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RELIABILITY MODELING OF 2-OUT-Of-3 REDUNDANT SYSTEM SUBJECT TO DEGRADATION AFTER REPAIR

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Abstract

This paper has been designed with an aim to study a reliability model for 2-out-of-3 redundant system in which unit becomes degraded after repair. There is a single server who plays the dual role of inspection and repair. The system is considered in up-state if any of two original and/or degraded units are operative. Server inspects the degraded unit at its failure to see the feasibility of repair. If repair of the degraded unit is not feasible, it is replaced by new one. The original (called new) unit gets priority in operation over the degraded unit. The distributions of failure time of the units follow negative exponential while that of inspection and repair times are taken as arbitrary with different probability density functions. Various reliability and economic measures are obtained by using semi-Markov process and regenerative point technique. Graphs are drawn to depict the behavior of MTSF, availability and profit of the model for a particular case.

Key Words: Reliability Model, Redundant System, Degradation, Priority and Regenerative Point.

2000 Mathematics Subject Classification: Primary 90 B25 and Secondary 60K10

1. Introduction

Stochastic models of redundant systems have widely been studied by the researchers including Srinivasan and Gopalan [1973], Murari and Goyal [1984], Singh [1989], Nakagawa [1989] and Dhillon [1992] under the assumptions that

(i) Unit works as new after repair
(ii) There is no need to give priority in operation to one unit over the other.
(iii) The repair of the unit is always feasible.

However, in practice these assumptions are not always true. Since the working capacity and efficiency of a repaired unit after complete failure depend more or less on the standard of the repair mechanism exercised. In case of being repaired by an ordinary server, the chances of its failure may be higher and thus such a unit is declared as degraded. Mokaddis et al. [1997] have proposed a reliability model for standby system subject to degradation. Also, some times it becomes necessary to give priority in operation to one unit over the other in order to increase reliability and availability of the system. Chander [2005] has analyzed reliability models with priority subject to arrival time of the server. Further, there are cases in which repair of the degraded unit is neither possible nor economical to the system due to its excessive use as well as high cost of maintenance. Under such situations the degraded failed unit may be replaced by new one after getting the necessary inspection.

In view of the practical applications of three-unit redundant systems, a reliability model for 2-out-of-3 redundant system of identical units is proposed by considering the concepts of inspection, priority and degradation of the unit after repair. Initially, two units work in parallel and one unit kept as cold standby. There is a single server who visits the system immediately whenever needed. The unit becomes degraded after repair. Server inspects only the degraded unit at its failure to see the feasibility of repair. If repair of the degraded unit is not feasible, it is replaced by new unit, that is, the original unit. The original unit gets priority in operation over the degraded unit. The system is considered in up-state if any of two original and/or degraded units are operative. The unit when not working can not fail. The switch devices are perfect. The failure and repair times of units are assumed to be mutually independent and uncorrelated random variables. The distributions of failure time of the units are taken as negative exponential while that of inspection and repair times are arbitrary with different probability density functions. By making use of simple probabilistic approach and regenerative point technique some reliability characteristics of interest such as mean sojourn times, mean time to system failure (MTSF), steady state availability, busy period and expected number of visits are obtained. The profit function is also derived to carry out the cost-benefit analysis. The numerical results for MTSF, availability and profit of the model are evaluated for a particular case. Graphs are plotted to highlight the results.

2. Notations

E	Set of regenerative states
No/$\overline{\text{No}}$	Original unit in normal mode and operative/ not working
Do/$\overline{\text{Do}}$	Degraded unit is operative/ not working
NCs / DCs	Original/degraded unit in cold standby
p/q	Probability that repair of degraded unit is feasible/not feasible
λ/λ_1	Constant failure rate of original /degraded unit
g(t)/G(t), $g_1(t)/G_1(t)$	p.d.f./c.d.f of repair time for original /degraded unit
h(t)/H(t)	p.d.f./c.d.f of inspection time
$NF_{Ur}/NF_{UR}/NF_{Wr}$	Original unit is failed and under repair/under continuously from previous state/waiting for repair.
$DF_{Ur}/DF_{UR}/DF_{Wr}$	Degraded unit is failed and under repair/under repair Continuously from previous state/waiting for repair.
$DF_{Ui}/DF_{Wi}/DF_{UI}/DF_{WI}$	Degraded unit is failed and is under inspection /waiting for inspection/under inspection continuously from the previous state/waiting for inspection continuously from previous state.
$q_{ij}(t), Q_{ij}(t)$	p.d.f and c.d.f of first passage time from regenerative to a regenerative state *j* or to a failed state *j* withou visiting any other regenerative state in (0,t].

$q_{ij.k}(t), Q_{ij.k}(t)$	p.d.f and c.d.f of first passage time from regenerative state *i* to a regenerative state *j* or to a failed state *j* visiting state k once in (0,t].
$q_{ij.kr}(t), Q_{ij.kr}(t)$	p.d.f and c.d.f of first passage time from regenera state *i* to a regenerative state *j* or to a failed state *j* visit state k, r once in (0,t].
$M_i(t)$	Probability that the system up initially in state S_i ε E is up at time *t* without visiting to any other regenerative sate
$W_i(t)$	Probability that the server is busy in the state S_i up to time *t* without making any transition to any other regenerative state or returning to the same via one or more non-regenerative states
Ⓢ/©	Symbols for Stieltjes convolution/Laplace convolution
~|*	Symbols for Laplace Stieltjes transform (LST)/ Laplace transform (LT)
'	Symbol for derivative of the function
•	A time point (called regenerative point) at which the system history prior to it, is irrelevant to the system conditions.

The following are the possible transition states of the system model

S_0 = (No, No, NCs),	S_1 = (No, No, NF_{Ur}),	S_2 = ($\overline{No}$, NF_{Wr} , NF_{UR})
S_3 = (No, No, DCs) ,	S_4 = (No, , Do, NF_{Ur}),	S_5 = (NF_{wr} , $\overline{Do}$, NF_{UR})
S_6 = (No, Do, DCs),	S_7 = (Do, Do, NF_{Ur})	S_8 = (No, Do, DF_{Ui}),
S_9 = ($\overline{Do}$,DF_{UI} , NF_{Wr}),	S_{10} = ($NF_{WR,}$ $\overline{Do}$, DF_{Ur}),	S_{11} = (No, Do, DF_{Ur}),
S_{12}= ($\overline{No}$, DF_{Wi} ,DF_{UI}),	S_{13} = (No, No, DF_{Ui}),	S_{14} = ($\overline{No}$, DF_{Wi},DF_{UR})
S_{15} =(NF_{Wr}, $\overline{Do}$,DF_{UR}),	S_{16} = (No, No, DF_{Ur}),	S_{17} = ($\overline{No}$, NF_{Wr}, DF_{UR})
S_{18}=($\overline{No}$, NF_{Wr}, DF_{UI}),	S_{19} = ($\overline{No}$, NF_{WR}, DF_{Ur})	S_{20} = ($\overline{No}$, DF_{WI} ,DF_{Ur})
S_{21}=($\overline{No}$,DF_{Wi}, NF_{UR}),	S_{22} = ($\overline{Do}$, DF_{Wi}, DF_{UI})	S_{23} = ($\overline{Do}$, DF_{WI} ,DF_{ur})
S_{24} = (Do, Do, DCs),	S_{25}= (Do, Do, DF_{Ui})	S_{26} = (Do , Do , DF_{Ur})
S_{27}=($\overline{Do}$,DF_{Wi},NF_{UR})	S_{28}=($\overline{Do}$,DF_{Wi},DF_{UR})	(1)

The states S_0, S_1, S_3, S_4, S_6 S_7, S_8, S_{11}, $S_{13,}$ S_{16}, $S_{24,}$ S_{25}, S_{26} are regenerative states while S_2, S_5, S_9, S_{10} $S_{12,}$ $S_{14,}$ S_{15} $S_{17,}$ $S_{18,}$ $S_{19,}$ $S_{20,}$ $S_{21,}$ $S_{22,}$ $S_{23,}$ $S_{27,}$ S_{28} are non-regenerative states.

