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STOCHASTIC ANALYSIS OF A WEB SERVER WITH DIFFERENT TYPES OF FAILURE

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Abstract

This paper investigates the reliability characteristics of a system based on the concept of a university web server. Whenever a browser brow a request different types of error can occur due to which system may suffer from partial to complete failure. Different types of repair as per requirements, including waiting have been incorporated while modeling this real life system. It is interesting to mention here that waiting has been taken as a type of repair when web server is busy. With the applications of Laplace transforms and supplementary variable technique different reliability measures like availability, reliability, M.T.T.F. and effective cost analysis have been evaluated. Some particular cases have been taken at last to highlight the results.

Keywords: Availability, Reliability, MTTF, Cost Analysis, Supplementary variable Techniques.

1. Introduction

A web server deals with the different types of tasks. While using a web server different types of errors can occur. A data intensive web document is created by a web server whenever a browser requests the document. When a request arrives, the web server runs an application program or a script that creates the data intensive web document. The server returns the output of the program or script as a response to the browser. The contents of document can vary from one request to another because a fresh document is created for each request

The present system considered is based on the concept of a university web server. The web server may be in partial failure state due to access forbidden error where it may work perfectly after waiting for sometime or it may fail completely from here or otherwise. At this state system may also suffer from too long URL request. If it happens then system will remain in partial failure state S_4. From this state system may fail completely if server is too busy or invalid application occur. Whenever there is a complete failure, system will be repaired. The system is studied by using supplementary variable technique and Laplace transforms. Different reliability measures such as availability, reliability, MTTF and cost effectiveness have been computed. At last some particular cases have been discussed to highlight the important characteristics of the system.

2. Error Description

The web server is a combination of hardware and software. It can suffer from various types of error which can lead to different types of failure of the system. In the present model following errors are taken into consideration while mathematically modeling the system.

HTTP 403.9 (Access forbidden error):

This error is caused when the server is busy and cannot process your request due to heavy traffic.

HTTP 500-13 (server is too busy):

Amount of traffic exceeds the website configuration capacity.

HTTP 500-14 (Invalid application error):

Part of the website is unavailable. The request can't be processed due to website configuration problem with the application.

HTTP 414 (Request URI is too long):

This error seldom occurs in most web traffic, particularly when the client system is a web browser. The URLs in this case are typically standard hyperlinks found on web pages. These links tend to be too large if they are simply wrong i.e. the Web page containing the link has been badly coded.

3. State Transition Diagram

Fig. 1 represents the state transition diagram of the system.

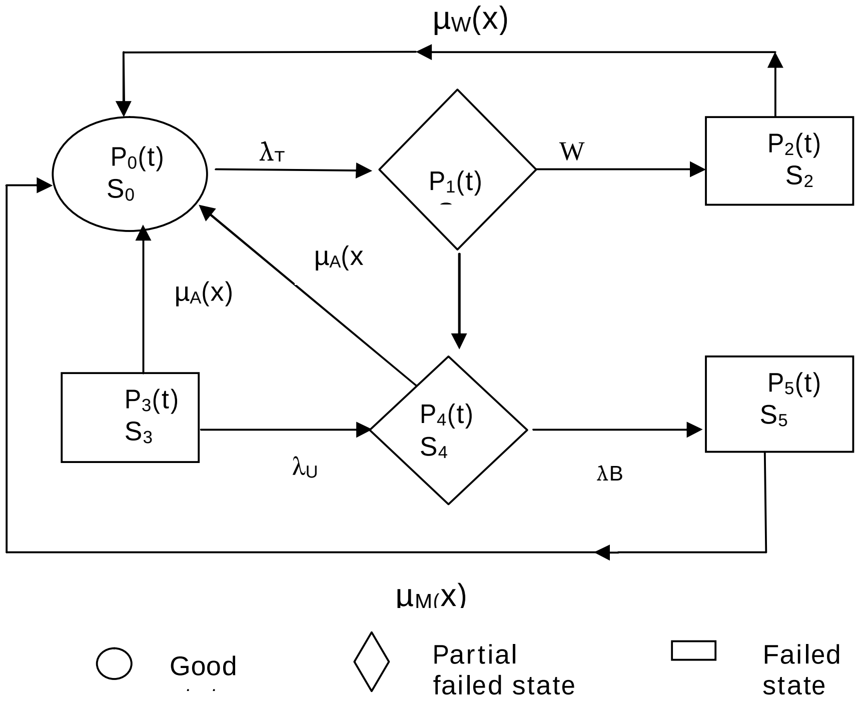


Fig. 1: State Transition Diagram

4. Notations

The following notations are associated with this model:

