Journal of Reliability and Statistical Studies (ISSN: 0974-8024)
Vol. 2, Issue 2(2009): 77-84

INFLATED PROBABILITY MODEL FOR RISK OF VULNERABILITY TO HIV/AIDS INFECTION AMONG FEMALE MIGRANTS

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Abstract

This paper is concerned in the development of the probability model which is based on the risk of sexual transmitted infection for the number of close boy friends to describe the distribution of single unmarried female migrants. The parameters involve in the model are estimated. The application of the model is illustrated through real data.

Key Words: Model, Probability Distribution, HIV/AIDS, Estimation Technique, Migrants.

1. Introduction

Modeling on migration is an essential and irreparable part of all scientific activity and a comparatively new area of activity involving the massage of ideas. Today, in an increasingly globalized economy, migration provides an employment opportunities giving rise to an unprecedented flow of migrants, including increasing numbers of female migrants.

The reason for migration is recognized that women move within countries in response to the inequitable distribution of resources, services and opportunities. In some countries, more than half of migrants are female. They are working in almost all types of jobs either that are in public or private sectors. So, the traditional role of women as housewives has gradually changed into working women (Reddy, 1986; Anand, 2003). Migrants to the city may bring HIV/AIDS with them or contract it after arrival, then spread it to others in the community. Women are especially vulnerable due to lack of access to information.

The number of female with HIV (Human Immunodeficiency Virus) infection and AIDS has (Acquired Immunodeficiency Syndrome) has increased steadily worldwide. In year 2005 more than 2.5 million people had died and 39 million were living with HIV. A large proportion of women in India with HIV appear to acquire the virus from regular partners who were infected during paid sex. In 2006, 2.9 million people died of AIDS related illnesses.

Two factors contributed an increased focus on female migrants, a quantitative increase in the number of women in the migratory flow and increasing evidence that a great number of these women had entered the labour market. In contrast to 60s and 70s, we can now no longer speak about male dominated migratory patterns (Jain, et al. 2007).

The main aim of this study is to indicate how women are vulnerable for STDs and HIV/AIDS transmitting among others. A study conducted among 379 HIV infected people in 1997, reported in the journal of the American Medical Association, observed an evidence of female-to-male transmission.

In this paper the probability model proposed is based on the risk for sexually transmitted infection for the number of close boy friend attached with unmarried female migrants.

2. Proposed Model

The probability model for the number of close boy friend to describe the distribution of single unmarried female migrants is developed under the following assumption.

1- Let α be the proportion of female migrants having at least one close boy friend.
2- Out of α proportion of female migrants, let β be the proportion of female migrants having only one close boy friend.
3- Remaining $(1-\beta)\alpha$ proportion of female's migrants having multiple close boy friends follows a displaced Poisson distribution with parameter θ .
4- According to the number of close boy friends, let θ be the average proportion of close boy friends attached with young unmarried female migrants, they are more vulnerable to HIV/AIDS infection.

Under the above assumptions, the probability distribution for number of close boy friends, X (Say) in given by.

$$\left.\begin{aligned} P[X=k] &= 1-\alpha && , \text{k=0} \\ P[X=k] &= \alpha\beta && , \text{k =1} \\ P[X=k] &= \frac{(1-\beta)\alpha[\theta^{k-1}(e^{\theta}-1)^{-1}]}{(k-1)!} && , \text{k = 2,3,....} \end{aligned}\right\} \qquad (2.1)$$

Where k is the number of boy friends having female. The above model involves three parameter α, β and θ to be estimated from observed distribution of female

3. Estimation

Method of Moments: The present model consists of three parameters α, β and θ . The parameters α and β are estimated by equating 0^{th} cell, First cell theoretical frequencies to observed frequencies, and theoretical mean to the observed mean. Pandey (2003).

$$\frac{n_0}{n} = 1-\alpha \qquad (3.1)$$

$$\frac{n_1}{n} = \alpha\beta \tag{3.2}$$

Mean = $E(X) = \overline{X} = \alpha\beta + (1-\beta)\alpha\{\theta e^{\theta}(e^{\theta}-1)^{-1} + 1\}$ (3.3)

Where $n_0 =$ Number of observations in Zero^th cell.

