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# BAYES ESTIMATORS OF THE SHAPE PARAMETERS OF A GENERALIZED GAMMA TYPE MODEL

**Gaurav Shukla**[1] **and Vinod Kumar**[2]

1. Deptt. of Statistics, G.F. (P.G.) College, Shahjahanpur, India.
E-Mail: saigaurav83@gmail.com

2. Deptt. of Maths, Stats. & Comp. Sc., G. B. Pant Univ. of Ag. & Tech., Pantnagar, India
E-Mail: vinod_kumarbcb@yahoo.com

## Abstract

Bayes estimators of the shape parameters of a generalized gamma type model are obtained for different priors using Lindley's approach. Bayes estimators of reliability and hazard rate functions have also been discussed. The calculations have been illustrated with the help of numerical example.

**Key Words**: Lifetime Model, Bayes estimator, Maximum Likelihood Estimator, Prior, Reliability Function, Hazard Rate Function.

## 1. Introduction

Besides exponential, Weibull, gamma, log normal, inverse gamma etc. distributions, the generalized gamma distribution proposed by Stacy (1962) is also suggested as a life-time model. Stacy and Mihram (1965) and Harter (1967) derived maximum likelihood estimators of generalized gamma distribution. Prentice (1974) obtained maximum likelihood estimators for generalized gamma distribution by using the technique of reparametrization. Lawless (1980) considered the problem of obtaining confidence intervals for the parameters of generalized and log gamma distributions. Upadhyay et al. (2000) have used Markov Chain Monte Carlo Simulation technique for drawing Bayes inferences in life testing and reliability. Shukla and Kumar (2006) have discussed the use of generalized gamma type distribution for many real life situations. Pandey and Rao (2006) have obtained Bayes estimators of scale parameter by using precautionary loss function whereas Shukla and Kumar (2008) have derived Bayes estimators of scale parameter for different priors by using Lindley approach for a generalized gamma type distribution.

In this paper we have obtained Bayes estimators of the shape parameters of the proposed model considering one at a time (keeping the other shape and scale parameters constant) under different priors viz. uniform, Jeffrey's, exponential, Mukharjee-Islam, Weibull, gamma etc. by using Lindley's (1980) approach. Bayes estimators of reliability and hazard rate functions under different priors have also been obtained.

The probability density function of the proposed generalized gamma type model is

$$f(t) = \frac{p}{\theta^{k}\Gamma k} t^{pk-1} e^{\left\{\frac{-t^{p}}{\theta}\right\}} I_{(0,\infty)}(t) \qquad p>0,\ \theta>0,\ k>0 \qquad (1)$$

where $\theta>0$ is a scale parameter and $k>0$ and $p>0$ are shape parameters.

The model includes widely used exponential (p=k=1), Weibull (k=1) and gamma (p=1) as special cases. The model introduced by Stacy (1962) may also be obtained from (1) simply by substituting $\theta = \alpha^{p}$.

The Likelihood function (L) of (1) is given by

$$L = \left[\frac{p}{\theta^{k}\Gamma k}\right]^{n} e^{-\sum_{i=1}^{n}\frac{t_i^{p}}{\theta}} \prod_{i=1}^{n} t_i^{pk-1} \tag{2}$$

## 1. Lindley's Approach

Bayes estimators are often obtained as the ratio of two integrals which can not be solved by using asymptotic expansion and calculus of difference. Lindley (1980) developed an asymptotic approximation to the ratio

$$I = \frac{\int_{\beta} u(\beta) e^{L(\beta)} g(\beta) d\beta}{\int_{\beta} e^{L(\beta)} g(\beta) d\beta} \tag{3}$$

where $L(\beta)$ is the logarithm of the likelihood function.
According to him

$$I \approx u(\beta^*) + \frac{\sigma^2}{2}\left[u_2(\beta^*) + 2u_1(\beta^*)p_1(\beta^*)\right] + \frac{\sigma^4}{2}\left[L_3(\beta^*)u_1(\beta^*)\right] \tag{4}$$

where $\beta^*$ is the MLE of $\beta$
Also,

$$L_k(\beta^*) = \left.\frac{\partial^k}{\partial\beta^k} L(\beta)\right|_{\beta=\beta^*} \tag{5}$$

$$u_k(\beta^*) = \left.\frac{\partial^k}{\partial\beta^k} u(\beta)\right|_{\beta=\beta^*} \tag{6}$$

$$\sigma^2 = -L_2^{-1}(\beta^*) \tag{7}$$

$$p_k(\beta^*) = \left.\frac{\partial^k}{\partial\beta^k} \log_e g(\beta)\right|_{\beta=\beta^*} \tag{8}$$