$P_0(t)$	:	The probability that at time t, the system is in the state S_0.
$P_i(x, t)$	:	The probability density function (system is in state S_i and is under repair; elapsed repair time is (x, t)) where i = 2…5
λ_T	:	Failure rate for HTTP 403.9 error
λ_B	:	Failure rate for HTTP 500.13 error
λ_U	:	Failure rate for HTTP 500.14 error
λ_R	:	Failure rate for HTTP 414 error
$\mu_W(x)$	:	Repair rate when W is waiting time
$\mu_A(x)$	:	Repair rate when application (web) is modified
$\mu_M(x)$	:	Repair rate when service server is modified
W	:	Waiting time

5. Formulation and Solution of Mathematical Model

By probability consideration and continuity argument the following difference- differential equations governing the behavior of the system seem to be good.

$$\left(\frac{d}{dt}+\lambda_T\right)P_0(t)=\int_0^\infty \mu_A(x)P_3(x,t)dx+\int_0^\infty \mu_A(x)P_4(x,t)dx+\int_0^\infty \mu_W(x)P_2(x,t)dx+\int_0^\infty \mu_S(x)P_S(x,t)dx \tag{1}$$

$$\left(\frac{d}{dt}+\mathrm{W}+\lambda_\mathrm{R}\right)P_1(\mathrm{t})=\lambda_\mathrm{T}\mathrm{P}_0(\mathrm{t}) \tag{2}$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\mu_W(x)\right)P_2(x,t)=0 \tag{3}$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial t}+\mu_A(x)\right)P_3(x,t)=0 \tag{4}$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\lambda_U+\lambda_B+\mu_A(x)\right)P_4(x,t)=0 \tag{5}$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\mu_S(x)\right)P_5(x,t)=0 \tag{6}$$

Boundary Conditions:

$$P_2(0,t)=WP_1(t) \tag{7}$$

$$P_3(0,t)=\lambda_U P_4(0,t) \tag{8}$$

$$P_4(0,t)=\lambda_R P_1(0,t) \tag{9}$$

$$P_5(0,t)=\lambda_B P_4(0,t) \tag{10}$$

Initial Condition:

$$P_0(t)=1 \tag{11}$$

Taking Laplace Transform of equation (1) - (11), we get

$$(s+\lambda_T)\overline{P}_0(s)=1+\int_0^\infty \mu_W(x)\overline{P}_2(x,s)dx+\int_0^\infty \mu_A(x)\overline{P}_3(x,s)dx+\int_0^\infty \mu_A(x)\overline{P}_4(x,s)dx+\int_0^\infty \mu_S(x)\overline{P}_5(x,s)dx \quad (12)$$

$$(s+W+\lambda_R)\overline{P}_1(s)=\lambda_T\overline{P}_0(s) \quad (13)$$

$$\left(s+\frac{\partial}{\partial x}+\mu_W(x)\right)\overline{P}_2(x,s)=0 \quad (14)$$

$$\left(s+\frac{\partial}{\partial x}+\mu_A(x)\right)\overline{P}_3(x,s)=0 \quad (15)$$

$$\left(s+\frac{\partial}{\partial x}+\lambda_U+\lambda_B+\mu_A(x)\right)\overline{P}_4(x,s)=0 \quad (16)$$

$$\left(s+\frac{\partial}{\partial x}+\mu_S(x)\right)\overline{P}_5(x,s)=0 \quad (17)$$

