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STOCHASTIC ANALYSIS OF A TWO-UNIT PARALLEL SYSTEM WITH PREVENTIVE MAINTENANCE

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Abstract

This paper presents the stochastic analysis of a two unit parallel system model with the concept of correlation between time to preventive maintenance and preventive maintenance time. The joint distribution of time to preventive maintenance and preventive maintenance time is taken bivariate exponential. Using regenerative point technique, various measures of system effectiveness useful to system managers are obtained. For a more concrete study of the system graphical behaviour of MTSF and profit function have also been studied.

Key Words: Parallel configuration, preventive maintenance, MTSF.

1. Introduction

Two unit parallel systems have attracted the attention of many researchers in the field of reliability theory due to their prevalence in modern business and industrial systems. Various authors including (3, 4) have analysed two-unit parallel system models under different sets of assumptions such as administrative delay in repair, subject to degradation, slow switching device, abnormal weather conditions, etc. A very few authors including (6, 8) have analysed system models with the concept of preventive maintenance i.e. after working for a random amount of time, a unit goes for its preventive maintenance. A common assumption in the analysis of these system models is that the time to preventive maintenance and preventive maintenance time are uncorrelated random variables. However, in real existing situations we observe that some sort of correlation exists between the time to preventive maintenance and preventive time of a unit i.e. if a unit is sent for its preventive maintenance after working for a long time then the server takes more time for its preventive maintenance and vice versa.

Keeping this fact in view, we, in the present paper analyse a two unit parallel system model introducing the concept of correlation between time to preventive maintenance and preventive maintenance time. The system description and assumptions are as follows:

(i) System consists of two identical units arranged in parallel configuration. Initially the system starts its operation from state S_0 in which both the units are operative.
(ii) After working for some time, the operative unit goes for preventive maintenance.
(iii) A single repairman is always with the system to repair a failed unit and for preventive maintenance of an operating unit. The repair and preventive maintenance is done on FCFS basis.
(iv) The failure and repair time distributions are taken exponential with different parameters.

(v) The time to preventive maintenance and time taken in preventive maintenance are assumed to be correlated random variables following the bivariate exponential distribution with the density given by

$$f(x, y) = \alpha\beta(1-r)\, e^{-\alpha x - \beta y} I_0\left(2\sqrt{\alpha\beta\, rxy}\right) \quad \alpha, \beta, x, y > 0;\ 0 \le r \le 1$$

Where, $$I_0(z) = \sum_{k=0}^{\infty} \frac{(z/2)^{2k}}{(k!)^2}$$

is the modified Bessel function of type-I and order zero.

2. Notations and States of the System

Notations:

E	:	set of regenerative states.
X	:	r.v. representing time to preventive maintenance of an operative unit.
Y	:	r.v. representing time taken in preventive maintenance of an operative unit.
$f(x, y)$	:	joint p.d.f. of (X, Y) $= \alpha\beta(1-r)\, e^{-\alpha x - \beta y} I_0\left(2\sqrt{\alpha\beta\, rxy}\right) \quad \alpha, \beta, x, y > 0;\ 0 \le r \le 1$
$g(.)$	:	marginal p.d.f. of $X = x$ $= \alpha(1-r)\, e^{-\alpha(1-r)x};\ x > r$
$k(y \vert x)$	:	conditional p.d.f. of $Y/X = x$ $= \beta e^{-\beta y - \alpha r x} I_0\left(2\sqrt{\alpha\beta\, rxy}\right)$
λ	:	constant failure rate of a unit.
μ	:	constant repair rate of a unit.
$q_{ij}(.)$	:	p.d.f. of direct transition from regenerative state S_i to S_j.
$q_{ij}^{(k)}(.)$	:	p.d.f. of transition from regenerative state S_i to S_j via non-regenerative state S_k.
p_{ij}	:	steady state direct probability of transition from state S_i to S_j such that $$p_{ij} = \int_0^{\infty} q_{ij}(u)\,du$$
$Z_i(t)$	:	probability that the system sojourns in state S_i up to time t.
ϕ_i	:	mean sojourn time in state S_i.
©, Ⓢ	:	symbols for ordinary and Stieltjes convolution.

$$A(t) © B(t) = \int_0^t A(u)B(t-u)\,du$$

$$\text{and} \qquad A(t)\,(s)\,B(t) = \int_0^t A(u)B(t-u)du$$

Symbols

We define the following symbols for the states of the system.

