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ON POWER FUNCTION OF A SOMETIMES POOL TEST PROCEDURE IN A ANOVA MODEL – II: A THEORETICAL / NUMERICAL INVESTIGATION

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Abstract

The present paper deals with a hypothesis testing problem based on conditional specification in a three-way random effect model. A sometimes pool test procedure using two preliminary tests has been proposed for testing the main hypothesis. The power of the proposed test has been proposed for test has been derived. Numerical study of the power and size has been made for certain sets of degrees of freedom. It is found that the power of the proposed test procedure is more than that of the test procedure proposed by Gupta and Singh (1977), for certain set of values of the nuisance parameters. Thus, the proposed method is an improvement over the existing test procedure, incorporating one preliminary test of significance.

Key words: Random-effects model (ANOVA model-Ii), test procedure, power nuisance parameter, preliminary test of significance.

1. Introduction

Suppose that an agricultural equipment(s) producing concern is producing some small parts to be used in the equipments say sprayer etc. The parts are being produced, using a large number of machines of same make and model. The concern may be interested in getting an answer to the questions: 'Is there any difference between the machines?' Since the total number of machines in use is very large, it is not possible to make such a study by taking samples of output of all machines. Therefore, keeping this and other related problems in mind the following experiment is performed.

A random sample of I machines from the lot of machine and J workers from the totality of workers has been selected independently. Each worker is assigned to work on a machine for one day. A random sample of K batches of materials produced by each worker on a machine from the total output is selected. Since for any machine or worker or batch of material, there may be considerable variation, we will treat the output as a continuous random variable. The situation is expressed by a model in equation (2.1).

Many investigations have been made in fixed and random effects model to study the power of the test procedures incorporating one or two preliminary tests by Paull (1950), Bechhofer (1956), Bozivich, Bancroft and Hartely (1956), Srivastava and Bozivich (1961), Gupta and Srivastava (1968), Saxena and Srivastava (1970), Gupta and Singh (1976), Gupta and Agarwal (1981).

Srivastava and Tanna (2001) have studied the problem of estimation of error variance in a three – way layout for ANOVA model –II incorporating two preliminary tests of significance for testing presence / absence of first order interaction(s).

In the present paper we have described the model in section 2, derived the expressions for the power function of the test procedure incorporating two preliminary tests of significance in section 3. Series formulae for power of the tests based on PTS are generally lengthy and tedious. An attempt has been made to derive approximate formulae for power of the test procedure which are much easier and give quite satisfactory results. In section 4, we have derived some mathematical results. Section 5 comprises of numerical computation for power under proposed set – up and a comparison has been made with test procedure incorporating one preliminary test of significance, proposed by Gupta and Singh (1977). Power has been computed numerically for some preliminary level(s) of significance and combinations of degrees of freedom, for certain range of nuisance parameters. A comparison of these values, have been made with the power of the test procedure proposed by Gupta and Singh.

2. The Model under Investigation and Conditional Specification

Let Y_{ijkl} denotes the l^{th} observation in the k^{th} batch of material produced by j^{th} worker if he uses i^{th} machine. The sample observations can well be represented by a complete three – way layout, designating machines as factor *A*, workers as factor *B* and batches as factor *C*. Thus, we can assume that

$$Y_{ijkl} = \mu + a_i^A + a_j^B + a_k^C + a_{ij}^{AB} + a_{jk}^{BC} + a_{ik}^{AC} + a_{ijk}^{abc} + e_{ijkl} \qquad (2.1)$$

i=1, …, I; j=1, …, k; l=1, … , L

The random variables a_i^A are uncorrelated and have $N\left(0,\sigma_A^2\right)$ distribution. Similarly a_j^B have $N\left(0,\sigma_B^2\right)$, … a_{ijk}^{ABC} have $N\left(0,\sigma_{ABC}^2\right)$ distributions. The error $eijkl$ are independently and identically distributed with mean zero and variance σ_e^2. We are interested in testing the main hypothesis $H_A : \sigma_A^2 = 0$ against the alternative $H_A^{'} : \sigma_A^2 > 0$ i.e. we are interested in examining whether there is any significant difference between the machines from which these *I* machines have been drawn at random beyond their variation from l^{th} one batch to another or in their use by different workers. If $\sigma_{AC}^2 \geq 0$ and $\sigma_{AB}^2 \geq 0$, then (2.1) is called an incompletely specified random model.

To test the hypothesis $H_A : \sigma_A^2 = 0$ against $H_A^{'} : \sigma_A^2 > 0$ about the effect A, the abridge ANOVA table is given below.