$n_1 =$ Number of observations in first cell.

$n =$ Total number of observations.

$\overline{X} =$ Observed mean.

Maximum Likelihood Method: - Consider a sample consisting of N observations of random variable X with probability function (2.1) in which n_0 designate the number of Zero observation; n_1 is the number of one observation and n_2 is the number of second observation and n is the total number of observations. Then the likelihood function for the given sample can be expressed as:

$$P(x_1, x_2, ..., x_n, \alpha, \beta, \theta) = L = (1-\alpha)^{n_0}(\alpha\beta)^{n_1}[(1-\beta)\alpha\theta(e^{\theta}-1)^{-1}]^{n_2}$$
$$[\alpha\{1-\beta(1-\beta)\theta(e^{\theta}-1)^{-1}\}]^{n-n_0-n_1-n_2} \tag{3.4}$$

Now taking logarithmic of above equation and partially differentiating w. r. to α, β and θ in turn, and equating to zero yields the estimating equation:

$$\frac{\partial LogL}{\partial\alpha} = \frac{-n_0}{1-\alpha} + \frac{n_1}{\alpha} + \frac{n_2}{\alpha} + \frac{(n-n_0-n_1-n_2)}{\alpha} = 0 \tag{3.5}$$

$$\frac{\partial LogL}{\partial\beta} = \frac{n_1}{\beta} - \frac{n_2}{1-\beta} - \frac{(n-n_0-n_1-n_2)}{1-\beta} = 0 \tag{3.6}$$

$$\frac{\partial LogL}{\partial\theta} = n_2[-(e^{\theta}-1)^{-1}e^{\theta} + 1] - \frac{(n-n_0-n_1-n_2)(e^{\theta}-1)^{-1}[-(e^{\theta}-1)^{-1}e^{\theta}\theta + 1]}{[1-\theta(e^{\theta}-1)^{-1}]} = 0 \tag{3.7}$$

Solution of equation (3.6) provides the estimate of α as:

$$\alpha = \frac{n-n_0}{n}$$

and solution of the remaining equation provides the estimate of β and θ as:

$$\beta = \frac{n_1}{n-n_0}; \qquad \theta(e^{\theta}-1)^{-1} = \frac{n_2}{n-n_0-n_1}$$

The second partial derivatives of $Log\, L$ can be obtained as:

$$\frac{\partial^2 LogL}{\partial\alpha^2} = \frac{-n_0}{(1-\alpha)^2} - \frac{n_1}{\alpha^2} - \frac{n_2}{\alpha^2} - \frac{(n-n_0-n_1-n_2)}{\alpha^2} \tag{3.8}$$

$$\frac{\partial^2 LogL}{\partial\beta^2}=\frac{-n_1}{\beta^2}-\frac{n_2}{(1-\beta)^2}-\frac{(n-n_0-n_1-n_2)}{(1-\beta)^2} \tag{3.9}$$

In case of θ we have taken approximation of e^{θ} at one place upto three terms and then partially differentiating of $Log\,L$ we obtained as:

$$\frac{\partial^2 LogL}{\partial\theta^2}=-n_2(e^{\theta}-1)^{-1}\{1-e^{\theta}(e^{\theta}-1)^{-1}\}-\frac{n_2}{\theta^2}-\frac{(n-n_0-n_1-n_2)[\frac{3\theta^2}{4}+\theta]}{[\frac{\theta^3}{4}+\frac{\theta^2}{2}]^2} \tag{3.10}$$

Now, $$\frac{\partial LogL}{\partial\alpha\partial\theta}=\frac{\partial^2 LogL}{\partial\theta\partial\alpha}=0 \tag{3.11}$$

$$\frac{\partial LogL}{\partial\alpha\partial\beta}=\frac{\partial^2 LogL}{\partial\beta\partial\alpha}=0 \tag{3.12}$$

$$\frac{\partial LogL}{\partial\beta\partial\theta}=\frac{\partial^2 LogL}{\partial\theta\partial\beta}=0 \tag{3.13}$$