N_0	:	Unit is in normal (N) mode and operative.
N_{pm}/N_{wpm}	:	Unit is in N-mode and under preventive maintenance/waiting for preventive maintenance.
Fr/Fw	:	Unit is in failure (F) mode and under repair/waiting for repair.

Using these symbols and assumptions stated earlier, the transition diagram of the system model along with all transition time variables/rates is shown in Fig. 1. The epochs of transition from S_2 to S_5, and S_2 to S_6 are non regenerative.

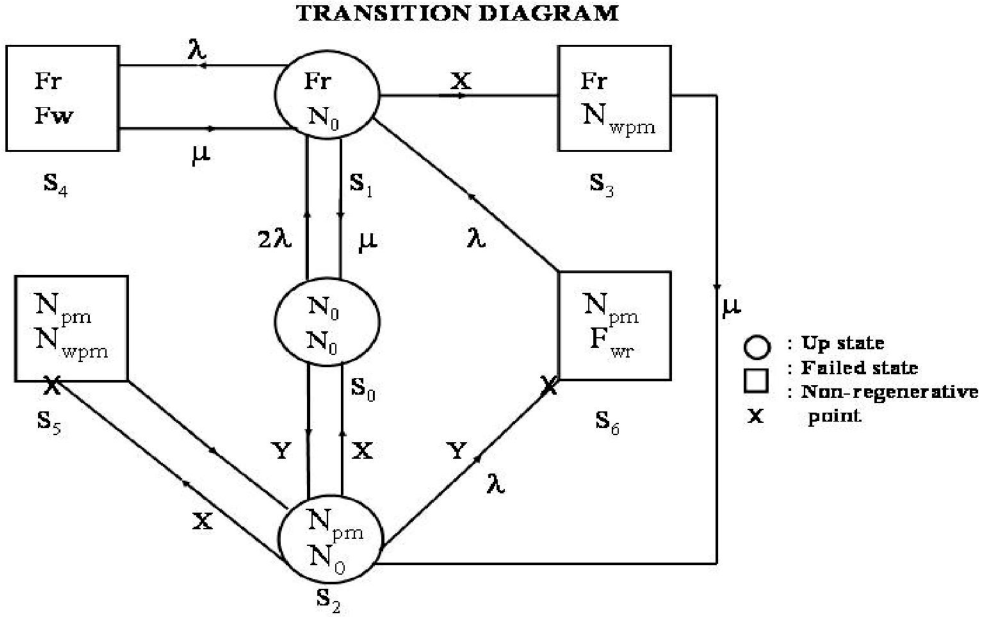

Fig. 1 : The transition diagram of the system model along with all transition time variables/ rates

3. Transition Probabilities and Sojourn Times

First, we obtain the direct and indirect conditional probabilities i.e. $p_{ij|x}$ and p_{ij} as follows:

$$p_{20|x} = \tilde{K}\,[\{\lambda + \alpha(1-r)\}|x] = K^*[\{\lambda + \alpha(1-r)\}|\,x]$$

Similarly,

$$p^{(6)}_{21|x} = \frac{\lambda}{\lambda+\alpha(1-r)}\;[1 - K^*\{\lambda + \alpha(1-r))|\,x\}]$$

$$p^{(5)}_{22|x} = \frac{\alpha(1-r)}{\lambda+\alpha(1-r)}\,[1 - K^{*}\{\lambda + \alpha(1-r))|\,x\}] \qquad (1-3)$$