### 2.1 Bayes Estimator of P Given $\theta$ and K

If $\theta$ and k are fixed quantities, Bayes estimators of p may be obtained by using Lindley's approach as follows:
Here,

$$u(p) = p, \quad u(p^*) = p^*, \quad u_1(p^*) = 1, \quad u_2(p^*) = 0$$

$$L_1(p^*) = \frac{n}{p^*} + k_0 \sum_{i=1}^{n} \log_e t_i - \sum_{i=1}^{n} \frac{t_i^{\ p^*}}{\theta_0} \log_e t_i \quad [\theta = \theta_0, k = k_0, fixed]$$

$$L_2(p^*) = -\frac{n}{p^{*2}} - \sum_{i=1}^{n} \frac{t_i^{\ p^*}}{\theta_0} (\log_e t_i)^2$$

$$L_3(p^*) = \frac{2n}{p^{*3}} - \sum_{i=1}^{n} \frac{t_i^{\ p^*}}{\theta_0} (\log_e t_i)^3$$

$$\sigma^2 = \frac{p^{*2}\theta_0}{n\theta_0 + p^{*2} \sum_{i=1}^{n} t_i^{\ p^*} (\log_e t_i)^2}, \sigma^4 = \frac{p^{*4}\theta_0^{\ 2}}{\left(n\theta_0 + p^{*2} \sum_{i=1}^{n} t_i^{\ p^*} (\log_e t_i)^2\right)^2} \quad (9)$$

The values of $p_1(p^*)$ and Bayes estimators ( $p^B$ ) of $p$ given $\theta = \theta_0$ and k= $k_0$ for different priors are given in Table 1.

| **Prior (1)** | **Density (2)** | $p_1(p^*)$ **(3)** |
|---|---|---|
| Uniform | $I_{(0,1)}(p)$ | 0 |
| Jeffrey's | $p^{-1} I_{(1,e)}(p)$ | $-\frac{1}{p^*}$ |
| Exponential | $e^{-p}$ ; $p > 0$ | $-1$ |
| Mukharjee-Islam | $\alpha\sigma^{-\alpha} p^{\alpha-1} I_{(0,\sigma)}(p)$ ; $\alpha, \sigma > 0$ | $\frac{\alpha - 1}{p^*}$ |
| Weibull | $\frac{\alpha}{\sigma} p^{\alpha-1} e^{-\theta^{\alpha}/\sigma}$ ; $\alpha, \sigma > 0, p > 0$ | $\frac{\alpha-1}{p^*} - \frac{\alpha}{\sigma} p^{*\alpha-1}$ |
| Gamma | $\frac{1}{\sigma^{\alpha} \overline{)\alpha}} p^{\alpha-1} e^{-\theta/\sigma}$ ; $\alpha, \sigma > 0, p > 0$ | $\frac{\alpha-1}{p^*} - \frac{1}{\sigma}$ |
| Proposed | $\frac{c-1}{p^c} I_{(1,\infty)}(p)$ | $\frac{-c}{p^*}$ |
| Generalized | $\frac{\alpha}{\sigma^{\beta} \overline{)\beta}} p^{\alpha\beta-1} e^{-p^{\alpha}/\sigma} I_{(0,\infty)}(p)$ | $\frac{\alpha\beta-1}{p^*} - \frac{\alpha p^{*\alpha-1}}{\sigma}$ |