Laplace Transform of Boundary conditions is given by

$$\overline{P}_2(0,s)=\overline{P}_1(s)W \quad (18)$$

$$\overline{P}_3(0,s)=\lambda_U\overline{P}_4(0,s)=\lambda_U\lambda_R\overline{P}_1(s) \quad (19)$$

$$\overline{P}_4(0,s)=\lambda_R\overline{P}_1(s) \quad ...(20)$$

$$\overline{P}_5(0,s)=\lambda_B\overline{P}_4(0,s)=\lambda_B\lambda_R\overline{P}_1(s) \quad (21)$$

Solving equation (14) to (17), we get

$$\overline{P}_2(x,s)=\overline{P}_2(0,s)\exp\left\{-sx-\int_0^x \mu_W(x)dx\right\} \quad (22)$$

$$\overline{P}_3(x,s)=\overline{P}_3(0,s)\exp\left\{-sx-\int_0^x \mu_A(x)dx\right\} \quad (23)$$

$$\overline{P}_4(x,s)=\overline{P}_4(0,s)\exp\left\{-(s+\lambda_U+\lambda_B)x-\int_0^x \mu_A(x)dx\right\} \quad (24)$$

$$\overline{P}_5(x,s)=\overline{P}_5(0,s)\exp\left\{-sx-\int_0^x \mu_S(x)dx\right\} \quad (25)$$

Putting all these values in equation (12)-(16) and subsequently simplifying we get the transition state probabilities of various states as

$$\overline{P}_0(s)=\frac{1}{D(s)} \quad (26)$$

$$\overline{P}_1(s)=\frac{\lambda_T}{(s+W+\lambda_R)}\overline{P}_0(s) \quad (27)$$

$$\overline{P}_2(s)=\frac{\lambda_T(1-\overline{S}_{\mu_W}(s))}{(s+W+\lambda_R)s}\overline{P}_0(s) \quad (28)$$

$$\bar{P}_3(s)=\frac{\lambda_U\lambda_R\lambda_T(1-\bar{S}_{\mu_A}(s+\lambda_U+\lambda_B))}{s(s+\lambda_R+W)}\bar{P}_0(s) \tag{29}$$

$$\bar{P}_4(s)=\frac{\lambda_U\lambda_R\lambda_T(1-\bar{S}_{\mu_A}(s+\lambda_U+\lambda_B))}{(s+\lambda_U+\lambda_B)(s+W+\lambda_R)}\bar{P}_0(s) \tag{30}$$

$$\bar{P}_5(s)=\frac{\lambda_B\lambda_R(1-\bar{S}_{\mu_M}(s))}{s(s+W+\lambda_R)}\bar{P}_0(s) \tag{31}$$

where

$$D(s)=\left(s+\lambda_T\right)-\frac{\lambda_T}{(s+W+\lambda_R)}\left(\lambda_U\lambda_R\bar{S}_{\mu_A}(s)+\lambda_R\bar{S}_{\mu_A}(s+\lambda_U+\lambda_B)+W\bar{S}_{\mu_W}(s)+\lambda_B\lambda_R\bar{S}_{\mu_M}(s)\right)$$

Transition state probability that the system is in up state is obtained by

$$\bar{P}_{up}(s)=\bar{P}_0(s)+\bar{P}_1(s)+\bar{P}_4(s)$$

$$=\bar{p}_0(s)\left[1+\frac{\lambda_T}{(s+W+\lambda_R)}+\frac{\lambda_U\lambda_R\lambda_T(1-\bar{S}_{\mu_A}(s+\lambda_U+\lambda_B))}{(s+\lambda_U+\lambda_B)(s+W+\lambda_R)}\right] \tag{32}$$

Transition state probability that the system is in down state is obtained by

$$\bar{P}_{\text{down}}(s)=\bar{P}_2(s)+\bar{P}_3(s)+\bar{P}_5(s)$$

$$=\bar{p}_0(s)\left[\frac{\lambda_T(1-\bar{S}_{\mu_W}(s))}{(s+W+\lambda_R)s}+\frac{\lambda_U\lambda_R\lambda_T(1-\bar{S}_{\mu_A}(s))}{s(s+W+\lambda_R)}+\frac{\lambda_B\lambda_R(1-\bar{S}_{\mu_M}(s))}{s(s+W+\lambda_R)}\right] \tag{33}$$

