for j^{th}, l^{th} and m^{th} strata is

$$E(T_c) = \frac{n}{N}\left[\left\{(C_{0jG_1} + C_{1jG_1})N_{jG_1} + C_{2jG_1}N'_{jG_1}\right\} + \left\{(C_{0lG_2} + C_{1lG_2})N_{lG_2} + C_{2lG_2}N'_{lG_2} + C_{3lG_2}\frac{N''_{lG_2}}{f_{lG_2}}\right\}\right.$$
$$\left. + \left\{(C_{0mG_3} + C_{1mG_3})N_{mG_3} + C_{3mG_3}\frac{N''_{mG_3}}{f^*_{mG_3}}\right\}\right]$$

with prefixed level of variances V_{0j}, V_{0l} and V_{0m}. For groups G_1, G_2 and G_3 respectively.

Step II: Define a function

$\phi(j,l,m)$ = [{Expected cost in j^{th} strata + Expected cost in l^{th} strata + expected cost in m^{th} strata} $+\lambda_j\{V\{\bar{y}(r_1,r_2,\alpha_1,\alpha_2,\alpha_3)\} - V_{0j}\}$ $+\lambda_l\{V\{\bar{y}(r_1,r_2,\alpha_1,\alpha_2,\alpha_3)\} - V_{0l}\}$ $+\lambda_m\{V\{\bar{y}(r_1,r_2,\alpha_1,\alpha_2,\alpha_3)\} - V_{0m}\}$]

where V_j, V_l, V_m are component of variance related to j^{th}, l^{th} and m^{th} strata only.

Step III: Differentiate ϕ *(j, l, m)* with respect to $\lambda_l, \lambda_m, f_{lG_2}, f^*_{mG_3}$ and n, to get five equations. Solution of these would provide cost optimal choice of $f_{lG_2}, f^*_{mG_3}$ and n restricted to l^{th} and m^{th} strata only.

Step IV: Repeat the above procedure, from step I to step III, one by one for all strata $l=1,2,\ldots..r_2$ and $m = 1,2,\ldots\ldots k - r_1 - r_2$.

Step V: There shall be several cost optimal choices of $f_{lG_2}, f^*_{mG_3}$ varying over *j*, *l* and *m*. For unique solution, choose maximum of them having greater than unity. Moreover, an integer value of the average within the group, if greater than unity, may be an ideal cost-optimal solution. Similarly the repetition of the procedure would generate many values of cost-optimal n. The best choice could be (a) maximum of them (b) minimum of them (c) total of all *n* and (d) average integer value of *n*.

7.1 Optimum Equations

$$n_{opt}(r_1, r_2, f_{lG_2}, f^*_{mG_3}, \alpha_1, \alpha_2, \alpha_3) = \frac{\left[-\gamma^*_2 \pm \sqrt{\gamma^{*2}_2 - 4\gamma^*_1\gamma^*_3}\right]}{2\gamma^*_1} \quad (7.1)$$

where0

$$\delta^*_1 = (M^*_{1m}NW_{lG_2}C_{3lG_2})\{\alpha^2_2L_{1l} + (1-\alpha_2)^2L_{2l} + (2\alpha_2 - \alpha^2_2)L_{4l} - 2\alpha_1\alpha_2L_{5jl} - \alpha_2\alpha_3L_{6lm}\}$$

$$\delta^*_2 = (M^*_{1m}NW_{lG_2}C_{3lG_2})\{(1-\alpha_2)^2L_{3l}\} + (M^*_{2m}NW_{lG_2}C_{3lG_2})$$
$$\{\alpha^2_2L_{1l} + (1-\alpha_2)^2L_{2l} + (2\alpha_2 - \alpha^2_2)L_{4l} - 2\alpha_1\alpha_2L_{5jl} - \alpha_2\alpha_3L_{6lm}\}$$

$$\delta^*_3 = M^*_{2m}W_{lG_2}NC_{3lG_2}(1-\alpha_2)^2L_{3l}$$

$$\delta^*_4 = (M^*_{1l}NN''_{mG_3}C_{3mG_3})\{\alpha^2_3L_{1m} + (1-\alpha_3)^2L_{2m} + (2\alpha_3 - \alpha^2_3)L_{4m} - 2\alpha_1\alpha_3L_{5jm} - \alpha_2\alpha_3L_{6lm}\}$$