| $p^B$ **(4)** |
|---|
| $p^*\left[1+\frac{\theta_0\left(2n\theta_0-p^{*3}\sum_{i=1}^{n}t_i^{p^*}(\log_e t_i)^3\right)}{2\left(n\theta_0+p^{*2}\sum_{i=1}^{n}t_i^{p^*}(\log_e t_i)^2\right)^2}\right]$ |
| $p^*\left[1-\frac{\theta_0}{\left(n\theta_0+p^{*2}\sum_{i=1}^{n}t_i^{p^*}(\log_e t_i)^2\right)}+\frac{\theta_0\left(2n\theta_0-p^{*3}\sum_{i=1}^{n}t_i^{p^*}(\log_e t_i)^3\right)}{2\left(n\theta_0+p^{*2}\sum_{i=1}^{n}t_i^{p^*}(\log_e t_i)^2\right)^2}\right]$ |
| $p^*\left[1-\frac{p^*\theta_0}{\left(n\theta_0+p^{*2}\sum_{i=1}^{n}t_i^{p^*}(\log_e t_i)^2\right)}+\frac{\theta_0\left(2n\theta_0-p^{*3}\sum_{i=1}^{n}t_i^{p^*}(\log_e t_i)^3\right)}{2\left(n\theta_0+p^{*2}\sum_{i=1}^{n}t_i^{p^*}(\log_e t_i)^2\right)^2}\right]$ |
| $p^*\left[1+\frac{(\alpha-1)\theta_0}{\left(n\theta_0+p^{*2}\sum_{i=1}^{n}t_i^{p^*}(\log_e t_i)^2\right)}+\frac{\theta_0\left(2n\theta_0-p^{*3}\sum_{i=1}^{n}t_i^{p^*}(\log_e t_i)^3\right)}{2\left(n\theta_0+p^{*2}\sum_{i=1}^{n}t_i^{p^*}(\log_e t_i)^2\right)^2}\right]$ |
| $=p^*\left[1+\frac{[\sigma(\alpha-1)-\alpha p^{*\alpha}]\theta_0}{\sigma\left(n\theta_0+p^{*2}\sum_{i=1}^{n}t_i^{p^*}(\log_e t_i)^2\right)}+\frac{\theta_0\left(2n\theta_0-p^{*3}\sum_{i=1}^{n}t_i^{p^*}(\log_e t_i)^3\right)}{2\left(n\theta_0+p^{*2}\sum_{i=1}^{n}t_i^{p^*}(\log_e t_i)^2\right)^2}\right]$ |
| $=p^*\left[1+\frac{[\sigma(\alpha-1)-p^*]\theta_0}{\sigma\left(n\theta_0+p^{*2}\sum_{i=1}^{n}t_i^{p^*}(\log_e t_i)^2\right)}+\frac{\theta_0\left(2n\theta_0-p^{*3}\sum_{i=1}^{n}t_i^{p^*}(\log_e t_i)^3\right)}{2\left(n\theta_0+p^{*2}\sum_{i=1}^{n}t_i^{p^*}(\log_e t_i)^2\right)^2}\right]$ |
| $=p^*\left[1-\frac{c\theta_0}{\left(n\theta_0+p^{*2}\sum_{i=1}^{n}t_i^{p^*}(\log_e t_i)^2\right)}+\frac{\theta_0\left(2n\theta_0-p^{*3}\sum_{i=1}^{n}t_i^{p^*}(\log_e t_i)^3\right)}{2\left(n\theta_0+p^{*2}\sum_{i=1}^{n}t_i^{p^*}(\log_e t_i)^2\right)^2}\right]$ |
| $p^*\left[1+\frac{(\alpha-1)\theta_0}{\left(n\theta_0+p^{*2}\sum_{i=1}^{n}t_i^{p^*}(\log_e t_i)^2\right)}-\frac{p^{*\alpha}\theta_0\alpha\beta}{\sigma\left(n\theta_0+p^{*2}\sum_{i=1}^{n}t_i^{p^*}(\log_e t_i)^2\right)}+\frac{\theta_0\left(2n\theta_0-p^{*3}\sum_{i=1}^{n}t_i^{p^*}(\log_e t_i)^3\right)}{2\left(n\theta_0+p^{*2}\sum_{i=1}^{n}t_i^{p^*}(\log_e t_i)^2\right)^2}\right]$ |

**Table 1 : The values of** $p_1(p^*)$ **and** $p^B$ **for different priors**

## 2.2 Bayes Estimators of Reliability Function Under Different Priors

We have found Bayes estimators of reliability function under different priors for k=1 and a fixed value ($\theta_0$) of $\theta$ .