Source of Variation	***Degrees of freedom***	***Mean Squares***	***Expected Mean Squares***
A	$n_4=(I-1)$	V_4	$\sigma_4^2=\sigma_e^2+L\sigma_{ABC}^2+KL\sigma_{AB}^2$ $+JL\sigma_{AC}^2+JKL\sigma_A^2$
AB	$n_3=(I-1)(J-1)$	V_3	$\sigma_3^2=\sigma_e^2+L\sigma_{ABC}^2+KL\sigma_{AB}^2$
AC	$n_2=(I-1)(K-1)$	V_2	$\sigma_4^2=\sigma_e^2+L\sigma_{ABC}^2+JL\sigma_{AC}^2$
ABC	$n_1=(I-1)(J-1)(K-1)$	V_1	$\sigma_4^2=\sigma_e^2+L\sigma_{ABC}^2$

Table 2.1: ***Analysis of Variance Table for*** $\sigma_{AB}^2>0$, $\sigma_{AC}^2>0$

From the table 2.1 it is evident that no interaction mean square is adequate to be taken as error mean square unless the interaction AC and/or AB are/is zero. So, first it is necessary to test the existence of AC and/or AB by testing two hypotheses, viz., $H_{01}:\sigma_{AC}^2=0$ and $H_{01}^{'}:\sigma_{AC}^2>0$ against $H_{02}:\sigma_{AB}^2=0$ and $H_{02}^{'}:\sigma_{AB}^2>0$. The final test depends upon the outcome of preliminary tests of significance. Such tests are called test based on conditional specification.

Test procedure:

$$A_1:\left\{\frac{V_2}{V_1}<\beta \quad and \quad \frac{V_3}{V_{12}}<\delta \quad and \quad \frac{V_4}{V_{123}}\geq\beta_1\right\}$$

$$A_2:\left\{\frac{V_2}{V_1}\geq\beta \quad and \quad \frac{V_3}{V_1}<\delta_1 \quad and \quad \frac{V_4}{V_2}\geq\beta_2\right\}$$

$$A_3:\left\{\frac{V_2}{V_1}<\beta \quad and \quad \frac{V_3}{V_{12}}\geq\delta \quad and \quad \frac{V_4}{V_3}\geq\beta_3\right\}$$

$$A_4:\left\{\frac{V_2}{V_1}\geq\beta \quad and \quad \frac{V_3}{V_1}\geq\delta_1 \quad and \quad \frac{V_4}{V_A}\geq\beta_4\right\} \tag{2.2}$$

where,

$$\beta=F\left(n_2,n_1,\alpha_1\right),\ \delta=F\left(n_3,n_{12},\alpha_2\right),\ \delta_1=F\left(n_3,n_1,\alpha_3\right)$$

$$\beta_1=F\left(n_4,n_{123},\alpha_4\right),\ \beta_2=F\left(n_4,n_2,\alpha_5\right),\ \beta_3=F\left(n_4,n_3,\alpha_6\right),\ \beta_3=F\left(n_4,n^*,\alpha_7\right)$$

$$V_{123}=\frac{n_1V_1+n_2V_2+n_3V_3}{n_1+n_2+n_3},\ V_{12}=\frac{n_1V_1+n_2V_2}{n_1+n_2},\ V_A=V_3+V_2-V_1$$

$$n_{ijk}=n_i+n_j+n_k \quad and \quad n^*=\frac{\left(\sigma_3^2+\sigma_2^2-\sigma_1^2\right)^2}{\left(\frac{\sigma_3^4}{n_3}+\frac{\sigma_2^4}{n_2}+\frac{\sigma_1^4}{n_1}\right)}$$

It may be noted that σ_i^2 $(i = 1, 2, 3 \text{ and } 4)$ are in general, unknown and in practice n^* is estimated by replacing σ_i^2's by their unbiased estimates V_i's. It is to be noted that n^* may be fractional, in this case F-value may be obtained by using Mathcad (version 7).

The mean squares V_i's ($i=1,2,3,$ *and* 4) are independently distributed as $\dfrac{\sigma_i^2\chi_i^2}{n_i}$, where χ_i^2 is a central chi – square statistic with n_i degrees of freedom.

The power of the test procedure 'P', the probability of rejecting H_A is the sum of the probabilities associated with the four mutually exclusive events which are given above.