It is noticeable that $\bar{P}_{\text{up}}(s)+\bar{P}_{\text{down}}(s)=\frac{1}{s}$

6. Asymptotic Behaviour of the System

Using Abel's lemma, viz. $\lim_{s\to 0} s\bar{F}(s)=\lim_{t\to\infty}F(t)=F$ (say) in equation through (26) - (31), provided the limit on R.H.S. exists, the time independent probabilities are obtained as follows

$$P_0=\frac{1}{D(0)}$$

$$P_1=\frac{\lambda_T}{D(0)(W+\lambda_R)}$$

$$P_2=\frac{\lambda_T}{D(0)(s+W+\lambda_R)}$$

$$P_3=\frac{\lambda_U\lambda_R}{D(0)(W+\lambda_R)}$$

$$P_4=\frac{\lambda_U\lambda_R\lambda_T}{D(0)(W+\lambda_R)\left(\lambda_U+\lambda_B\right)}$$

$$P_5=\frac{\lambda_B\lambda_R}{D(0)(W+\lambda_R)}$$

where

$$D(s)=(s+\lambda_T)-\frac{\lambda_T}{(s+W+\lambda_R)}\left(\lambda_U\lambda_R\overline{S}_{\mu_A}(s)+\lambda_R\overline{S}_{\mu_A}(s+\lambda_U+\lambda_B)+W\overline{S}_{\mu_W}(s)+\lambda_B\lambda_R\overline{S}_{\mu_M}(s)\right)$$

7. Particular Cases

Since in the present system waiting is considered to be a type of repairs which is not a regular phenomenon in real life. Therefore, it is worth checking the system behavior when there is no waiting. This can be done by putting W=0 in equation (26) to (31).

8. Numerical Computations

8.1 Availability Analysis

(a) Assuming repair rate to be exponential then

$$S_{\mu_A}(s)=\frac{\mu_A}{s+\mu_A},\ S_{\mu_W}(s)=\frac{\mu_W}{s+\mu_W},\ S_{\mu_M}(s)=\frac{\mu_M}{s+\mu_M} \tag{34}$$

Let the failure rates of system have values $\lambda_B = 0.45$, $\lambda_U = 0.18$, $\lambda_R = 0.25$, $\lambda_T = 0.35$, waiting W = 0.85 and repair rates $\mu_A = \mu_W = \mu_M = 1$. Substituting all these values in equation (32), we get

$$P_{up}(t)=0.05943396226e^{(-1.365000000t)}\sinh(0.26500000t)-0.006843701460e^{(-1.556111180t)}+0.2168977206e^{(-1.26897706t)}\cos(0.5885373950t+0.4488433965\,e^{(-1.267725102t)}\sin(0.5685373950)+0.7899459810e^{(-0.01156138426t)} \tag{35}$$

(b) Let the failure rates of system have values $\lambda_B = 0.45$, $\lambda_U = 0.18$, $\lambda_R = 0.25$, $\lambda_{T=}$ 0.35, waiting W = 0 and repair rates $\mu_A = \mu_W = \mu_M =1$. Putting all these values in equation (32) and using equation (34), we get

$$P_{up}(t) = 0.02282608696e^{(0.940000t)}\sinh(0.690000t)0.01743209251\,e^{(1.591305791t)}+0.3883216471e^{(0.9548205346t)}\cos(0.54181677t)+0.3185250107\,e^{(0.9548205346t)}\sin(.541816t)+0.6291104454\,e^{(-0.2709468603t)} \tag{36}$$

8.2 Reliability Analysis

Assuming repair rates to be exponential and letting failure rates as $\lambda_B = 0.45$, $\lambda_U = 0.18$, $\lambda_R = 0.25$, $\lambda_T = 0.35$, waiting W = 0.85 and repair rates $\mu_A = \mu_W = \mu_M =1$. Putting all these values in equation (32) one can obtain Table 3 and Fig.3 which show the variation of reliability with respect to time.