Here, R (t) = $e^{-t^p/\theta_0}$ => $u(p) = e^{-t^p/\theta_0}$ ,

$$u(p^*) = e^{-t^{p^*}/\theta_0}, u_1(p^*) = -\frac{t^{p^*}}{\theta_0}(\log_e t)e^{-t^{p^*}/\theta_0}$$

$$u_2(p^*) = \frac{e^{-t^{p^*}/\theta_0}}{\theta_0} t^{p^*}(\log_e t)^2\left[\frac{t^{p^*}}{\theta_0} - 1\right] \tag{10}$$

Bayes estimators [ $R(t)_p{}^B$ ] of Reliability Function [R(t)] given $\theta = \theta_0$ and k=1 under different priors in the abovesaid order are given in Table 2.

## 2.3 Bayes Estimators of Hazard Rate Function Under Different Priors

We have found Bayes estimators [$H(t)_p{}^B$] of hazard rate function [H (t)] for k=1 and a fixed value $\theta_0$ of $\theta$ under different priors, as for k=1, it is possible to evaluate H (t) exactly.

Here, H (t) = $\frac{f(t)}{R(t)}$ = $\frac{p}{\theta_0}t^{p-1}$ , $u(p^*) = \frac{p^*}{\theta_0}t^{p^*-1}$,

$$u_1(p^*) = \left[\frac{p^*}{\theta_0}t^{p^*-1}(\log_e t) + \frac{t^{p^*-1}}{\theta_0}\right],$$

$$u_2(p^*) = \left[\frac{p^*}{\theta_0}t^{p^*-1}(\log_e t)^2 + \frac{2t^{p^*-1}(\log_e t)}{\theta_0}\right] \tag{11}$$

Bayes estimator **(** $H(t)_p{}^B$ **)** of Hazard Rate Function [H(t)] given p and k under different priors in the abovesaid order are given in Table 3.

## 2.4 Bayes Estimators of k for Given p and $\theta$

If $\theta$ and p are fixed quantities, Bayes estimators of p may be obtained by using Lindley's approach as follows:.

Here; $u(k) = k$ , $u(k^*) = k^*$, $u_1(k^*) = 1$, $u_2(k^*) = 0$

$$L_1(k^*) = -n\log_e\theta_0 - n\left[\log_e k - \frac{1}{2k}\right] + p_0\sum_{i=1}^{n}\log_e t_i$$

$$L_2(k^*) = -n\left[\frac{1}{k^*} + \frac{1}{2k^{*2}}\right]. \quad L_3(k^*) = \frac{n}{k^{*2}}\left(1 + \frac{1}{k^*}\right)$$

$$\sigma^2 = \frac{1}{n}\left[\frac{1}{k*} + \frac{1}{2k*^2}\right]^{-1}, \qquad \sigma^4 = \frac{1}{n^2}\left[\frac{1}{k*} + \frac{1}{2k*^2}\right]^{-2} \tag{12}$$

Bayes estimators ($k^B$) of k for fixed values $\theta = \theta_0$ and $p = p_0$ under different priors are given in Table 4.

**Illustration**

A random sample of size 25 was generated from the proposed model with k=1, p = 2 and $\theta$ = 4. Table 5 shows Bayes estimators of p for $\theta$ = 4, k = 1.and Bayes estimators of k for $\theta$ = 4, p = 2.

Table 5 reveals that Bayes estimator of k for $\theta$ = 4, p = 2 seems to be closer to its true value under gamma prior. The calculations for reliability and hazard rate functions may also be performed in a similar manner at different values of t considering different priors.