Now, $$P = \sum_{i=1}^{4} \Pr.(A_i),$$

Since $A_i \cap A_j = \phi$ for all $i \neq j$

$$P(A)_1 : \ \Pr.\left\{ \frac{V_2}{V_1} < \beta \ \ and \ \ \frac{V_3}{V_{12}} < \delta \ \ and \ \ \frac{V_4}{V_{123}} \geq \beta_1 \right\}$$

$$P(A_2) : \ \Pr.\left\{ \frac{V_2}{V_1} \geq \beta \ \ and \ \ \frac{V_3}{V_1} < \delta_1 \ \ and \ \ \frac{V_4}{V_2} \geq \beta_2 \right\}$$

$$P(A_3) : \ \Pr.\left\{ \frac{V_2}{V_1} < \beta \ \ and \ \ \frac{V_3}{V_{12}} \geq \delta \ \ and \ \ \frac{V_4}{V_3} \geq \beta_3 \right\}$$

$$P(A_4) : \ \Pr.\left\{ \frac{V_2}{V_1} \geq \beta \ \ and \ \ \frac{V_3}{V_1} \geq \delta_1 \ \ and \ \ \frac{V_4}{V_A} \geq \beta_4 \right\} \tag{2.3}$$

3. Approximate Power of the Test Procedure

The sum of squares $\dfrac{n_i V_i}{\sigma_i^2}$ (i=1,2,3 and 4) are independently distributed as central χ_i^2 with n_i degrees of freedom. The joint probability distribution function of V_1, V_2, V_3 and V_4 is given by

$$f(V_1, V_2, V_3, V_4) = K_1 V_1^{\frac{n_1}{2}-1} V_2^{\frac{n_2}{2}-1} V_3^{\frac{n_3}{2}-1} V_4^{\frac{n_4}{2}-1} .$$
$$\exp\left[-\frac{1}{2}\left(\frac{n_1 V_1}{\sigma_1^2} + \frac{n_2 V_2}{\sigma_2^2} + \frac{n_3 V_3}{\sigma_3^2} + \frac{n_4 V_4}{\sigma_4^2} \right) \right] dV_1 dV_2 dV_3 dV_4 \tag{3.1}$$

where, $K_1 = \dfrac{\left(\dfrac{n_1}{\sigma_1^2}\right)^{\frac{n_1}{2}}\left(\dfrac{n_2}{\sigma_2^2}\right)^{\frac{n_2}{2}}\left(\dfrac{n_3}{\sigma_3^2}\right)^{\frac{n_3}{2}}\left(\dfrac{n_4}{\sigma_4^2}\right)^{\frac{n_4}{2}}}{2^{\frac{n_{1234}}{2}}\left|\left(\dfrac{n_1}{2}\right)\right|\left(\dfrac{n_2}{2}\right)\left|\left(\dfrac{n_3}{2}\right)\right|\left(\dfrac{n_4}{2}\right)}$, $n_{1234} = n_1 + n_2 + n_3 + n_4$

Making the following transformations in (3.1)

$$y_4 = \frac{n_1 V_1}{\sigma_1^2},\ y_3 = \frac{n_4 V_4}{n_1 V_1}\theta_{14},\ y_2 = \frac{n_3 V_3}{n_1 V_1}\theta_{13},\ y_1 = \frac{n_2 V_2}{n_1 V_1}\theta_{12} \tag{3.2}$$

where, $\theta_{12} = \dfrac{\sigma_1^2}{\sigma_2^2}$, ,$\theta_{13} = \dfrac{\sigma_1^2}{\sigma_3^2}$, $\theta_{14} = \dfrac{\sigma_1^2}{\sigma_4^2}$ are the nuisance parameters.

The joint probability distribution function of $y_1,\ y_2,\ y_3\ and\ y_4$ is given by

$$f\left(y_1, y_2, y_3, y_4\right) = K y_1^{\frac{n_2}{2}-1} y_2^{\frac{n_3}{2}-1} y_3^{\frac{n_4}{2}-1} y_4^{\frac{n_{1234}}{2}-1}.$$

$$\exp\left[-\frac{y_4}{2}\left(1 + y_1 + y_2 + y_3\right)\right] dy_1 dy_2 dy_3 dy_4 \tag{3.3}$$

where, $K = \dfrac{1}{2^{\frac{n_{1234}}{2}}\left|\left(\dfrac{n_1}{2}\right)\right|\left(\dfrac{n_2}{2}\right)\left|\left(\dfrac{n_3}{2}\right)\right|\left(\dfrac{n_4}{2}\right)}$

Let the probability of the four steps be denoted by $P\left(A_i\right)\ \left(i = 1, 2, 3\ and\ 4\right)$ respectively. The probability of test procedure which in turn represents the power of the same is given by

$$P = P\left(A_1\right) + P\left(A_2\right) + P\left(A_3\right) + P\left(A_4\right) \tag{3.4}$$

Derivation of the approximate power formulae is based on the following assumptions as suggested by Bozivich (1956).