$$\delta^*_5 = (M^*_{1l}NN''_{mG_3}C_{3mG_3})\{(1-\alpha_3)^2L_{3m}\} + (M^*_{2l}NN''_{mG_3}C_{3mG_3})$$
$$\{\alpha^2_3L_{1m} + (1-\alpha_3)^2L_{2m} + (2\alpha_3 - \alpha^2_3)L_{4m} - 2\alpha_1\alpha_3L_{5jm} - \alpha_2\alpha_3L_{6lm}\}$$

$$\delta^*_6 = M^*_{2l}N''_{mG_3}NC_{3mG_3}(1-\alpha_3)^2L_{3m} \text{ and } \gamma^*_1 = \{(M^*_{1l}M^*_{1m}X^*_1) - \delta^*_1 - \delta^*_4\};$$

$$\gamma^*_2 = \{(M^*_{1l}M^*_{2m} + M^*_{1m}M^*_{2l})X^*_1 - \delta^*_2 - \delta^*_5\}; \gamma^*_3 = \{M^*_{2l}M^*_{2m}X^*_1 - \delta^*_3 - \delta^*_6\}$$

Remark 7.0: The (7.1) provides cost-optimal choice of n as a function of r_1, r_2 $f_{lG_2}, f^*_{mG_3}, \alpha_1, \alpha_2$ and α_3. For varying j, l and m values within the group, this provides several values of n_{opt}, the best choice of whom shall be as per step V of the proposed strategy.

$$T_{1l} - \alpha_2^2 T_{2l} + 2\alpha_1\alpha_2 T_{3jl} + \alpha_2\alpha_3 T_{4lm} = \left(A_l^* + S^2_{lG_2}\right)\left\{T_{5l} + (1-\alpha_2)^2 T_{6l}\right\} \qquad (7.2)$$

$$T_{1m} - \alpha_3^2 T_{2m} + 2\alpha_1\alpha_3 T_{3jm} + \alpha_2\alpha_3 T_{4lm} = \left(B_m + S^2_{mG_3}\right)\left\{T_{5m} + (1-\alpha_3)^2 T_{6m}\right\} \qquad (7.3)$$

where $T_{1l} = n^2(N-1)\left\{V_{0l} + \dfrac{W_{lG_2} S^2_{lG_2}}{N}\right\}$, $\mathrm{T}_{2l} = n(N-n)\left(1-W_{lG_2}\right)\left(\bar{Y}^2_{lG_2} - \dfrac{S^2_{lG_2}}{N_{lG_2}}\right)$

$T_{3jl} = n(N-n)W_{jG_1}W_{lG_2}\bar{Y}_{jG_1}\bar{Y}_{lG_2}$, $\mathrm{T}_{4lm} = n(N-n)W_{lG_2}W_{mG_3}\bar{Y}_{lG_2}\bar{Y}_{mG_3}$; $T_{5l} = n(N-1)W_{lG_2}$;

$T_{6l} = (N-n)\left(1-W_{lG_2}\right)$; $T_{1m} = n^2(N-1)\left\{V_{0m} + \dfrac{W_{mG_3} S^2_{mG_3}}{N}\right\}$; $\mathrm{T}_{2m} = n(N-n)W_{mG_3}\left(1-W_{mG_3}\right)\left(\bar{Y}^2_{mG_3} - \dfrac{S^2_{mG_3}}{N_{mG_3}}\right)$

$T_{3jm} = n(N-n)W_{jG_1}W_{mG_3}\bar{Y}_{jG_1}\bar{Y}_{mG_3}$; $T_{5m} = n(N-1)W_{mG_3}$, $T_{6m} = (N-n)\left(1-W_{mG_3}\right)$

The above defined term are the functions of n not containing terms $f^*_{mG_3}, \alpha_1, \alpha_2$ and α_3. On solving (7.2) and (7.3), the cost-optimal solution of $f_{lG_2}, f^*_{mG_3}$ is

$$\left[f_{lG_2}(r_1, r_2, n, \alpha_1, \alpha_2, \alpha_3)\right]_{opt} = \left[1 + \left(\frac{N_{lG_2}}{N''_{lG_2} S^2_{2lG_2}}\right)\left\{\frac{T_{1l} - \alpha_2^2 T_{2l} + 2\alpha_1\alpha_2 T_{3jl} + \alpha_2\alpha_3 T_{4lm}}{T_{5l} + (1-\alpha_2)^2 T_{6l}} - S^2_{lG_2}\right\}\right] \qquad (7.4)$$

$$\left[f^*_{mG_3}(r_1, r_2, n, \alpha_1, \alpha_2, \alpha_3)\right]_{opt} = \left[1 + \left(\frac{1}{S^2_{2mG_3}}\right)\left\{\frac{T_{1m} - \alpha_3^2 T_{2m} + 2\alpha_1\alpha_3 T_{3jm} + \alpha_2\alpha_3 T_{4lm}}{T_{5m} + (1-\alpha_3)^2 T_{6m}} - S^2_{mG_3}\right\}\right] \qquad (7.5)$$