Thus E={ S_0, S_1, S_3, S_4, $S_{6,}$ S_7, S_8, S_{11}, $S_{13,}$ S_{16}, $S_{24,}$ S_{25}, S_{26} }

The possible transition between states along with transition rates for the model is shown in figure 1.

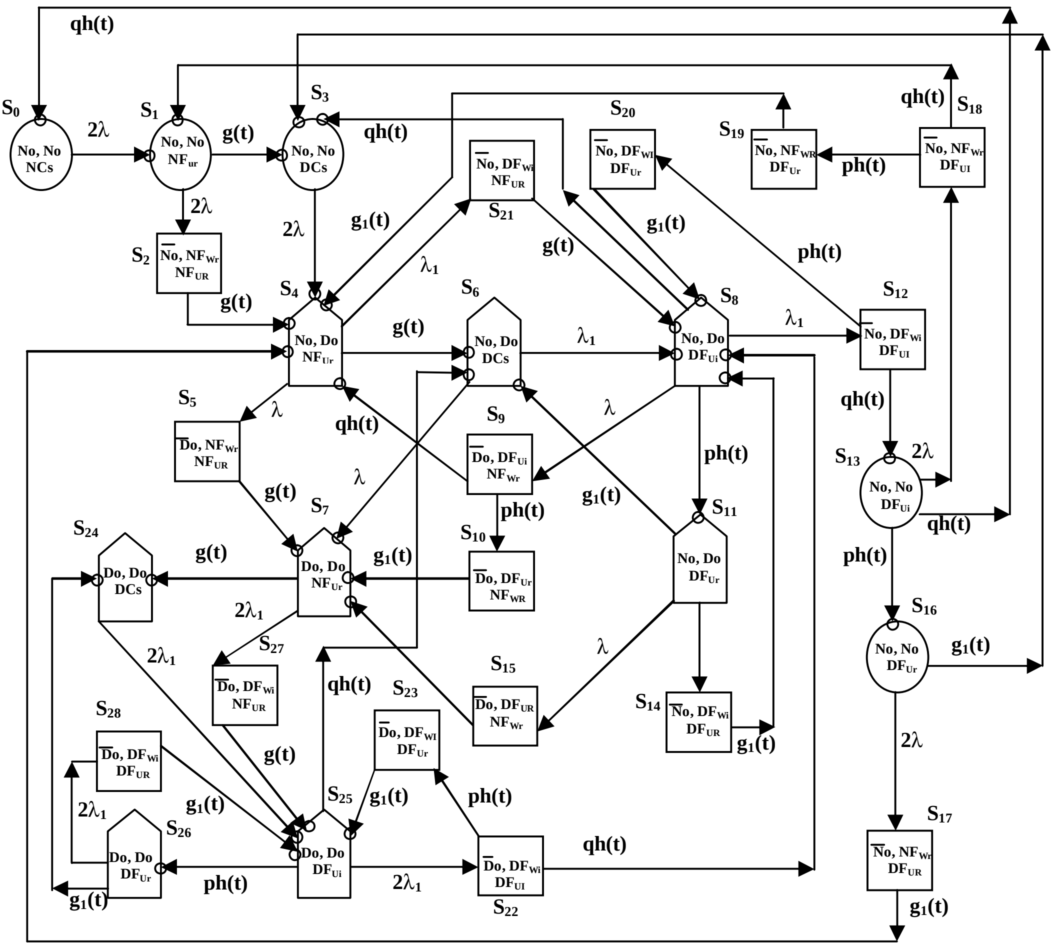
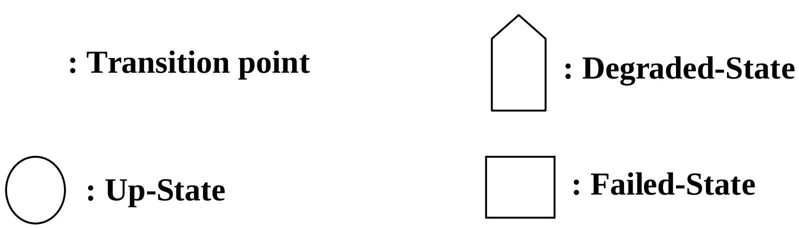


Fig. 1: State Transition Diagram



3. Transition Probabilities and Mean Sojourn Times

Simple probabilistic consideration yield the following expressions for the non-zero elements $p_{ij} = Q_{ij}(\infty) = \int q_{ij}(t)\,dt$ as

$p_{01} = p_{34} = p_{24,25}$, $\quad p_{12} = 1-g^*(2\lambda) = p_{14.2}$, $\quad p_{13} = g^*(2\lambda)$, $\quad p_{46} = g^*(\lambda+\lambda_1)$,

$p_{4,21} = \frac{\lambda_1}{\lambda+\lambda_1}[1- g^*(\lambda+\lambda_1)] = p_{48.21}$, $p_{47.5} = \frac{\lambda}{\lambda+\lambda_1}[1- g^*(\lambda+\lambda_1)] = p_{45}$,

$p_{68} = \frac{\lambda_1}{\lambda+\lambda_1}$, $p_{67} = \frac{\lambda}{\lambda+\lambda_1}$, $p_{7,27} = 1-g^*(2\lambda_1) = p_{7,25.27}$,

$p_{7,24} = g^*(2\lambda_1)$, $p_{83} = q\, h^*(\lambda+\lambda_1)$, $p_{8,11} = p\, h^*(\lambda+\lambda_1)$,

$p_{89} = \frac{\lambda}{\lambda+\lambda_1}[1- h^*(\lambda+\lambda_1)]$ $p_{8,12} = \frac{\lambda_1}{\lambda+\lambda_1}[1- h^*(\lambda+\lambda_1)]$,

$p_{84.9} = \frac{q\lambda}{\lambda+\lambda_1}[1- h^*(\lambda+\lambda_1)]$, $p_{8,13.12} = \frac{q\lambda_1}{\lambda+\lambda_1}[1- h^*(\lambda+\lambda_1)]$,

$p_{8,8.12,20} = \frac{p\lambda_1}{\lambda+\lambda_1}[1- h^*(\lambda+\lambda_1)]$, $p_{87.9,10} = \frac{p\lambda}{\lambda+\lambda_1}[1- h^*(\lambda+\lambda_1)]$,

$p_{13,16} = p\, h^*(2\lambda)$, $p_{13,0} = q\, h^*(2\lambda)$, $p_{13,18} = 1- h^*(2\lambda)$

$p_{13,1.18} = q[1- h^*(2\lambda)]$, $p_{13,4.18,19} = p[1- h^*(2\lambda)]$ $p_{16,3} = g_1^*(2\lambda)$,

