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PROFIT ANALYSIS OF A COMPLEX SYSTEM WITH CORRELATION IN TIME TO PREVENTIVE MAINTENANCE AND TIME TAKEN IN PREVENTIVE MAINTENANCE

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Abstract: This paper deals with the cost benefit analysis of a complex system with correlation in time to preventive maintenance and time taken in preventive maintenance. The system consists of two subsystems, say A & B, connected in series. Subsystem A consists of two identical units while subsystem B consist only one unit. The operation of only one unit of subsystem A with subsystem B is sufficient to do the job. The joint distribution of time to preventive maintenance and time taken in preventive maintenance are taken bivariate exponential. Each repaired unit works as good as new. Various measures of system effectiveness of interest to system designers and operation managers are obtained by using regenerative point technique. The graphical behaviors of MTSF and profit function have also been studied.

Keywords: Mean sojourn time, Reliability, Availability, Busy period, Profit function.

1. Introduction

Complex redundant systems have attracted many researchers including [1, 4, 6, 7] in the field of reliability theory. Some of them obtained the availability of a complex system with two sub-systems taking constant failure and general repair time distributions for each sub-system. Both sub-systems are repaired according to one of the two repair policies – FCFS repair and repair on priority basis. Gupta et. al. [3] analysed a complex system consisting of two sub-systems A' and B' arranged in series. Sub-system A' has two identical units – one operative and the other cold standby, while sub-system B' is a single unit system and has two types of mal functions – degraded and total failure. Degradation reduces the efficiency of the system whereas in total failure, the system either stops working or the efficiency of the system goes below the specified tolerance limit. The failure and repair times of units for both sub-systems follow exponential and general distributions respectively. Using supplementary variable technique, they obtained transition state probabilities. In the above system model they have considered an assumption that the random variable denoting the failure and repair times are uncorrelated, which does not seem feasible always in real situations. In most of the systems, it can be observed that an early (late) failure leads to an early (delayed) repair.

Keeping this concept in view, the purpose of the present paper is to analyse a different type of complex system. The system composed of two sub-systems A and B, which are connected in series. Sub-system A contains two identical units and subsystem B consist only one unit. The operation of only one unit of subsystem A with subsystem B is sufficient to do the job. Each unit of subsystem A has two modes normal and total failures while unit of subsystem B has three modes normal, preventive maintenance and total failure. The time to preventive maintenance and time taken in preventive maintenance are taken to be correlated random variables having their joint distribution as Bivariate exponential.

The following system characteristics are obtained by identifying the system at suitable regenerative epochs with the help of regenerative point technique:

(i) State transition probabilities and mean sojourn times in different states.
(ii) Reliability of the system and mean time to system failure (MTSF).
(iii) Pointwise and steady state availabilities of the system during (0, t).
(iv) Expected busy period of repair facility during (0, t) and in steady state.
(v) Expected profit incurred by the system during (0, t) and in steady state.

2. System Description and Assumptions

The assumptions about the model under study are as under:

(i) The system consists of two subsystems, say A & B, arranged in series configuration.
(ii) Subsystem A comprise of two identical units and subsystem B comprise of only one unit. The operation of only one unit of subsystem A with subsystem B is sufficient to do the job.
(iii) Each unit of subsystem A has two modes – normal (N) and total failure (F) while the subsystem B has three modes – normal (N), preventive maintenance (PM) and total failure (F).
(iv) Initially the system starts from state S_0, in which one unit of subsystem A with subsystem B is operative and other unit of subsystem A is kept as cold standby.
(v) After working some significant time, the operative unit oft subsystem B goes for preventive maintenance.
(vi) A perfect and instantaneous switching device is used to detect the failed unit and put the standby unit into operation.
(vii) A repairman is always available with the system to repair a failed unit and for preventive maintenance of an operative unit.
(viii) The priority is given to do preventive maintenance or repair of unit of subsystem B over the repair of failed unit of subsystem A.
(ix) The switching device, used to detect the failed unit and to put the standby unit into operation, is perfect and instantaneous.
(x) The time to preventive maintenance and time taken in preventive maintenance are considered to be correlated and follow the bivariate exponential distribution.
(xi) The failure and repair times distribution are taken as exponential with different parameters.