| $R(t)_p^{\ B}$ |
|---|
| $e^{\frac{-t^{p*}}{\theta_0}} + \frac{e^{-t^{p*}/\theta_0}\, t^{p*} (\log_e t)\, p*^2}{2\left(n\theta_0 + p*^2 \sum_{i=1}^{n} t_i^{\ p*} (\log_e t_i)^2\right)}\left[(\log_e t)\left(\frac{t^{p*}}{\theta_0} - 1\right)\right]$ $e^{-t^{p*}/\theta_0}\, p*\, t^{p*} (\log_e t)\left\lvert \frac{\left(2n\theta_0 - p*^3 \sum_{i=1}^{n} t_i^{\ p*} (\log_e t_i)^3\right)}{2\left(n\theta_0 + p*^2 \sum_{i=1}^{n} t_i^{\ p*} (\log_e t_i)^2\right)^2} \right\rvert$ |
| $e^{\frac{-t^{p*}}{\theta_0}} + \frac{e^{-t^{p*}/\theta_0}\, t^{p*} (\log_e t)\, p*^2}{2\left(n\theta_0 + p*^2 \sum_{i=1}^{n} t_i^{\ p*} (\log_e t_i)^2\right)}\left[(\log_e t)\left(\frac{t^{p*}}{\theta_0} - 1\right) + \frac{2}{p*}\right]$ $- e^{-t^{p*}/\theta_0}\, p*\, t^{p*} (\log_e t)\left[ \frac{\left(2n\theta_0 - p*^3 \sum_{i=1}^{n} t_i^{\ p*} (\log_e t_i)^3\right)}{2\left(n\theta_0 + p*^2 \sum_{i=1}^{n} t_i^{\ p*} (\log_e t_i)^2\right)^2} \right]$ |
| $e^{\frac{-t^{p*}}{\theta_0}} + \frac{e^{-t^{p*}/\theta_0}\, t^{p*} (\log_e t)\, p*^2}{2\left(n\theta_0 + p*^2 \sum_{i=1}^{n} t_i^{\ p*} (\log_e t_i)^2\right)}\left[(\log_e t)\left(\frac{t^{p*}}{\theta_0} - 1\right) + 2\right]$ $- e^{-t^{p*}/\theta_0}\, p*\, t^{p*} (\log_e t)\left[ \frac{\left(2n\theta_0 - p*^3 \sum_{i=1}^{n} t_i^{\ p*} (\log_e t_i)^3\right)}{2\left(n\theta_0 + p*^2 \sum_{i=1}^{n} t_i^{\ p*} (\log_e t_i)^2\right)^2} \right]$ |

$$e^{\frac{-t^{p^*}}{\theta_0}} + \frac{e^{-t^{p^*}/\theta_0} t^{p^*} (\log_e t) p^{*2}}{2\left(n\theta_0 + p^{*2}\sum_{i=1}^{n} t_i^{\,p^*}(\log_e t_i)^2\right)} \left[(\log_e t)\left(\frac{t^{p^*}}{\theta_0}-1\right) - \frac{2(\alpha-1)}{p^*}\right]$$
$$- e^{-t^{p^*}/\theta_0} p^* t^{p^*}(\log_e t)\left[\frac{\left(2n\theta_0 - p^{*3}\sum_{i=1}^{n} t_i^{\,p^*}(\log_e t_i)^3\right)}{2\left(n\theta_0+p^{*2}\sum_{i=1}^{n} t_i^{\,p^*}(\log_e t_i)^2\right)^2}\right]$$

$$e^{\frac{-t^{p^*}}{\theta_0}} + \frac{e^{-t^{p^*}/\theta_0} t^{p^*} (\log_e t) p^{*2}}{2\left(n\theta_0 + p^{*2}\sum_{i=1}^{n} t_i^{\,p^*}(\log_e t_i)^2\right)} \left[(\log_e t)\left(\frac{t^{p^*}}{\theta_0}-1\right) - \frac{2(\alpha-1)}{p^*} + \frac{2\alpha p^{*\alpha-1}}{\sigma}\right]$$
$$- e^{-t^{p^*}/\theta_0} p^* t^{p^*}(\log_e t)\left[\frac{\left(2n\theta_0 - p^{*3}\sum_{i=1}^{n} t_i^{\,p^*}(\log_e t_i)^3\right)}{2\left(n\theta_0+p^{*2}\sum_{i=1}^{n} t_i^{\,p^*}(\log_e t_i)^2\right)^2}\right]$$

$$e^{\frac{-t^{p^*}}{\theta_0}} + \frac{e^{-t^{p^*}/\theta_0} t^{p^*} (\log_e t) p^{*2}}{2\left(n\theta_0 + p^{*2}\sum_{i=1}^{n} t_i^{\,p^*}(\log_e t_i)^2\right)} \left[(\log_e t)\left(\frac{t^{p^*}}{\theta_0}-1\right) - \frac{2(\alpha-1)}{p^*} + \frac{2}{\sigma}\right]$$
$$- e^{-t^{p^*}/\theta_0} p^* t^{p^*}(\log_e t)\left[\frac{\left(2n\theta_0 - p^{*3}\sum_{i=1}^{n} t_i^{\,p^*}(\log_e t_i)^3\right)}{2\left(n\theta_0+p^{*2}\sum_{i=1}^{n} t_i^{\,p^*}(\log_e t_i)^2\right)^2}\right]$$