Using the fact $E(n_0)=N(1-\alpha)$; $E(n_1)=N\alpha\beta$;

$E(n_2)=N(1-\beta)\alpha(e^{\theta}-1)^{-1}\theta$;

$E(n-n_0-n_1-n_2)=N\alpha\{1-\beta-(1-\beta)\theta(e^{\theta}-1)^{-1}\}$

Where E denote for the expectation.
The expected value of second partial derivatives of $Log\,L$ can be obtained by using three different cases as:

Case 1: When β is known, then from the method of moments we have,

$$\phi_{11}=E(\frac{-\partial^2 LogL/\partial\alpha^2}{N})=[\frac{1}{1-\alpha}+\frac{1}{\alpha}] \tag{3.14}$$

$$\phi_{22}=E(\frac{-\partial^2 LogL/\partial\theta^2}{N})=(1-\beta)\alpha\theta e^{\theta}(e^{\theta}-1)^{-2}\{1-e^{\theta}(e^{\theta}-1)^{-1}\}+\frac{(1-\beta)\alpha(e^{\theta}-1)^{-1}}{\theta}$$
$$+\frac{\alpha\{1-\beta-(1-\beta)\theta(e^{\theta}-1)^{-1}\}[\frac{3\theta^2}{4}+\theta]}{[\frac{\theta^3}{4}+\frac{\theta^2}{2}]^2} \tag{3.15}$$

$$\phi_{12} = \phi_{21} = E(\frac{-\partial^2 LogL / \partial\alpha\partial\theta}{N}) = 0 \tag{3.16}$$

$$V(\hat{\alpha}) = \frac{1}{N}[\frac{\phi_{22}}{\phi_{11}\phi_{22} - \phi_{12}^{2}}] \tag{3.17a}$$

$$V(\hat{\theta}) = \frac{1}{N}[\frac{\phi_{11}}{\phi_{11}\phi_{22} - \phi_{21}^{2}}] \tag{3.17b}$$

Case 2: When θ is known, then from the method of moments we have,

$$\phi_{11} = E(\frac{-\partial^2 LogL / \partial\alpha^2}{N}) = [\frac{1}{1-\alpha} + \frac{1}{\alpha}] \tag{3.18}$$

$$\phi_{22} = E(\frac{-\partial^2 LogL / \partial\beta^2}{N}) = [\frac{1}{\beta} + \frac{1}{1-\beta}] \tag{3.19}$$

and

$$\phi_{12} = \phi_{21} = E(\frac{-\partial^2 LogL / \partial\alpha\partial\beta}{N}) = 0 \tag{3.20}$$

$$V(\hat{\alpha}) = \frac{1}{N}[\frac{\phi_{22}}{\phi_{11}\phi_{22} - \phi_{12}^{2}}] \tag{3.21a}$$

$$V(\hat{\beta}) = \frac{1}{N}[\frac{\phi_{11}}{\phi_{11}\phi_{22} - \phi_{21}^{2}}] \tag{3.21b}$$

Case 3: When α is taking know from the method of moment then.

$$\phi_{11} = E(\frac{-\partial^2 LogL / \partial\beta^2}{N}) = [\frac{1}{\beta} + \frac{1}{1-\beta}] \tag{3.22}$$

$$\phi_{22} = E(\frac{-\partial^2 LogL / \partial\theta^2}{N}) = (1-\beta)\alpha\theta e^{\theta}(e^{\theta}-1)^{-2}\{1 - e^{\theta}(e^{\theta}-1)^{-1}\} + \frac{(1-\beta)\alpha(e^{\theta}-1)^{-1}}{\theta}$$
$$+ \frac{\alpha\{1-\beta-(1-\beta)\theta(e^{\theta}-1)^{-1}\}[\frac{3\theta^2}{4}+\theta]}{[\frac{\theta^3}{4}+\frac{\theta^2}{2}]^2} \tag{3.23}$$