3. Notations and States of the System

(a) Notations

X : time to preventive maintenance of an operative unit of subsystem B.
Y : time taken in preventive maintenance of unit of subsystem B.
$f(x,y)$: joint probability density function of (X,Y) for subsystem B respectively.

$$=\alpha\beta(1-r)e^{-(\alpha x+\beta y)}\, I_0\left(2\sqrt{\alpha\beta r x y}\right);\ x,y,\alpha,\beta>0;\ 0\le r\le 1$$

where, $I_0\left(2\sqrt{\alpha\beta r x y}\right)=\sum_{k=0}^{\infty}\frac{(\alpha\beta r x y)^k}{(k!)^2}$

is the modified Bessel function of type-I and order zero.

$g(\cdot)$: marginal p.d.f. of $X = \alpha(1-r)e^{-\alpha(1-r)x}$

$h(\cdot)$: marginal p.d.f. of $Y = \beta(1-r)e^{-\beta(1-r)y}$

$k(y|X=x)$: conditional p.d.f. of Y_i given $X=x$.

$= \beta e^{-(\beta y+\alpha r x)} I_0\left(2\sqrt{\alpha\beta r x y}\right)$

$K(y|X=x)$: conditional c.d.f. of Y given $X=x$.

θ_1/θ_2 : constant failure rates of a unit from N to F-mode of subsystem A/subsystem B.

λ : constant failure rate of a unit from PM (preventive maintenance) mode to F-mode of subsystem B.

μ_1 : repair rate of a unit of subsystem A.

μ_2 : repair rate of unit of subsystem B.

(b) Symbols for the States of the System

We define the following symbols for the states of the system:

NA_o/NA_s : unit is in N-mode and operative and standby of the subsystem A.

NB_o/NB_{PM} : unit is in N-mode and operative and under preventive maintenance of the subsystem B.

FA_{wr}/FA_r : unit is in F-mode and waiting for repair/under repair of subsystem A.

FB_r : unit is in F-mode under repair of subsystem B.

Using these symbols, the possible states of the system are shown in the transition diagram of the system model (figure–1). In this figure, we observe that the epochs of entrance from S_1 to S_4 and S_4 to S_7 are non-regenerative as the future probabilistic behaviour at these epochs depends upon the previous states. The all other entrance epochs are regenerative.

4. Transition Probabilities and Sojourn Times

First, we obtained the direct conditional and unconditional transition probabilities as follows:

$$p_{01}=\frac{\alpha(1-r)}{[\theta_1+\theta_2+\alpha(1-r)]};\qquad p_{02}=\frac{\theta_2}{[\theta_1+\theta_2+\alpha(1-r)]};$$

$$p_{03}=\frac{\theta_1}{[\theta_1+\theta_2+\alpha(1-r)]};\qquad p_{20}=1;$$

$$p_{30}=\frac{\mu_1}{[\mu_1+\theta_1+\theta_2+\alpha(1-r)]};\qquad p_{34}=\frac{\alpha(1-r)}{[\mu_1+\theta_1+\theta_2+\alpha(1-r)]};$$

$$p_{35}=\frac{\theta_1}{[\mu_1+\theta_1+\theta_2+\alpha(1-r)]};\qquad p_{36}=\frac{\theta_2}{[\mu_1+\theta_1+\theta_2+\alpha(1-r)]};$$

$p_{53}=1$; $p_{63}=1$;

$p_{10\,|\,x}=p_{43\,|\,x}=\beta' e^{-\alpha r x(1-\beta')}$; where $\beta'=\beta/(\lambda+\beta+\theta_1)$

$p_{12\,|\,x}=p_{46\,|\,x}=\lambda\left[1-\beta' e^{-\alpha r x(1-\beta')}\right]\Big/(\lambda+\theta_1)$;