8.3 M.T.T.F. Analysis

M.T.T.F. of the system can be obtained as

$$\text{M. T.T. F.} = \lim_{s\to 0}\overline{P}\text{up}(s)$$

When repair follows exponential distribution:

(a) Putting $\lambda_B = 0.45$, $\lambda_R = 0.25$, $\lambda_U = 0.18$, x = 1 and varying the value of λ_T as 0.10, 0.20, 0.30, 0.40, 0.50, 0.60, 0.70, 0.80, 0.90, 1 one can obtain Fig. 4 which demonstrates variation of M.T.T.F. with respect to $\lambda_{T.}$

(b) Setting $\lambda_T = 0.35$, $\lambda_R = 0.25$, $\lambda_U = 0.18$, x = 1 and changing the value of λ_B as 0.10, 0.20, 0.30, 0.40, 0.50, 0.60, 0.70, 0.80, 0.90, 1, we can observe from Fig. 5 the variation of M.T.T.F. with respect to λ_B.

(c) Keeping $\lambda_T = 0.35$, $\lambda_B = 0.45$, $\lambda_U = 0.18$, x = 1 and varying the value of λ_R as 0.10, 0.20, 0.30, 0.40, 0.50, 0.60, 0.70, 0.80, 0.90, 1 one can obtain Table 6 and Fig. 6 which represent variation of M.T.T.F. with respect to λ_R.

(d) Taking $\lambda_T = 0.35$, $\lambda_B = 0.45$, $\lambda_R = 0.25$, x = 1 and varying the value of λ_U as 0.10, 0.20, 0.30, 0.40, 0.50, 0.60, 0.70, 0.80, 0.90, 1 one can obtain Table 7 together with Fig. 7 which show variation of M.T.T.F. with respect to λ_U.

8.4 Cost Analysis

Let the failure rates of system have values $\lambda_B = 0.45$, $\lambda_T = 0.35$, $\lambda_R = 0.25$, $\lambda_U = 0.18$ and waiting time W = 0.85. Repair rate $\mu_A = \mu_W = \mu_M = 1$, x = 1. Also let the repair follow exponential distribution. Putting all these values in equation (32) and using equation (34) then taking inverse Laplace transform, one can obtain equation (35). Let the service facility be always available, then expected profit during the interval (0, t] is given by

$$Ep(t) = k_1 \int_0^t P_{up}(t)dt - k_2 t \tag{37}$$

where k_1 and k_2 are revenue rate per unit time and service cost per unit time respectively.
Using (34) and (35) one can obtain (38)

$$E_p(t)=K_1[0.018231276677\sinh(1.63000000t)+0.027015437339\sinh(1.100000t)+ 0.0183127677\cosh(1.63000000t)0.02701543739\cosh(1.100000t)+ 0.0043979 \times e^{(1.556111180t)}\, 0.274638136\, e^{(1.26772510t)} \cos(0.5685373950t)\, 0.2308870792 \times e^{(1.267725102t)}\sin(0.5685373950t)-68.32624565\, e^{(.011561384t)} +68.6052700]-K_2 t \tag{38}$$

Time	Pup
0	1.000000000
1	.9029576373
2	.8132262664
3	.7730935876
4	.7558219859
5	.7455537372
6	.7368808898
7	.7284804113
8	.7201456044
9	.7118798000
10	.7036992736

Table.1: Time vs. Availability

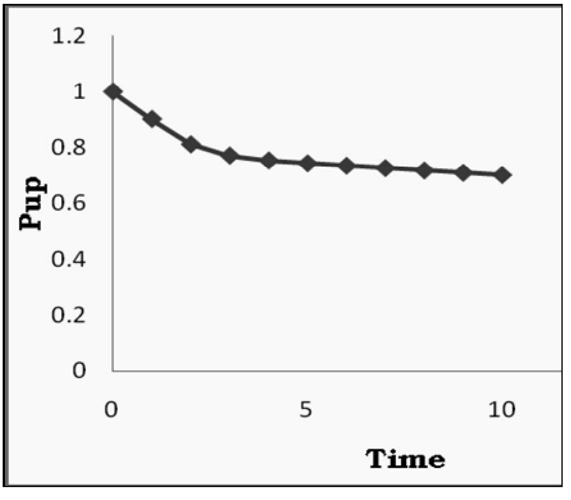


Fig1: Time vs. Availability

Time	Pup
0	1.0000000000
1	.67417004670
2	.44030356560
3	.30115284080
4	.21798335500
5	.16373089210
6	.12497185680
7	.09576796284
8	.07333771075
9	.05607133826
10	.04281875999