$$\phi_{12} = \phi_{21} = E(\frac{-\partial^2 LogL / \partial\beta\partial\theta}{N}) = 0 \tag{3.24}$$

$$V(\hat{\beta}) = \frac{1}{N}[\frac{\phi_{22}}{\phi_{11}\phi_{22} - \phi_{12}^{2}}] \tag{3.25a}$$

$$V(\hat{\theta}) = \frac{1}{N}\left[\frac{\phi_{11}}{\phi_{11}\phi_{22} - \phi_{21}^{2}}\right] \qquad (3.25b)$$

Application

For the application of the model, the data are based on the survey of 362 unmarried working women randomly selected from 12 working women's hostels in Delhi. The list of the hostels was obtained from Social Welfare Department, YWCA and Wardens of the hostels. Details about the data are given in Jain, et al, 2007. This study (Jain et al 2007) was basically intended to explore the sexual risk behaviours among unmarried young female workers staying in working women's hostels in Delhi for not less than six months and was specially focused on the unmarried women migrants who were involved in white-collar jobs for many reasons.

In Table 1, the value of χ^2 statistic has been calculated by grouping some last cells together. The calculated value of χ^2 is insignificant at 5% level of significance for the proposed model. This suggests that the proposed model fits well in the number of close boy friends to describe the distribution of single unmarried female migrants under the given situations.

No. of close boy friends	Observed No. of unmarried single female migrants	Expected No. of unmarried single female migrants	
		Method of Moments	Maximum Likelihood Method
0	71	70.9882	70.9882
1	127	126.9975	126.9975
2	80	82.4688	79.9991
3	55	51.4234	51.8994
4	19	21.3767	22.4465
5	10	8.74544	9.6693
Total	362	362	362
Estimate of α		0.8039	0.8039
Estimate of β		0.4364	0.4364
Estimate of θ		1.2471	1.2975
χ^2		0.7668	0.7257
$d.f.$		2	2

Table 1:Observed and expected number of unmarried single female migrants according to their closed boy friends.

An estimate of proportion of female migrants having only one close boy friend is found very low $(\hat{\beta} = 0.4364)$ in comparison to having at least one boy friend $(\hat{\alpha} = 0.8039)$. The higher value of α indicates that the risk of HIV/AIDS among female migrant having at least one close boy friend is greater. It is two times more than female migrant having one close boy friend. The average proportion of close boy friends $\hat{\theta}$ is found 1.2471 and 1.2975 by the method of moments and method of maximum likelihood respectively. This indicates that the young unmarried female migrant attached with close boy friend are more vulnerable to HIV/AIDS. The exact variances of the estimators obtained by maximum likelihood method as well as the method of moments are also given.

Hence by increasing the lifestyle of living, working condition and by providing the adequate support to unmarried female migrant, the vulnerability to STDs and HIV/AIDS infections in them can be reduced.

Case I : When β is known	
$\phi_{11} = 9.34343$ $\phi_{22} = 0.3587$ $\phi_{11}\phi_{22} = 2.2754$	$V(\hat{\theta}) = 0.007701$ $V(\hat{\alpha}) = 0.0004355$
Case II : When the θ is known	
$\phi_{11} = 6.3434$ $\phi_{22} = 03.2685$ $\phi_{11}\phi_{22} = 20.7334$	$V(\hat{\alpha}) = 0.0004355$ $V(\hat{\beta}) = 0.0008452$
Case III : When the α is known	
$\phi_{11} = 3.2685$ $\phi_{22} = 0.3587$ $\phi_{11}\phi_{22} = 1.1724$	$V(\hat{\beta}) = 0.0008452$ $V(\hat{\theta}) = 0.007701$

Table 2: The variances of the estimators of α, β and θ using method of moments

Acknowledgement

The authors are very thankful to the referees for their valuable suggestions. This research work was supported by the Rajiv Gandhi National Fellowship (HRD Ministry) U.G.C. New Delhi.

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