7.2 Approximate Expressions

The cost-optimal solutions incorporate terms $\bar{Y}_{lG_2}$, $\bar{Y}_{mG_3}$ which could never be known. Therefore, some other form of solution, closer to original is required.

Remark 7.1: Assume $\bar{Y}^2_{lG_2} \approx \dfrac{S^2_{lG_2}}{N_{lG_2}}$, $\bar{Y}^2_{mG_3} \approx \dfrac{S^2_{mG_3}}{N_{mG_3}}$ or assume best values of α_1, α_2 and a_3 terms are very small (which is actually true through data-base support). Then L_{1l}, L_{1m}, L_{5jl}, L_{5jm}, L_{6lm}, T_{2l}, T_{3jl}, T_{4lm}, T_{2m} and T_{3jm} could be neglected. So, we can rewrite equation (7.1) as

$$n^2\left\{\left(M^*_{1l}M^*_{1m}X^*_1\right) - \delta^*_{1l} - \delta^*_{4m}\right\} + n\left\{\left(M^*_{1l}M^*_{2m} + M^*_{1m}M^*_{2l}\right)X^*_1 - \delta^*_{2l} - \delta^*_{5m}\right\} + \left\{M^*_{2l}M^*_{2m}X^*_1 - \delta^*_3 - \delta^*_6\right\} = 0 \qquad (7.6)$$

$n^2\gamma^*_4 + n\gamma^*_5 + \gamma^*_6 = 0$ which provides solution for approximate cost-optimal n as.

$$\left[n_{opt}(r_1, r_2, \alpha_1, \alpha_2, \alpha_3)\right] = \frac{\left[-\gamma^*_5 \pm \sqrt{\gamma^{*2}_5 - 4\gamma^*_4\gamma^*_6}\right]}{2\gamma^*_4} \qquad (7.7)$$

where

$$\delta^*_{1l} = \left(M^*_{1m}NW_{lG_2}C_{3lG_2}\right)\left\{\left(1-\alpha_2^2\right)L_{2l} + \left(2\alpha_2 - \alpha_2^2\right)L_{4l}\right\}$$

$\delta^*_{2l} = \left(M^*_{1m}NW_{lG_2}C_{3lG_2}\right)\left\{(1-\alpha_2)^2 L_{3l}\right\} + \left(M^*_{2m}NW_{lG_2}C_{3lG_2}\right)\left\{(1-\alpha_2)^2 L_{2l} + \left(2\alpha_2 - \alpha_2^2\right)L_{4l}\right\}$; $\delta^*_{3l} = \delta^*_3$,

$\delta^*_{4m} = \left(M^*_{1l}NN''_{mG_3}C_{3mG_3}\right)\left\{\left(1-\alpha_3^2\right)L_{2m} + \left(2\alpha_3 - \alpha_3^2\right)L_{4m}\right\}$;

$\delta^*_{5m} = \left(M^*_{1l}NN''_{mG_3}C_{3mG_3}\right)\left\{(1-\alpha_3)^2 L_{3m}\right\} + \left(M^*_{2l}NN''_{mG_3}C_{3mG_3}\right)\left\{(1-\alpha_3)^2 L_{2m} + \left(2\alpha_3 - \alpha_3^2\right)L_{4m}\right\}$, $\delta^*_{6l} = \delta^*_6$

$\gamma_4^* = \{(M_{1l}^* M_{1m}^* X_1^*) - \delta_{1l}^* - \delta_{4m}^*\}; \gamma_5^* = \{(M_{1l}^* M_{2m}^* + M_{1m}^* M_{2l}^*) X_1^* - \delta_{2l}^* - \delta_{5m}^*\}; \gamma_6^* = \{M_{2l}^* M_{2m}^* X_1^* - \delta_3^* - \delta_6^*\}$