Now the transition probabilities via one or more non-regenerative states are given by

$$p_{13|x}^{(4)}=\theta_1\beta'(1+\alpha\beta' r x)e^{-\alpha r x(1-\beta')}\Big/(\lambda+\beta+\theta_1)$$

$$p_{16|x}^{(4)}=\frac{\lambda\theta_1}{(\lambda+\theta_1)^2}\left[1-\beta' e^{-\alpha r x(1-\beta')}-\frac{(\lambda+\theta_1)\beta' e^{-\alpha r x(1-\beta')}}{(\lambda+\beta+\theta_1)}(1+\alpha\beta' r x)\right]$$

$$p_{45|x}^{(7)}=\theta_1\left[1-\beta' e^{-\alpha r x(1-\beta')}\right]\Big/(\lambda+\theta_1)$$

$$p_{15|x}^{(4,7)}=\frac{\theta_1^2}{(\lambda+\theta_1)^2}\left[1-\beta' e^{-\alpha r x(1-\beta')}-\frac{(\lambda+\theta_1)\beta' e^{-\alpha r x(1-\beta')}}{(\lambda+\beta+\theta_1)}(1+\alpha\beta' r x)\right]$$

It can be easily verified that,

$p_{01}+p_{02}+p_{03}=1$; $p_{10|x}+p_{12|x}+p_{13|x}^{(4)}+p_{16|x}^{(4)}+p_{15|x}^{(4,7)}=1$

$p_{20}=p_{53}=p_{63}=1$; $p_{30}+p_{34}+p_{35}+p_{36}=1$;

$p_{43|x}+p_{45|x}^{(7)}+p_{46|x}=1$.

From the above conditional steady state transition probabilities, the unconditional steady state transition probabilities can be obtained by using the result:

$$p_{ij}=\int p_{ij\,|\,x}\; g(x)\,dx$$

Thus, $p_{10}=\int p_{10\,|\,x}\; g(x)\,dx =\beta'(1-r)/(1-r\beta') =p_{43}$

Similarly,

$$p_{12}=\frac{\lambda}{(\lambda+\theta_1)}\left[1-\frac{\beta'(1-r)}{1-r\beta'}\right]=p_{46};$$

$$p_{13}^{(4)}=\frac{\theta_1\beta'(1-r)}{(\lambda+\beta+\theta_1)(1-r\beta')^2} \quad ;$$

$$p_{16}^{(4)}=\frac{\lambda\theta_1}{(\lambda+\theta_1)^2}\left[1-\frac{\beta'(1-r)}{1-r\beta'}-\frac{(\lambda+\theta_1)\beta'(1-r)}{(\lambda+\beta+\theta_1)(1-r\beta')^2}\right];$$

$$p_{45}^{(7)}=\frac{\theta_1}{(\lambda+\theta_1)}\left[1-\frac{\beta'(1-r)}{1-r\beta'}\right];$$

$$p_{15}^{(4,7)}=\frac{\theta_1^2}{(\lambda+\theta_1)^2}\left[1-\frac{\beta'(1-r)}{1-r\beta'}-\frac{(\lambda+\theta_1)\beta'(1-r)}{(\lambda+\beta+\theta_1)(1-r\beta')^2}\right]$$

Thus we have,

$p_{01}+p_{02}+p_{03}=1$; $p_{10}+p_{12}+p_{13}^{(4)}+p_{15}^{(4,7)}+p_{16}^{(4)}=1$

$p_{20}=1$; $p_{30}+p_{34}+p_{35}+p_{36}=1$

$$p_{43}+p_{45}^{(7)}+p_{46}=1 \qquad ; \qquad p_{53}=p_{63}=1$$

The mean sojourn times in various states are as follows:

$$\psi_0=\int e^{-\alpha(1-r)t}\cdot e^{-(\theta_1+\theta_2)t}\,dt=1/[\theta_1+\theta_2+\alpha(1-r)]$$

Similarly,

$$\psi_{1|x}=\left[1-\beta' e^{-\alpha r x(1-\beta')}\right]/(\lambda+\theta_1)=\psi_{4|x}$$