It can be easily verified that

$$p_{20|x} + p^{(6)}_{21|x} + p^{(5)}_{22|x} = 1 \qquad (4)$$

The unconditional direct and indirect steady state transition probabilities are given by

$$p_{01} = 2\lambda \int_0^t e^{-\{2\lambda+\alpha(1-r)\}u}\, du$$

$$= \frac{2\lambda}{2\lambda + \alpha(1-r)}$$

Similarly,

$$p_{02} = \frac{\alpha(1-r)}{2\lambda+\alpha(1-r)}$$

$$p_{10} = \frac{\mu}{\mu+\lambda+\alpha(1-r)}$$

$$p_{13} = \frac{\alpha(1-r)}{\mu+\lambda+\alpha(1-r)}$$

$$p_{14} = \frac{\lambda}{\mu+\lambda+\alpha(1-r)}$$

$$p_{20} = \frac{\beta(1-r)}{\lambda+(\alpha+\beta)(1-r)}$$

$$p^{(6)}_{21} = \frac{\lambda}{\lambda+(\alpha+\beta)(1-r)}$$

$$p^{(5)}_{22} = \frac{\alpha(1-r)}{\lambda+(\alpha+\beta)(1-r)}$$

$$p_{32} = 1 = p_{41} \qquad (5-14)$$

It can be easily verified that

$$p_{01} + p_{02} = 1; \qquad p_{10} + p_{13} + p_{14} = 1$$

$$p_{20} + p^{(6)}_{21}\,(= p_{26}) + p^{(5)}_{22}\,(= p_{25}) = 1 \qquad (15-17)$$

The mean sojourn times in various states are as follows:

$$\phi_0 = \int e^{-\{2\lambda+\alpha(1-r)\}t}\, dt = [2\lambda + \alpha(1-r)]^{-1}$$

Similarly, $\phi_1 = [\lambda + \alpha(1-r)]^{-1}$

$\phi_{2|x} = [\lambda + \alpha(1-r)]^{-1}[1 - K^{*}\{\lambda + \alpha(1-r))|\,x\}]$

So that $\phi_2 = [\lambda + (\alpha+\beta)\,(1-r)]^{-1}$

$$\phi_3 = \mu^{-1} = \phi_4$$

$$\phi_{5|x} = \frac{1+\alpha r x}{\beta}$$

So that $$\phi_5 = [\beta\,(1-r)]^{-1} \qquad (18-25)$$

4. Analysis of Characteristics

(a) Reliability and MTSF

Using the technique of regenerative point, expression of reliability, in terms of its Laplace transform (L.T.), is given by

$$R_0^*(s) = \frac{Z_0^*(s) + q_{01}^*(s) Z_1^*(s) + q_{02}^*(s)\, Z_2^*(s)}{1 - q_{01}^*(s)\, q_{10}^*(s) - q_{02}^*(s) - q_{02}^*(s)\, q_{20}^*(s)} \qquad (26)$$

Where, $Z_0^*(s)$, $Z_1^*(s)$ and $Z_2^*(s)$ are the Laplace transforms of

$$Z_0(t) = e^{-[2\lambda + \alpha(1-r)]t}$$

$$Z_1(t) = e^{-[\mu + \lambda + \alpha(1-r)]t}$$

and $$Z_2(t) = e^{-[\lambda + \alpha(1-r)]t}\, \overline{K}(t/x).$$

Taking inverse Laplace transform of relations (26), we can get the reliability of the system when it initially starts from state S_0. Now the expression of mean time to system failure (MTSF) is given by

$$E(T) = \lim_{s\to 0} R_0^*(s) = \frac{\phi_0 + p_{01}\phi_1 + p_{02}\phi_2}{1 - p_{01}p_{10} - p_{02}p_{20}} \qquad (27)$$

(b) Availability Analysis

Let us define $A_i(t)$ as the probability that the system is up (operative) at epoch 't' when initially the system starts from the state $S_i \in E$. Using the regenerative point technique and the tools of Laplace transform one can obtain the value of $A_0(t)$ in terms of its Laplace transform i.e. $A_0^*(t)$.

The steady state availability (probability in the long run that the system is operative) of the system when it initially starts from state S_0, is given by

$$A_0 = \lim_{t\to\infty} A_0(t) = \lim_{s\to 0} s\, A_0^*(s) = N_1/D_1 \qquad (28)$$