$$e^{\frac{-t^{p^*}}{\theta_0}} + \frac{e^{-t^{p^*}/\theta_0} t^{p^*} (\log_e t) p^{*2}}{2\left(n\theta_0 + p^{*2}\sum_{i=1}^{n} t_i^{\,p^*}(\log_e t_i)^2\right)} \left[(\log_e t)\left(\frac{t^{p^*}}{\theta_0}-1\right) + \frac{2c}{p^*}\right]$$
$$- e^{-t^{p^*}/\theta_0} p^* t^{p^*}(\log_e t)\left[\frac{\left(2n\theta_0 - p^{*3}\sum_{i=1}^{n} t_i^{\,p^*}(\log_e t_i)^3\right)}{2\left(n\theta_0+p^{*2}\sum_{i=1}^{n} t_i^{\,p^*}(\log_e t_i)^2\right)^2}\right]$$

$$e^{\frac{-t^{p^*}}{\theta_0}} + \frac{e^{-t^{p^*}/\theta_0}\, t^{p^*}(\log_e t)\, p^{*2}}{2\left(n\theta_0 + p^{*2}\sum_{i=1}^{n} t_i^{p^*}(\log_e t_i)^2\right)}\left[(\log_e t)\left(\frac{t^{p^*}}{\theta_0}-1\right) - \frac{2(\alpha\beta-1)}{p^*} + \frac{2\alpha p^{*\alpha-1}}{\sigma}\right]$$
$$- e^{-t^{p^*}/\theta_0}\, p^*\, t^{p^*}(\log_e t)\left[\frac{\left(2n\theta_0 - p^{*3}\sum_{i=1}^{n} t_i^{p^*}(\log_e t_i)^3\right)}{2\left(n\theta_0 + p^{*2}\sum_{i=1}^{n} t_i^{p^*}(\log_e t_i)^2\right)^2}\right]$$

**Table 2: Bayes estimators [ $R(t)_p^{\,B}$ ] of Reliability Function [R(t)] given $\theta = \theta_0$ and k=1 for different priors**

| $H(t)_p^{\,B}$ |
|---|
| $\frac{p^*}{\theta_0} t^{p^*-1} + \frac{p^{*2}\, t^{p^*-1}(\log_e t)}{2\left[n\theta_0 + p^{*2}\sum_{i=1}^{n} t_i^{p^*}(\log_e t_i)^2\right]}\left(p^*(\log_e t) + 2\right) + p^* t^{p^*-1}\left(1 + p^*(\log_e t)\right)\left[\frac{\left(2n\theta_0 - p^{*3}\sum_{i=1}^{n} t_i^{p^*}(\log_e t_i)^3\right)}{2\left(n\theta_0 + p^{*2}\sum_{i=1}^{n} t_i^{p^*}(\log_e t_i)^2\right)^2}\right]$ |
| $\frac{p^*}{\theta_0} t^{p^*-1} + \frac{p^{*2}\, t^{p^*-1}}{2\left[n\theta_0 + p^{*2}\sum_{i=1}^{n} t_i^{p^*}(\log_e t_i)^2\right]}\left(p^*(\log_e t)^2 + 2\log_e t - \frac{2}{p^*}(p^*\log_e t + 1)\right) + p^* t^{p^*-1}\left(1 + p^*(\log_e t)\right)\left[\frac{\left(2n\theta_0 - p^{*3}\sum_{i=1}^{n} t_i^{p^*}(\log_e t_i)^3\right)}{2\left(n\theta_0 + p^{*2}\sum_{i=1}^{n} t_i^{p^*}(\log_e t_i)^2\right)^2}\right]$ |
| $\frac{p^*}{\theta_0} t^{p^*-1} + \frac{p^{*2}\, t^{p^*-1}}{2\left[n\theta_0 + p^{*2}\sum_{i=1}^{n} t_i^{p^*}(\log_e t_i)^2\right]}\left(p^*(\log_e t)^2 + 2\log_e t - 2(p^*\log_e t + 1)\right) + p^* t^{p^*-1}\left(1 + p^*(\log_e t)\right)\left[\frac{\left(2n\theta_0 - p^{*3}\sum_{i=1}^{n} t_i^{p^*}(\log_e t_i)^3\right)}{2\left(n\theta_0 + p^{*2}\sum_{i=1}^{n} t_i^{p^*}(\log_e t_i)^2\right)^2}\right]$ |
| $\frac{p^*}{\theta_0} t^{p^*-1} + \frac{p^{*2}\, t^{p^*-1}}{2\left[n\theta_0 + p^{*2}\sum_{i=1}^{n} t_i^{p^*}(\log_e t_i)^2\right]}\left(p^*(\log_e t)^2 + 2\log_e t + 2\frac{(\alpha-1)}{p^*}(p^*\log_e t + 1)\right)$ |