$p_{16,17} = 1- g_1^*(2\lambda) = p_{16,4.17}$, $p_{11,6} = g_1^*(\lambda+\lambda_1)$,

$p_{11,14} = \frac{\lambda_1}{\lambda+\lambda_1}[1- g_1^*(\lambda+\lambda_1)] = p_{11,8.14}$, $p_{25,26} = ph^*(2\lambda_1)$,

$p_{11,15} = \frac{\lambda}{\lambda+\lambda_1}[1- g_1^*(\lambda+\lambda_1)] = p_{11,7.15}$, $p_{25,6} = qh^*(2\lambda_1)$,

$p_{25,22} = 1-h^*(2\lambda_1)$, $p_{25,8.22} = q[1-h^*(2\lambda_1)]$, $p_{25,25.22,23} = p[1-h^*(2\lambda_1)]$,

$p_{26,24} = g_1^*(2\lambda_1)$, $p_{26,28} = 1- g_1^*(2\lambda_1) = p_{26,25.28}$ (2)

For these transition probabilities, it can be verified that

$$p_{01} = p_{34} = p_{24,25} = p_{12} + p_{13} = p_{14.2} + p_{13} = p_{45} + p_{46} + p_{4,21} = p_{47} + p_{48.21} + p_{47.5} = p_{67}$$
$$+ p_{68} = p_{7,24} + p_{7,27} = p_{7,24} + p_{7,25.27} = p_{83} + p_{8,11} + p_{8,12} + p_{89} = p_{83} + p_{8,11} + p_{84.9} + p_{87.9,10}$$
$$+ p_{8.8.12,20} + p_{8,13.12} = p_{11,6} + p_{11,15} + p_{11,14} = p_{11,6} + p_{11,7.15} + p_{11,8.14} = p_{13,16} + p_{13,0} + p_{13,18}$$
$$= p_{13,16} + p_{13,0} + p_{13,1.18} + p_{13,4.18,19} = p_{16,3} + p_{16,17} = p_{16,3} + p_{16,4.17} = p_{25,26} + p_{25,6} + p_{25,22}$$
$$= p_{25,26} + p_{25,6} + p_{25,25.22,23} + p_{25,8.22} = p_{26,24} + p_{26,28} = p_{26,24} + p_{26,25.28} = 1 \quad (3)$$

The unconditional mean time taken by the system to transit from any regenerative state S_i when time is counted from epoch at entrance into state S_j is stated as:

$m_{ij} = \int_0^\infty t\,dQij\,(t) = -q_{ij}^{*\prime}(0)$ and the mean sojourn times μ_i in states S_i are given by

$$\mu_i = \int_0^\infty P(T > t)\,dt \quad (4)$$

where T denotes the time to system failure

We have

$\mu_0 = \frac{1}{2\lambda} = \mu_3$, $\mu_1 = \frac{1}{2\lambda}[1-g^*(2\lambda)]$, $\mu_4 = \frac{1}{\lambda+\lambda_1}[1- g^*(\lambda+\lambda_1)]$,

$\mu_6 = \frac{1}{\lambda+\lambda_1}$ $\mu_7 = \frac{1}{2\lambda_1}[1-g^*(2\lambda_1)]$, $\mu_8 = \frac{1}{\lambda+\lambda_1}[1- h^*(\lambda+\lambda_1)]$,

$\mu_{11} = \frac{1}{\lambda+\lambda_1}[1- g_1^*(\lambda+\lambda_1)]$, $\mu_{13} = \frac{1}{2\lambda}[1-h^*(2\lambda)]$, $\mu_{16} = \frac{1}{2\lambda}[1-g_1^*(2\lambda)]$,

$\mu_{24} = \frac{1}{2\lambda_1}$ $\mu_{25} = \frac{1}{2\lambda_1}[1-h^*(2\lambda_1)]$, $\mu_{26} = \frac{1}{2\lambda_1}[1- g_1^*(2\lambda_1)$ (5)

and

$m_{01} = \mu_0$ $m_{12} + m_{13} = \mu_1$ $m_{13} + m_{14.2} = \mu^1{}_1$ (say)

$m_{34} = \mu_3$ $m_{45} + m_{46} + m_{4,21} = \mu_4$ $m_{46} + m_{48.21} + m_{47.5} = \mu^1{}_4$ (say)

$m_{68} + m_{67} = \mu_6$ $m_{7,24} + m_{7,27} = \mu_7$ $m_{7,24} + m_{7,25.27} = \mu^1{}_7$

$m_{83} + m_{8,11} + m_{8,9} + m_{8,12} = \mu_8$,

$m_{83} + m_{88.12,20} + m_{84.9} + m_{8,11} + m_{87.9,10} + m_{8,13.12} = \mu^1{}_8$ (say)

$m_{11,6} + m_{11,14} + m_{11,15} = \mu_{11}$ $m_{11,6} + m_{11,8.14} + m_{11,7.15} = \mu^1{}_{11}$(say)

$m_{13,16} + m_{13,0} + m_{13,18} = \mu_{13,}$ $m_{13,16} + m_{13,0} + m_{13,1.18} + m_{13,4..18,19} = \mu^1{}_{13}$(say)

$m_{16,17} + m_{16,3} = \mu_{16}$, $m_{16,4.17} + m_{16,3} = \mu^1{}_{16}$(say),

$m_{24,25} = \mu_{24,}$ $m_{25,26} + m_{25,6} + m_{25,22} = \mu_{25}$

$m_{25,26} + m_{25,6} + m_{25,8.22} + m_{25,25.22,23} = \mu^1{}_{25}$(say) (6)

4. Reliability and Mean Time to System Failure (MTSF)

Let $\phi_i(t)$ be the cdf of the first passage time from regenerative state *i* to a failed state. Regarding the failed state as absorbing state. we have the following recursive relations for $\phi_i(t)$:

$\phi_0(t) = Q_{01}(t) \circledS \phi_1(t)$, $\phi_1(t) = Q_{13}(t) \circledS \phi_3(t)+Q_{12}(t)$

$\phi_3(t) = Q_{34}(t) \circledS \phi_4(t)$, $\phi_4(t) = Q_{46}(t) \circledS \phi_6(t)+ (Q_{45}(t)+Q_{4,21}(t))$

$\phi_6(t) = Q_{68}(t) \circledS \phi_8(t) + Q_{67}(t) \circledS \phi_7(t)$, $\phi_7(t) = Q_{7,24}(t) \circledS \phi_{24}(t) + Q_{7,27}(t)$

$\phi_8(t) = Q_{83}(t) \circledS \phi_3(t) + Q_{8,11}(t) \circledS \phi_{11}(t) + (Q_{8,9}(t)+ Q_{8,12}(t))$

$\phi_{11}(t) = Q_{11,6}(t) \circledS \phi_6(t) + (Q_{11,14}(t)+ Q_{11,15}(t))$

$\phi_{24}(t) = Q_{24,25}(t) \circledS \phi_{25}(t)$,

$\phi_{25}(t) = Q_{25,26}(t) \circledS \phi_{26}(t) + Q_{25,6}(t) \circledS \phi_6(t)+ Q_{25,22}(t)$

$\phi_{26}(t) = Q_{26,24}(t) \circledS \phi_{24}(t) + Q_{26,28}(t)$ (7)

Taking LST of relations (7), solving for $\tilde{\phi}_0(s)$ and using this, we have

$$R^*(s) = \left(1-\tilde{\phi}_0(s)\right)/s \quad (8)$$

The reliability R(t) can be obtained by taking Laplace inverse transform of (8).