So that, $\psi_1=\int \psi_{1|x}\ g(x)\ dx=\dfrac{1}{(\lambda+\theta_1)}\left[1-\dfrac{\beta'(1-r)}{1-r\beta'}\right]=\psi_4$

$$\psi_2=1/\mu_2=\psi_6; \qquad \psi_3=1/[\mu_1+\theta_1+\theta_2+\alpha(1-r)]$$

$$\psi_5=1/\mu_1\ ; \qquad \psi_{7|x}=(1+\alpha r x)/\beta$$

So that, $\psi_7=\int \psi_{7|x}\ g(x)\ dx=1/[\beta(1-r)]$

5. Analysis of Results

(a) Reliability of the System and MTSF

Using the technique of regenerative point the expression of reliability, in terms of its Laplace transform, is given by

$$R_0^*(s)=\frac{\left[Z_0^*+q_{01}^*\left(Z_1^*+q_{14}^*Z_4^*\right)\right]\left(1-q_{34}^*q_{43}^*\right)+\left(q_{03}^*+q_{01}^*q_{13}^{(4)*}\right)\left(Z_3^*+q_{34}^*Z_4^*\right)}{\left(1-q_{01}^*q_{10}^*\right)\left(1-q_{34}^*q_{43}^*\right)-q_{01}^*q_{13}^{(4)*}q_{30}^*+q_{03}^*q_{30}^*}$$

where Z_0^*, Z_1^*, Z_3^* and Z_4^* are the L.T. of

$Z_0(t)=e^{-\{\theta_1+\theta_2+\alpha(1-r)\}t}$; $\quad Z_1(t)=Z_4(t)=e^{-(\lambda+\theta_1)t}\,\overline{k}(t|x)$ and

$Z_3(t)=e^{-\{\mu_1+\theta_1+\theta_2+\alpha(1-r)\}t}$

The expression of mean time to system failure (MTSF) is given by

$$E(T_0)=\lim_{s\to 0} R_0^*(s)=\frac{\left[\psi_0+p_{01}(\psi_1+p_{14}\psi_4)\right](1-p_{34}p_{43})+\left(p_{03}+p_{01}p_{13}^{(4)}\right)(\psi_3+p_{34}\psi_4)}{(1-p_{01}p_{10})(1-p_{34}p_{43})-p_{01}p_{13}^{(4)}p_{30}-p_{03}p_{30}}$$

(b) Availability Analysis

Let us define $A_i(t)$ as the probability that the system is up (operative) at epoch ‘t’, when initially the system starts from state $S_i \in E$. Using the technique of Laplace transforms, one can obtain the value of $A_0(t)$ in terms of its Laplace transforms *i.e.* $A_0^*(s)$.

The steady state availability of the system is given by

$$A_0=\lim_{t\to\infty} A_0(t)=\lim_{s\to 0} s\,A_0^*(s)=N_1/D_1$$

where,

$$N_1=p_{30}\left[\psi_0+p_{01}(\psi_1+p_{14}\psi_4)\right]-(\psi_3+p_{34}\psi_4)\left[p_{03}+p_{01}\left(p_{13}^{(4)}+p_{15}^{(4,7)}+p_{16}^{(4)}\right)\right]$$

and $D_1=\psi_0 p_{30}+\psi_1 p_{01}p_{30}+\psi_2 p_{30}(p_{01}p_{12}+p_{02})+\left[\psi_5 p_{15}^{(4,7)}+\psi_6 p_{16}^{(4)}\right]p_{01}p_{30}$

$$+\left[\psi_3+\psi_4 p_{34}+\psi_5\left(p_{34}p_{45}^{(7)}+p_{35}\right)+\psi_6\left(p_{36}+p_{34}p_{46}\right)\right]\left[1-p_{02}-p_{01}\left(p_{10}+p_{12}\right)\right]$$

The expected up time of the system during (0, t) is given by,

$$\mu_{up}^{*}(s)=A_0^{*}(s)/s$$

(c) Busy Period Analysis

Let $B_i^P(t)$ and $B_i^r(t)$ be the probability that the repairman is busy in the preventive maintenance of subsystem ‘B’ and repair of failed unit of subsystem ‘A’ or subsystem ‘B’ respectively at epoch t, when initially the system starts from state $S_i \in E$. Using the technique of Laplace transforms, one can obtain the value of $B_0^P(t)$ and $B_0^r(t)$ in terms of its Laplace transforms *i.e.* $B_0^{P*}(s)$ and $B_0^{r*}(s)$.