Letting n_2 and $n_3 \to \infty$ in such a way that $\dfrac{n_2}{n_1}$ and $\dfrac{n_3}{n_1}$ are finite. Hence V_i tends to σ_i^2 (i=1,2,3, and 4).

Thus, the probability of A_1 is

$$P\left(A\right)_1 :\ \Pr.\left\{\frac{V_2}{V_1} < \beta\ \ and\ \ \frac{V_3}{V_{12}} < \delta\ \ and\ \ \frac{V_4}{V_{123}} \geq \beta_1\right\} \tag{3.5}$$

Now, on making use of the assumptions given above $\dfrac{V_2}{V_1} \to \dfrac{\sigma_2^2}{\sigma_1^2}$, $\dfrac{V_3}{V_{12}} \to \dfrac{\sigma_3^2}{\sigma_1^2}$ and $\dfrac{V_4}{V_{123}} \to \dfrac{\sigma_4^2}{\sigma_1^2}$. Obviously the three solitary test statistics are independent.

Therefore, $$P(A_1) = P\left(\frac{V_2}{V_1} < \beta\right) P\left(\frac{V_3}{V_{12}} < \delta\right) P\left(\frac{V_4}{V_{123}} \geq \beta_1\right) \tag{3.6}$$

To evaluate the above probabilities we make use of the following standard results:

$$P\left(F_{p,q} < F_0\right) = I_{X_0}\left(X; \frac{p}{2}, \frac{q}{2}\right) \tag{3.7}$$

$$P\left(F_{p,q} \geq F_0\right) = 1 - I_{X_0}\left(X; \frac{p}{2}, \frac{q}{2}\right) \tag{3.8}$$

where,

$X_0 = \dfrac{pF_0}{q + pF_0}$ and $I_{X_0}\left(X; \dfrac{p}{2}, \dfrac{q}{2}\right)$ is the normalised incomplete beta function. The probabilities given in (3.6) can be reduced to the form of (3.7) and (3.8) in the following manner.

$$P(A_1) = P\left(\frac{V_2/\sigma_2^2}{V_1/\sigma_1^2} < \beta\theta_{12}\right) P\left(\frac{V_3/\sigma_3^2}{V_{12}/\sigma_1^2} < \delta\theta_{13}\right) P\left(\frac{V_4/\sigma_4^2}{V_{123}/\sigma_1^2} \geq \beta_1\theta_{14}\right) \tag{3.9}$$

or,

$$P(A_1) = P\left\{F(n_2, n_1) < \beta\theta_{12}\right\} P\left\{F(n_3, n_{12}) < \delta\theta_{13}\right\} P\left\{F(n_4, n_{123}) \geq \beta_1\theta_{14}\right\}$$

$$= I_{X_1}\left(\frac{n_2}{2}, \frac{n_1}{2}\right) I_{X_2}\left(\frac{n_3}{2}, \frac{n_{12}}{2}\right) \left\{1 - I_{X_3}\left(\frac{n_4}{2}, \frac{n_{123}}{2}\right)\right\} \tag{3.10}$$

where,

$$X_1 = \frac{n_2\beta\theta_{12}}{n_1 + n_2\beta\theta_{12}};\quad X_2 = \frac{n_3\delta\theta_{13}}{n_{12} + n_3\delta\theta_{13}};\quad X_3 = \frac{n_4\beta_1\theta_{14}}{n_{123} + n_4\beta_1\theta_{14}} \tag{3.11}$$

Proceeding in the same manner and making similar assumptions as in the case of $P(A_1)$, we obtain the probabilities $P(A_2)$, $P(A_3)$ and $P(A_4)$.