The mean time to system failure (MTSF)n is given by

$$\mathrm{MTSF}(T_1)=\lim_{s\to 0}(1-\phi_0(s))/s=\frac{N_{11}}{D_{11}} \tag{9}$$

where

$N_{11} = [(1- p_{25,26}\, p_{26,24})(1- p_{46}\, p_{68}\, p_{83}- p_{68}\, p_{8,11}\, p_{11,6})- p_{67}\, p_{7,24}\, p_{25,6}][\mu_0 + \mu_1 + p_{13}(\mu_3+ \mu_4)]$
$\quad + p_{13}\, p_{46}(1- p_{25,26}\, p_{26,24})\, (\mu_6 + p_{67}\mu_7+ p_{68}\mu_8+ p_{68}\, p_{8,11})$

And $D_{11} = (1- p_{25,26}\, p_{26,24})(1- p_{46}\, p_{68}\, p_{83}- p_{68}\, p_{8,11}\, p_{11,6})- p_{67}\, p_{7,24}\, p_{25,6}$

5. Availability Analysis

Let $A_i(t)$ be the probability that the system is in up state at instant *t* given that the system entered regenerative state *i* at *t=0*. The recursive relations for $A_i(t)$ are given by :

$A_0(t) = M_0(t) + q_{01}(t)©A_1(t)$,

$A_1(t) = M_1(t) + q_{13}(t)©A_3(t) + q_{14.2}(t)©A_4(t)$

$A_3(t) = M_3(t) + q_{34}(t)©A_4(t)$

$A_4(t) = M_4(t) + q_{46}(t)©A_6(t) + q_{47.5}(t)©A_7(t)+ q_{48.21}(t)©A_8(t)$

$A_6(t) = M_6(t) + q_{68}(t)©A_8(t) + q_{67}(t)©A_7(t)$

$A_7(t) = M_7(t) + q_{7,24}(t)©A_{24}(t) + q_{7,25.27}(t)©A_{25}(t)$

$A_8(t) = M_8(t) + q_{83}(t)©A_3(t) + q_{88.12,20}(t)©A_8(t) + q_{8,4.9}(t)©A_4(t) +q_{87.9,10}(t)©A_7(t)$
$\quad + q_{8,11}(t)©A_{11}(t) + q_{8,13.12}(t)©A_{13}(t)$

$A_{11}(t) = M_{11}(t) + q_{11,6}(t)©A_6(t) + q_{11,8.14}(t)©A_8(t) + q_{11,7.15}(t)©A_7(t)$

$A_{13}(t) = M_{13}(t) + q_{13,0}(t)©A_0(t) + q_{13,16}(t)©A_{16}(t) + q_{13,1.18}(t)©A_1(t)$
$\quad +q_{13,4.18.19}(t)©A_4(t)$

$A_{16}(t) = M_{16}(t) + q_{16,3}(t)©A_3(t) + q_{16,4.17}(t)©A_{14}(t)$

$A_{24}(t) = M_{24}(t) + q_{24,25}(t)©A_{25}(t)$

$A_{25}(t)=M_{25}(t)+q_{25,26}(t)©A_{26}(t)+q_{25,25.22,23}(t)©A_{25}(t)+ q_{25,8.22}(t)©A_8(t)$
$\quad + q_{25,6}(t)©A_6(t)$

$A_{26}(t) = M_{26}(t) + q_{26,24}(t)©A_{24}(t) + q_{26,25.28}(t)©A_{25}(t)$ (10)

where

$M_0(t)=e^{-2\lambda t} = M_3(t)$ $\quad M_1(t)=\bar{G}(t)\ e^{-2\lambda t}$ $\quad M_4(t) = e^{-(\lambda+\lambda_1)t}\,\bar{G}(t)$

$M_6(t) = e^{-(\lambda+\lambda_1)t}$ $\quad M_7(t) = e^{-2\lambda_1 t}\,\bar{G}(t)$ $\quad M_8(t) = e^{-(\lambda+\lambda_1)t}\,\bar{H}(t)$

$M_{11}(t) = e^{-(\lambda+\lambda_1)t}\,\bar{G}_1(t)$ $\quad M_{13}(t) = e^{-2\lambda t}\,\bar{H}(t)$ $\quad M_{16}(t) = e^{-2\lambda t}\,\bar{G}_1(t)$

$M_{24}(t) = e^{-2\lambda_1 t}$ $\quad M_{25}(t) = e^{-2\lambda_1 t}\,\bar{H}(t)$ $\quad M_{26}(t) = e^{-2\lambda_1 t}\,\bar{G}_1(t)$ (11)

Now taking L.T of relations (10) and solving for $A_0^*(s)$.

The steady-state availability of the system is given by

$$A_0(\infty) = \lim_{s\to 0} s \ A_0^*(s) = \frac{N_{12}}{D_{12}} \qquad (12)$$

where

$N_{12} = (p_{25,6}\ p_{68}+p_{25,8,22})[(\mu_0 p_{13,0}+(p_{13,0}+p_{13,1.18})(\mu_1+p_{13}\mu_3))p_{8,13.12}+\mu_4(p_{83}+ p_{84.9}+p_{8,13.12})$
$+\mu_8+ p_{8,11}\mu_{11}+ p_{8,13.12}(\mu_{13}+ p_{13,16}\mu_{16})]+\mu_6 [p_{25,6}(1- p_{88.12,20}- p_{8,11}\ p_{11,6}- p_{8,11}\ p_{11,8.14})$
$+ p_{8,11}\ p_{11,6}\ p_{25,8.22} + (p_{83} + p_{84.9}+ p_{8,13.12})(p_{46}\ p_{25,8.22} - p_{25,6}\ p_{48.21})]$
$+ (1- p_{88.12,20} - p_{8,11}\ p_{11,6}- p_{8,11}\ p_{11,8.14}- p_{8,11}\ p_{11,6}\ p_{68}-(p_{83} + p_{84.9}$
$+ p_{8,13.12})(p_{46}\ p_{68}+ p_{48.21}))[(p_{25,8.22}\ p_{8,13.12}+\mu^1{}_4(p_{83}+p_{84.9}+ p_{8,13.12})$
$+ p_{26,24}) + p_{25,26}\mu_{26}+ \mu_{25}]$

$D_{12} = (p_{25,6}\ p_{68} + p_{25,8,22})[(\mu_0\ p_{13,0} + (p_{13,0} + p_{13,1.18})(\mu^1{}_1 + p_{13}\mu_3))\ p_{8,13.12}$
$+\mu^1{}_4(p_{83}+p_{84.9}+ p_{8,13.12}) + \mu^1{}_8 + p_{8,11}\mu^1{}_{11} + p_{8,13.12}(\mu^1{}_{13}+ p_{13,16}\mu^1{}_{16})]$
$+ \mu_6[p_{25,6}\ (1- p_{88.12,20} - p_{8,11}\ p_{11,6}- p_{8,11}\ p_{11,8.14}) + p_{8,11}\ p_{11,6}\ p_{25,8.22}$
$+ (p_{83} + p_{84.9}+ p_{8,13.12})(p_{46}\ p_{25,8.22} - p_{25,6}\ p_{48.21})] + (1-p_{88.12,20} - p_{8,11}\ p_{11,6}$
$- p_{8,11}\ p_{11,8.14}- p_{8,11}\ p_{11,6}\ p_{68}-(p_{83} + p_{84.9}+ p_{8,13.12})(p_{46}\ p_{68} + p_{48.21}))$
$[(p_{25,8.22} + p_{25,6,}) \mu^1{}_7 + \mu_{24}(p_{25,8.22} + p_{25,6}+ p_{25,26} + p_{26,24}) + p_{25,26}\mu^1{}_{26} + \mu^1{}_{25}]$