Similarly the approximate optimum values of f_{lG_2} and $f_{mG_3}^*$ are

$$\left[f_{lG_2}(n,\alpha_1,\alpha_2,\alpha_3,r_1,r_2)_{opt}\right]_{approx} = \left[1 + \left(\frac{N_{lG_2}}{N_{lG_2}'' S_{2lG_2}^2}\right)\left\{\frac{T_{1l}}{T_{5l} + (1-\alpha_2)^2 T_{6l}} - S_{lG_2}^2\right\}\right] \quad (7.8)$$

$$\left[f_{mG_3}^*(n,\alpha_1,\alpha_2,\alpha_3,r_1,r_2)_{opt}\right]_{approx} = \left[1 + \left(\frac{1}{S_{2mG_3}^2}\right)\left\{\frac{T_{1m}}{T_{5m} + (1-\alpha_3)^2 T_{6m}} - S_{mG_3}^2\right\}\right] \quad (7.9)$$

8. Numerical Illustration

Strata	**I**	**II**	**III**	**IV**	**V**	**VI**	**VII**
Res. (R)	$N_1' = 15$	$N_2' = 20$	$N_3' = 25$	$N_4' = 30$	$N_5' = 35$	$N_6' = 40$	$N_7' = 45$
Non-Res. (NR)	$N_1'' = 15$	$N_2'' = 20$	$N_3'' = 25$	$N_4'' = 30$	$N_5'' = 35$	$N_6'' = 40$	$N_7'' = 45$
Total	$N_1 = 30$	$N_2 = 40$	$N_3 = 50$	$N_4 = 60$	$N_5 = 70$	$N_6 = 80$	$N_7 = 90$
Mean $\bar{Y}_i$	9.76	27.32	45.22	69.28	90.44	109.35	125.24
Res. (R) S_{1i}^2	30.46	50.89	95.36	201.37	180.02	166.25	237.03
Non-Res (NR) S_{2i}^2	41.35	74.54	104.23	134.69	164.83	154.68	242.88
S_i^2	35.63	61.51	97.97	165.49	173.70	158.71	239.17
Weight (W_i)	0.07	0.10	0.12	0.14	0.17	0.19	0.21

Group I [Strata I, II]; *r* = 2, Group II [Strata III, IV, V, VI, VII] ; *k* - *r* = 5]

Table 8.1: Population-I

Group – I			**Group –II**					
Strata	**I**	**II**	**III**	**IV**	**V**	**VI**	**VII**	**Total**
n_i	8	8	10	12	13	18	21	90
Under case I	-	-	$f_{1G_2} = 2.00$	$f_{2G_2} = 2.33$	$f_{3G_2} = 2.00$	$f_{4G_2} = 2.20$	$f_{5G_2} = 2.17$	-
Under case II	$f_{1G_1}^* = 3.00$	$f_{2G_1}^* = 2.50$	$f_{1G_2} = 2.00$	$f_{2G_2} = 2.33$	$f_{3G_2} = 2.00$	$f_{4G_2} = 2.20$	$f_{5G_2} = 2.17$	-
Under case III	-	-	$f_{1G_2} = 2.00$	$f_{2G_2} = 2.33$	$f_{3G_2} = 2.00$	$f_{4G_2} = 2.20$	$f_{5G_2} = 2.17$	-

Table 8.2: About Post-Stratified Sample *N*=90

α_3 \ α_2	0.05	0.10	0.30	0.50	0.75
0.05	2.8657	1.3020	1.9840	3.6262	7.0291
0.10	3.0008	2.9677	3.3998	4.792	7.8828
0.30	5.0453	4.7536	4.1864	4.5794	6.4208
0.50	9.4165	8.8751	7.3084	6.7021	7.2942
0.75	18.2091	17.3113	14.4959	12.6399	11.6707

Table 8.3: At fixed α_1= 0.05.

α_3 \ α_2	0.05	0.10	0.30	0.50	0.75
0.05	1.2735	1.2895	1.9534	3.5775	6.9577
0.10	2.9946	2.9482	3.3622	4.7365	7.8044
0.30	5.0025	4.7362	4.1209	4.4958	6.3145
0.50	9.4716	8.897	7.2149	6.5904	7.1599
0.75	18.0592	17.2308	14.3669	12.4932	11.5013

Table 8.4: At fixed α_1 = 0.10

α_3 \ α_2	0.05	0.10	0.30	0.50	0.75
0.05	1.27729	1.2708	1.8621	3.4138	6.7034
0.10	2.9661	2.9018	3.2430	4.5449	7.5222
0.30	4.8621	4.5477	3.8898	5.6913	5.9204
0.50	9.0939	8.5293	6.7089	6.1751	6.6540
0.75	17.6672	16.7907	13.8842	11.9381	10.8556