$$P(A_2) = P\left(\frac{V_2}{V_1} \geq \beta\right) P\left(\frac{V_3}{V_1} < \delta_1\right) P\left(\frac{V_4}{V_2} \geq \beta_2\right) \tag{3.12}$$

$$= P\left(\frac{V_2/\sigma_2^2}{V_1/\sigma_1^2} \geq \beta\theta_{12}\right) P\left(\frac{V_3/\sigma_3^2}{V_1/\sigma_1^2} < \delta_1\theta_{13}\right) P\left(\frac{V_4/\sigma_4^2}{V_2/\sigma_2^2} \geq \beta_2\theta_{24}\right)$$

$$P(A_2) = P\left\{F(n_2, n_1) \geq \beta\theta_{12}\right\} P\left\{F(n_3, n_1) < \delta_1\theta_{13}\right\} P\left\{F(n_4, n_2) \geq \beta_2\theta_{24}\right\}$$

making use of relation (3.7) we obtain

$$P(A_2) = \left\{1 - I_{X_1}\left(\frac{n_2}{2}, \frac{n_1}{2}\right)\right\} I_{X_4}\left(\frac{n_3}{2}, \frac{n_1}{2}\right) \left\{1 - I_{X_3}\left(\frac{n_4}{2}, \frac{n_2}{2}\right)\right\} \tag{3.13}$$

In (3.13) X_1 is same as given in (3.11) and

$$X_4 = \frac{n_3\delta_1\theta_{13}}{n_{12} + n_3\delta_1\theta_{13}};\quad X_5 = \frac{n_4\beta_2\theta_{24}}{n_2 + n_4\beta_2\theta_{24}};\qquad \text{where } \theta_{24} = \frac{\sigma_2^2}{\sigma_4^2} \tag{3.14}$$

The probability of A_3 under the similar assumption as discussed in case of $P(A_1)$ is

$$P(A_3) = P\left(\frac{V_2}{V_1} < \beta\right) P\left(\frac{V_3}{V_{12}} \geq \delta\right) P\left(\frac{V_4}{V_3} \geq \beta_3\right) \tag{3.15}$$

$$P(A_3) = P\left(\frac{V_2/\sigma_2^2}{V_1/\sigma_1^2} < \beta\theta_{12}\right) P\left(\frac{V_3/\sigma_3^2}{V_{12}/\sigma_1^2} \geq \delta\theta_{13}\right) P\left(\frac{V_4/\sigma_4^2}{V_3/\sigma_3^2} \geq \beta_3\theta_{34}\right)$$

or,

$$P(A_3) = P\{F(n_2, n_1) < \beta\theta_{12}\} P\{F(n_3, n_{12}) \geq \delta\theta_{13}\} P\{F(n_4, n_3) \geq \beta_3\theta_{34}\} \tag{3.16}$$

Applying the relation (3.7) and (3.8) we get

$$P(A_3) = I_{X_1}\left(\frac{n_2}{2}, \frac{n_1}{2}\right)\left\{1 - I_{X_2}\left(\frac{n_3}{2}, \frac{n_{12}}{2}\right)\right\}\left\{1 - I_{X_6}\left(\frac{n_4}{2}, \frac{n_3}{2}\right)\right\} \tag{3.17}$$

where X_1 and X_2 are given earlier and

$$X_6 = \frac{n_4\beta_3\theta_{34}}{n_3 + n_4\beta_3\theta_{34}} \tag{3.18}$$

where, $\theta_{34} = \dfrac{\sigma_3^2}{\sigma_4^2}$

Similarly the probability of A_4 is

$$P(A_4) = P\left(\frac{V_2}{V_1} \geq \beta\right) P\left(\frac{V_3}{V_1} \geq \delta_1\right) P\left(\frac{V_4}{V_A} \geq \beta_4\right) \tag{3.19}$$

$$= P\left(\frac{V_2/\sigma_2^2}{V_1/\sigma_1^2} \geq \beta\theta_{12}\right) P\left(\frac{V_3/\sigma_3^2}{V_1/\sigma_1^2} \geq \delta_1\theta_{13}\right).$$

$$P\left(\frac{V_4/\sigma_4^2}{V_A/\left(\sigma_3^2 + \sigma_2^2 - \sigma_1^2\right)} \geq \beta_4(\theta_{34} + \theta_{24} - \theta_{14})\right)$$

$$P(A_4) = P\{F(n_2, n_1) \geq \beta\theta_{12}\} P\{F(n_3, n_1) \geq \delta_1\theta_{13}\}.$$
$$P\{F(n_4, n^*) \geq \beta_4(\theta_{34} + \theta_{24} - \theta_{14})\} \tag{3.20}$$