6. Busy Period Analysis for Server

Let $B_i(t)$ be the probability that the server is busy at an instant t given that the system entered regenerative state i at $t = 0$. The following are the recursive relations for $B_i(t)$:

$B_0(t) = q_{01}(t)©B_1(t)$, $\qquad B_1(t) = W_1(t)+ q_{13}(t)©B_3(t) + q_{14.2}(t)©B_4(t)$

$B_3(t) = q_{34}(t)©B_4(t)$

$B_4(t) = W_4(t) + q_{46}(t)©B_6(t) + q_{47.5}(t)©B_7(t) + q_{48.21}(t)©B_8(t)$

$B_6(t) = q_{68}(t)©B_8(t) + q_{67}(t)©B_7(t)$

$B_7(t) = W_7(t) + q_{7,24}(t)©B_{24}(t) + q_{7,25.27}(t)© B_{25}(t)$

$B_8(t) = W_8(t) + q_{83}(t)©B_3(t) + q_{88.12,20}(t)©B_8(t) + q_{8,4.9}(t)©B_4(t) + q_{87.9,10}(t)©B_7(t)$
$+ q_{8,11}(t)©B_{11}(t) + q_{8,13.12}(t)©B_{13}(t)$

$B_{11}(t) = W_{11}(t) + q_{11,6}(t)©B_6(t) + q_{11,8.14}(t)©B_8(t) + q_{11,7.15}(t)©B_7(t)$

$B_{13}(t) = W_{13}(t) + q_{13,0}(t)©B_0(t) + q_{13,16}(t)©B_{16}(t) + q_{13,1.18}(t)©B_1(t)$
$+ q_{13,4.18.19}(t)©B_4(t)$

$B_{16}(t) = W_{16}(t) + q_{16,3}(t)©B_3(t) + q_{16,4.17}(t)©B_{14}(t)$, $\qquad B_{24}(t) = q_{24,25}(t)©B_{25}(t)$

$B_{25}(t)=W_{25}(t)+q_{25,26}(t)©B_{26}(t)+q_{25,25.22,23}(t)©B_{25}(t) +q_{25,8.22}(t)©B_8(t)$
$+q_{25,6}(t)©B_6(t)$

$B_{26}(t) = W_{26}(t) + q_{26,24}(t)©B_{24}(t) + q_{26,25.28}(t)©B_{25}(t) \qquad (13)$

where

$W_1(t) = [e^{-2\lambda t}+(2\lambda e^{-2\lambda t}©1)]\bar{G}(t)$,

$$W_4(t) = e^{-(\lambda+\lambda_1)t}\bar{G}(t) + [(\lambda_1 e^{-(\lambda+\lambda_1)t} © 1)]\bar{G}(t) + (\lambda e^{-(\lambda+\lambda_1)t} © 1)\bar{G}(t)$$

$$W_7(t) = [e^{-2\lambda_1 t} + (2\lambda_1 e^{-2\lambda_1 t} © 1)]\bar{G}(t),$$

$$W_8(t) = e^{-(\lambda+\lambda_1)t}\bar{H}(t) + [(\lambda_1 e^{-(\lambda+\lambda_1)t} © 1)]\bar{H}(t) + (\lambda_1 e^{-(\lambda+\lambda_1)t}\bar{H}(t) © ph(t) © 1)]\bar{G}_1(t)$$
$$+ (\lambda e^{-(\lambda+\lambda_1)t} © 1)\bar{H}(t) + (\lambda e^{-(\lambda+\lambda_1)t}\bar{H}(t) © ph(t) © 1)]\bar{G}_1(t)$$

$$W_{11}(t) = e^{-(\lambda+\lambda_1)t}\bar{G}_1(t) + [(\lambda_1 e^{-(\lambda+\lambda_1)t} © 1)]\bar{G}_1(t) + (\lambda e^{-(\lambda+\lambda_1)t} © 1)\bar{G}_1(t)$$

$$W_{13}(t) = e^{-2\lambda t}\bar{H}(t) + [(2\lambda e^{-2\lambda t} © 1)]\bar{H}(t) + (2\lambda e^{-2\lambda t}\bar{H}(t) © ph(t) © 1)]\bar{G}_1(t)$$

$$W_{16}(t) = e^{-2\lambda t}\bar{G}_1(t) + [(2\lambda e^{-2\lambda t} © 1)]\bar{G}_1(t)$$

$$W_{25}(t) = e^{-2\lambda_1 t}\bar{H}(t) + [(2\lambda_1 e^{-2\lambda_1 t} © 1)]\bar{H}(t) + (2\lambda_1 e^{-2\lambda_1 t} © ph(t) © 1)]\bar{G}_1(t)$$

$$W_{26}(t) = e^{-2\lambda_1 t}\bar{G}_1(t) + [(2\lambda_1 e^{-2\lambda_1 t} © 1)]\bar{G}_1(t) \qquad (14)$$

Taking L.T. of relations (13) and solving for $B_0^*(s)$ and using this, we can obtain the fraction of time for which the repairman is busy in steady state

$$B_0 = \lim_{s\to 0} s\ B_0^*(s) = \frac{N_{13}}{D_{12}} \qquad (15)$$

$N_{13} = (p_{25,6}\, p_{68} + p_{25,8,22})[W_1^*(0)\,(p_{13,0} + p_{13,1.18})\, p_{8,13.12} + W_4^*(0)\{(p_{83} + p_{84.9} + p_{8,13.12})$
$+ W_8^*(0) + p_{8,11}\mu^1_{11} + p_{8,13.12}(W_{13}^*(0) + p_{13,16} W_{16}^*(0))] + (1- p_{88.12,20} - p_{8,11}\, p_{11,6}$
$- p_{8,11}\, p_{11,8.14} - p_{8,11}\, p_{11,6}\, p_{68} - (p_{83} + p_{84.9} + p_{8,13.12})(p_{46}\, p_{68} + p_{48.21}))$
$[(p_{25,8.22} + p_{25,6,})\, W_7^*(0) + p_{25,26}\, W_{26}^*(0) + W_{25}^*(0)]$

and D_{12} is already mentioned.