Where,

$$N_1 = \phi_0\left[\left(1 - p_{14}p_{41}\right)\left(1 - p_{22}^{(5)}\right) - p_{13}p_{21}^{(6)}p_{32}\right] + \phi_1\left[p_{01}\left(1 - p_{22}^{(5)}\right) - p_{02}p_{21}^{(6)}\right] + \phi_2\left[p_{01}p_{13}p_{32} + p_{02}(1 - p_{14}p_{41})\right] \qquad (29)$$

and

$$D_1 = \phi_0\left[p_{20}(1-p_{14})-p_{10}p_{21}^{(6)}\right]+\phi_1\left(p_{21}^{(6)}-p_{02}p_{20}\right)$$
$$+\left(\phi_2+p_{26}\phi_6+p_{25}\phi_5\right)\left(p_{13}+p_{02}p_{10}\right)$$
$$+\ \phi_3p_{13}p_{21}^{(6)}+\phi_4p_{14}\left(p_{01}p_{20}+p_{21}^{(6)}\right) \tag{30}$$

The expected uptime of the system during (0, t) is given by

$$\mu_{up}(t) = \int_0^t A_0\ (u)\ du, \qquad \text{so that } \mu_{up}^{*}(s) = A_0^{*}(s)/s$$

(c) Busy Period Analysis

Let B_i^R (t) and B_i^P (t) be the respective probabilities that the repairman is busy at time 't' in repair of a failed unit and in preventive maintenance of the operative unit when initially system starts functioning from state $S_i \in E$. Using the regenerative point technique and the tools of Laplace transform one can obtain the values of B_0^R (t) and B_0^P (t) in terms of their Laplace transforms i.e. B_0^{R*} (s) and B_0^{P*} (s).

In the long run, the probabilities that the repair facility will be busy in the repair of failed unit and in preventive maintenance of operative unit are respectively given by

$$B_0^R = N_2/D_1 \quad \text{and} \quad B_0^P = N_3/D_1 \tag{31 – 32}$$

Where, $$N_2 = (\phi_1 + p_{14}\phi_4 + p_{13}\phi_3)\left[p_{01}\left(1-p_{22}^{(5)}\right)-p_{02}p_{21}^{(6)}\right] \tag{33}$$

and $$N_3 = \phi_2\ [\ p_{01}\phi_{13} + p_{02}(\ 1 - p_{14})] \tag{34}$$

The value of D_1 is same as in (30).

The expected busy period of repairman when he is busy in repair of the failed unit during (0, t) is given by

$$\mu_b^R(t) = \int_0^t B_0^R\ (u)\ du, \qquad \text{so that } \mu_b^{R*}(s) = B_0^{R*}(s)/s$$

Similarly, the expected busy period of repairman when he is busy in the preventive maintenance of operative unit during internal (0, t) is given by

$$\mu_b^P(t) = \int_0^t B_0^P\ (u)\ du, \qquad \text{so that } \mu_b^{P*}(s) = B_0^{P*}(s)/s$$

(d) Profit Function Analysis

The expected profit incurred the system during (0, t) is given by

P(t) = Expected total revenue in (0, t) – Expected total expenditure (0, t)

$$= C_0\mu_{up}(t) - C_1\mu_b^R(t) - C_2\mu_b^P(t) \tag{35}$$

Where C_0 is the revenue per unit up time by the system due to operation while C_1 and C_2 are the amounts paid to the repairman per unit time when he is busy in repair of failed unit and in preventive maintenance of the operative unit respectively.

The expected profit per unit time in steady state is give by

$$P = C_0A_0 - C_1 B_0^R - C_2 B_0^P \tag{36}$$

Where, A_0, B_0^R and B_0^P have been already defined.

5. Graphical Study of the System Behaviour

For a more concrete study of the system we plot the graphs for MTSF and profit function w.r.t. α for different values of r (= 0.25, 0.50, 0.75) while the other parameters are kept fixed as $\lambda = .002$, $\beta = .003$, $\mu = .10$. The curves so obtained are shown in Figures 2 and 3 respectively.

From Fig. 2 it is clear that MTSF of the system decreases w.r.t. α irrespective of other parameters. Also, for fixed value of α, MTSF is higher for higher values of r. So we conclude that the high correlation (r) between time to preventive maintenance and time taken in preventive maintenance of a unit increases the expected life time of the system.