Making use of the relation (3.7) we obtain

$$P(A_4) = \left\{1 - I_{X_1}\left(\frac{n_2}{2}, \frac{n_1}{2}\right)\right\}\left\{1 - I_{X_4}\left(\frac{n_3}{2}, \frac{n_1}{2}\right)\right\}\left\{1 - I_{X_7}\left(\frac{n_4}{2}, \frac{n^*}{2}\right)\right\} \tag{3.21}$$

where X_1 and X_4 are given earlier and $X_7 = \dfrac{n_4\beta_4(\theta_{34} + \theta_{24} - \theta_{14})}{n^* + n_4\beta_4(\theta_{34} + \theta_{24} - \theta_{14})}$

Thus the power of the test procedure is the sum of the probabilities $P(A_1)$, $P(A_2)$, $P(A_3)$ and $P(A_4)$ given by (3.10), (3.13), (3.17) and (3.21) respectively.

Special Checks:

To check the final expression of power we consider the following cases:

I. Taking the limits that β, δ_1 and β_4 tend to 0 in (3.10), (3.13), (3.17) and (3.21) we obtain

$$\begin{aligned} P(A_i) &= 0 \quad ; \quad i = 1,2,3 \\ &= 1 \quad ; \quad i = 4 \end{aligned}$$

Then from (3.4) we obtain *P*=1.

The case when we let β, δ_1 and β_4 tend to 0 i.e. we always reject the hypothesis $H_A : \sigma_A^2 = 0$ and in this case we use: V_A as an estimator of σ^2.

II. Taking the limits that β and δ tend to ∞ and β_1 tends to 0 in (3.10), (3.13), (3.17) and (3.21) we obtain

$$\begin{aligned} P(A_i) &= 1 \quad ; \quad i = 1 \\ &= 0 \quad ; \quad i = 2,3,4 \end{aligned}$$

Then from (3.4) we obtain *P*=1.

The case when we let β and δ tend to ∞ and β_1 tends to 0 i.e. we never reject the hypothesis $H_A : \sigma_A^2 = 0$ and in this case we use: V_{123} as an estimator of σ^2.

4. Mathematical Results

Result 1: For a given set of degrees of freedom, as β, δ and δ_1 tend to ∞, the sometimes pool test procedure approaches the exact F-test, the power which depends only on the values of $\theta_{14}, \theta_{24}, \theta_{34}$ and the final level of significance α_4, is always greater or equal to α_4.

Proof: As β, δ and δ_1 tend to ∞ the power 'P' which has the components as $P(A_2)$, $P(A_3)$ and $P(A_4)$ of the sometimes pool test procedure tends to zero and $P(A_1)$ tends to P_{1e} (say).

Where

$$P_{1e} \geq P\left\{ \frac{V_2/\sigma_2^2}{V_1/\sigma_1^2} < \infty, \ \frac{V_3/\sigma_3^2}{V_{12}/\sigma_1^2} < \infty, \ \frac{V_4/\sigma_4^2}{V_{123}/\sigma_1^2} \geq \beta_1 \right\}$$

Using the relation (3.8) we get

$$P_{1e} \geq \alpha_4 \tag{4.1}$$

Result 2: For $\theta_{12} = \theta_{13} = 1$, the lower bound and upper bound for size of the sometimes pool test procedure are respectively $(1-\alpha_1)(1-\alpha_2)\alpha_4$ and $(1-\alpha_1)(1-\alpha_2)\alpha_4 + (1-\alpha_3)\alpha_5 + (1-\alpha_1)\alpha_6 + \alpha_7$.

Proof: Under the conditions of the theorem, the size of the sometimes pool test procedure is given by $S = \sum_{i=1}^{4} S_i$.

Since S_i (i=1,2,3,4); hence, $S \ge S_1$.

We have

$$S_1 \ge P\left(\frac{V_2/\sigma_2^2}{V_1/\sigma_1^2} < \beta\right) P\left(\frac{V_3/\sigma_3^2}{V_{12}/\sigma_1^2} < \delta\right) P\left(\frac{V_4/\sigma_4^2}{V_{123}/\sigma_1^2} \ge \beta_1\right)$$

$$S_1 \ge (1-\alpha_1)(1-\alpha_2)\alpha_4 \tag{4.2}$$

By using the relations (3.7) and (3.8), we have

$S \ge (1-\alpha_1)(1-\alpha_2)\alpha_4$, which is the lower bound of the test procedure.