Table 2: Time vs. Availability (W = 0)

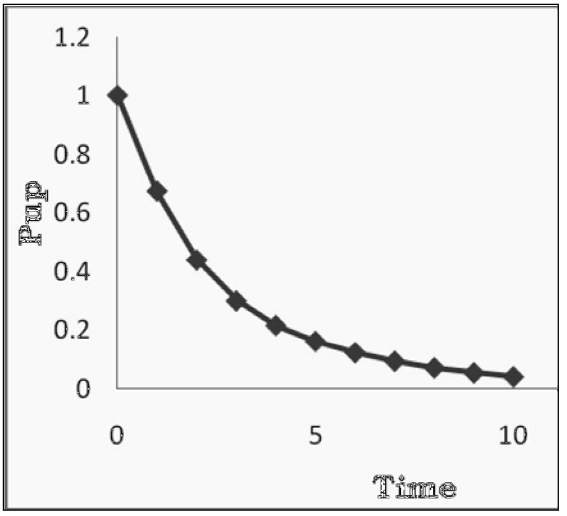


Fig. 2: Time vs. Availability (W = 0)

Time	Pup
0	1.0000000000
1	.88218622330
2	.68486369370
3	.50584278710
4	.36535998190
5	.26104835770
6	.18548149860
7	.13139011720
8	.09291045564
9	.06562987916
10	.04632748105

Table 3: Time vs. Reliability

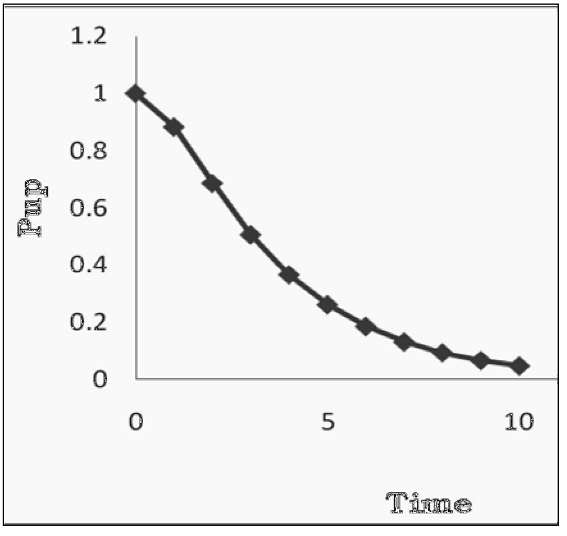


Fig. 3: Time vs. Reliability

λ_T	M.T.T.F.
0.1	10.97403
0.2	5.974026
0.3	4.307359
0.4	3.474026
0.5	2.974026
0.6	2.640693
0.7	2.402597
0.8	2.224026
0.9	2.085137
1.0	1.974026

Table.4: λ_T vs. M.T.T.F.

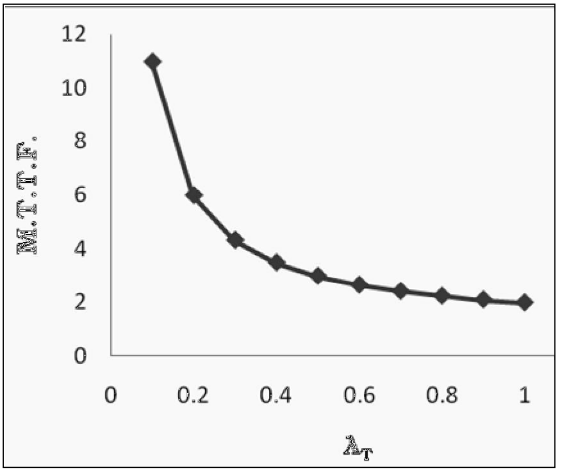


Fig..4: λ_T vs. M.T.T.F.