Table 8.5: At fixed α_1 = 0.30

α_3 \ α_2	0.05	0.10	0.30	0.50	0.75
0.05	1.3213	1.3011	1.8200	3.2991	6.4981
0.10	2.9865	2.9039	3.1729	4.5281	6.7491
0.30	4.7707	4.4382	3.7079	3.9378	5.5753
0.50	8.8903	8.3080	6.5783	5.8087	6.1977
0.75	17.3242	17.3875	13.4506	11.4319	10.2589

Table 8.6: At fixed α_1 = 0.50

α_3 \ α_2	0.05	0.10	0.30	0.50	0.75
0.05	1.4509	1.3204	1.8363	3.2248	6.3105
0.10	3.0811	2.8882	3.1542	4.2929	7.0663
0.30	4.7255	4.2827	3.5493	3.6887	5.2129
0.50	8.7053	8.0127	6.2791	5.4163	5.6949
0.75	16.9945	15.9596	12.9777	10.8684	9.5820

Table 8.7: At fixed α_1 = 0.75

α - Values	Variance
$(\alpha_1)_{opt}$ = 0.0496	$[V(r_1, r_2, \alpha_1, \alpha_2 a_3)]_{opt}$ =1.2503
$(\alpha_2)_{opt}$ = 0.0158	
$(\alpha_3)_{opt}$ = 0.0172	

Table 8.8: Optimum Variance

8.1 Calculation for Alternative Cost Strategy

The calculations based on equations (7.4), (7.5), (7.8) and (7.9) are given in Tables (8.1.1 - 8.1.4), (8.1.5 - 8.1.10), (8.1.11 - 8.1.14) and (8.1.15 - 8.1.20) respectively.

Strata	f_{lG_2} for given n ; j=1, m =6		
	n=15	20	25
III	2.6630	4.296	7.8730
IV	1.0291	2.0901	3.1798
V	1.000	1.4240	2.2066

Table 8.1.1

Strata	f_{lG_2} for given n ; j=2, m =6		
	n=15	20	25
III	2.6678	4.2981	5.9731
IV	1.0313	2.0928	3.1823
V	1.000	1.4266	2.2095

Table 8.1.2

Strata	f_{lG_2} for given n ; j=1, m =7		
	n=15	20	25
III	2.6730	4.2930	5.9630
IV	1.0141	2.0911	3.1810
V	1.000	1.4960	2.2080

Table 8.1.3

Strata	f_{lG_2} for given n ; j=2, m =7		
	n=15	20	25
III	2.6692	4.2997	5.9820
IV	1.0320	2.0939	3.1834
V	1.000	1.4279	2.2108

Table 8.1.4

Strata	$f^*_{mG_3}$ for given n ; j=1, l = 3		
	n=15	20	25
VI	1.01365	1.4448	1.8828
VII	1.000	1.000	1.1556

Table 8.1.5

Strata	$f^*_{mG_3}$ for given n ; j=2, l = 3		
	n=15	20	25
VI	1.0151	1.4415	1.8848
VII	1.000	1.000	1.1573

Table 8.1.6

Strata	$f^*_{mG_3}$ for given *n* ; *j*=1, *l* = 4		
	n=15	20	25
VI	1.0142	1.4455	1.8834
VII	1.000	1.000	1.1567

Table 8.1.7

Strata	$f^*_{mG_3}$ for given n ; j=2, *l* = 4		
	n=15	20	25
VI	1.0313	2.0928	3.1823
VII	1.000	1.000	1.0000

Table 8.1.8

Strata	$f^*_{mG_3}$ for given *n* ; *j*=1, *l* = 5		
	n=15	20	25
VI	1.0150	1.4463	1.8844
VII	1.000	1.000	1.1567

Table 8.1.9

Strata	$f^*_{mG_3}$ for given *n* ; *j*=2, *l* = 5		
	n=15	20	25
VI	1.0169	1.4483	1.8863
VII	1.000	1.000	1.1585

Table 8.1.10

Strata	f_{lG_2} for given *n* ; *j*=1, *m* =6		
	N=15	20	25
III	2.6630	4.292	7.8369
IV	1.0246	2.085	3.1749
V	1.000	1.418	2.2009