Now, by taking the limit β tends to zero in (3.12), the S_2 may be written as:

$$S_2 \le P\left(\frac{V_2/\sigma_2^2}{V_1/\sigma_1^2} \ge 0\right) P\left(\frac{V_3/\sigma_3^2}{V_1/\sigma_1^2} < \delta_1\right) P\left(\frac{V_4/\sigma_4^2}{V_2/\sigma_2^2} \ge \beta_2\right)$$

By using the relations (3.7) and (3.8), we have

$$S_2 \le (1-\alpha_3)\alpha_5 \tag{4.3}$$

Now, by taking the limit δ tends to zero in (3.15), the S_3 may be written as:

$$S_3 \le P\left(\frac{V_2/\sigma_2^2}{V_1/\sigma_1^2} < \beta\right) P\left(\frac{V_3/\sigma_3^2}{V_{12}/\sigma_1^2} \ge 0\right) P\left(\frac{V_4/\sigma_4^2}{V_3/\sigma_3^2} \ge \beta_3\right)$$

By using the relations (3.7) and (3.8), we have

$$S_3 \le (1-\alpha_1)\alpha_6 \tag{4.4}$$

Now, by taking the limits β and δ tends to zero in (3.19), the S_4 may be written as:

$$S_4 \le P\left(\frac{V_2/\sigma_2^2}{V_1/\sigma_1^2} \ge 0\right) P\left(\frac{V_3/\sigma_3^2}{V_1/\sigma_1^2} \ge 0\right) P\left(\frac{V_4/\sigma_4^2}{V_A/\left(\sigma_3^2+\sigma_2^2-\sigma_1^2\right)} \ge \beta_4\right)$$

By using the relation (3.8) we have

$$S_4 \le \alpha_7 \tag{4.5}$$

On combining (4.2), (4.3), (4.4) and (4.5) we have

$$S \le (1-\alpha_1)(1-\alpha_2)\alpha_4 + (1-\alpha_3)\alpha_5 + (1-\alpha_1)\alpha_6 + \alpha_7.$$

5. Numerical Study of the Behaviour of the Size and Power of the Proposed Test Procedure

Seven sets of degrees of freedom (d.f.) which have been taken for evaluation of size and power of the test procedure are given in Table 5.1.

d.f.	Set 1	Set 2	Set 3	Set 4	Set 5	Set 6	Set 7
n_1	8	12	12	24	24	24	48
n_2	4	6	4	12	8	12	16
n_3	4	4	6	8	12	12	24
n_4	2	2	2	4	4	6	8

Table 5.1: Seven sets of degrees of freedom for evaluation of size and power of test

The values of size and power for set 5, set 6 and set 7 are summarised in the Appendix. The values for the other sets are not given here for the want of space.

From (3.10), (3.13), (3.17) and (3.21) we observe that the power of sometimes pool test procedure is a function of 14 parameters, viz. 4 degrees of freedom n_1, n_2, n_3 and n_4; in all seven level of significance: out of which the preliminary level of significance are α_1, α_2 and α_3 and the final level of significance are α_4, α_5, α_6 and α_7; and the three nuisance parameters are θ_{14}, θ_{24} and θ_{34}.

We have considered the values of θ_{14}, θ_{24} and θ_{34} ranging from .2(.2)1.0. However, the nuisance parameters θ_{14}, θ_{24} and θ_{34} must satisfy the following inequalities

$$\theta_{34} + \theta_{24} - \theta_{14} \le 1,\ \theta_{24} \ge \theta_{14},\ \theta_{34} \ge \theta_{14},$$

which is evident from the analysis of variance table 2.1 and considering the alternative hypothesis:

The power of the sometimes pool test procedure has been calculated only for those combinations of values of the above relations. Besides these, three additional sets of values of the nuisance parameters which satisfy the above inequalities, have also been taken. These values are $\theta_{14} = .002,\ \theta_{24} = .006,\ \theta_{34} = .006, \theta_{14} = .01$; $\theta_{24} = .05, \theta_{34} = .01$ and $\theta_{14} = .02,\ \theta_{24} = .06, \theta_{34} = .08$. For these values power is more. The values of preliminary level of significance α_p which have been considered for studying the behaviour of the size and power of sometimes pool test procedure are (in %) 0, .01, .05, .10, .25, and 1.0. However the final levels of significance have been fixed at 1%.

First we consider the size of the sometimes pool test procedure summarised in tables 1 - 2. We observe from these tables that for $\alpha_p = 0$, the size, which is the size of the exact F – test procedure, is always greater than .05. For $\alpha_p = 1.0$, the size of the sometimes pool test procedure reduces to the approximate F – test and its size is equal to the pre – fixed nominal size. If we exclude these two extreme cases, we observe that the size of sometimes pool test procedure decreases monotonically as we increase the level of significance for given values of θ_{14}, θ_{24} and θ_{34}.