7. Expected Number of Visits

Let $N_i(t)$ be the expected number of visits by the server in (0,t] given that the system entered the regenerative state *i* at *t=0*. We have the following recursive relations for $N_i(t)$:

$$N_0(t) = Q_{01}(t) \circledS [1 + N_1(t)], \qquad N_1(t) = Q_{13}(t) \circledS N_3(t) + Q_{14.2}(t) \circledS N_4(t)$$

$$N_3(t) = Q_{34}(t) \circledS [1 + N_4(t)],$$

$$N_4(t) = Q_{46}(t) \circledS N_6(t) + Q_{47.5}(t) \circledS N_7(t) + Q_{48.21}(t) \circledS N_8(t)$$

$$N_6(t) = Q_{67}(t) \circledS [1 + N_7(t)] + Q_{68}(t) \circledS [1 + N_8(t)]$$

$$N_7(t) = Q_{7,24}(t) \circledS N_{24}(t) + Q_{7,25.27}(t) \circledS N_{25}(t)$$

$$N_8(t) = Q_{83}(t) \circledS N_3(t) + Q_{88.12,20}(t) \circledS N_8(t) + Q_{8,4.9}(t) \circledS N_4(t) + Q_{87.9,10}(t) \circledS N_7(t)$$
$$+ Q_{8,11}(t) \circledS N_{11}(t) + Q_{8,13.12}(t) \circledS N_{13}(t)$$

$$N_{11}(t) = Q_{11,6}(t) \circledS N_6(t) + Q_{11,8.14}(t) \circledS N_8(t) + Q_{11,7.15}(t) \circledS N_7(t)$$

$$N_{13}(t) = Q_{13,0}(t) \circledS N_0(t) + Q_{13,16}(t) \circledS N_{16}(t) + Q_{13,1.18}(t) \circledS N_1(t) + Q_{13,4.18.19}(t) \circledS N_4(t)$$

$$N_{16}(t) = Q_{16,3}(t) \circledS N_3(t) + Q_{16,4.17}(t) \circledS N_{14}(t)$$

$$N_{24}(t) = Q_{24,25}(t) \circledS [1 + N_{25}(t)]$$

$N_{25}(t) = Q_{25,26}(t) Ⓢ N_{26}(t) + Q_{25,25.22,23}(t) Ⓢ N_{25}(t) + Q_{25,8.22}(t) Ⓢ N_8(t)$
$+ Q_{25,6}(t) Ⓢ N_6(t)$

$N_{26}(t) = Q_{26,24}(t) Ⓢ N_{24}(t) + Q_{26,25.28}(t) Ⓢ N_{25}(t)$ (16)

Taking LST of relations (16) and solving for $\tilde{N}_0(s)$.

The expected number of visits per unit time can be obtained as

$$N_0 = \underset{s\to 0}{\mathrm{Lt}}\ s\ \tilde{N}_0(s) = \frac{N_{14}}{D_{12}}, \quad (17)$$

where

$N_{14} = (p_{25,6}\, p_{68} + p_{25,8,22})[(p_{01}\, p_{13,0} + p_{34}\, p_{13}(p_{13,0} + p_{13,1.18}))\, p_{8,13.12}]$
$+ [p_{25,6}(1- p_{88.12,20} - p_{8,11}\, p_{11,6} - p_{8,11}\, p_{11,8.14}) + p_{8,11}\, p_{11,6}\, p_{25,8.22} + (p_{83} + p_{84.9}$
$+ p_{8,13.12})(p_{46}\, p_{25,8,22} - p_{25,6}\, p_{4,8.21})] + p_{24,25}(1- p_{88.12,20} - p_{8,11}\, p_{11,6} - p_{8,11}\, p_{11,8.14}$
$- p_{8,11}\, p_{11,6}\, p_{68} -(p_{83} + p_{84.9} + p_{8,13.12})(p_{46}\, p_{68} + p_{48.21}))$

and D_{12} is already specified.

8. Cost- Benefit Analysis

Profit incurred to the system model in steady state is given by

$$P_1 = K_1A_0 - K_2B_0 - K_3 N_0$$

where

K_1 = Revenue per unit up time of the system
K_2 = Cost per unit time for which server is busy
K_3 = Cost per visit by the server

and A_0, B_0, N_0 are already mentioned

9. Particular Case

Let us take $g(t) = \theta e^{-\theta t}$, $g_1(t) = \alpha e^{-\alpha t}$ and $h(t) = \theta_1 e^{-\theta_1 t}$

By using the non-zero elements p_{ij}, we get the following results:

MTSF $(T_1) = N_{11}/D_{11}$, Availability $(A_0) = N_{12}/D_{12}$

Busy Period $(B_0) = N_{13}/D_{12}$, Expected no. of visits $(N_0) = N_{14}/D_{12}$

where

$D_{11} = [[(\theta_1+2\lambda_1)(\alpha+2\lambda_1)- p\theta_1\alpha][(\lambda+\lambda_1+\theta_1)(\lambda+\lambda_1+\alpha)(\lambda+\lambda_1+\theta)(\lambda+\lambda_1)-p\theta_1\alpha(\lambda+\lambda_1+\theta)$
$-q\theta\lambda_1\theta_1\lambda+\lambda_1+\alpha)](\theta+2\lambda_1)-q\theta\lambda\theta_1(\alpha+2\lambda_1)(\lambda+\lambda_1+\alpha)(\lambda+\lambda_1+\theta_1)(\lambda+\lambda_1+\theta)]$
$/[(\theta_1+2\lambda_1)(\alpha+2\lambda_1)\ (\lambda+\lambda_1+\theta_1)(\lambda+\lambda_1+\alpha)(\lambda+\lambda_1+\theta)(\lambda+\lambda_1)(\theta+2\lambda_1)]$

$N_{11} = [D_{11}][(\theta+2\lambda)(\lambda+\lambda_1+\theta)+\lambda\theta](\theta_1+2\lambda_1)(\alpha+2\lambda_1)(\lambda+\lambda_1+\theta_1)(\lambda+\lambda_1)(\theta+2\lambda_1)$
$+\theta^2[(\theta_1+2\lambda_1)(\alpha+2\lambda_1)-p\alpha\theta_1][(\theta+2\lambda_1)(\lambda+\lambda_1+\theta_1)+\lambda(\lambda+\lambda_1+\theta_1)+\lambda_1(\theta+2\lambda_1)$
$(1+p\theta_1)]/[\lambda(\theta+2\lambda)(\lambda+\lambda_1+\theta)(\theta_1+2\lambda_1)(\alpha+2\lambda_1)(\lambda+\lambda_1)(\theta+2\lambda_1)(\lambda+\lambda_1+\theta_1)]$

$D_{12}=[q.C[q\lambda_1[\theta_1\theta(\theta+2\lambda)+2\lambda(\theta+2\lambda)(\theta_1+2\lambda)+\theta^2(\theta_1+2\lambda)]+2\lambda(\theta+2\lambda)(\theta_1+2\lambda)(\lambda+\lambda_1+\theta_1)]$

$$/[\theta.2\lambda.(\theta+2\lambda)(\theta_1+2\lambda)(\lambda+\lambda_1+\theta_1)]]+[(q/(\lambda+\lambda_1))[D.\theta_1(\lambda+\lambda_1+\theta_1)(\lambda+\lambda_1+\alpha)(\lambda+\lambda_1+\theta)$$

$$+p.\alpha\theta_{1.}2\lambda_1(\lambda+\lambda_1+\theta)+q\theta 2\lambda_1(\lambda+\lambda_1+\theta_1)(\lambda+\lambda_1+\alpha)-q\theta_{1.}\lambda_1(\lambda+\lambda_1+\theta_1)(\lambda+\lambda_1+\alpha)]$$

$$/[(\theta_1+2\lambda_1)(\lambda+\lambda_1+\theta_1)(\lambda+\lambda_1+\alpha)(\lambda+\lambda_1+\theta)]]+C[(\theta_1+2\lambda)[(\lambda+\lambda_1+\theta_1)\alpha+p\theta_1^2]$$