$$+\,p^*t^{p^*-1}\left(1+p^*(\log_e t)\right)\left[\frac{\left(2n\theta_0-p^{*3}\sum_{i=1}^{n}t_i^{\,p^*}(\log_e t_i)^3\right)}{2\left(n\theta_0+p^{*2}\sum_{i=1}^{n}t_i^{\,p^*}(\log_e t_i)^2\right)^2}\right]$$

$$\frac{p^*}{\theta_0}t^{p^*-1}+\frac{p^{*2}\,t^{p^*-1}}{2\left[n\theta_0+p^{*2}\sum_{i=1}^{n}t_i^{\,p^*}(\log_e t_i)^2\right]}\left(p^*(\log_e t)^2+2\log_e t+2\left(\frac{\alpha-1}{p^*}-\frac{p^{*\alpha-1}\alpha}{\sigma}\right)(p^*\log_e t+1)\right)$$

$$+\,p^*t^{p^*-1}\left(1+p^*(\log_e t)\right)\left[\frac{\left(2n\theta_0-p^{*3}\sum_{i=1}^{n}t_i^{\,p^*}(\log_e t_i)^3\right)}{2\left(n\theta_0+p^{*2}\sum_{i=1}^{n}t_i^{\,p^*}(\log_e t_i)^2\right)^2}\right]$$

$$\frac{p^*}{\theta_0}t^{p^*-1}+\frac{p^{*2}\,t^{p^*-1}}{2\left[n\theta_0+p^{*2}\sum_{i=1}^{n}t_i^{\,p^*}(\log_e t_i)^2\right]}\left(p^*(\log_e t)^2+2\log_e t+2\left(\frac{\alpha-1}{p^*}-\frac{1}{\sigma}\right)(p^*\log_e t+1)\right)$$

$$+\,p^*t^{p^*-1}\left(1+p^*(\log_e t)\right)\left[\frac{\left(2n\theta_0-p^{*3}\sum_{i=1}^{n}t_i^{\,p^*}(\log_e t_i)^3\right)}{2\left(n\theta_0+p^{*2}\sum_{i=1}^{n}t_i^{\,p^*}(\log_e t_i)^2\right)^2}\right]$$

$$\frac{p^*}{\theta_0}t^{p^*-1}+\frac{p^{*2}\,t^{p^*-1}}{2\left[n\theta_0+p^{*2}\sum_{i=1}^{n}t_i^{\,p^*}(\log_e t_i)^2\right]}\left(p^*(\log_e t)^2+2\log_e t-\frac{2c}{p^*}(p^*\log_e t+1)\right)$$

$$+\,p^*t^{p^*-1}\left(1+p^*(\log_e t)\right)\left[\frac{\left(2n\theta_0-p^{*3}\sum_{i=1}^{n}t_i^{\,p^*}(\log_e t_i)^3\right)}{2\left(n\theta_0+p^{*2}\sum_{i=1}^{n}t_i^{\,p^*}(\log_e t_i)^2\right)^2}\right]$$

$$\frac{p^*}{\theta_0}t^{p^*-1}+\frac{p^{*2}\,t^{p^*-1}}{2\left[n\theta_0+p^{*2}\sum_{i=1}^{n}t_i^{\,p^*}(\log_e t_i)^2\right]}\left(p^*(\log_e t)^2+2\log_e t+2\left(\frac{\alpha\beta-1}{p^*}-\frac{p^{*\alpha-1}\alpha}{\sigma}\right)(p^*\log_e t+1)\right)$$

$$+\,p^*t^{p^*-1}\left(1+p^*(\log_e t)\right)\left[\frac{\left(2n\theta_0-p^{*3}\sum_{i=1}^{n}t_i^{\,p^*}(\log_e t_i)^3\right)}{2\left(n\theta_0+p^{*2}\sum_{i=1}^{n}t_i^{\,p^*}(\log_e t_i)^2\right)^2}\right]$$