In a long run, the probability that the repairman will be busy in the preventive maintenance and repair of a failed unit of subsystem ‘A’ or subsystem ‘B’ respectively is given by

$$B_0^P=N_2/D_1 \text{ and } B_0^r=N_3/D_1$$

where, $N_2=p_{01}p_{03}\left(\psi_1+p_{14}\psi_4\right)+\left[p_{03}+p_{01}\left(p_{13}^{(4)}+p_{15}^{(4,7)}+p_{16}^{(4)}\right)\right]\left(\psi_4+p_{47}\psi_7\right)p_{34}$

and $N_3=p_{30}\left[p_{02}\psi_2+p_{01}\left(p_{12}\psi_2+p_{15}^{(4,7)}\psi_5+p_{16}^{(4)}\psi_6\right)\right]+\left[p_{03}+p_{01}\left(p_{13}^{(4)}+p_{15}^{(4,7)}+p_{16}^{(4)}\right)\right]$

$$\times\left[\psi_3+p_{36}\psi_6+p_{34}\left(p_{45}^{(7)}\psi_5+p_{46}\psi_6\right)\right]$$

The value of D_1 is same as in case of availability analysis.

The expected busy period of the repair facility in repair of a unit of subsystem ‘A’ and subsystem ‘B’ respectively during (0, t) is given by

$$\mu_b^{P*}(s)=B_0^{P*}(s)/s \text{ and } \mu_b^{r*}(s)=B_0^{r*}(s)/s$$

(d) Profit Function Analysis

The expected profit incurred by the system during (0, t) is given by

P(t) = Expected total revenue in (0, t) – Expected total repair cost in (0, t)

$$= K_0\,\mu_{up}(t)-K_1\mu_b^P(t)-K_2\,\mu_b^r(t)$$

where, K_0 is the revenue per unit up time by the system and K_1 and K_2 are respectively the amounts paid to the repairman per unit of time when the system is under repair due to the failure of any unit of subsystem ‘A’ and subsystem ‘B’.

The expected profit per unit time in a steady state is given by

$$P = K_0\,A_0 - K_1\,B_0^P - K_2\,B_0^r$$

where A_0, B_0^A, and B_0^B has been already defined.

6. Graphical Study of the System

For a more concrete study of system behaviour of the model, we plot the curves for MTSF and Profit function in figure–2 and figure–3 respectively for different values of r (= 0.50, 0.25) and μ_2 (= 0.07, 0.05, 0.03) against the failure parameter

θ_2 while the other parameters are kept fixed as: $\theta_1 = 0.05$, $\mu_1 = 0.08$, $\lambda = 0.04$, $\alpha = 0.04$, $\beta = 0.06$, $K_0 = 2500$, $K_1 = 1000$, $K_2 = 700$.

From the curves, we observe that both the MTSF and profit decrease as the failure rate θ_2 increases and these characteristics increase with the increase in repair rate μ_2 and correlation coefficient r. Thus, we conclude that the high correlation between the time to preventive maintenance and time taken in preventive maintenance results the better system performances.