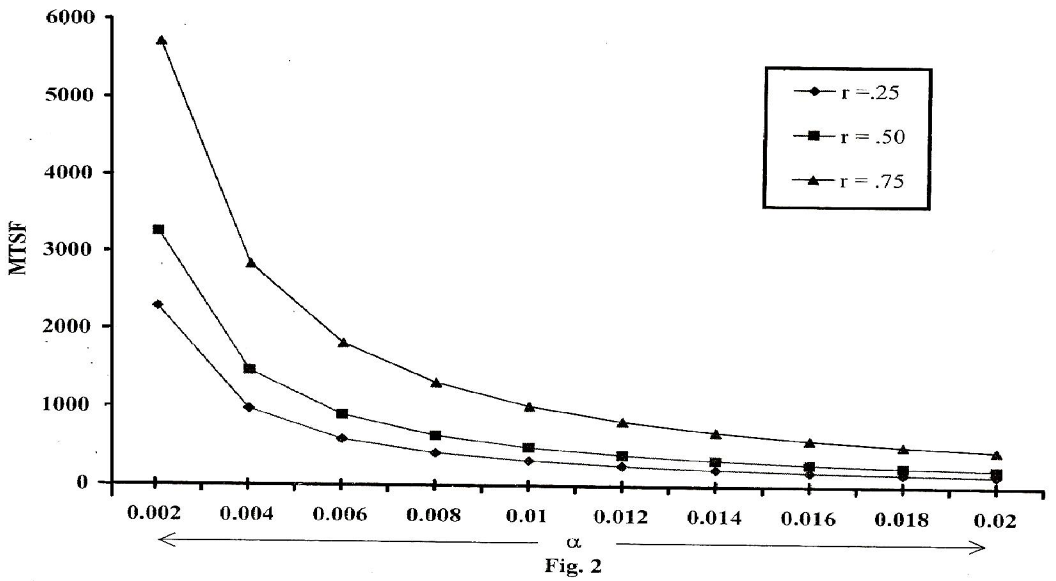

Fig. 2

Fig.3 represents the variation in profit w.r.t. α for different values of r (= 0.25, 0.50, 0.75) while in addition to the above parameters we fix $C_0 = 10000$, $C_1 = 2000$ and $C_2 = 1000$. From figure it is clear that profit decreases as α increases. Also for the fixed value of α, the profit is higher for high correlation (r). Thus, finally we conclude that the high correlation between time to preventive maintenance and time taken in preventive maintenance of a unit yields the better system performance.

Behaviour of Profit Function w.r.t. α For different values of r

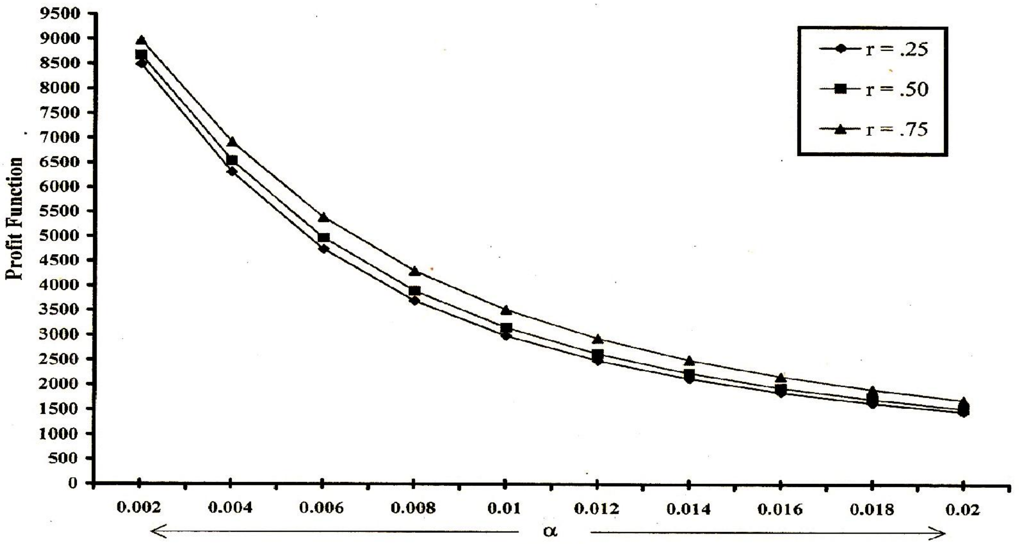

Fig. 3

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