**Table 3: Bayes estimator ( $H(t)_p^{\,B}$ ) of Hazard Rate Function [H(t)] given p and k under different priors**

| **Priors** | **Density** | $p_1(k^*)$ | $k^B$ |
|---|---|---|---|
| Uni-form | $I_{(0,1)}(k)$ | 0 | $k^* + \frac{2k^*}{n}\left[\frac{(k^*+1)}{(2k^*+1)^2}\right]$ |
| Jeffrey's | $\frac{1}{k}$ | $-\frac{1}{k^*}$ | $k^* - \frac{2}{n}\left[\frac{k^*}{(2k^*+1)}\right]^2$ |
| Expo-nential | $e^{-k}; \quad k>0$ | $-1$ | $k^* - \frac{2k^*}{n}\left[\frac{2k^{*2}-1}{(2k^*+1)^2}\right]$ |
| Mukharjee-Islam | $\frac{\alpha}{\sigma^{\alpha}} k^{\alpha-1}$ ; $0<k<\alpha$ | $\frac{\alpha-1}{k^*}$ | $k^* + \frac{2k^*}{n}\left[\frac{k^*(2\alpha-1)+\alpha}{(2k^*+1)^2}\right]$ |
| Weibull | $\frac{\alpha}{\sigma} k^{\alpha-1} e^{-k^{\alpha}/\sigma}$ ; $\alpha,\sigma>0$, k>0 | $\frac{\alpha-1}{k^*} - \frac{\alpha}{\sigma} k^{*\alpha-1}$ | $k^* + \frac{2k^*}{n(2k^*+1)}\left[\frac{k^*+1}{(2k^*+1)} + (\alpha-1) - \frac{\alpha k^{*\alpha}}{\sigma}\right]$ |
| Gamma | $\frac{1}{\sigma^{\alpha}\Gamma(\alpha)} k^{\alpha-1} e^{-k/\sigma}$ ; $\alpha,\sigma>0$, k>0 | $\frac{\alpha-1}{k^*} - \frac{1}{\sigma}$ | $k^* + \frac{2k^*}{n(2k^*+1)}\left[\frac{k^*+1}{(2k^*+1)} + (\alpha-1) - \frac{k^*}{\sigma}\right]$ |
| Proposed | $\frac{c-1}{k^c} I_{(1,\infty)}(k)$ | $\frac{-c}{k^*}$ | $k^* - \frac{2k^{*2}}{n}\left[\frac{c}{(2k^*+1)^2}\right]$ |
| Genera-lized | $\frac{\alpha}{\sigma^{\beta}\Gamma(\beta)} k^{\alpha\beta-1} e^{-k^{\alpha}/\sigma} I_{(0,\infty)}(k)$ ; $\alpha,\beta,\sigma>0$ | $\frac{\alpha\beta-1}{k^*} - \frac{\alpha k^{*\alpha-1}}{\sigma}$ | $k^* + \frac{2k^*}{n(2k^*+1)}\left[\frac{k^*+1}{(2k^*+1)} + (\alpha\beta-1) - \frac{\alpha k^{*\alpha}}{\sigma}\right]$ |

**Table 4: Bayes estimators ( $k^B$ ) of k for fixed values $\theta = \theta_0$ and $p = p_0$ under different priors**

| **Prior** | $p^B$ | $k^B$ |
|---|---|---|
| Uniform | 2.341264448 | 0.976817076 |
| Jeffrey's | 2.32282788 | 0.95015563 |
| Exponential | 2.297923764 | 0.951597227 |
| Mukharjee-Islam $\alpha=1$ | 2.341264448 | 0.976817076 |
| $=2$ | 2.359701017 | 1.003110611 |
| $=3$ | 2.378137585 | 1.029404145 |

| | | |
|---|---|---|
| Weibull $\alpha = 1, \sigma = 1$<br>$= 2$<br>$= 3$ | 2.297923764<br>2.319594106<br>2.326817554 | 0.951597227<br>0.990500686<br>1.020997529 |
| Gamma $\alpha = 1, \sigma = 1$<br>$= 2$<br>$= 3$ | 2.297923764<br>2.319594106<br>2.326817554 | 0.951597227<br>0.990500686<br>1.020997529 |

**Table 5: Bayes estimators of p for $\theta = 4$, k = 1.and Bayes estimators of k for $\theta = 4$, p = 2.**

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