$$+q.\lambda_1[(2\lambda+\theta_1)\alpha+p\theta_1^2]]/[\alpha\theta_1(\theta_1+2\lambda)[(\lambda+\lambda_1+\theta_1)]+F[2q\alpha\theta_1\lambda_1(\theta_1+2\lambda_1)(\alpha+2\lambda_1)$$

$$+q\alpha\theta_1\theta(\theta_1+2\lambda_1)(\alpha+2\lambda_1)+p\theta_1^2\alpha^2\theta+2\theta_1^2\lambda_1 p\theta(\alpha+2\lambda)+2\alpha\theta\lambda_1(\theta_1+2\lambda_1)(\alpha+2\lambda_1)]$$

$$/[2\alpha\theta\theta_1\lambda_1(\theta_1+2\lambda_1)(\alpha+2\lambda_1)]$$

$$N_{12}=[qC[q\lambda_1[\theta_1(\theta+2\lambda)(\lambda+\lambda_1+\theta_1)+2\lambda(\theta_1+2\lambda)(\lambda+\lambda_1+\theta_1)(\lambda+\lambda_1+\theta)+\theta(\theta_1+2\lambda)(\lambda+\lambda_1+\theta)]$$

$$+2\lambda(\theta+2\lambda)(\theta_1+2\lambda)(\lambda+\lambda_1+\theta_1)]/[2\lambda.(\theta+2\lambda)(\theta_1+2\lambda)(\lambda+\lambda_1+\theta_1)(\lambda+\lambda_1+\theta)]]$$

$$+[(q/(\lambda+\lambda_1))[D.\theta_1(\lambda+\lambda_1+\theta_1)(\lambda+\lambda_1+\alpha)(\lambda+\lambda_1+\theta)+p.\alpha\theta_{1.}2\lambda_1(\lambda+\lambda_1+\theta)+q\theta 2\lambda_1$$

$$(\lambda+\lambda_1+\theta_1)(\lambda+\lambda_1+\alpha)-q\theta_{1.}\lambda_1(\lambda+\lambda_1+\theta_1)(\lambda+\lambda_1+\alpha)]$$

$$/[(\theta_1+2\lambda_1)(\lambda+\lambda_1+\theta_1)(\lambda+\lambda_1+\alpha)(\lambda+\lambda_1+\theta)]]+C[(\theta_1+2\lambda)(\lambda+\lambda_1+\alpha)(\alpha+2\lambda)+p\theta_1(\theta_1+2\lambda)$$

$$(\alpha+2\lambda)+q.\lambda_1[(\lambda+\lambda_1+\alpha)(\alpha+2\lambda)+p\theta_1(\lambda+\lambda_1+\alpha)]]$$

$$/[(\lambda+\lambda_1+\alpha)(\lambda+\lambda_1+\theta_1)(\theta_1+2\lambda)(\alpha+2\lambda)]$$

$$+F[q.2\lambda_1(\theta_1+2\lambda_1)(\alpha+2\lambda_1)+q.(\theta+2\lambda_1)(\alpha+2\lambda_1)(\theta+2\lambda_1)+p\theta_1\alpha(\theta+2\lambda_1)+2\lambda_1(\theta+2\lambda_1)$$

$$(\alpha+2\lambda_1)+p\theta_1 2\lambda_1(\theta+2\lambda_1)]/[2\lambda_1(\theta+2\lambda_1)(\alpha+2\lambda_1)(\theta_1+2\lambda_1)]$$

$$N_{13}=C[q^2\lambda_1\alpha(\theta_1+2\lambda)(\theta_1+2\lambda_1)+q\alpha(\theta_1+2\lambda)(\theta_1+2\lambda_1)(\lambda+\lambda_1+\theta_1)+w_8\alpha\theta(\theta_1+2\lambda)$$

$$(\theta_1+2\lambda_1)(\lambda+\lambda_1+\theta_1)+q\lambda_1\theta(\theta_1+2\lambda_1)(w_{13}\alpha(\theta_1+2\lambda)+p\theta_1)+p\theta_1\theta(\theta_1+2\lambda_1)$$

$$(\theta_1+2\lambda)]+F[(\theta_1+2\lambda)(\lambda+\lambda_1+\theta_1)(q\alpha(\theta_1+2\lambda_1)+p\theta_1\theta+w_{25}\theta\alpha(\theta_1+2\lambda_1))]$$

$$/[\alpha\theta(\theta_1+2\lambda)(\theta_1+2\lambda_1)(\lambda+\lambda_1+\theta_1)]$$

$$N_{14}=Cq^2\lambda_1[\theta_1(\theta+2\lambda)+\theta(\theta_1+2\lambda)]/[(\lambda+\lambda+\theta)(\theta_1+2\lambda)(\theta+2\lambda)]+[D\theta_1(\lambda+\lambda_1+\theta_1)(\lambda+\lambda_1+\alpha)$$

$$(\lambda+\lambda_1+\theta)+p.\alpha\theta_{1.}2\lambda_1(\lambda+\lambda_1+\theta)+q\theta 2\lambda_1(\lambda+\lambda_1+\theta_1)(\lambda+\lambda_1+\alpha)$$

$$-q\theta_{1.}\lambda_1(\lambda+\lambda_1+\theta_1)(\lambda+\lambda_1+\alpha)]+F$$

$$w_8=[\alpha(\alpha+\theta_1))(\lambda+\lambda_1+\alpha+\theta_1)+\theta_1^2(\lambda+\lambda_1)p]/[\alpha\theta_1(\alpha+\theta_1))(\lambda+\lambda_1+\alpha+\theta_1)]$$

$$w_{13}=[\alpha(\alpha+\theta_1)(\theta_1+\alpha+2\lambda)+\theta_1^2 2\lambda p]/[\theta_1(\alpha+\theta_1)(\theta_1+2\lambda+\alpha)\alpha]$$

$$w_{25}=[(\theta_1+\alpha+2\lambda_1)\alpha(\alpha+\theta_1)+2\theta_1^2\lambda_1 p]/[(\theta_1+\alpha+2\lambda_1)\theta_1(\alpha+\theta_1)\alpha]$$

$$C=q\lambda_1[\theta_1+2(\lambda+\lambda_1)]/(\theta_1+2\lambda_1)(\lambda+\lambda_1)$$

$$D=[(\lambda+\lambda_1+\theta_1)(\lambda+\lambda_1+\alpha)-p\lambda_1(\lambda+\lambda_1+\alpha)-p\theta_1(\alpha-\lambda_1)]/[(\lambda+\lambda_1+\theta_1)(\lambda+\lambda_1+\alpha)]$$

$$F=[(\lambda+\lambda_1+\theta_1)(\lambda+\lambda_1+\alpha)(\lambda+\lambda_1+\theta)(\lambda+\lambda_1)-p\lambda_1(\lambda+\lambda_1+\alpha)(\lambda+\lambda_1+\theta)(\lambda+\lambda_1)-p\theta_1\alpha\lambda_1(\lambda+\lambda_1+\theta)$$

$$-p\theta_1(\lambda_1+\alpha)(\lambda+\lambda_1+\theta)(\lambda+\lambda_1)-[\theta\lambda_1(\lambda+\lambda_1+\theta_1)(\lambda+\lambda_1+\alpha)+(\lambda+\lambda_1+\theta_1)(\lambda+\lambda_1+\alpha)\lambda_1(\lambda+\lambda_1)]]$$

$$/[(\lambda+\lambda_1+\theta_1)(\lambda+\lambda_1+\alpha)(\lambda+\lambda_1+\theta)(\lambda+\lambda_1)]$$

10. Graphical Study

The mean time to system failure (MTSF) and availability of the system model decrease more rapidly with the increase of failure rates λ and λ_1 for fixed values of other parameters as shown in figure 2 and 3. However, their values increase as repair rate (α) of the degraded unit increases. The behavior of profit of the system model with respect to failure rate λ is shown in figure 4 this figure indicate that profit of the system goes on decreasing with the increase of failure rates λ and λ_1. But system becomes more profitable when repair rate by increasing the repair rate α and revenue per unit up time increase.