The four degrees of freedom n_1, n_2, n_3 and n_4 are determined completely by the numbers of levels of factors *A*, *B* and *C*, therefore to study the behaviour of size for variations in degrees of freedom we have considered the effect of variations in the number of levels of the factors on the size. Comparing values of size for different data sets (the tables of size not given), we notice that for $\alpha_p = 0.25$ if we increase the number of levels of factor *A* then the size of sometimes pool test procedure decreases for small values of the nuisance parameters (i.e $\theta_{14} < .4$) however it increases for the other values. Further, an increase in the preliminary level of significance widens the range of nuisance parameters where the size decreases and/or comes closer (equal) to the prefixed nominal size. A similar comparison for the other factors viz. *B*, *C*, *A* and *B* and *A* and *A* and *C* can also be made.

Tables of the data sets 1, 2, 3 and 4 (not shown here) and the Table 5.2 (where the values within parenthesis are those obtained by Gupta and Singh) shows that the power of the proposed test procedure is more than that of the test procedure proposed by Gupta and Singh. The other tables (not shown here) of the power of the proposed test procedure for the 7 data sets reveal that, it in general decreases as α_p increases. In general, as we increase number of levels for factor A, it is observed that power increases with it for all the α_p considered here. We conclude that:

1. Power is more whenever $\alpha_p \leq 0.25$ for all the sets of degrees of freedom.
2. Size remains more or less under control whenever $\alpha_p \geq 0.25$ for certain range of nuisance parameters, we recommend $\alpha_p = 0.25$ for a better control over size and adequate power.
3. Power formulae incorporating <u>two</u> preliminary tests of significance though difficult to compute, can be easily computed with the proposed method.
4. Power comparison demonstrates superiority of the proposed method over the existing one.

			Preliminary level of significance						
θ_{14}	θ_{24}	θ_{34}	0.0	.01	.05	.10	.25	.50	1.0
.002	.006	.006	.9999	.9990	.9983	.9980	.9979	.9986	.9978
			(.9987)		(.9981)	(.9978)	(.9986)		(.9978)
.001	.005	.001	.9987	.9541	.9943	.9409	.9974	.9354	.9935
			(.9359)		(.9358)	(.9357)	(.9955)		(.9935)
.002	.006	.008	.9948	.8961	.8646	.8519	.8376	.8278	.8257
			(.9138)		(.8378)	(.8328)	(.8282)		(.8257)
.002	.004	.002	.7238	.6544	.5643	.5044	.4057	.3232	.2533
			(.2804)		(.2790)	(.2776)	(.2735)		(.2528)
.002	.004	.004	.7238	.5782	.4352	.3588	.2587	.1963	.1588
			(.2804)		(.2306)	(.2140)	(.1888)		(.1588)
.002	.004	.006	.7235	.3923	.2491	.1934	.1348	.1057	.0893
			(.2804)		(.1444)	(.1252)	(.1041)		(.0893)
.002	.006	.002	.7238	.5012	.3640	.2990	.2181	.1693	.1412
			(.1466)		(.1463)	(.1461)	(.1452)		(.1411)
.002	.006	.004	.7238	.4440	.2534	.2161	.1429	.1056	.0872
			(.1466)		(.1223)	(.1142)	(.1019)		(.0872)
.002	.008	.002	.7238	.3537	.2246	.0174	.1213	.0944	.0819
			(.0829)		(.0828)	(.0827)	(.0826)		(.0818)
.004	.006	.004	.4007	.3846	.3501	.3202	.2588	.1944	.1275
			(.1466)		(.1456)	(.1447)	(.1417)		(.1270)
.004	.006	.004	.4007	.3677	.3077	.2632	.1874	.1268	.0825
			(.1466)		(.1332)	(.1258)	(.1111)		(.0825)
.004	.008	.004	.4007	.3509	.2861	.2434	.1743	.1194	.0793
			(.0829)		(.0827)	(.0825)	(.0819)		(.0791)
.006	.008	.006	.2044	.1994	.1864	.1739	.1456	.1129	.0756
			(.0829)		(.0825)	(.0821)	(.0810)		(.0754)

Table 5.2 : Power of sometimes pool estimation test procedure n_1=24, n_2=8, n_3=12, n_4=4, α_4=25=α_6=α_7=.05

The values within parentheses were given by Gupta and Singh (1977).

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