λ_B	M.T.T.F.
0.1	3.912338
0.2	3.873889
0.3	3.851461
0.4	3.836767
0.5	3.826394
0.6	3.818681
0.7	3.812721
0.8	3.807978
0.9	3.804113
1.0	3.800902

Table.5: λ_B vs. M.T.T.F.

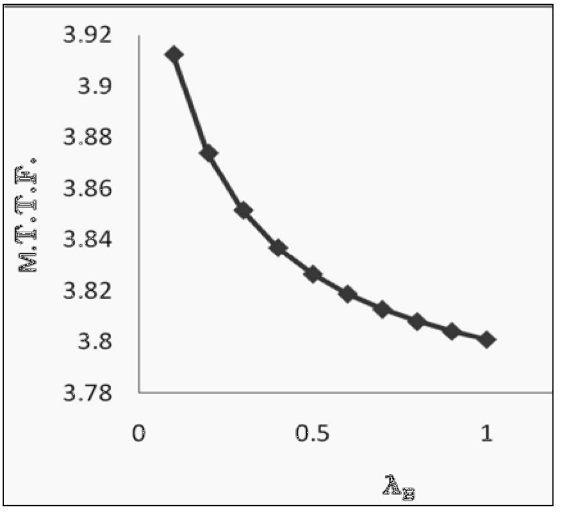
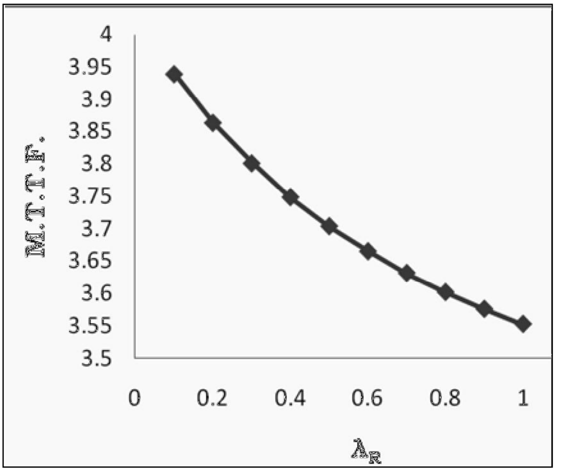


Fig..5: λ_B vs. M.T.T.F.

λ_R	M.T.T.F.
0.1	3.939850
0.2	3.863946
0.3	3.801242
0.4	3.748571
0.5	3.703704
0.6	3.665025
0.7	3.631336
0.8	3.601732
0.9	3.575510
1.0	3.552124

Table 6: λ_R vs. M.T.T.F.



F ig. 6: λ_R vs. M.T.T.F.

λ_U	M.T.T.F.
.1	3.807556
.2	3.836164
.3	3.857143
.4	3.873186
.5	3.885851
.6	3.896104
.7	3.904574
.8	3.911688
.9	3.917749
1	3.922974