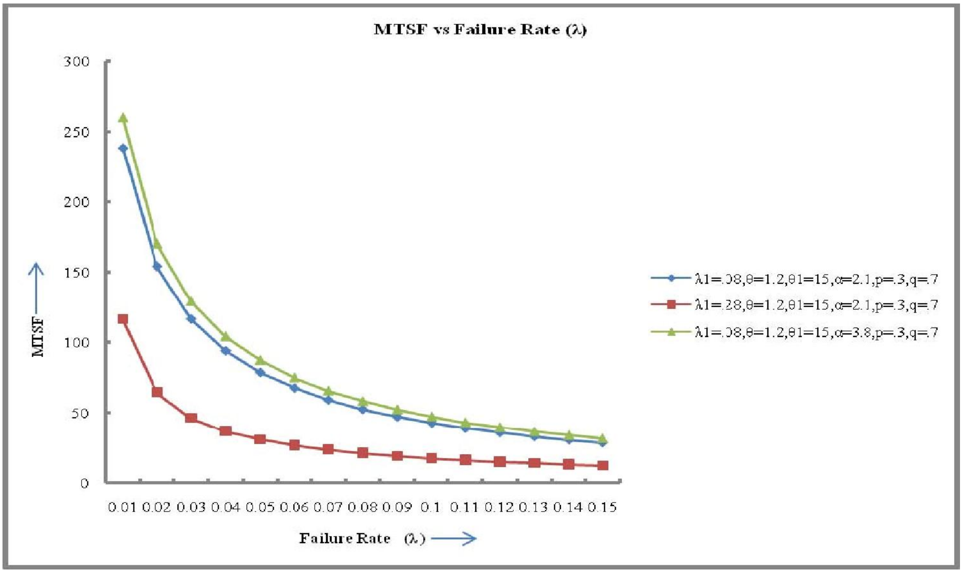


Fig.2

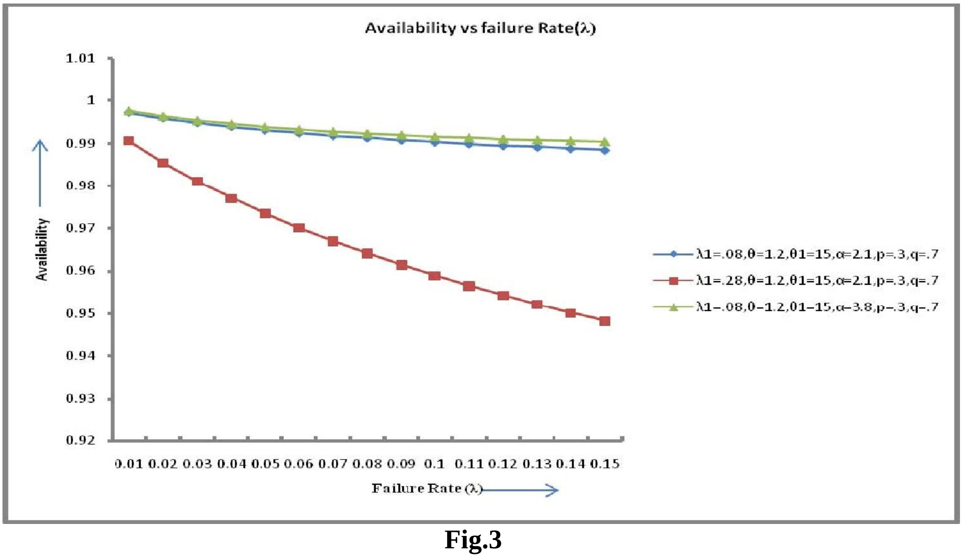


Fig.3

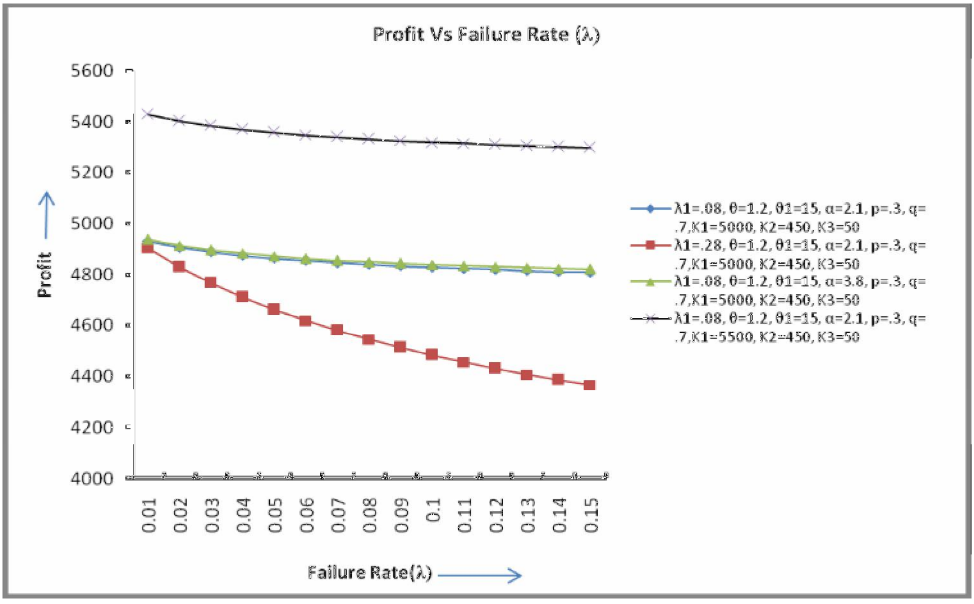


Fig.4

11. Application of the Study

The application of the present study can be visualized in various practical situations. For example, consider a communication amplifier in which redundancy is used as means of increasing the reliability. Redundant systems first appeared in communication systems with the introduction of Klystron amplifier. Generating high power microwave energy places tremendous electrical stress on the amplification device. Microwave Klystron and traveling waves tubes operate at extra ordinary high cathode temperatures. These high operating temperatures result in relatively low mean time between failures and thus a corresponding high failure rate. The high voltage power supplies required to operate traveling wave tubes also have a history of high constant failure rates. In recent years solid state power amplifiers have made significant improvement in mean time between failure but still typically fall short of meeting the reliability expectations of satellite communication links. Satellite transponder time is extremely expensive and operators cannot afford to have a satellite link off the air for any period of time, no matter how short in duration. In many instances satellite equipment is installed in remote locations which are not easily accessed for maintenance. Therefore, it is imperative that any amplifier system used in satellite communication be equipped with some form of automatic backup or redundancy. The goal of any redundant amplifier system is to achieve a system reliability that is greater than the reliability of an individual amplifier. Hence, a stochastic model for 2-out-of-3 modular amplifier system is under taken for study.

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