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Transition Diagram

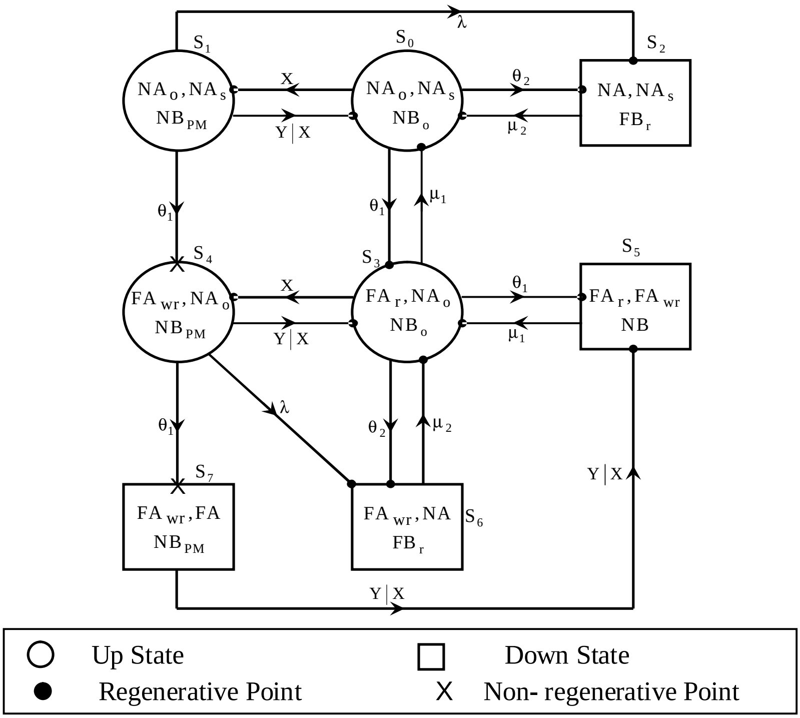
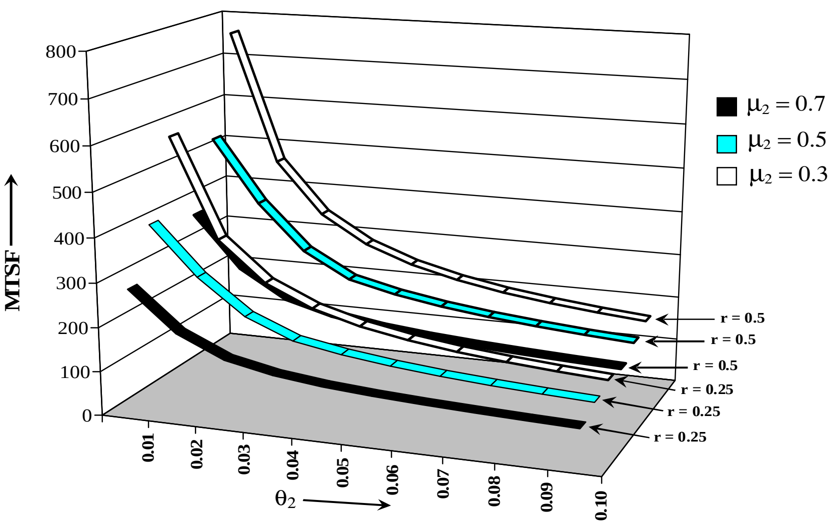
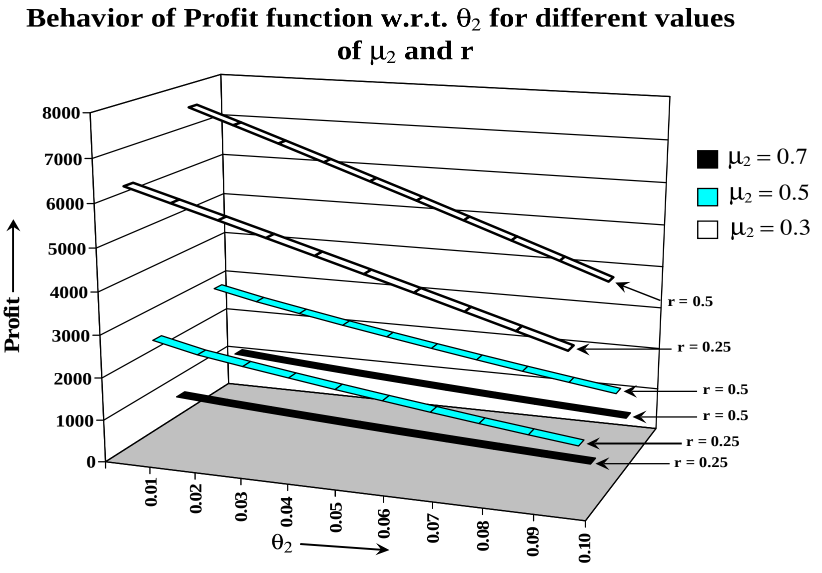


Figure – 1

Behavior of MTSF w.r.t. θ_2 for different values of μ_2 and r



Figure–2


Behavior of Profit function w.r.t. θ_2 for different values of μ_2 and r
8000
7000
6000
5000
4000
3000
2000
1000
0
Profit
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.09
0.10
θ_2
$\mu_2 = 0.7$
$\mu_2 = 0.5$
$\mu_2 = 0.3$
r = 0.5
r = 0.25
r = 0.5
r = 0.5
r = 0.25
r = 0.25

Figure–3