Table 8.1.11

Strata	f_{lG_2} for given *n* ; *j*=2, *m* =6		
	n=15	20	25
III	2.6607	4.2900	5.9701
IV	1.0300	2.0918	3.1813
V	1.000	1.4153	2.2071

Table 8.1.12

Strata	f_{lG_2} for given *n* ; *j*=1, *m* =7		
	n=15	20	25
III	2.6701	4.2903	5.9513
IV	1.0117	2.0908	3.1806
V	1.000	1.4916	2.2008

Table 8.1.13

Strata	f_{lG_2} for given n ; j=2, m =7		
	n=15	20	25
III	2.6581	4.2911	5.9718
IV	1.2921	2.0891	3.1814
V	1.000	1.4193	2.0007

Table 8.1.14

Strata	$f^*_{mG_3}$ for given n ; j=1, l = 3		
	n=15	20	25
VI	1.01299	1.4437	1.8791
VII	1.000	1.000	1.5502

Table 8.1.15

Strata	$f^*_{mG_3}$ for given n ; j=2, l = 3		
	n=15	20	25
VI	1.0102	1.3945	1.8804
VII	1.000	1.000	1.1537

Table 8.1.16

Strata	$f^*_{mG_3}$ for given n ; j=1, l = 4		
	n=15	20	25
VI	1.0142	1.4441	1.8729
VII	1.000	1.000	1.1496

Table 8.1.17

Strata	$f^*_{mG_3}$ for given n ; j=2, l = 4		
	n=15	20	25
VI	1.0311	2.0917	3.1791
VII	1.000	1.000	1.000

Table 8.1.18

Strata	$f^*_{mG_3}$ for given n ; j=1, l = 5		
	n=15	20	25
VI	1.0149	1.4396	1.8839
VII	1.000	1.000	1.1493

Table 8.1.19

Strata	$f^*_{mG_3}$ for given n ; j=2, l = 5		
	n=15	20	25
VI	1.0159	1.4383	1.8835
VII	1.000	1.000	1.1459

Table 8.1.20

9. Data Presentation

Population I

Strata I : CR		6	8	5	15	5	18	3	6
14	7	13	2	1	13	16	**NR**	4	15
14	9	1	12	20	3	10	17	11	18
6	2	19							
Strata II: CR		32	26	39	36	29	18	31	15
38	20	18	39	26	24	32	23	24	32
30	27	**NR**	40	20	15	38	16	29	19
17	28	24	36	26	33	33	17	40	19
37	20	27							
Strata III: CR		54	46	32	36	57	39	36	55
60	38	30	56	31	47	53	39	49	52
60	37	40	38	53	49	32	**NR**	33	58
52	36	60	32	33	59	56	58	45	44
35	45	38	50	42	32	48	58	41	45
30	60	52							
Strata IV: CR	85	61	82	62	88	90	55	78	75
53	59	52	83	77	67	63	75	86	68
51	89	41	57	83	50	54	57	87	65
69	**NR**	81	84	64	73	52	90	75	75
66	67	61	82	67	82	64	57	62	56
70	55	76	87	65	82	87	66	55	67
53	84								
Strata V: CR		79	88	75	111	91	95	70	100
72	107	83	97	77	88	103	84	87	98
96	75	77	85	74	108	99	82	74	105
76	119	92	76	106	68	81	**NR**	98	77
78	86	105	120	83	114	91	97	81	99
89	108	93	94	87	74	91	77	96	82
86	70	83	86	104	86	100	79	105	118
93	88	115							
Strata VI: CR		96	111	98	134	104	98	114	130
124	109	105	113	93	91	124	120	101	119
106	108	124	128	94	107	100	128	115	101
105	130	92	99	115	99	91	130	105	93
104	95	**NR**	93	116	124	119	105	106	97
100	124	109	127	115	92	98	90	94	130
118	128	99	109	128	101	98	104	111	106
95	109	123	90	112	127	105	124	100	128
110	106	125							
Strata VII: CR		107	103	109	123	142	124	107	102
144	142	107	127	105	129	132	141	105	116
133	111	142	109	147	115	127	139	125	136
101	150	125	125	107	132	128	100	149	123
145	117	104	129	126	149	115	**NR**	108	133
102	122	149	133	106	150	118	123	141	135
125	150	129	107	122	143	104	131	107	125
147	109	121	112	128	128	131	123	113	127
111	142	103	149	129	147	105	144	102	137
148	129	150							

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