Table 7: M.T.T.F. vs. λ_U

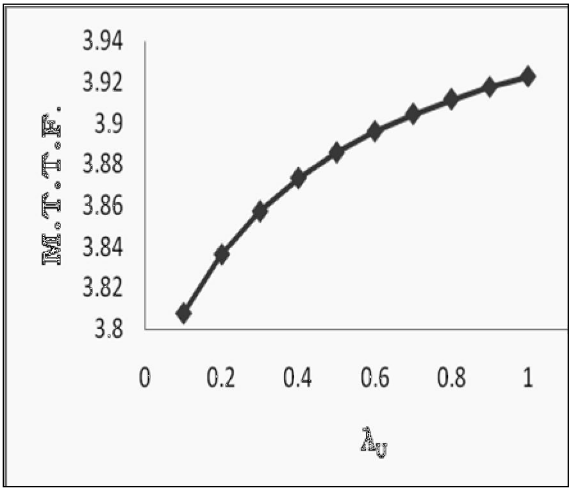


Fig. 7: M.T.T.F. vs. λ_U

Time	$E_p(t)$				
	$K_2 = 0.1$	$K_2 = 0.2$	$K_2 = 0.3$	$K_2 = 0.4$	$K_2 = 0.5$
0	0	0	0	0	0
1	0.726313	0.626313	0.526313	0.426313	0.326313
2	1.305047	1.105047	0.905047	0.705047	0.505047
3	1.835253	1.535253	1.235253	0.935253	0.635253
4	2.340839	1.940839	1.540839	1.140839	0.740839
5	2.830619	2.330619	1.830679	1.330619	0.830619
6	3.308670	2.708670	2.108670	1.508670	0.908670
7	3.776976	3.076976	2.376976	1.676976	0.976976
8	4.236962	3.436462	2.636462	1.836462	1.036462
9	4.687495	3.787495	2.887495	1.987495	1.087495
10	5.130168	4.130168	3.130168	2.130680	1.130168

Table 8: Time vs. expected profit

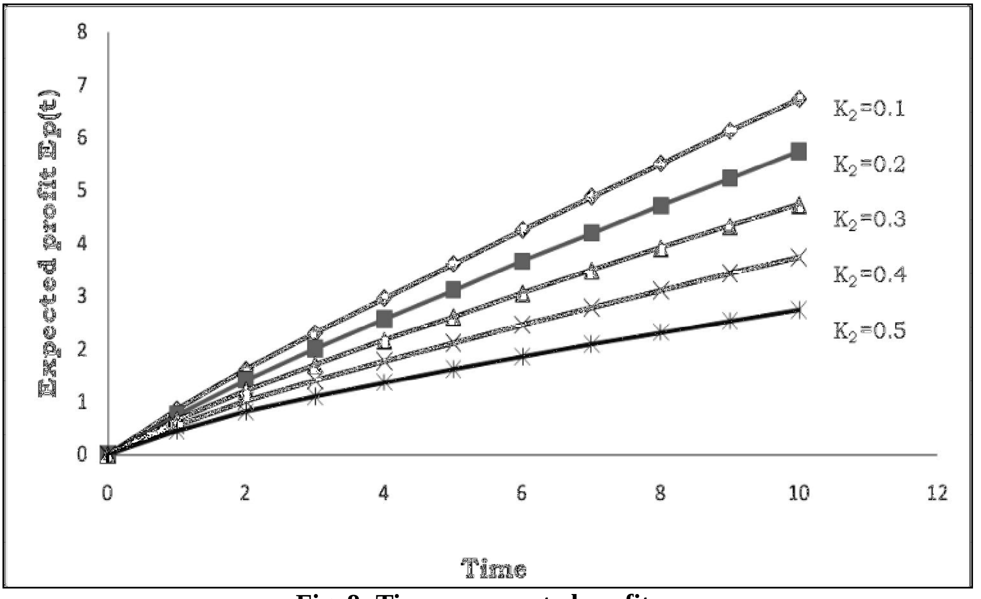


Fig. 8: Time vs. expected profit

9. Interpretation of the Result and Conclusion

In the present study various reliability measures have been computed for the considered web server system for different cases. Table 1 shows the availability of the system at instant *t* and the variation of availability is depicted in Fig.1. Critical examination of these reveals one of the interesting facts that the decrement in availability is more in the case of waiting in comparison to the case when one has not to wait. Fig. 3 represents the variation of reliability with respect to time. It is obvious from this Figure that the reliability of the system decreases as the time passes. Figs. 4, 5, 6 and 7 give the variation of M.T.T.F. with respect to variation in λ_T, λ_B, λ_R and λ_U respectively. These graphs show that M.T.T.F. of decreases with the failure rates except in the case of λ_U in which M.T.T.F. is increasing with respect to λ_U. Table 8 demonstrates the expected profit of the system when revenue cost is fixed at 1.0 and service cost varies. One can draw an important conclusion from the Fig. 8 that expected profit decreases when service cost increases with the increment of time.

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